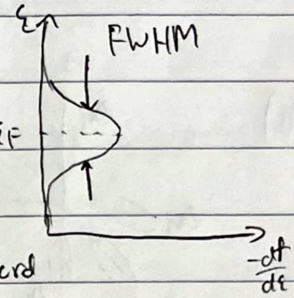
HW 3

1) a) FWHM ~ $7k_BT$

I'm going to call $\frac{\varepsilon-\varepsilon_F}{kT} = x$ to simplify solutions

$f = \frac{1}{e^x+1}$, $\frac{df}{d\varepsilon} = \frac{d}{d\varepsilon}(e^x+1)^{-1} \rightarrow -1(e^x+1)^{-2}\cdot(\varepsilon-\varepsilon_F)x\, e^x$

$\frac{df}{d\varepsilon} = \frac{-e^x}{k_BT(e^x+1)^2} \rightarrow \frac{-df}{d\varepsilon} = \frac{e^x}{k_BT(e^x+1)^2}$



This plot from class shows $\max\left(\frac{-df}{d\varepsilon}\right)$ is at $\varepsilon = \varepsilon_F$. Then, we can find $\frac{1}{2}\max\left(\frac{-df}{d\varepsilon}\right)$, and get the ε values, and find the difference, $\Delta\varepsilon_{1/2\,max}$

$-\frac{df}{d\varepsilon}(\varepsilon_F) = \frac{e^0}{k_BT(e^0+1)^2} = \frac{1}{4k_BT} \rightarrow \frac{1}{2}\max = \frac{1}{8k_BT}$

$\frac{1}{8\cancel{k_BT}} = \frac{e^x}{\cancel{k_BT}(e^x+1)^2} \rightarrow \frac{1}{8} = \frac{e^x}{(e^x+1)^2}$, $(e^x+1)^2 = 8e^x$

$(e^x)^2 + 2e^x + 1 = 8e^x \rightarrow (e^x)^2 - 6e^x + 1 = 0$ Quadratic

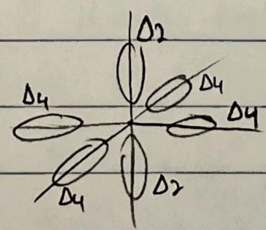
$\frac{6 \pm \sqrt{(6)^2 - 4(1)(1)}}{2} = 3 \pm \frac{\sqrt{32}}{2} = 3 \pm 2\sqrt{2} = e^x$

$\ln(e^x) = \ln(3\pm 2\sqrt{2}) \rightarrow x = 1.7627$ AND -1.7627, $\Delta x = 2(1.7627) = 3.525$

remember $x = \frac{\varepsilon-\varepsilon_F}{k_BT}$, $\Delta\frac{\varepsilon-\varepsilon_F}{k_BT} = 3.5 \rightarrow \boxed{\varepsilon-\varepsilon_F = 3.5\,k_BT}$

b) The inequality must now focus on the valence band compared to the fermi-level. Since the fermi-level has higher energy than the valence band, we now have $\boxed{\varepsilon_F - \varepsilon_V > 3.5\,k_BT}$

2) a)

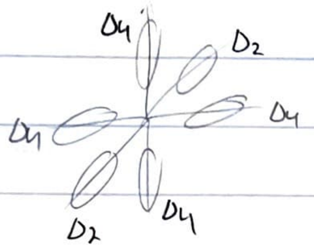


3)a) Now 1D, so confinement in y, z.

$$\boxed{E = E_c + \frac{\hbar^2}{2}\frac{k_x^2}{m_x} + \frac{\hbar^2}{2m_y}\left(\frac{n\pi}{L_y}\right)^2 + \frac{\hbar^2}{2m_z}\left(\frac{n\pi}{L_z}\right)^2}$$

confinement

b)



y and z both have confinement, should have same degeneracy, Δ4

x is the lone Δ2

5) $n = \int g_{2D}(E) f_0 \, dE = g_{2D\,well}(E) \int \frac{1}{1+e^{(E-E_F)/kT}} dE \rightarrow \ln(1+e^{(E_F-E)/kT})$

$$\underline{n = \frac{m_{DOS}}{\pi\hbar^2} kT \ln\left(1+e^{(E_F-E)/kT}\right)}$$