University of Wisconsin-Madison
Dept. of Electrical and Computer Engineering

Homework Assignment #3
Due Friday, March 21, 11:59 pm

Problem 1.

a) It is often said that an *n*-type semiconductor is nondegenerate if $\varepsilon_C - \varepsilon_F > 3.5 k_B T$. Derive the above inequality based on finding the full width at half-maximum (FWHM) of $-\frac{df}{d\varepsilon}$. [The above inequality measures when exactly the Fermi-Dirac (FD) distribution function can be approximated as an exponential, i.e. when it no longer bears the step-like feature of the FD distribution; 2 x FWHM is the width of the energy interval that separates the mostly full states (which are at least FWHM below the Fermi level) from the mostly empty states (which are at least FWHM above the Fermi level).]

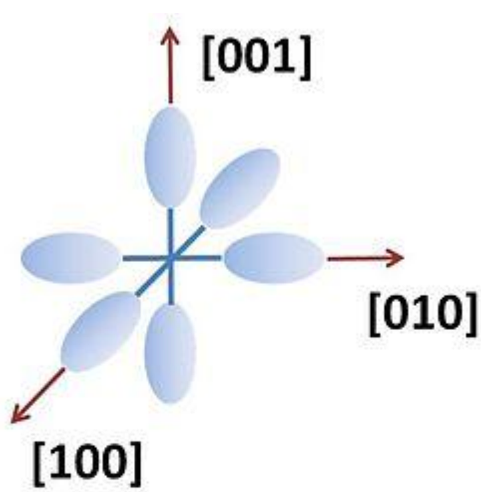
b) What is the analogous inequality that describes when a *p*-type semiconductor can be considered nondegenerate? Explain.

Quantum confinement in silicon

In bulk Si under equilibrium conditions, electrons occupy 6 equivalent Δ valleys (often denoted as Δ_6 valleys). In each valley, energy can be written as

$$\varepsilon_{\sigma,\mathbf{q}} = \varepsilon_C + \frac{\hbar^2}{2}\left(\frac{q_x^2}{m_x} + \frac{q_y^2}{m_y} + \frac{q_z^2}{m_z}\right),$$

where m_x, m_y, and m_z are either m_l=0.98m_0 or m_t=0.19 m_0, depending on the valley, and wave vector $\mathbf{q}$ is measured with respect to that of the valley bottom.

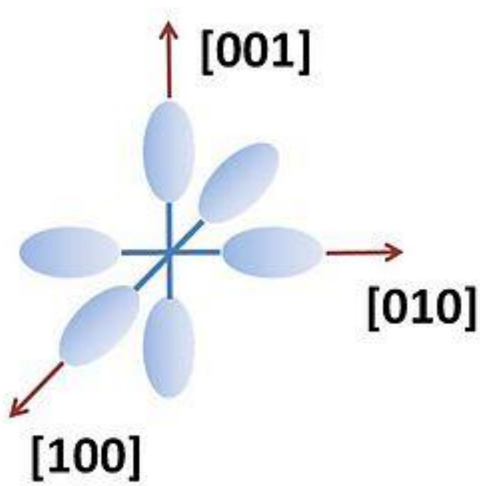


Problem 2. A thin layer of silicon, of thickness L, is sandwiched between two layers of SiO_2, making the electrons in Si confined in a deep quantum well. Upon confinement, subbands form. Assuming that the well is formed along the [001] (z-direction), the single electron energy now looks

$$\varepsilon_{\sigma,q_x,q_y,n} = \varepsilon_C + \frac{\hbar^2}{2}\left(\frac{q_x^2}{m_x} + \frac{q_y^2}{m_y}\right) + \frac{\hbar^2}{2m_z}\left(\frac{n\pi}{L}\right)^2$$

Because of the different effective masses along the z-directions, there are two different ladders of 2D subbands forming. One is the twofold-degenerate Δ_2-ladder, corresponding to the two valleys whose z-axis mass is m_l, and the fourfold-degenerate Δ_4-subband, corresponding to the four valleys whose z-axis mass is m_t.

a) Denote the Δ_2 and Δ_4 valleys on the schematic below.



b) Plot the energy dispersions ε vs. $\mathbf{q}$ in the plane of unrestricted motion, i.e., along (100) and (010) directions, on the same graph. Clearly denote which curves belong to which ladder.

c) For a given subband n in one of the Δ_2 valleys, plot one of the constant energy surfaces. What is the density of states mass and the density of states (DOS) for this subband?

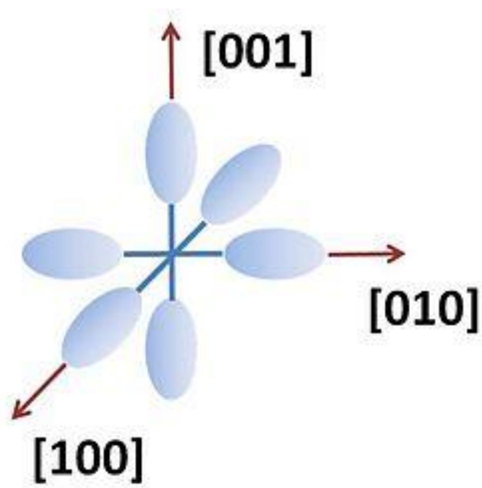
d) For a given subband n in one of the Δ_4 valleys, plot one of the constant energy surfaces. What is the density of states mass and the density of states (DOS) for this subband?

e) Plot the total DOS (per unit area) for this quasi-2D system.

Problem 3. Upon additional lithography and etching on the Si layer from problem 4, a square Si nanowire with cross-sectional side L is formed along [100] (the x-direction) (in other words, there is confinement along y and z directions).

a) What are the quantum numbers for an electron in a wire? Write the expression for energy vs relevant quantum numbers.

b) There are Δ_2 and Δ_4 subband ladders, twofold and fourfold degenerate, respectively, but they are now 1D subbands. Which bulk valleys contribute to Δ_2 and which to Δ_4, and which subband ladder has the lowest energy overall? (Clearly denote on the plot of the Δ_6 valleys.)



c) Plot the dispersion relationships for relevant subband ladders.

d) Plot the total DOS (per unit length of the nanowire).

Problem 4. On the same graph, plot the density of states per unit volume for bulk Si, Si quantum well as in problem 10, and silicon nanowire as in problem 11. Make sure that the appropriate DOS you are plotting have the correct units.

Problem 5: Consider a thin silicon-on-insulator (SOI) nanomembrane of thickness 20 nm, doped n-type. On both sides of the membrane is SiO_2, whose conduction band offset with respect to bulk Si is high enough that you can think of the electrons in the nanomembrane as being in an infinitely deep potential well. Write a MATLAB code to calculate the position of the Fermi level with respect to the bottom of the bulk conduction band as a function of the doping density. Plot the position of the Fermi level vs doping density in the doping density range $10^{14} - 10^{21}$ cm^{-2}.