Multiple particles colliding random walks

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Abstract

We solve the problem of collisions for multiple particles random walks by treating it as a binary differential equation with finite time steps, finite space, and random transitions. No other paper generalizes random walks for three or more particles. Collision is resolved by requiring that the transition rule is Boolean self-dual, similarity under logical negation for particles and antiparticles. It also exhibits nonlocality such as in quantum mechanics because the first few switches dictate what switches are left over in the current time step. The state space grows according to the combination function. Simulation of a 4x4 system implies that the state space is not uniform and not unimodal, implying convergent and divergent states, unlike single-particle random walks. A Fast Fourier transform appears to have a radial concentration of states towards lower frequencies, suggesting the Fourier transform is Normally distributed like in Brownian motion. It directly explains why a low entropy state is unlikely but inevitable. Colliding random walks were conceived by considering the Planck length as the length of a universe's infinitesimal unit cube. Observing the wavefunction may be interpreted as a Bayesian information update rule of a Markov process that converges to its stationary distribution.

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1 Background and Introduction

In this paper, multiple particles colliding random walks describes the collision of multiple random walks processes. Despite the titles "Multiparticle Random Walks" and "Multiple Random Walks" appearing in literature, it is clear that none of them are referring to same topic. One paper titled "Multiparticle random walks" by Acedo et al. is an example of a paper with a similar name, but it is different conceptually [1]. Chen's paper "Two particles' repelling random walks on the complete graph" is of interest because we sample for the collision distribution of two particles, but it does not generalize to multiple particles [3]. We use multiple particle random walk (MPRW) to refer to our topic throughout this paper.

Resolving collisions requires relaxing the constraint of locality and synchronicity. Since each cell depends on the adjacencies of other cells, it is difficult to run parallel computations on the system as a whole. The problem of globally conditional probability only becomes more difficult as system size increases. Only when a smaller system is cut off from the rest is parallel computation possible.

According to the Law of Large Numbers, a Binomial distribution converges in distribution to a Gaussian distribution as the sample size approaches infinity. This fact is important as random walks approximate Brownian motion at the limit. Squares are circles, so long as you convolve uniform or Bernoulli distributions repeatedly over time. Random walks, like Brownian motion, have their position variance equal to the time t since the last known position. The Multinomial distribution and the Multivariate Normal distribution are equivalently related for higher dimensions.

Random walks, Brownian motion, and fluid simulation are stochastic processes that are often used to model physical phenomena in the real world. Wave-like objects appear everywhere in nature such as in the diffusion of gas, the waves of the ocean, or even the slow-rolling oscillations of mountains and hills. Brownian motion is named after Robert Brown who described the phenomenon 1827. Einstein published a paper modeling pollen particles in water in 1905, and Karl Pearson first introduced the term *random walk* in the same year.

The following conjecture is possibly relevant to the issue of phenomena of waves and diffusion. This may be of use when dealing with the alternating, colliding Multinomial distributions that occur in MPRWs.

Conjecture 1 (Alternating Normal Sum Conjecture) *Let the Normal distribution be given by* $\phi(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \sim N(\mu,\sigma^2)$*. Then:*

$$\sum_{\mu=-\infty}^{\infty} \phi(x|\mu\cdot\sigma,\sigma^2)(-1)^{\mu} = \frac{2\omega}{\sigma}cos(\frac{\pi x}{\sigma})$$

$\forall x \in \mathbb{R}$. *It is called the wavebound equation, where* $\omega = e^{-\frac{\pi^2}{2}} \approx 0.00719188$ *is the wavebound constant,* $\omega^{-1} = e^{\frac{\pi^2}{2}} \approx 139.046$.

Quantum mechanics describes the nature of the universe at subatomic scales. "Quanta" refers to the discrete nature of measurements. At this scale of the universe, the Law of Large

Numbers does not apply as individual random events have large consequences. Quantum entanglement refers to the dependency of one quantum system's state on another quantum system.

Particles in real- life quantum systems are identical, meaning that you could not tell them apart. If a virtual particle-antiparticle pair came into existence, the antiparticle annihilated the original, and the virtual particle replaced the original, it is unlikely that you would notice unlike you were constantly measuring the system. It is similar to asking the question, "Where is the sea?". We see the phenomenon of waves, but are waves real objects?

The Ship of Theseus is a thought experiment of whether or not a ship, gradually having all of its parts replaced, is the same ship or not. It is also a question of identity, should you reassemble a second ship using all or some of the original parts. Many physical objects are subject to the Theseus paradox. Atomic particles, subatomic particles, and colliding random walk particles are no different.

Cellular automata have been studied extensively since the 1940s but were first popularized by John Horton Conway's *Game of Life*. It is Turing complete, and can simulate logic gates and thus Turing machines. It is entirely possible to relax the constraint Turing completeness by allowing for probabilistic Turing machines, thus creating probabilistic logic gates, which shall be referred to as chaos gates here informally. The name chaos gate is chosen because is illogical to distinguish between transformer and transformee because one way interactions logically contradict the definition of interaction. Globally conditional probability, or non-locality, is itself not proof of the emergence of chaos gates or a Class 4 cellular automaton.

A Boolean function is said to be self-dual iff negating the inputs and negating the outputs is the same as the original function. Since self-dual may refer to many types of dual function, Boolean self-dual or negation-symmetric are used instead. The following formula describes a Boolean self-dual function:

$$f(x_1, x_2, ...x_n) = \neg f(\neg x_1, \neg x_2, ...\neg x_n)$$

This is the requirement that we place on the probability function of our Markov process. We could not locate any stochastic processes that are Boolean negation-symmetric or anti-symmetric. Boolean self-duality is relevant if chaos gates exist because the gate is entwined with the input and output. NAND and NOR gates are functionally complete, meaning they can construct any other logical function in series. Functional completeness under logical negation has unsettling implications if taken as a model of the universe. Physical relevance is not necessary for the stochastic process to be useful.

The Pauli exclusion principle states that fermions, such as electrons, cannot occupy roughly the same position and quantum state at the same time. This is somewhat applicable to the research presented. Not only are probability density functions well-defined, the identity of which particle is which is still probabilistically defined. Two particles in an antiparticle sea may be indistinguishable, but each still is more likely to be closer to where it started and less likely to be where the other started. The collision property is the main reason why multiple particle random walks cannot be described as a martingale, a key property of single particle random walks.

2 Motivation

The original goal of the project was to interpret the Planck length. The Planck length ℓ_P is a constant in physics that can be solved by cancelling known fundamental constants of nature. It appeared at first to be an oddity. A definite length for objects in the universe? Then came the Planck time m_P, the Planck mass m_P, and the Planck temperature T_P. Physics has continuously shown that there was a smaller fundamental object than before, and string theory was acclaimed to be the theory to overtake the Standard Model. So easy is it that we forget that our screen is composed of a dense collection of pixels and our vision is composed of a dense collection of cells. The idea of the Planck length ℓ_P as the infinitesimal unit cube length ϵ had not been explored before, despite the abundance of squares and matrices throughout mathematics.

That original goal evolved to developing a theory to compete with string theory, and a year long search for a suitable transition rule. If the universe did work on the principle of Planck-length pixels and randomness, what would that be? Why are Gaussians everywhere in the universe? What was a random walk? How should collisions be handled? Why are discrete numbers fundamental to quantum mechanics? Why do antiparticles exist? Is there an implied orientation to the universe that vanishes at larger scales?

The perspective of logical negation and Boolean self-duality started to become more apparent. The mistake was assuming that empty space should be treated as differently from full space. If grains are subject to Brownian motion in water, so too is empty space. The grains of empty space are just smaller than the grains of water molecules. Most of space is empty, yet that empty space contains fluctations in the vaccum, irreducible motion. Multiple random walks, colliding and dancing, condensing and expanding. Is reality reducible to random Boolean systems? Even if it was not true, the idea that none had considered precisely *what* collding random walks should be was perplexing.

The solution to multiple particle colliding random walks was solved by considered a logical contradiction. Blocks that could switch with any neighbor according to the exponential distribution. It took time to rule out that particles could not could not switch with other particles, a "false switch" being a waste of time. If blocks could switch with themselves, then we are merely shuffling blocks with no respect to their value. Then, it was considering an exponential distribution for 1s and 0s to switch. The Planck time rears its ugly head. Is time discrete? A particle should not move more than once per time step. Lastly, it was realizing that switches could be sampled and cells could be *locked* until the next time step. If blocks were not locked, then they could move infinitely per time step.

The concept of locking blocks that have been designated to switch, or undergo logical negation, is what defines a cellular automata as asynchronous. Cellular automata (CA) are assumed to be synchronous, deterministic, and local. Random walks are clearly not deterministic, so what random function determines who goes first when two particles are at risk of collision? Is it even possible to have multiple random walkers that rely only on local information? It is exactly this particle bias that veiled the fundamental that particles and antiparticles are subject to the same rules. Lattice gas automata simulate fluid flow but are anisotropic, preferring certain orientations more than others. MPRWs are anisotropic in singular time steps but are statistically isotropic over long stretches of time. Multinomial distributions are approximately Gaussian over long stretches of time.

Physics often describes the problem of balls in a box to model entropy. One side being purely blue balls and the other side being purely red balls is considered a state of low entropy. Information content of an event E is given by $I(E) = -log_2(p(E))$. However, you cannot actually measure the information of a continuous state space. By definition, you need to know the exact probability. For continuous functions, entropy requires measuring an integral in which a single state is an infinitesimal. How can you know for sure that such an ordered state is surprising? MPRW simulation provide a clear answer because of the discrete nature of the system and its state space. What makes an ordered state unlikely?

3 Planar Case Study

It would be misleading to say that the following example is what caused the discovery of the transition rule, but it is what solidified it. It provides an interpretation of the unlikelihood of gas particles in a box allocating on one side of the box. It provides a clear diffusion equation. It shows the switches aligned perfectly in parallel. See Figure 1 to see the relationship to differential equations. Waves emerge naturally because switches are perpindicular to the boundary. The relationship with the imaginary number i is more obvious when shown the boundary approximates a sine wave.

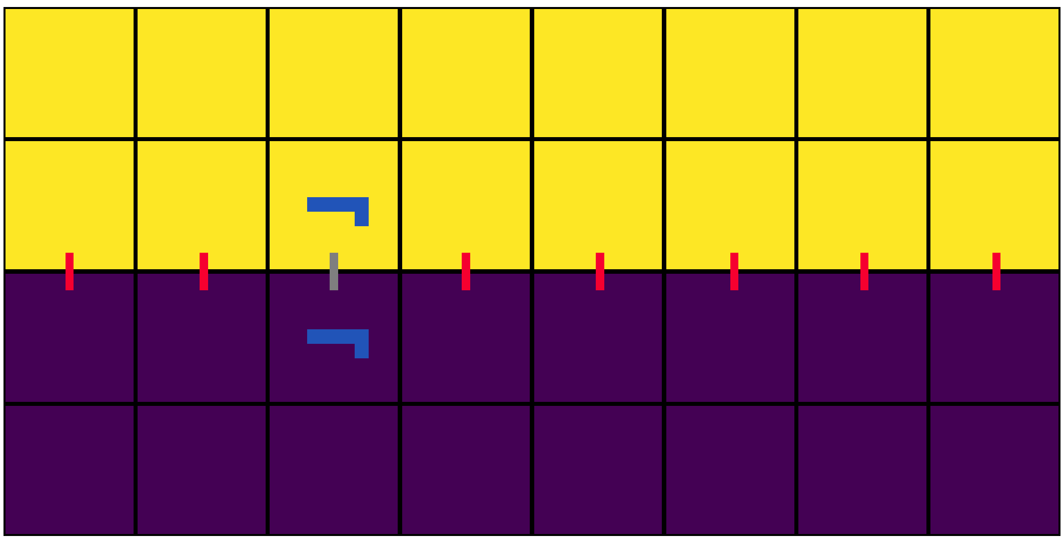

Figure 1: 2D planar case. Stochastic binary diffusion equations require a difference between cells to sample. Cells are marked for logical negation.

The balls in a box problem, and the gas particles problem, is far easier to analyze because of the finite state space. In MPRWs, the planar problem represents a vertex in the state space where there is clearly only *one way in and one way out.* If viewed under the lens of graph theory, the most unlikely state is a leaf vertex, a vertex of degree of one. Some states, when represented as vertices, are even or odd vertices based on their degree mod 2. The three-dimensional case is similar, except it is far more difficult to visualize an animation of the diffusion due to particles obscuring each other. See Figure 2 for a 3D example of binary diffusion.

4 Definitions and Notation

This section is important as it merges and modifies various notations across multiple branches of mathematics. Each branch of mathematics has its own unique notation that was developed in relative isolation, and thus they sometimes conflict. This paper crosses the leaves

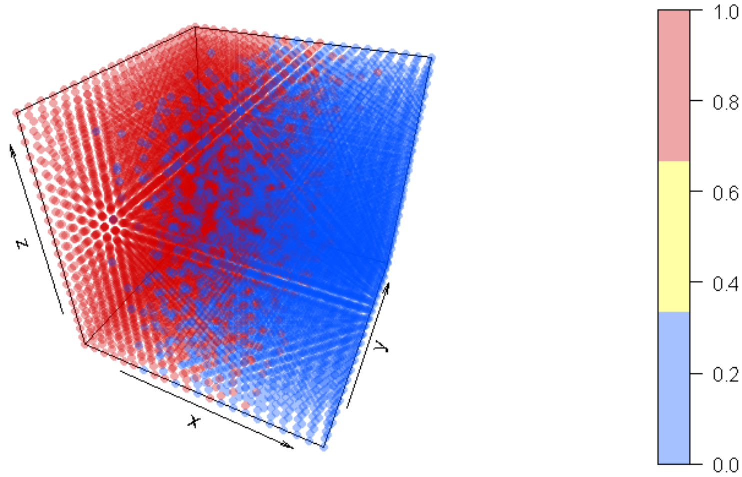

Figure 2: 3D planar case. Diffusion after 10 time steps in a 22^3 system. Particles and antiparticles are translucent to allow for visualization.

and branches of stochastic processes, differential equations, mathematical logic, cellular automata, and graph theory.

The *system tensor* X, which may be clarified as the state system tensor, is the d-dimensional array containing values from the binary set

$$\mathbb{B} = \{0, 1\}$$

It is called dynamical system because it is a space that evolves over time according to a transition rule. We are typically dealing with matrices for visualization reasons, thus we may refer to it as a state matrix. Note that $\neg X$ refers to the logical negation of X, corresponding to flipping all ones and zeroes.

The system has d *dimensions*, such that $d \in \mathbb{N}$. The l*ength of each dimension* is thus $n_1, n_2, ..., n_d$ such that the *total number of cells* N is given by their product

$$\prod_{i=1}^{d} n_i = N$$

The *time* variable t indexes the progression of states as the evolve according to the transition rule. The system tensor evolves over time, so it may be indexed using X_t. X_0 is implied to be a particular realization at time $t = 0$ while X is understood to be any system matrix one.

The *state space of* X is denoted using Ω_X or $\mathcal{X}$. The state space of X denotes all binary strings of X and is of size $\theta = 2^N$. The *reduced state space of* X is denoted using $\mathcal{X}^{(k)}$. It refers all states to having k particles and $(N-k)$ antiparticles. The reduced state space is of size

$$\theta^{(k)} = \frac{N}{k!(N-k)!}$$

This reduced state space is more often used when dealing with finite, bounded systems.

The *graph* of the state space of X is referred to as G_X. This representation is not used often.

The *probability density function* p maps the conditional probability of one state transitioning to another at the next time step

$$p(X_{t+1} = A_j | X_t = A_i), i \neq j$$

This representation is important when understood as a Markov chain or Markov process. Each row of the transition matrix is a probability density function.

The *transition function* q samples a realization of p for a given input X. The idea of sampling is important as the state space $\mathcal{X}$ grows according to the exponential function or $\mathcal{X}^{(k)}$ to the combination function. We specify in the transition function of the boundary walls are looping or reflecting.

The *state vector* y, which may be clarified as the state indexing vector, is the θ length vector containing all states in the state space. It is a realization of the system, meaning that all values of y_i sum to 1. Eachy_i represents the indicator function $1_{X=i}$ or the probability function $P(X = i)$. A state vector represents a specific state if only one value equals 1 and the rest equal 0, and it represents a distribution or superposition if it has multiple values not equal to 0.

The *stochastic transition matrix* P describes the probability that a state transitions into another state. It is of size θ^2 for the full state space. A 10^2 system with $10^2/2$ particles quickly far exceeds the number of electrons in the observable universe. Although a transition matrix theoretically exists, even a reduced transition matrix $P^{(k)}$ is practically impossible to compute. It is assumed to be symmetric such that $P = P^T$, meaning that it is both left stochastic and right stochastic. This is because the probability of originating equals the probability of becoming. It is worth noting that the diagonal equals 0 at all times, meaning that it is periodic. P is thus a *hollow matrix.* It is assumed that eigenvalues do exist and the determinant is positive.

The *adjacency matrix* A defines whether or not states are connected to each other. When a probabilility measure is applied to it, then it is a stochastic transition matrix.

This *indicator function* $1_{\{X=i\}} = A_i$ referring to a state being state i. Each unique state in a finite system X with N cells has a unique binary value that may be converted to decimal. The use of the adjacency matrix term A_i helps to avoid confusion in identifying states across all representations such as $p(y_t = A_i)$, $p(X_{t+1} = A_j | X_t = A_i)$, or $A_\alpha = \neg A_\omega$.

The *limiting distribution* λ is given by$\lim_{n\to\infty} yP = \lambda$ for any initial distribution y. The limiting distribution is not guaranteed to exist, but if it does it is a stationary distribution. Common reasons why limiting distributions may not exist are periodicity such oscillating between two states. This is guaranteed to not exist since no state chains into itself, a minimum periodicity of 2.

The *limiting transition matrix* Λ is given by

$$\lim_{n\to\infty} P^\infty = \Lambda$$

which represents running an infinite amount of transitions. It is a transition matrix where all row are equal to λ^T. It only exist if λ exists. This is not possible since P is a hollow matrix, implying periodicity.

The *stationary distribution* π is the state vector such that $\pi P = \pi$. The stationary distribution are thus the eigenvalues of P since $(\pi P - \pi I) = 0$ is a linear system of equations, where I is the identity matrix. It is guaranteed to exist, unlike the limiting distribution. It unlikely to be computable due to the growth rate of the choose function.

The *finite forward difference operator* Δ operates on the system tensor X. If X a d-dimensional tensor, then ΔX is a d^2-dimensional tensor. If X is a d dimensional tensor, it can be represented with $d+1$ columns with the first d columns representing the dimensional coordinate and the last column indicating the value of $x \in \mathbb{B}$. Likewise, ΔX can be represented as a $d+d$ columns. Thus, ΔX can simply attach d more columns to X in order to store the forward difference information in a unified way. If the system X is allowed to loop, then we may use modular arithmetic according to the length of each dimension $n_1, ..., n_d$, we can calculate the looping forward difference of endpoints. Looping means that X and $DeltaX$ are isomorphic to a graph and its dual graph. This representation is useful when we are realizing the transition function.

5 Solution to Colliding Random Walks

Suppose that you have a finite system tensor X_t. Obtain the forward difference ΔX. This data from ΔX will be copied on D and is mutable. Then use one of the two following methods:

- Sample each dimension according to total number of switches. Select a random cell from that dimension.
- Sample each cell weighted to the number of forward switches. Select a random dimension from that cell.

Cell $x_{a_1,a_2,...,a_d}$ has a switch corresponding to each dimension, $\delta x_1, \delta x_2, ...\delta x_d$. . Each dimensional switch corresponds to one block ahead along that dimension. Sample switches from x where $\delta x \neq 0$, and ignore switches where $\delta x = 0$. It may be practical to take $D = |\Delta X|$ for computational purposes. Once the cell and the forward cell are chosen, mark down the unique position of each cell to be negated.

The unique position of the cell is given by:

$$a_1 * 1 + a_2 * n_1 + a_3 * n_2 + ... + a_n * n_{d-1} = \sum_{i=1}^{d} a_i * n_{i-1}$$

where $a_0 = 1$. This indexing protocol is useful when it is easier to compute and store the forward cell in a table instead of compute it every single time. The number of switches asympototically approaches the upper bound of $min(k, N-k)$ switches per time step.

Once the original cell and the forward cell are marked for logical negation, all switches related to those cells must be disabled on D. The number of switches to coerce to 0 is given by $2f-1$, where f is the number of faces on the cube. Those cells are now *locked* for the rest of the time step since all switches are disabled. Disable all switches on the forward cell, and disable the forward switch of the cells that are before the original cell.

Repeat the process using D until the sum of all forward differences is 0. All cells x marked for logical negation are negated. This new tensor is X_{t+1}. The simulation may be repeated as many simulation time steps, usually denoted *nsim*.

6 Colliding versus Independent Walks Case Study

Colliding random walks give us an interesting interpretation of the "collision" of two (maximum entropy) Multinomial(t) distributions. A simulation with two particles with one cell between them, a distance of two apart, is run 1000 times. This helps generate a decent sample of the distribution of particles at time $t = 10$. See Figure 3 for the initial state and difference plot.

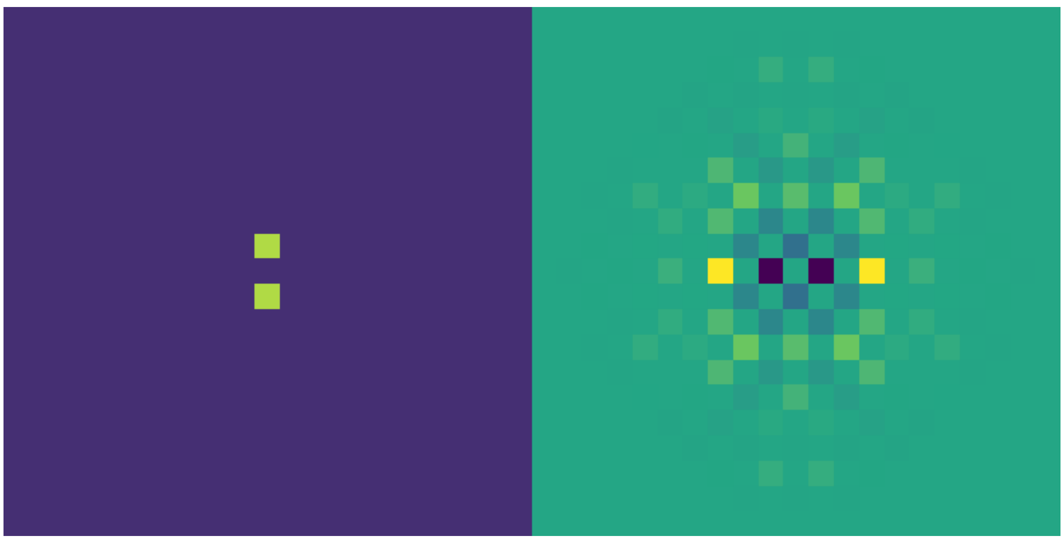

Figure 3: The left image is initial state of two particles at a distance of 2 apart. The right image is the colliding distribution density minus the noncolliding distribution after 10 time steps.

The viridis color palette is used for the purpose of aiding the color blind as well as allowing for black and white print. In the initial state, the yellow color indicates the initial particle positions while the purple color represents the antiparticle sea. In the difference graph, the yellow indicates spots where the colliding particles were more likely, the purple indicates spots where the colliding particles were less likely, and the green indicates approximately neutral or the same probability.

The yellow spots indicate a higher probability of a particle being there while the blue spots indicate a higher probability of an antiparticle being there. As a reminder, particles are identical meaning that distinguishing between them is a probabilistic measure. Likewise, the negation-symmetric nature of the simulation means that distinguishing between antiparticles is *exceedingly* difficult. It is easier to ask where the needle is in a haystack than to distinguish hay in a haystack. Interpreted using the uncertainty principle, less frequent objects have a more certain position. When in the ocean, it is more coherent to ask where the water is *not*.

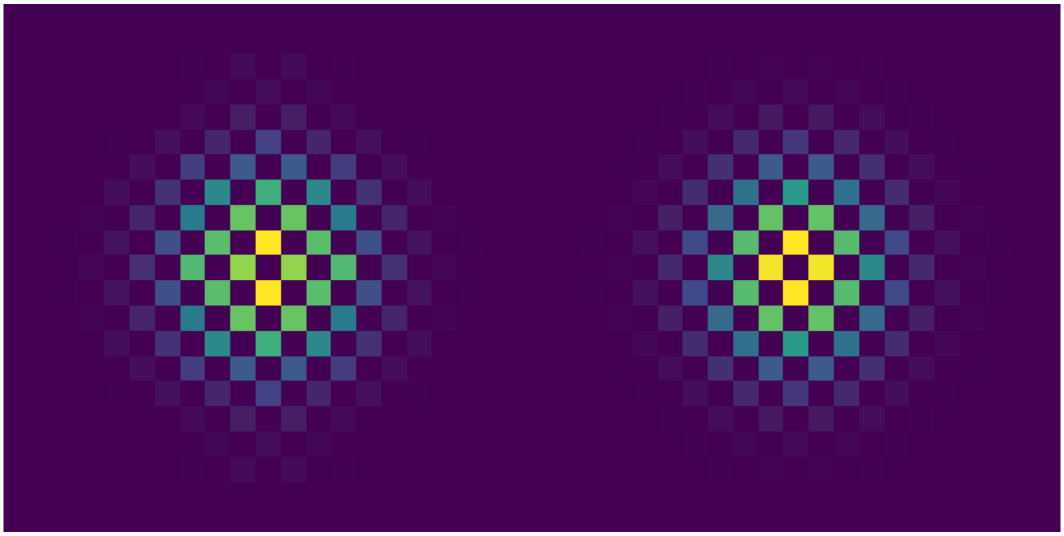

Figure 4: The left image is the colliding distribution density, forcing symmetry of the data to reduce sampling needed. The right image is the noncolliding distribution density by convolving Multinomial(10).

7 4x4 State Space Case Study

A 4x4 system the smallest even reflecting system that is nontrivial, which 2x2 is. Reflecting 2x2 systems, when set to have an even amount of particles and antiparticles, strictly oscillate between two states. A 4x4 system on the other hand has

$$\frac{(4^2)!}{(4^2/2)!(4^2/2)!} = 12870$$

unique states. A large state space, but it is small enough that we can directly sample simulations of every single permutation on a personal classical computer. This provides us a question that we can prove true or false. Is the state space unimodal or uniform? Are there convergent and divergent states? Both of these questions ask the same thing: Are some states preferred more than others in the short term and the long term?

A single particle random walk in a finite box can is unimodal as time progresses from a known position and one major plateau, not quite uniform, in the stationary distribution. It is unclear what the Euclidean dimensionality of the state space is, but is assumed to be at most $min(k, N-k)$. The dimension of a graph in graph theory is said to be no more than twice the degree the vertex with the maximal degree plus one:

$$EdimG \leq 2\Delta(G) + 1$$

Paul Erdős and Miklós Simonovits proved the result in 1980 [4]. Deeper research into graph theory is necessary to analyze the exact dimensionality of a MPRW system. Analyzing the dimension of a graph is an NP-hard problem, even if the Euclidean dimension of a graph is two. The state space graph G_X is at least a partial cube of Euclidean dimenson N if the goal is to track the identity of each particle and antiparticle, but it is unclear how many dimensions it may be reduced to when not tracking.

We run 100 1-step simulation of each of the 12870 states. We obtain each state's unique index number by converting the binary sequence into a base-10 number. In doing this, we clearly see that some initial states exit into more unique states than others and some states are exited into than others.

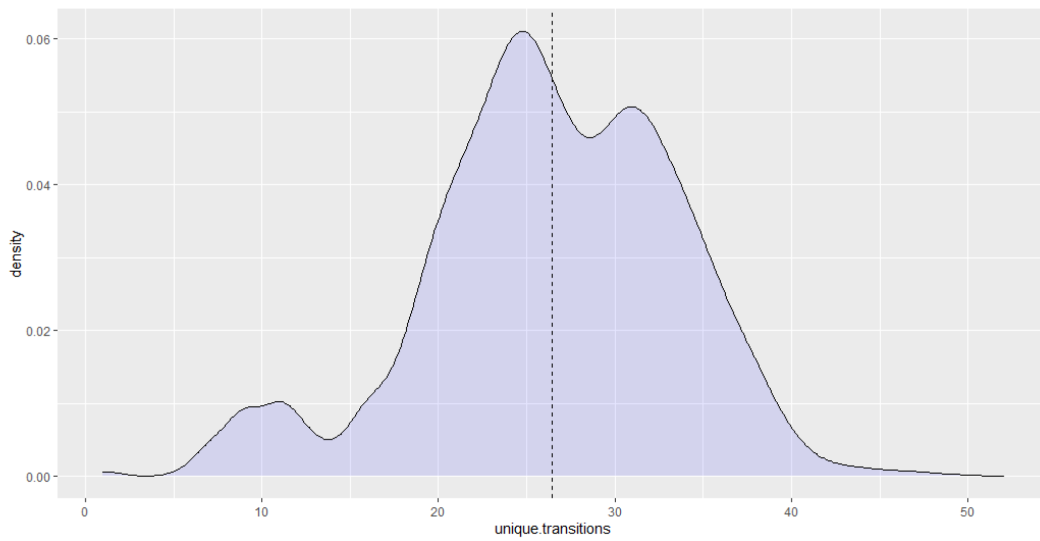

Figure 5: The density plot of the number of unique exits differs per state. This is the first hint of a nonuniform and nonunimodal distribution. The vertical line indicates the average.

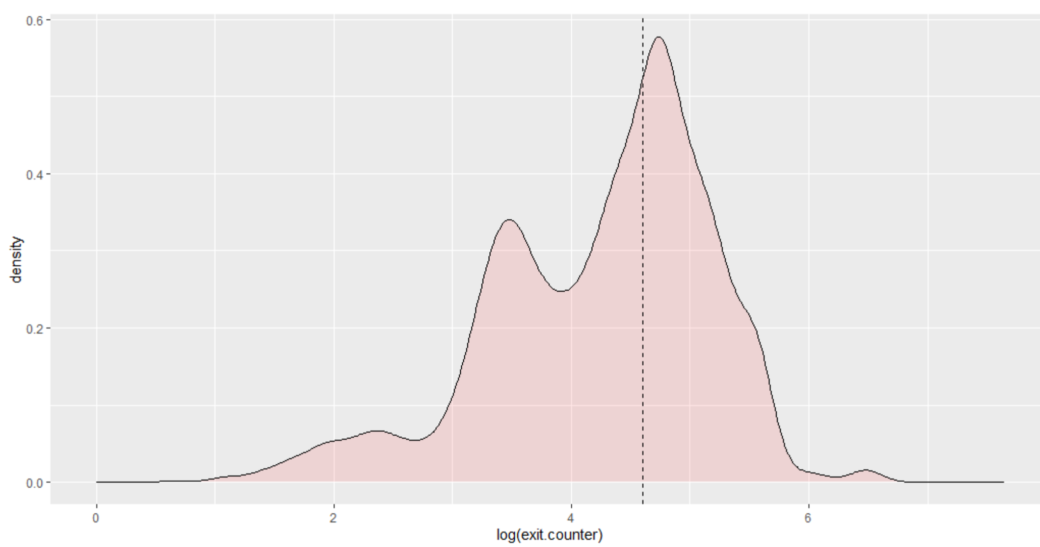

Figure 6: The density plot of the number of times a state is used as an exit, with an average of 100 due to 100 simulations. The plot is log-transformed due to the severe preference for checkerboard state as an exit.

There are two states of interest that represent convergent and divergent states. The first is what is referred to as the yin-yang state, and second is the checkerboard state. The yin-yang state has among the most unique exits of all the states, and it is rarely entered (or used as an exit) relative to other states. The checkerboard state has an above average number of unique exits, and it is the most common state for another to enter. Thus we have sources and sinks in the state space. It is worth noting that 100 samples of each state is not enough to almost surely acquire all the unique exits of the yin-yang state, but the size is sufficient to perform rank statistics. See Figure 7 for a visualization of these important states.

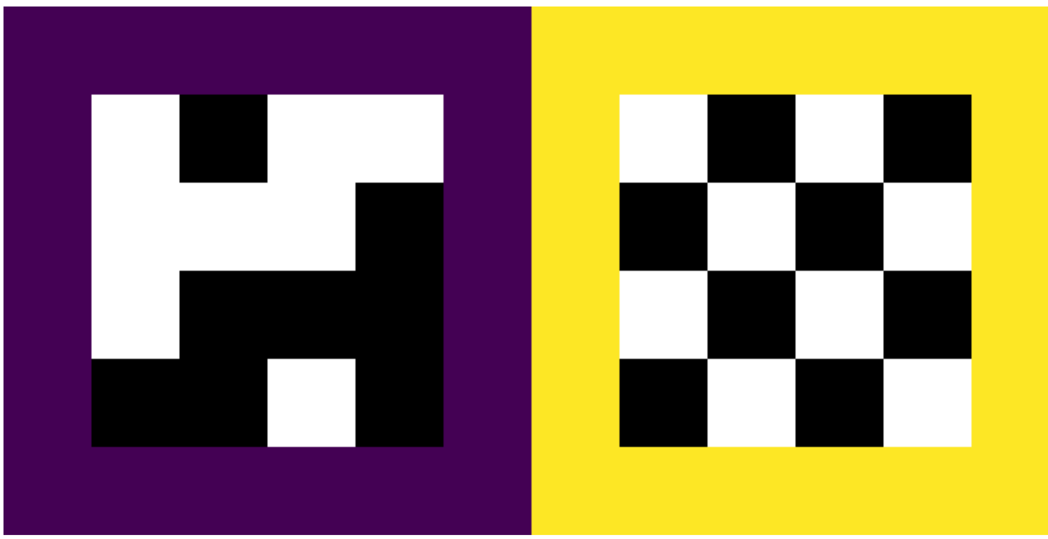

Figure 7: A visual comparison of the most divergent state against the most convergent state. Some states are almost as divergent, but none are quite as symmetric or obviously attention-grabbing.

The concept of *basins of attraction* for the nodes/vertices was of relevance to the 4x4 state space analysis. Basins of attraction, attractors, sinks, or convergent states are important because it is unclear how many there should be in a 4x4 system. What do such states even look like? How can you analyze the state space when there are too many states to count? How can you compare some states to others when it looks like noise?

A 2x2 system with 2 particle and 2 antiparticles is only capable of oscillating between two states: the initial state the the negated state. This forms a sort of loop or cycle, although no direction or preference exists in a period of length 2. A rough estimate of basins of attaction can be made for the 4x4 state space by looking for states that tend to loop into themselves.

The algorithm checks through all states, and it only allows each state to transition into its most likely state. This modal transition state is guaranteed to loop at some point. The intial assumption was that states have a preference to loop in certain directions or have a period length greater than 2, but that is not the case in the relatively small 4x4. Redundancies are checked for, making sure that only one state of a transition loop is accounted for. A total of 3233 basins of attraction were estimated to exist out of 12870 states. These states tended to be reflections, rotations, or negations of the current state. For example, the yin-yang state and the checkerboard state have a tendency to loop into an alternate version of themselves.

We cannot rely on state space analysis for any sort of large system. The system is far too entropic, and there are too many variables to keep track of. How can you compare the relationship between states when the state space is uncomputable and transitions are NP hard?

8 2x2 Substates in 100x100 System

We come up with a method to identify the distribution of 2x2 subsystems in a 100x100 system. If the system had a Fourier transform of white noise, then we should expect to see a distribution of 2x2s that is identical to independent Bernoulli simulations per cell. We initialize the system using sampling without replacement, roughly equivalent to white noise or Uniform error. We run the simulation for 100 time steps and compare against the empirical distribution of Uniform noise against our, presumably, Gaussian noise.

As we can see, the count of each 2x2 pattern differs dramatically from the start and end distribution. The simulation diverges from the distribution generated by Bernoulli noise extremely quickly. Checkerboard state is still clearly the most frequent. Analyses of larger states should incorporate scans of substates since the state space of almost every finite system uncomputable.

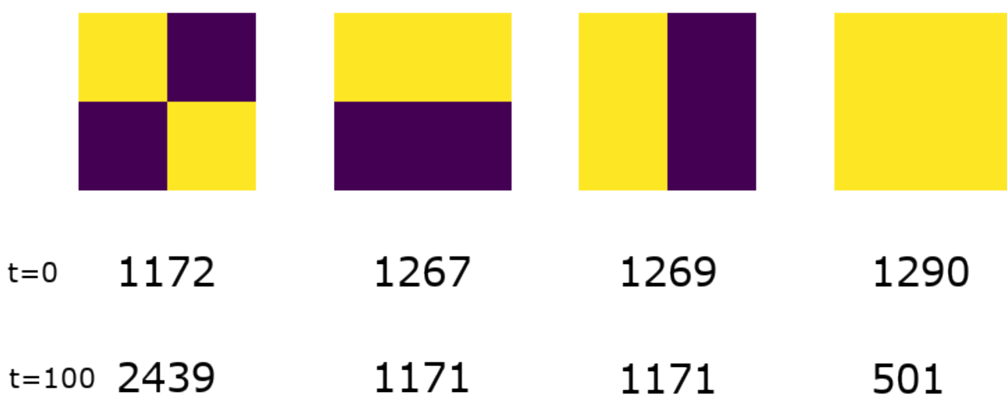

Figure 8: A count of the various 2x2 substates, with negations included.

9 100x100 System Fast Fourier Transform

A Fast Fourier Transform of the 100x100 system can be performed in order to analyze the spectral density. A Uniform random variable of the spectral densities implies white noise. Otherwise, a different form of noise is implied. This is more of an empirical method than a purely analytical method of comparing states. The first state was initialized using sampling with replacement, approximately Bernoulli per cell or white noise.

In Figure 10, the simulation after 100 simulations has a circular visual to it. The FFT of the end state seems to be concentrated in the lower frequencies. While not a formal

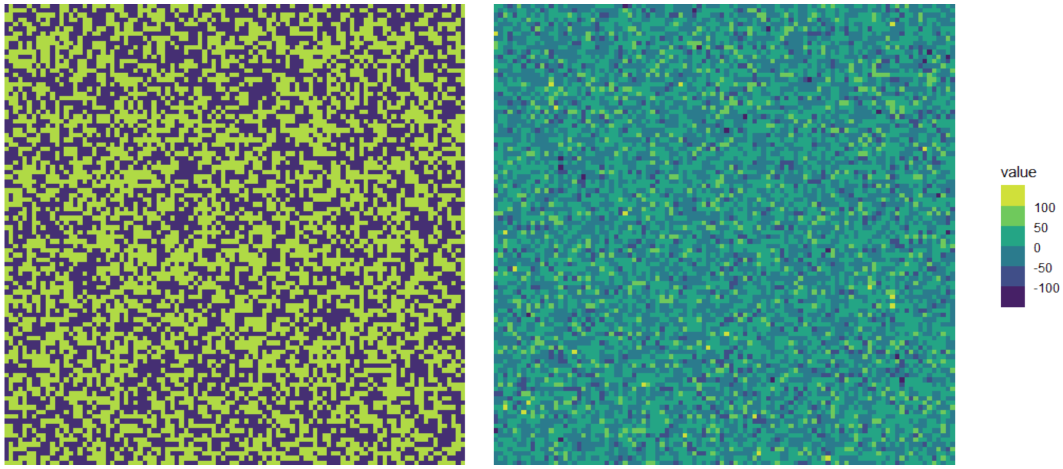

Figure 9: The 0th state of a 100x100 and its Fast Fourier transform.

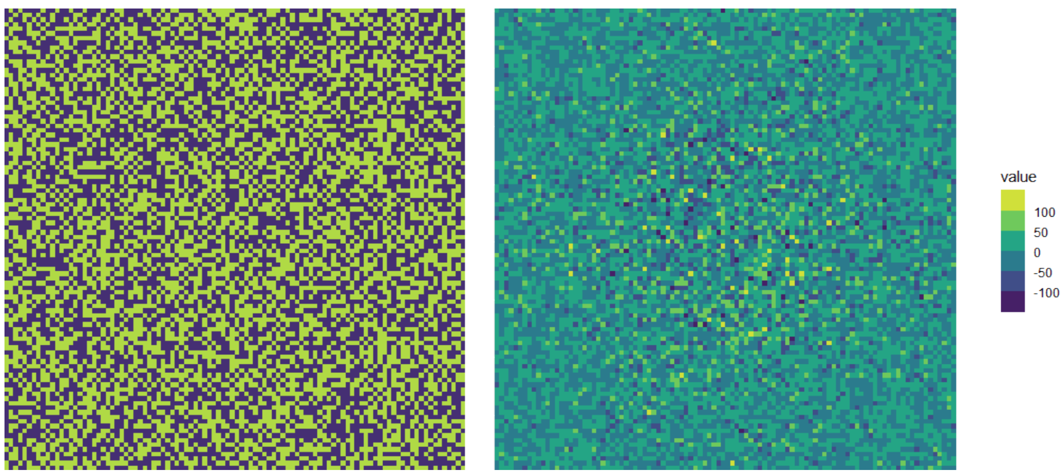

Figure 10: The 100th state of a 100x100 and its Fast Fourier transform.

proof, it implies that this is the most symmetrical transition rule just as a Gaussian is the most symmetrical distribution. It also means that a conditional probability function for an unknown space, given that you know a certain amount of information in another space, is not independent.

10 Conclusion and Discussion

MPRWs in dimensions of three or higher have a few interesting properties. Pólya's random walk constants are important because single particle random walks are not guaranteed to return to their original position in dimensions $d \geq 3$ on infinite systems [7]. There is also the idea of particle position estimation. A lone particle is, on average where it was last seen. This is not true in the case of multiple particles that collide. Variance is a quadratic loss function pertinent to Gaussian distributions, yet using it as maximum likelihood estimator of the particle position is not an admissable decision rule. The relationship to the random walks constants is deep because of the large chance that a particle does not return to its original position in the 3D case, yet all particles and antiparticles are identical. This begs the question: is a particle more or less likely to return to its original position in an infinite 3D MPRW system?

Andrey Markov Jr. encountered a problem with four-dimensional manifolds, specifically distinguishing between them. Four dimensional manifolds, in essence, ahve the ability to embed any algorithm in them. Distinguishing and classifying all four manifolds is equivalent to solving the Alan Turing's halting problem. The exact connection to four-dimensional MPRW simulations in unclear. It does imply that if chaos gates exist in four or higher dimensions, then the halting problem is solvable on higher-dimensional infinite systems.

If chaos gates are proven to exist and MPRWs are assumed to be fundamental to the universe, it implies that no computational system will ever have the computing power to understand its own complexity. It can perform information reduction techinques, but not understand in full. This is because the choose function of one system far outpaces the ability of a finite array of logic gates or chaos gates to interpret another system. A strange property, should chaos gates exist, is the idea of finite computational dimensions. Should a subsystem store data, it is analogous to storing extra, though finite, dimenisons in it. It is similar to how higher dimensions may be simulated on a classical computer.

A real life phenomenon of similar interest involves phonons and quasiparticles. In the paper "Random Walks in Multidimensional Spaces, Especially on Periodic Lattices", the imperfections on a crystalline lattice is bound to adhere to the same principles of collding random walks [5]. A lack of a particle in particle sea is subject to the same properties as the particle itself, assuming that particle was in a lattice too. The dynamics of a crystalline structure are different since adjacent lattice defects provide no structural integrity, thus deforming.

A metastable state, in the case of MPRW, is any local maxima state in the stationary distribution. In physics, they are semi-permanent states that are temporarily inhabited. The information of a MPRW stochastic process can by using less vertexes that represent these basins of attraction, these metastable states. If G_X is the original state space graph with transition matrix P, then $G_X^{[2]}$ is the shrinked graph. What that algorithm should be for shrinking the representation for states is unknown. The relationship to *shrinkage* in statistical modeling is intentional.

According to the formal definition of entropy

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) log(p(x))$$

a MPRW system is estimated to have relatively low entropy since some states are more frequent than others. Brown noise is certainly less entropic than white noise. The information content, or surprisal, t of a planar state A is given precisely by $I(A) = -log_2(\pi_A)$ where π_A is the stationary probability of the A. Ordered states are improbable, surprising, but not impossible.

The second law of thermodynamics states that systems tend towards their most likely states. This is often misinterpreted as entropy always increasing. This is partially correct as they tend towards a stationary distribution for finite systems, meaning that unlikely states have nonzero probability. Time in a Markov chain merely advances states according to some probability, and it is ignorant to the idea of time being reversible. In colliding random walks, the probability of becoming equals the probability of originating. Time however may still be perceived by an outside observer that assumes that any system tends towards its most probable state. Even as the heat death of the universe is inevitable, so is its rebirth.

Observing a wavefunction in physics is a misnomer. Gathering data on a particle requires interacting with the particle directly to gain information on it. In physics, when a wavefunction collapses, and it is *confusing* as if the particle's position becomes a singularity instead of a distribution. In statistics, sampling from y_t collapses the state to a single state, and it is *obvious* why it time mixes a single state towards the stationary distribution.

It isn't so much that observing collapses the wavefunction to a singularity, it is more like unit singularities of the universe has a stationary distribution that limits their variability. Multinomials are discrete and random, but smooth over time. Is it correct to say the wavefunction collapsed? I would argue that the observer was not spending energy to make an interaction observation at every single time step. Information on a segregated system decays until an interaction is made or the measurement devices burns out from overuse, a firm reminder that measurement is not a one way street.

Wolfgang Pauli is accredited for the phrase, "Das ist nicht nur einmal falsch", which translate roughly to "That is not even wrong" [6]. The phrase is often used as an insult for theories that are so poorly constructed that they are unfalsifiable. Whereas string theory cannot be proven true or false as theorists continuously add patches to a bad theory, cube theory is possible to prove true or false. String theory continuously adds to make the theory work *a posteriori* a failure. Cube theory holds *a priori* assumptions and simply allows itself to be validated or invalidated.

It also seems no mistake that the Schwarzschild radius, or gravitational radius, given by

$$r_s = \frac{2GM}{c^2}$$

means that a Planck mass m_P has a gravitational radius of $2\ell_P$. Alternatively a mass of $\frac{1}{2}m_P$ has a gravitational radius of ℓ_P. It is worth noting that the relationship between Schwarzschild's gravitational field equation and the geometric series formula bear a striking resemblance. The Grandi's series specifically has a sum of $1 - 1 + 1 - 1... = \frac{1}{2}$. Of course, this would require MPRW working with the set $S = \{-1, 1\}$.

I offer a falsifiable theory in the form of the hypothesis. A MPRW system's tendency towards low frequencies implies that a closed system moves towards neural or galactic clusters. MPRWs also stand as a reasonable candidate for a quantum field theory for dark matter's existence. A hypothesis that may be tested with cube theory is that the FFT of mass distribution in the universe is, on average, given by the Normal distribution as in Brownian motion. Likewise, the FFT of quantum fluctations or vacuum fluctuations are, on average, given by the Normal distribution as in Brownian motion. It would be more accurate to reference the properties of Wiener processes, commonly known as Brownian motion, and their random Fourier series.

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