Laptime Simulation of a Formula SAE Racecar

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1 Introduction

Formula SAE (FSAE) is an international design competition for collegiate engineering students. Teams of students build open-wheel racers simalar in form to Formula 1 or Formula Ford racers, typically powered by motorbike or ATV engines, which race in four different events: a drag race, a figure eight-course, an autocross course, and an endurance track (Which is simalar to the autocross course but ran approximately 20 times). The objective of this competition is to engineer a faster racecar, which means balancing several aspects of performance, since many components are already optimized for specific use cases.

Rose-Hulman Institute of Technology's FSAE team, RoseGPE, has employed laptime simulation in limited capacity, mostly to prove efficacy of specific design changes or general trends. Laptime simulation is simulating vehicle performance on a racetrack based on simplified models of various components of the car and driver. Such simulations have the power to drive design based on the ultimate metric: reducing time on the track. Laptime simulation poses the ability to make engineering decisions which truly balance each aspect of the car.

Many professional and open-source pieces of software already exist to do this very thing, such as OptimumLap (http://www.optimumg.com/software/optimumlap/) and FSAESim (http://fsaesim.com/). However, these have limitations and modeling assumptions which are not readily clear, so GPE seeks to develop their own package they understand and can enhance as time progresses. This project is not finished and will likely be under continuous development, but currently represents a useful model to drive design choices for some systems of the car, namely engine, aerodynamics, and drivetrain.

2 Source code

The entire package is available on GitHub: https://github.com/Thaddeus-Maximus/RoseLap.

3 Vehicle Physics, Limits, and Assumptions

The physics behind a racecar are rather complicated. However, they can be simplified with one main assumption: the point mass assumption. This is the assumption that the car acts as a single point where all loads are applied, and there is no suspension to complicate tire grip and steering characteristics. In addition, we assume a perfect driver- one that pushes the limits of the vehicle to their absolute maximum. This driver does not exist, but this is a useful model as it allows us to push the bounds to which a driver can handle the car. However, we must be sure to evaluate the feasability of the driving style that is found to be "perfect". If it involves constant gear shifting or oscillatory throttle and braking, we may need to re-evaluate our model. Perhaps, in the future, a stochastic model of driver may be applied to assess a design's ease of modeling.

Now, we can begin the vehicle physics. The forces in the vertical direction, with no vertical acceleration are:

$$-F_{down} - mg + N = ma_z = 0, \tag{1}$$
$$N = mg + F_{down} \tag{2}$$

where F_{down} is the downforce due to aerodynamic effects, m is the vehicle mass, g is the local acceleration due to gravity, and N is the normal force of the track on the tires.

The grip of the tires can be divided into longitudinal and lateral force. The total magnitude must not exceed the friction coefficient times the normal force; the limit of static friction.

$$\sqrt{F_{tire,lat}^2 + F_{tire,long}^2} \leq \mu N \tag{3}$$
$$F_{tire,long,remaining} = \sqrt{(\mu N)^2 - F_{tire,lat}^2}, \tag{4}$$

where μ is the coefficient of friction of the tires, and $F_{tire,lat}$ and $F_{tire,long}$ are the forces exerted on the tires by the track.

Given a specific vehicle path, we will prescribe a curvature the vehicle must pass through. The sum of forces in the lateral direction is, then,

$$\Sigma F_{lateral} = ma_{lateral} \tag{5}$$
$$F_{tire,lat} = mv^2 k, \tag{6}$$

where $a_{lateral}$ is lateral acceleration, v is instantaneous vehicle velocity, and k is curvature.

The longitudinal forces acting on the vehicle are that of the tire and drag:

$$a_{longitudinal} = \frac{\Sigma F_{longitudinal}}{m} \tag{7}$$
$$a_{longitudinal} = \frac{F_{long,tire} - F_{drag}}{m} \tag{8}$$

The force of the tire is, though, can vary. In the case of braking, we assume the driver has the ability to lock up the tires, but does not; i.e., the tires apply all remaining grip:

$$F_{long,tire,braking} = -F_{tire,long,remaining}. \tag{9}$$

Acceleration, though, is more complicated, as the vehicle could be limited by either tire grip or engine power:

$$F_{long,tire,acceleration} = min \begin{cases} HP(v)/v \\ F_{tire,long,remaining} \end{cases}. \tag{10}$$

The $HP(v)$ function is a linear interpolation of a power curve to represent the nonlinear nature of internal combustion engines, and gives the output power of the engine. This is maximized over the gears of the car, so time taken to shift gears is neglected (which is a bad assumption). If the velocity is less than the lowest specified RPM on the curve, the torque out of the engine is prescribed to be the same as at the lowest specified RPM, simulating the clutch on the car in some fashion. This behavior should be scrutinized, and maybe even omitted in favor of starting the vehicle simulation at a speed within the dynamometer curve.

An alternative method of dealing with launch is to run the simulation twice, and feed in the final velocity from the first run of a simulation as the initial velocity for the final one. This model should be sufficient if there are ever braking events or speed-limiting turns. Since it should result in a single lap of a driver running a course continuously, this will be referred to as a steady state solution. This is a poor and nonsensical modeling technique for drag races, but suffices when evaluating a vehicles' performance in longer events with multiple laps. This steady state solution is what will be implemented.

For velocities that cause the engine to hit its 'rev limiter', the engine is modeled as producing zero power.

The aerodynamic effects of the car have been discussed but not modeled thus far. These are:

$$F_{down} = \alpha_{down} v^2 \tag{11}$$

$$F_{drag} = \alpha_{drag} v^2 \tag{12}$$

While not a perfect model, this captures the ideal relationship between velocity and downforce.

4 Track Setup

Tracks are loaded from DXF files, a standard introduced by Autodesk in late 1982 as a medium of interchange between their programs that has since become a widely adopted standard for exchange of CAD geometry. Although lots of complicated geometry is supported by this format, only lines and arcs are supported by the parser written for this package. After loading all shapes, they are linked to form a track, then divided into segments, where the lengths and curvature are calculated:

$$l_m = \sqrt{(x_{n-1} - x_n)^2 + (y_{n-1} - y_n)^2} \tag{13}$$

$$l_p = \sqrt{(x_{n+1} - x_n)^2 + (y_{n+1} - y_n)^2} \tag{14}$$

$$l_{secant} = \sqrt{(x_{n+1} - x_{n-1})^2 + (y_{n+1} - y_{n-1})^2} \tag{15}$$

$$L = (l_m + lp)/2 \tag{16}$$

$$p = (l_m + l_p + l_{secant})/2 \tag{17}$$

$$A = \sqrt{p(p - l_m)(p - l_p)(p - l_{secant})} \tag{18}$$

$$k = \frac{4A}{l_m l_p l_{secant}}, \tag{19}$$

where x and y are the coordinates of points on the track, l_m, l_p, and l_{secant} are the distances from the current point to previous point, current point to next point, and previous point to next point. L is the effective length of this segment. p is half of the perimeter of the triangle defined by the current, previous, and next points, while A is the area of this triangle. k is the curvature of the segment. The last two equations are derived from Heron's formula and Menger curvature.

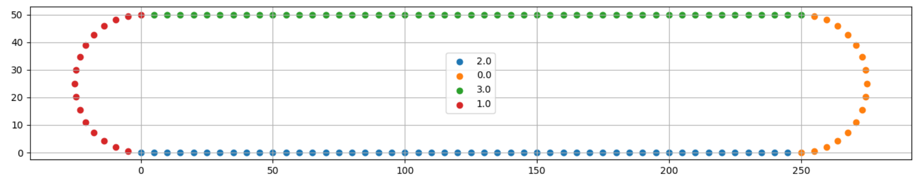

Figure 1: Points for a loop track, spaced at 5 ft

5 Simulation Step for Each Segment

With physics established and a track broken into discrete segments, we can create a function describing the step taken through every segment of the track.

Each step takes in the following arguments: initial vehicle velocity v_0, distance travelled x_0, elapsed time t_f, segment information (curvature k, segment length L), and a flag on whether or not to brake. For all calculations, unless specified, the value of v used will be v_0, as we will be using something akin to a forward-euler method.

The first calculations to be done are computing available tire grip and required lateral force, as described in the equations leading up to and including Equation 3.

if $F_{tire,lateral} > F_{tire,available}$, then the car will not be able to make the turn for this segment of the track. We'll throw an error for this step of the track, and how we handle braking will be discussed later, as this requires thinking ahead.

If we haven't thrown an error, however, we will continue with the program as normal. The remaining tire grip can be calculated with equation 4, which can be used for braking or acceleration. These are covered in Equations 9 and 10.

Now, we can compute acceleration from Equation 8, which will then be used to compute final vehicle velocity. We will assume that acceleration is fairly constant throughout each step, and so we can do simple integration throughout the step.. Recall that acceleration is the rate of change of velocity (which is the rate of change of position), so:

$$a = \frac{dv}{dt} \tag{20}$$

$$v = \frac{dx}{dt} \tag{21}$$

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}\text{by chain rule} \tag{22}$$

$$a = v\frac{dv}{dx} \tag{23}$$

$$\int_0^L a dx = \int_{v_0}^{v_f} v dv \text{by substitution} \tag{24}$$

$$aL = (v_f^2 - v_0^2)/2\text{separation and integration} \tag{25}$$

$$v_f = \sqrt{v_0^2 + 2aL}, \tag{26}$$

where v_f is the final velocity of the car after this step. The final elapsed distance can be calculated as $x_f = x_0 + L$, and the final elapsed time can be calculated as $t_f = t_0 + \frac{L}{(v_0+v_f)/2}$.

6 Solution process

The solution process for acceleration is now straightforward: start at some initial velocity, solve for final velocity out of the first segment, then feed the final velocity in as initial velocity for the next step, and so on.

However, we noted previously that steps can throw errors due to the car going off course from insufficient lateral grip. Ideally, the driver must stop as late as possible to maximize speed. This behavior is modeled with an iterative solver.

When failure occurs, the index at which it occurs is marked and the index to restart from; the "safe point" is set to the index prior this. We re-simulate from this safe point up to the index of failure, but with the brakes applied. If there is still failure at this point, we decrement the safe point and repeat this process from the new safe point. This is repeated until success. From the previously found point of failure though, the driver lets off the brakes, free to sustain and accelerate.

7 Evaluation of simulation

The loop track from Figure 1 will be evaluated in this section. Vehicle parameters used are:

Local gravity g	32.2 ft/s^2
Vehicle mass m	13.97 $slug$
Downforce at 35 mph	65 lbf
Drag at 35 mph	44 lbf
Tire coefficient of friction	2.0
Tire radius	0.75 ft
Engine RPMs for dyno curve	4500, 6000, 8000, 9000
Engine HPs for dyno curve	10, 25, 45, 30
Engine gear reduction	2.81
Final drive reduction	2.769
Shifting gear ratios	2.416, 1.920, 1.562, 1.277, 1.05

7.1 Segment size convergence

Figure 2 shows a segment-size convergence study for the loop track. From left to right are segment sizes of 8.0, 5.0, 2.0, 1.0, 0.5, 0.35, 0.2, 0.15, and 0.1 feet. The track itself is approximately 657 feet long.

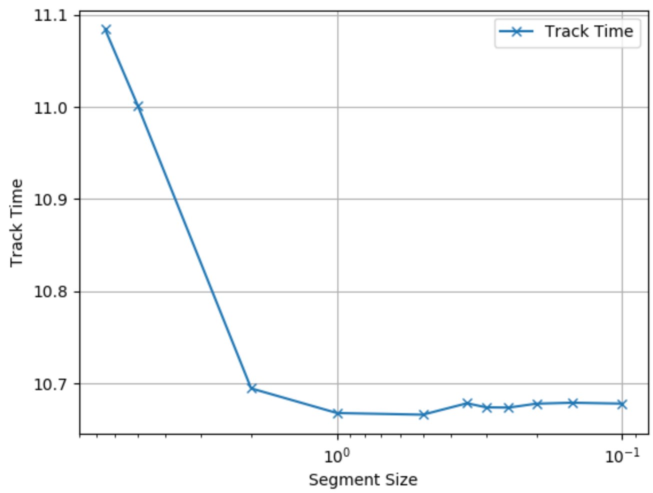

Figure 2: Mesh convergence for loop track

This shows good convergence at a segment length of 0.35 feet, so for further studies we will use a segment length of 0.25 feet to balance length independence while ensuring good computational speeds.

7.2 Sanity check of solution

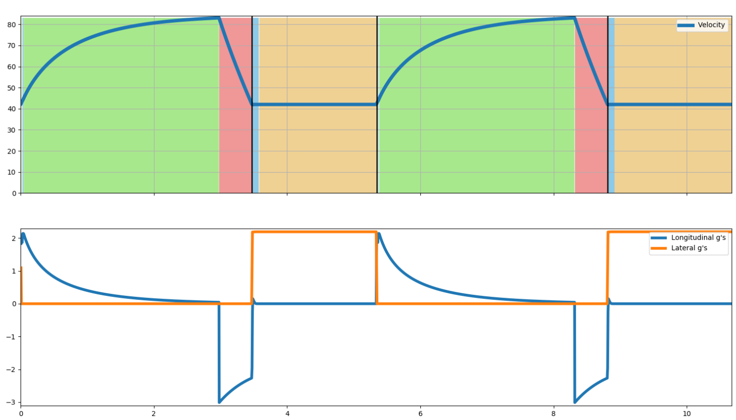

Figure 3: Vehicle speed and condition for loop track. Region colors are as follows: Red: Braking, Green: Engine-limited acceleration, Blue: Tire-limited acceleration, Orange: Tire-limited velocity sustaining.

The behavior shown in figure 3 is much what should be expected for this track- the driver accelerates as much as possible, and the acceleration tapers off since engine power does not keep up with increasing velocity. The driver experiences near-constant braking (there are aerodynamic effects, so the curve is actually nonlinear), before entering the curve at a constant velocity. This then repeats for the rest of the track which is laid out in the same way. Recall that this is a steady-state solution; there is no launch period. Lateral acceleration (in g's) spikes around the corners, and longitudinal acceleration is highest at braking, and declines due to decreased downforce.

7.3 Sanity check of solution - autocross course

The autocross track shown in Figure 4 is simulated, as shown in Figure 5.

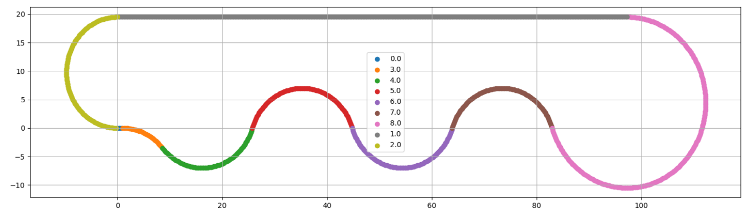

Figure 4: Autocross track.

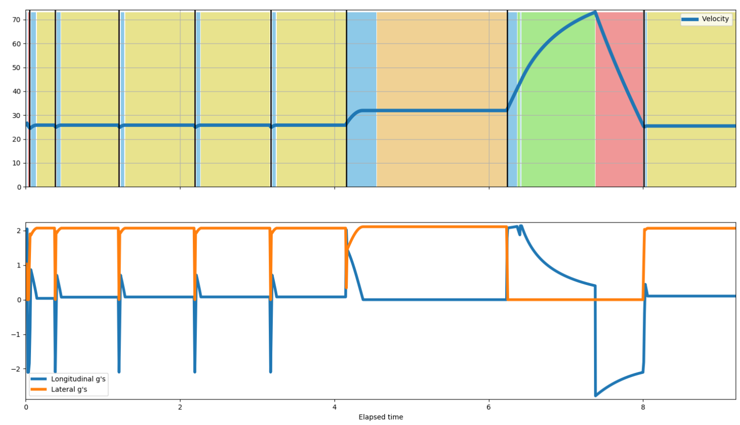

Figure 5: Vehicle speed and condition for autocross track. Region colors are as follows: Red: Braking, Green: Engine-limited acceleration, Blue: Tire-limited acceleration, Orange: Tire-limited velocity sustaining.

These results qualitatively make sense: the vehicle sustains a steady velocity throughout the slalom on the bottom, then accelerates to a engine-limited top speed around the right loop, then accelerates along the straightaway before braking for the left turn. The bursts of longitudinal acceleration at the apexes of the slalom are partly due to the zero curvature at these points, where the simulation believes the driver can accelerate further, but is soon stopped. This behavior may need to be mitigated, but does not appear to present significant issues at the moment.

8 Sample Design Studies

In these studies, vehicle parameters will be modified from the ones used previously in order to learn the effect of various parameters on track time.

8.1 Aerodynamic scaling factor

We will examine the effect of aerodynamic scaling factor, that is, a multiplier on both downforce and drag effects.

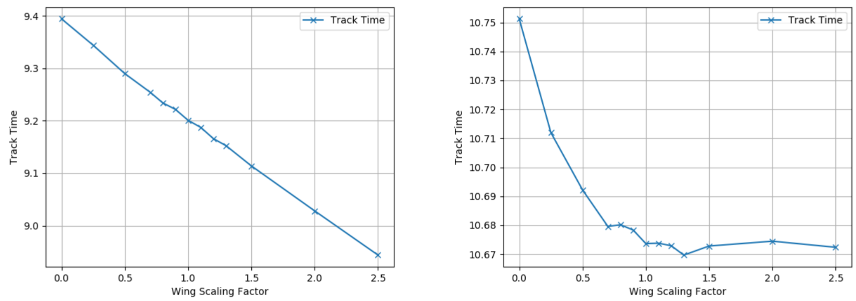

Figure 6: Track time (in seconds) for autocross (left) and loop (right) courses, as a function of aerodynamic scaling factor

For an autocross course, with lots of turns that are limited by tire grip rather than engine power/drag, producing more downforce has a significant impact on track time- doubling the aerodynamic characteristics gives a about 2% decrease in laptime. Although this may seem like a small margin, these gains are what can determine the difference between first and tenth place. The loop results show that adding any significant aerodynamic effects helps track time, but anything past a scale factor of one is a moot point since gains from the turns are negated by losses on the straights.

This should help to guide design decisions- if these were the only tracks we are designing towards, it's (somewhat) clear that adding more aerodynamic effects is beneficial.

8.2 Aerodynamic efficiency

We will now examine the effect of aerodynamic efficiency, that is, a ratio of downforce to drag.

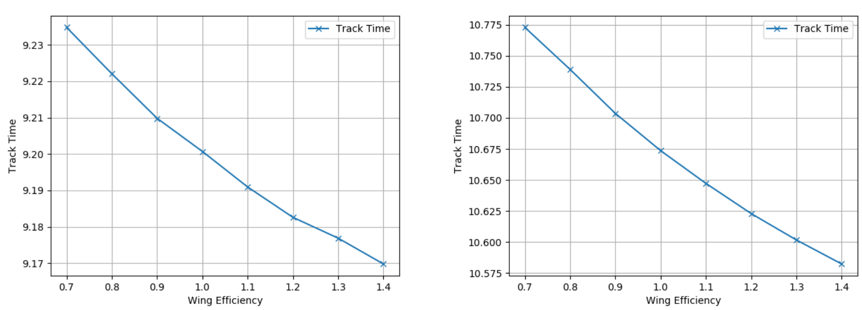

Figure 7: Track time for autocross (left) and loop (right) courses, as a function of aerodynamic efficiency

Unsurprisingly, increasing efficiency decreases track time. This is made more evident in the loop course, however; the gains on the loop are 1.89% while the gains from autocross are 0.8%.

8.3 Final drive ratio

We will now examine the effect of final drive ratio, which is a bidirectional scaling factor on the engine. The higher the ratio, the faster the engine spins.

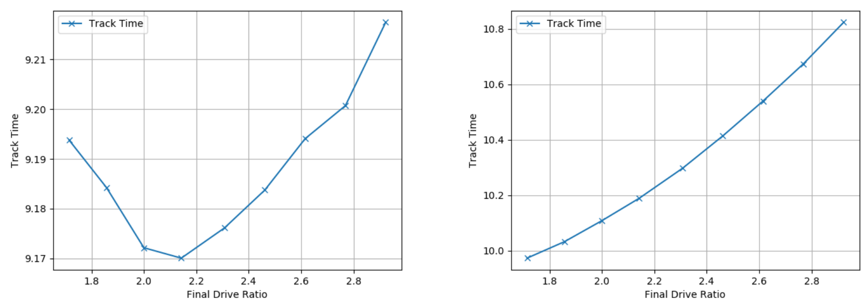

Figure 8: Track time for autocross (left) and loop (right) courses, as a function of final drive ratio

This shows a relatively small (0.6%) gain from optimizing final drive ratio within this range for autocross, and fairly large (8.5%) gain from optimizing final drive ratio for the loop track. This is likely due to the vehicle being out of operating velocity on the loop track in the first place.

9 Conclusions and Future Plans

The benefits to designing and setting up a vehicle to the track via simulation can be realized by the results already shown in this paper. Although the models are not yet validated, the trends are consistent with anecdotal and professional advice. Attempting to validate these models with on-track testing will be an important step in further development.

As mentioned before, this simulator is not yet superior to alternatives. To make this a fully-featured and highly useful tool, a number of improvements are to be added. Among these are:

- Shifting time: currently the best gear is selected, but the time taken to shift can and has influenced significant design decisions for teams, ranging from opting to use less gears to only using one.
- Suspension characteristics: oftentimes, understeer and oversteer impact the ability to maintain a vehicle along a path; the modeling of a suspension into the vehicle dynamics may prove to be crucial.
- Weight distribution: the effort required to rotationally accelerate a car is nonzero and must come from somewhere. Additionally, acceleration causes weight to shift forwards or rearwards, affecting braking and acceleration.
- Brake bias: the distribution of braking force can serve to limit effective braking.
- Tire models: tires have effective coefficient of friction dependent on normal force and heat, and grip is very frequently the limiting factor.
- Stochastic behavior of drivers: making a car easier to drive based on error bounds for track times can help make a car not only with higher performance bounds, but higher on-track performance.