Quantum measurement theory

Projection & sampling noise

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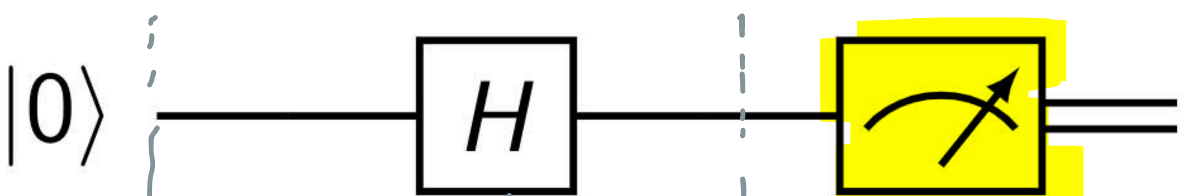
Qubit example

Pauli Operators

$$\hat{I} \rightarrow \begin{array}{c|cc} & \langle 0| & \langle 1| \\ \hline |0\rangle & 1 & 0 \\ |1\rangle & 0 & 1 \end{array}, \quad \hat{X} \rightarrow \begin{array}{cc} \langle 0| & \langle 1| \\ \end{array} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{Y} \rightarrow \begin{array}{cc} \langle 0| & \langle \end{array} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

t_0 t_1



Hadamard

$$|\psi_0\rangle = \begin{array}{c} \\ \end{array}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{array}{c} |0\rangle \\ |1\rangle \end{array} = |0\rangle$$

$$\langle \hat{X} \rangle = \langle \psi | \hat{X} | \psi \rangle = 0$$

$$\langle \hat{Y} \rangle = \langle \psi | \hat{Y} | \psi \rangle = 0$$

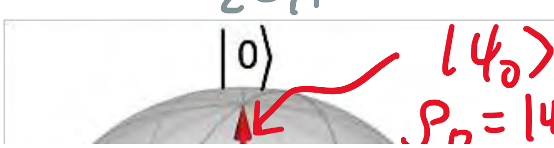
$$\langle \hat{Z} \rangle = \langle \psi | \hat{Z} | \psi \rangle = 1$$

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{array}{c} |0\rangle \\ |1\rangle \end{array} \begin{array}{c} \langle 0| \quad \langle 1| \\ \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \end{array}$$

$$= \frac{1}{\sqrt{2}} \left(\hat{X} + \hat{Z} \right.$$

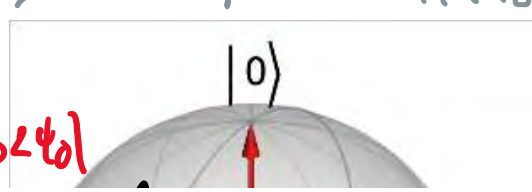
$$\hat{H} |\psi_0\rangle = |\psi_1\rangle = \frac{1}{\sqrt{\ }}$$

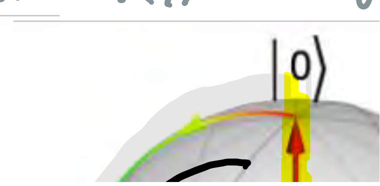
$z = +1$



$|\psi_0\rangle$

$\rho_0 = |\psi_0\rangle\langle\psi_0|$





$$\ldots\Big), \quad \hat{Z} \to \begin{array}{c} \begin{array}{cc} \langle 0| & \langle 1| \end{array} \\ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array}$$

$$\hat{Z} = 1\,|0\rangle\langle 0| + (-1)\,|1\rangle\langle 1|$$

$$\hat{Z}\,|0\rangle = (+1)\,|0\rangle$$

$$\hat{Z}\,|1\rangle = (-1)\,|1\rangle$$

time t_1

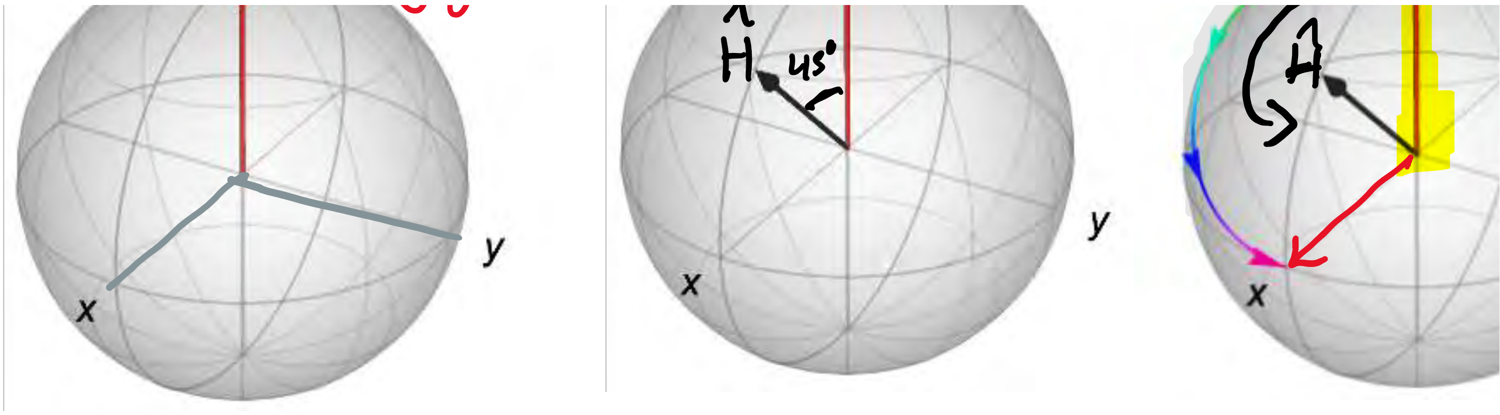
$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\rho_1 := |\psi_1\rangle\langle\psi_1| = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\left(\hat{I} + \hat{Z}\right)$$

$\ldots\frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

time t_1



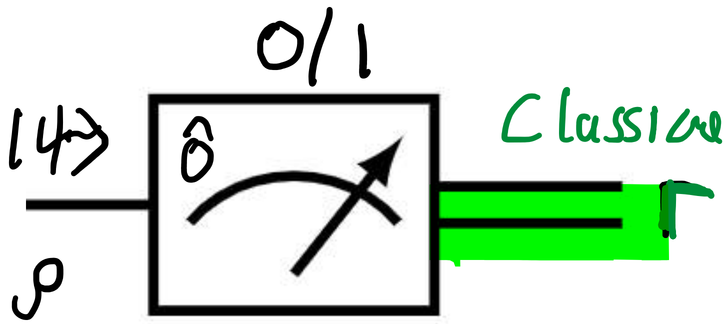
Deterministic state evolution

Measurement: probabilistic

The standard (von Neumann) measurement of a quantum syst

von Neumann measurement is efficient, strong, and projec

Q: Is the qubit in the 1 state or n



$$\hat{O} = \sum_{r \in \{0,1\}} r$$

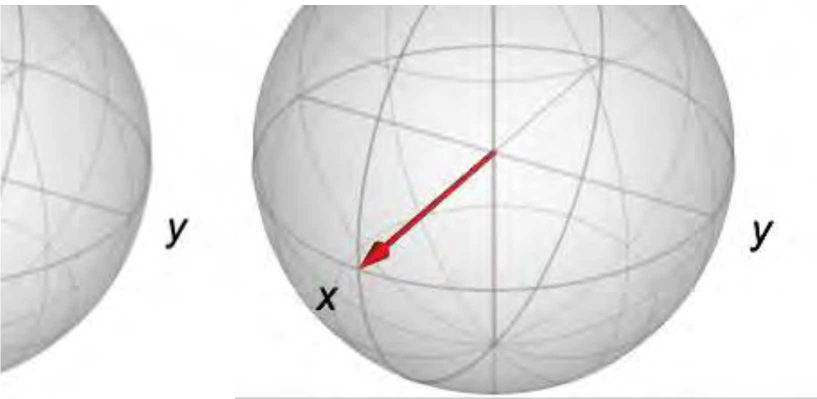
Bernoulli Distribution

$$\hat{\Pi}_r = |r\rangle\langle r|$$

$$\Pi_0 = |0\rangle\langle 0|$$

$$\Pi_1 = |1\rangle\langle 1|$$

$$I = \Pi_0 + \Pi$$



n

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classical variable

ot ? result: $\begin{cases} r = 0 & \rightarrow \quad |0\rangle \\ r = 1 & \rightarrow \quad |1\rangle \end{cases}$

$|r\rangle\langle r|$ $\qquad$ $r \in \{0, 1\}$

resolution of the identity

$\underline{1} \quad \underline{1}$

$\hat{O} = 0|0\rangle$

result $r=0$: $\Pr(r=0) = |\langle 0|$

result $r=1$: $\Pr(r=1) = |\langle 1|$

Projection noise

classical

$$\underbrace{\mathbb{E}[r]}_{\text{classical}} = \sum_r r\, \mathbb{P}(r) = \sum_r$$

classical expect.

$$= \mathrm{T}$$

$$= \mathrm{T}$$

$$= \langle$$

$$\mathbb{E}[r^2] = \sum_r r^2\, \mathbb{P}(r) = \mathrm{T}$$

$$\mathbb{V}[r] = \mathbb{E}[(r - \mathbb{E}[r])^2] =$$

For $\hat{O} = |1\rangle\langle 1|$

$$\langle 0| + |1\rangle\langle 1| = |1\rangle\langle 1| = \frac{I - Z}{2}$$

$$|\psi\rangle|^2 = \mathrm{Tr}(|0\rangle\langle 0| \rho) = \mathrm{Tr}(\hat{\Pi}_0 \rho) =$$

$$\psi\rangle|^2 = \mathrm{Tr}(|1\rangle\langle 1| \rho) = \mathrm{Tr}(\hat{\Pi}_1 \rho) =$$

$$r \, \mathrm{Tr}(|r\rangle\langle r| \rho)$$

$$\mathrm{Tr}(\sum_r r |r\rangle\langle r| \rho)$$

$$\mathrm{Tr}(\hat{O} \rho)$$

$$\langle \psi | \hat{O} | \psi \rangle \quad \text{for pure state.}$$

Quantum

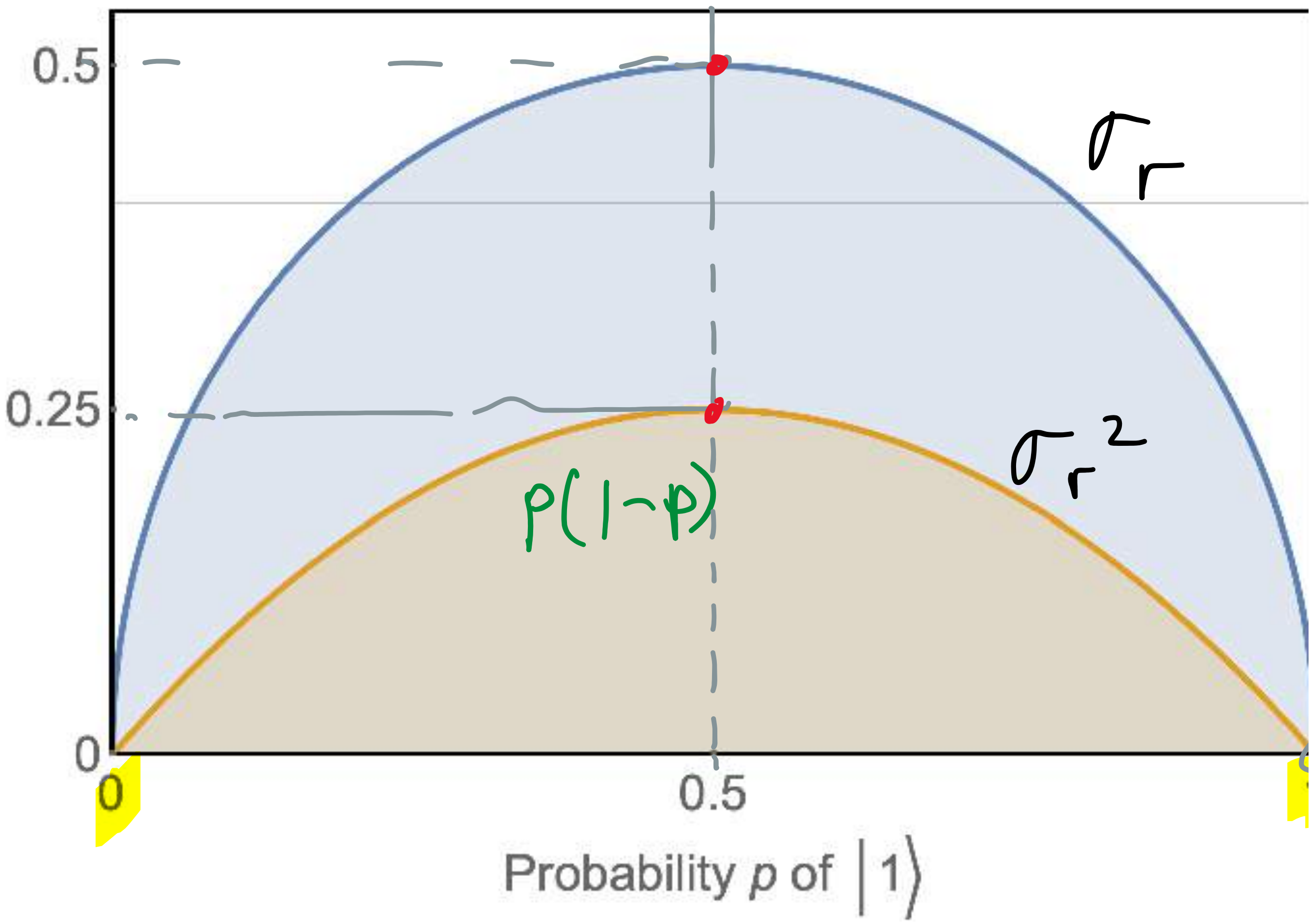
$$\mathrm{Tr}(\hat{O}^2 \rho)$$

$$= \mathbb{E}[r^2] - \mathbb{E}[r]^2$$

$$\mathbb{E}[r] = \mathrm{Tr}(|1\rangle\langle 1|\,\rho) = p$$

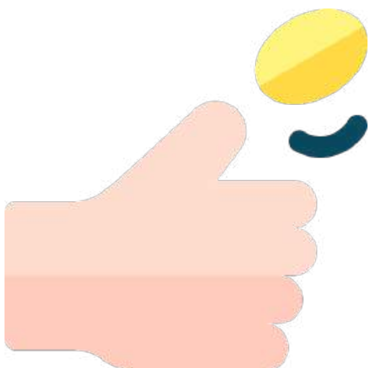
$$\mathbb{E}[r^2] = \mathrm{Tr}[|1\rangle\langle 1|\,\rho) = p$$

$$\mathbb{V}[r] = p - p^2 = p$$



Prob to be excited)

$p(1-p) \simeq: \sigma_r^2$



Random coin toss
Bernoulli distribution
Parameters determined by quantum algorithm

σ_r — std deviation

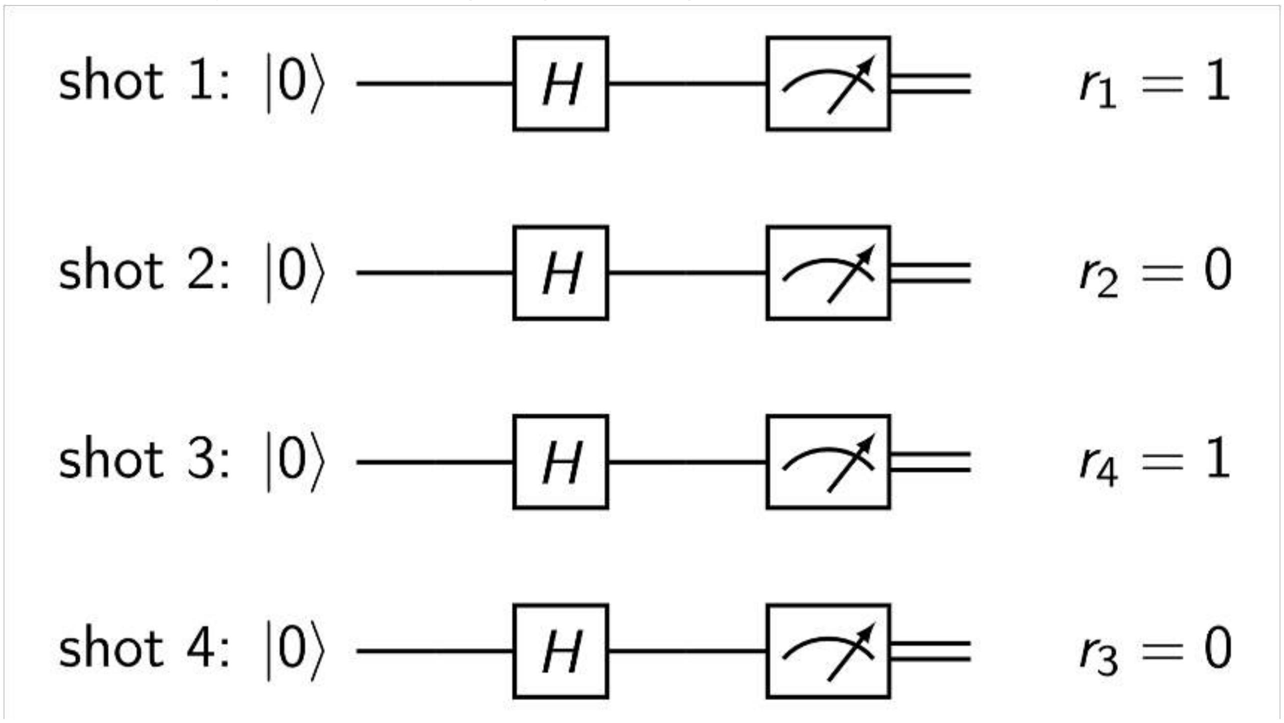
σ_r^2 — variance

1 → Deterministic answer

Projection noise and sampling error

Let's turn to the example of finite number of shots we execute for our experiment.

Perform N experiment, each giving us a single shot result 0 or 1



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Sample mean

$$S = \frac{1}{N}\sum_{n=1}^{N} r_n$$

random variable

random variable

Sample. : $\{r_1, r_2, r_3, \dots r_N\}$

$r_n \in \{0, 1\}$ $n \in \{1, 2, \dots N\}$

$r_n \sim$ i.i.d. $B(p = Tr(\hat{O} \rho))$

For the k-th circuit $\rho \to Tr(|1\rangle\langle 1| \rho) = p$

$N = 3$

For example with 3 samples 2^3 possible sequences

000	100
001	101
010	110
011	111

$\{r_1, r_2, r_3\}$

2^N result sequences

Recall