

## Problem 1

a)

Two lattice branches, one optical and one acoustic.

b)

$$\omega^2 = \gamma \frac{M_A + M_B}{M_A M_B} \pm \frac{\gamma}{M_A M_B} \sqrt{(M_A + M_B)^2 - 4(M_A M_B) \sin^2\left(\frac{ka}{2}\right)}$$

```
clc
clear
close all

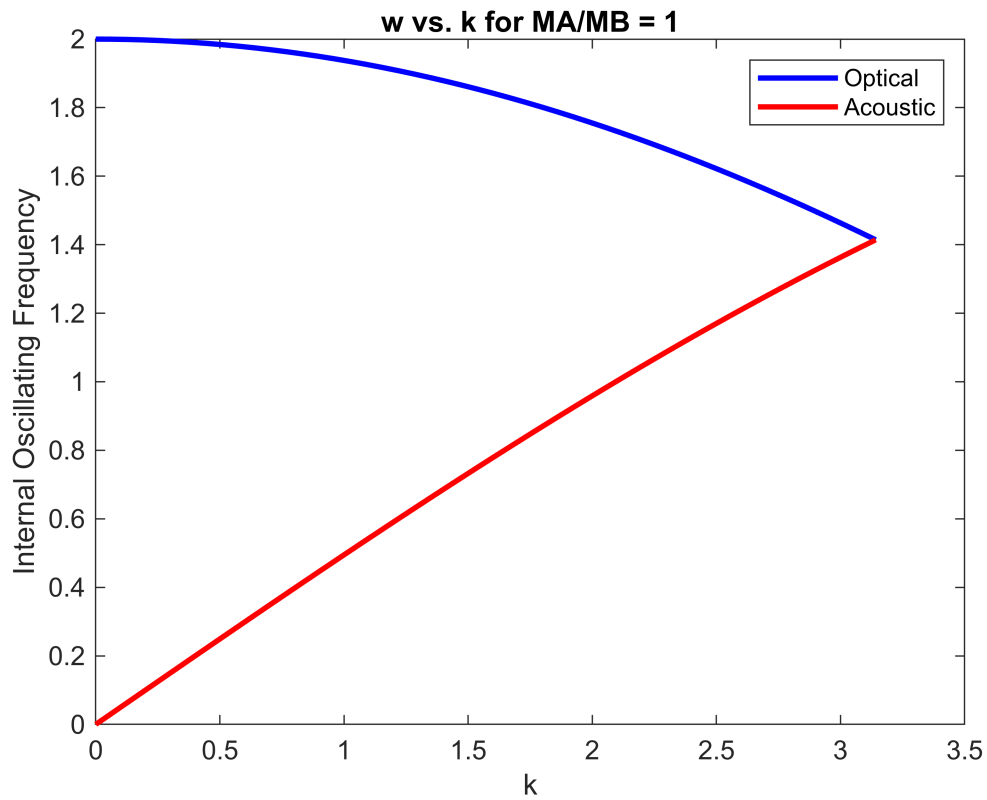
a = 1;
gamma = 1;
k = linspace(0, pi/a, 50);
w_plus = zeros(1, length(k));
w_minus = zeros(1, length(k));
```

$$M_A/M_B = 1$$

```
MA = 1;
MB = 1;

w_plus = gamma * (MA + MB) / (MA * MB) + (gamma / (MA * MB)) * ...
    sqrt((MA + MB) ^ 2 - 4 * MA * MB * sin(k * a / 2) .^ 2);
w_minus = gamma * (MA + MB) / (MA * MB) - (gamma / (MA * MB)) * ...
    sqrt((MA + MB) ^ 2 - 4 * MA * MB * sin(k * a / 2) .^ 2);

figure;
plot(k, sqrt(w_plus), 'b-', 'LineWidth', 2);
hold on;
plot(k, sqrt(w_minus), 'r-', 'LineWidth', 2);
ylabel('Internal Oscillating Frequency');
xlabel('k');
title('w vs. k for MA/MB = 1');
legend('Optical', 'Acoustic');
```

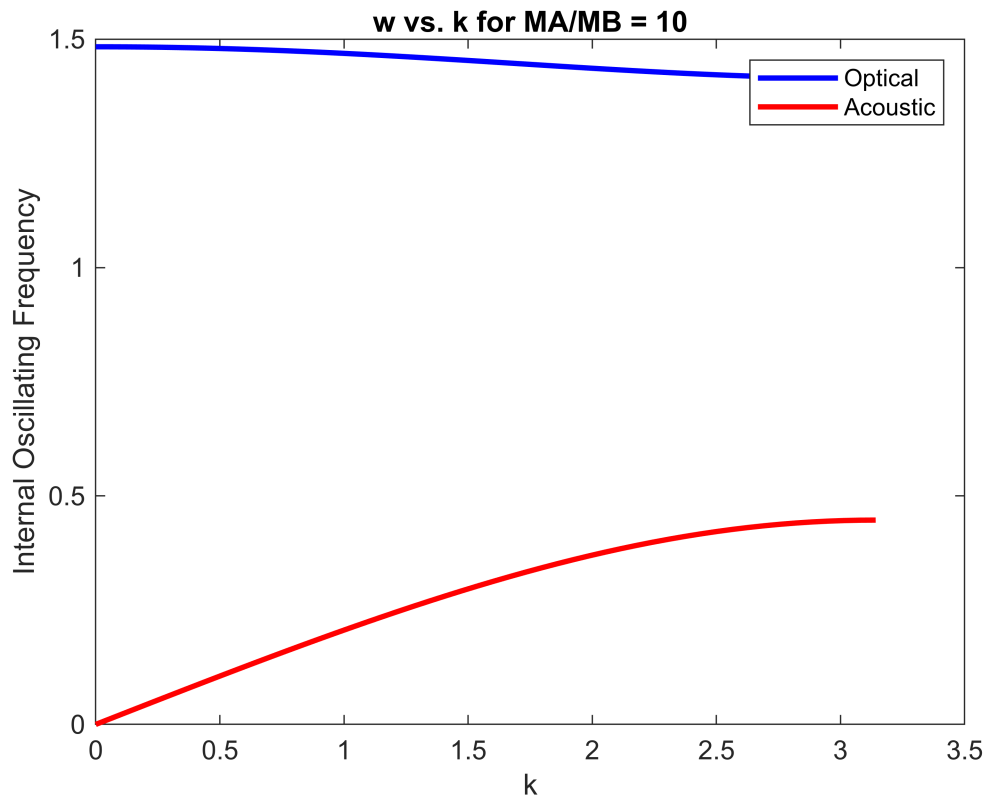


$M_A/M_B = 10$

```
MA = 10;
MB = 1;

w_plus = gamma * (MA + MB) / (MA * MB) + (gamma / (MA * MB)) * ...
    sqrt((MA + MB) ^ 2 - 4 * MA * MB * sin(k * a / 2) .^ 2);
w_minus = gamma * (MA + MB) / (MA * MB) - (gamma / (MA * MB)) * ...
    sqrt((MA + MB) ^ 2 - 4 * MA * MB * sin(k * a / 2) .^ 2);

figure;
plot(k, sqrt(w_plus), 'b-', 'LineWidth', 2);
hold on;
plot(k, sqrt(w_minus), 'r-', 'LineWidth', 2);
ylabel('Internal Oscillating Frequency');
xlabel('k');
title('w vs. k for MA/MB = 10');
legend('Optical', 'Acoustic');
```



**c)**

$$\begin{bmatrix} M_A \omega^2 - 2\gamma & \gamma(1 + e^{-ika}) \\ \gamma(1 + e^{ika}) & M_B \omega^2 - 2\gamma \end{bmatrix} \begin{bmatrix} u_n^A \\ u_n^B \end{bmatrix} = 0$$

$$\frac{u_n^A}{u_n^B} = \frac{-\gamma(1 + e^{-ika})}{M_A \omega^2 - 2\gamma}$$

**For Acoustic**

$$k \rightarrow 0$$

$$\omega^2 = 0$$

$$\frac{u_n^A}{u_n^B} = \frac{(1 + e^0)}{2} = 1$$

$$k \rightarrow \frac{\pi}{a}$$

$$\frac{u_n^A}{u_n^B} = 0$$

**For Optical**

$$k \rightarrow 0$$

$$\omega^2 = \frac{2\gamma(M_A + M_B)}{M_A M_B}$$

$$\frac{u_n^A}{u_n^B} = \frac{-(1+e^0)}{\frac{2(M_A + M_B)}{M_B} - 2} = \frac{-M_B}{M_A}$$

$$k \rightarrow \frac{\pi}{a}$$

$$\frac{u_n^A}{u_n^B} = 0$$