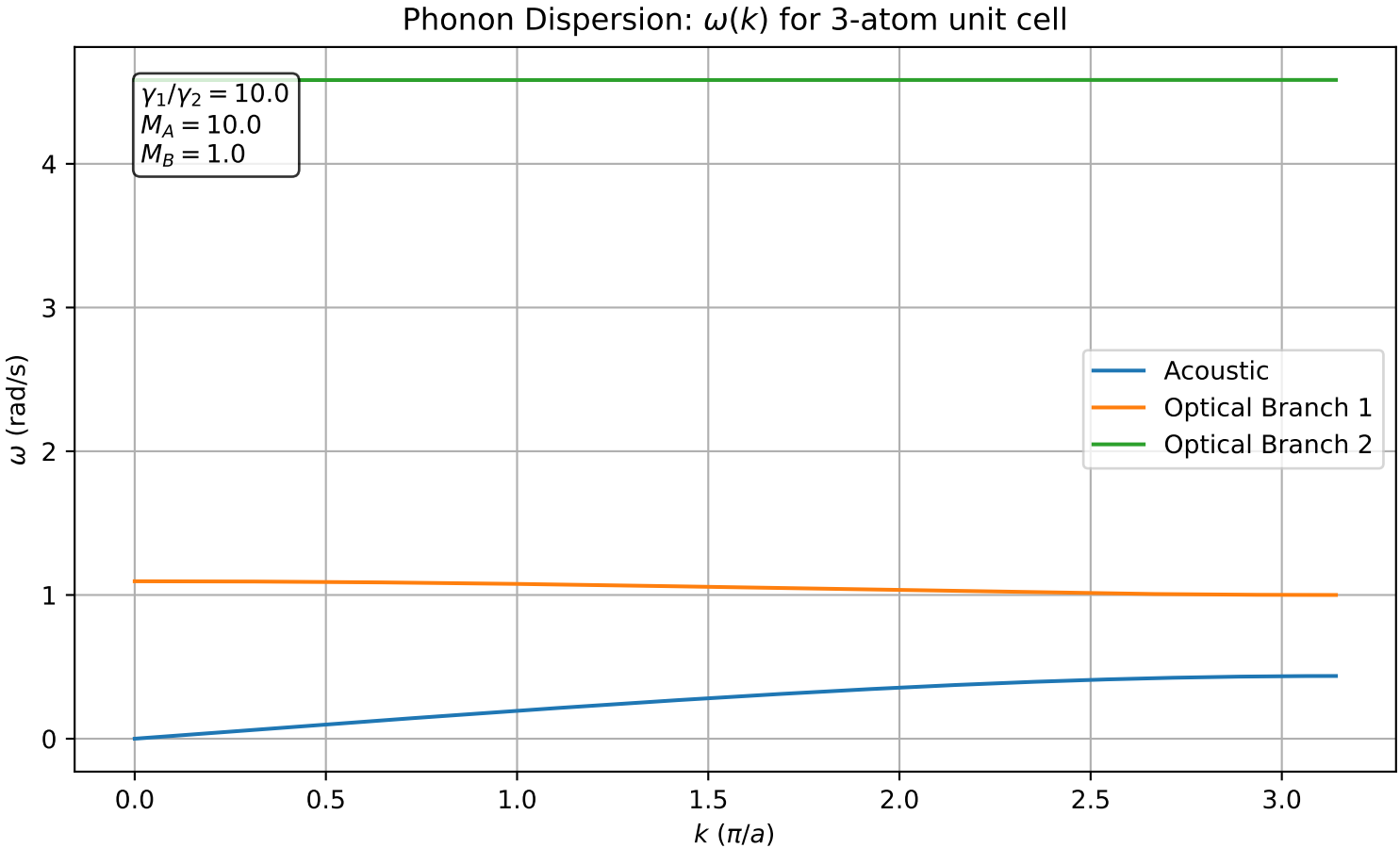
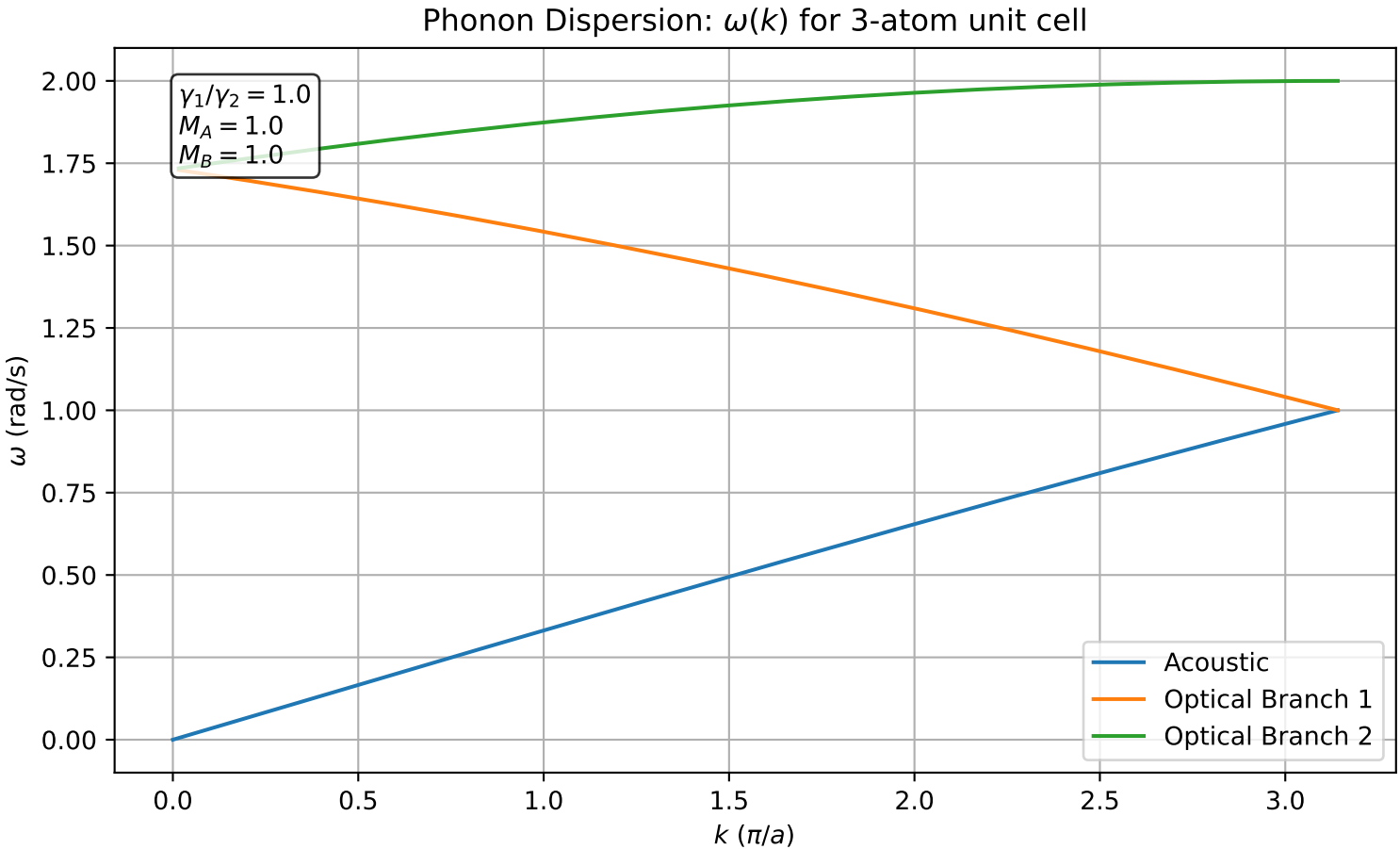
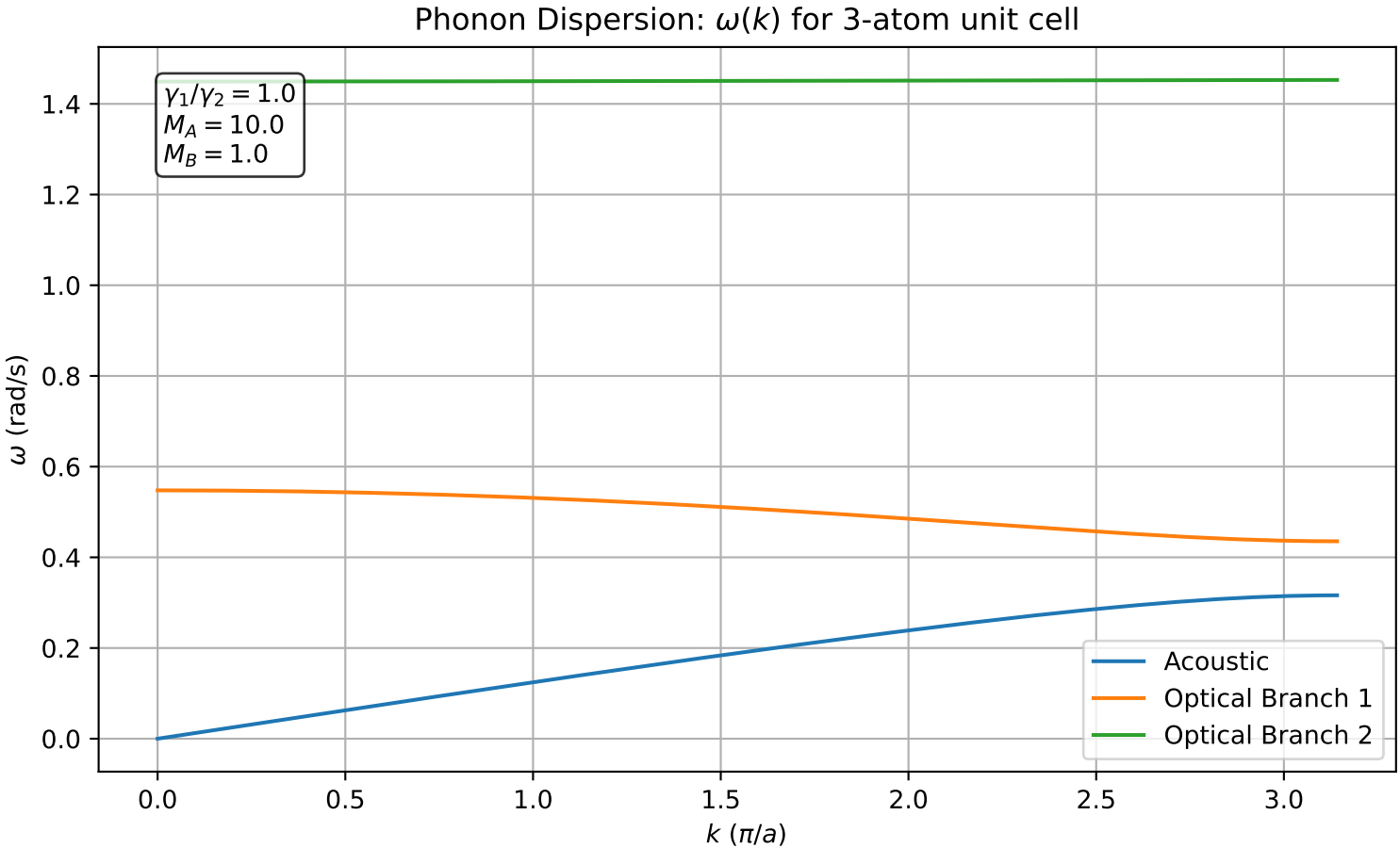
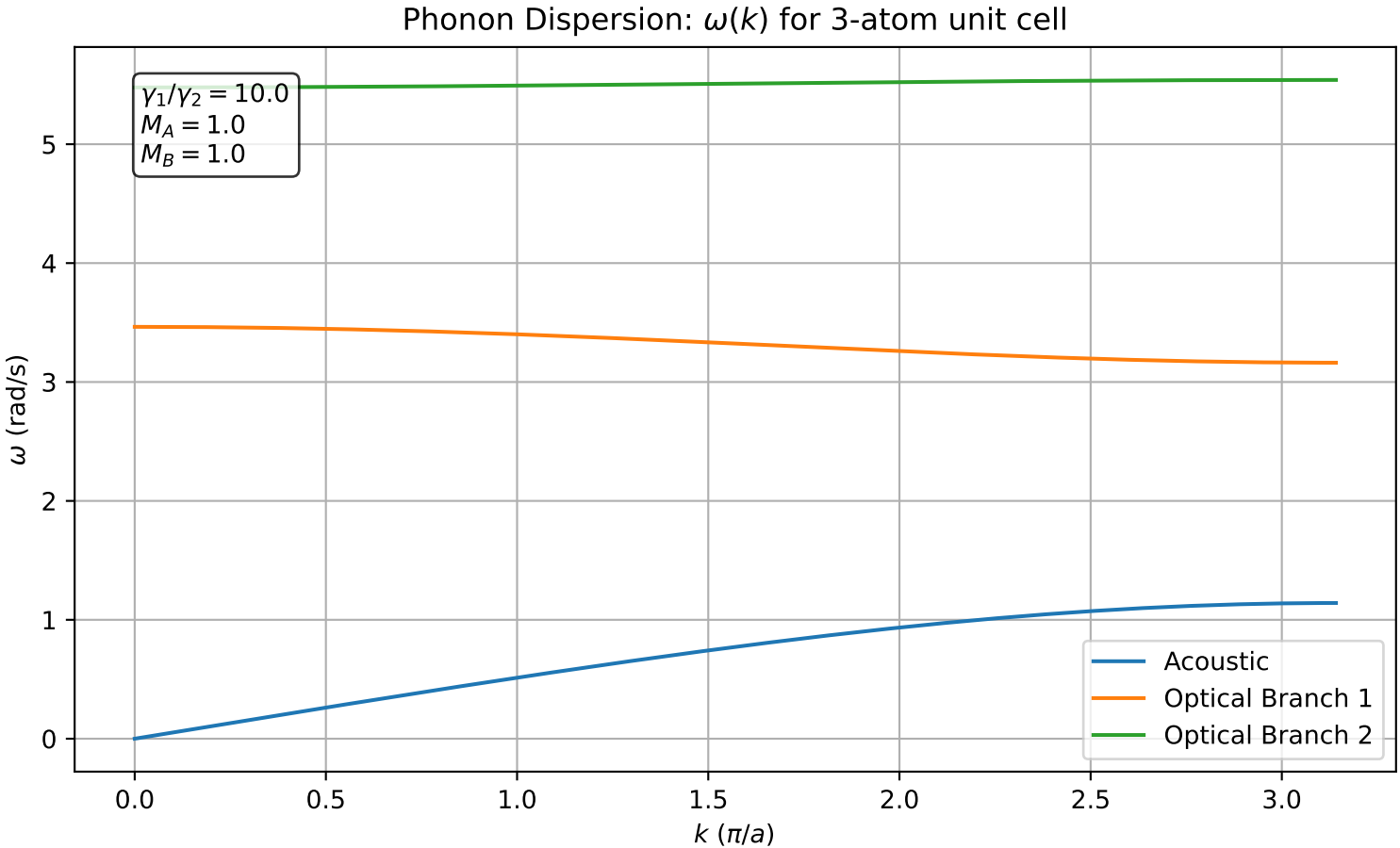

Phonon Dispersion: $\omega(k)$ for 3-atom unit cell
$\gamma_1/\gamma_2 = 10.0$
$M_A = 10.0$
$M_B = 1.0$
Acoustic
Optical Branch 1
Optical Branch 2
ω (rad/s)
k (π/a)
0
1
2
3
4
0.0
0.5
1.0
1.5
2.0
2.5
3.0


Phonon Dispersion: $\omega(k)$ for 3-atom unit cell
$\gamma_1/\gamma_2 = 1.0$
$M_A = 1.0$
$M_B = 1.0$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
ω (rad/s)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
k (π/a)
Acoustic
Optical Branch 1
Optical Branch 2


Phonon Dispersion: $\omega(k)$ for 3-atom unit cell
$\gamma_1/\gamma_2 = 1.0$
$M_A = 10.0$
$M_B = 1.0$
Acoustic
Optical Branch 1
Optical Branch 2
ω (rad/s)
k (π/a)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.0
0.5
1.0
1.5
2.0
2.5
3.0


Phonon Dispersion: $\omega(k)$ for 3-atom unit cell
$\gamma_1/\gamma_2 = 10.0$
$M_A = 1.0$
$M_B = 1.0$
Acoustic
Optical Branch 1
Optical Branch 2
ω (rad/s)
k (π/a)
0
1
2
3
4
5
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Problem 1

a)

Two lattice branches, one optical and one acoustic.

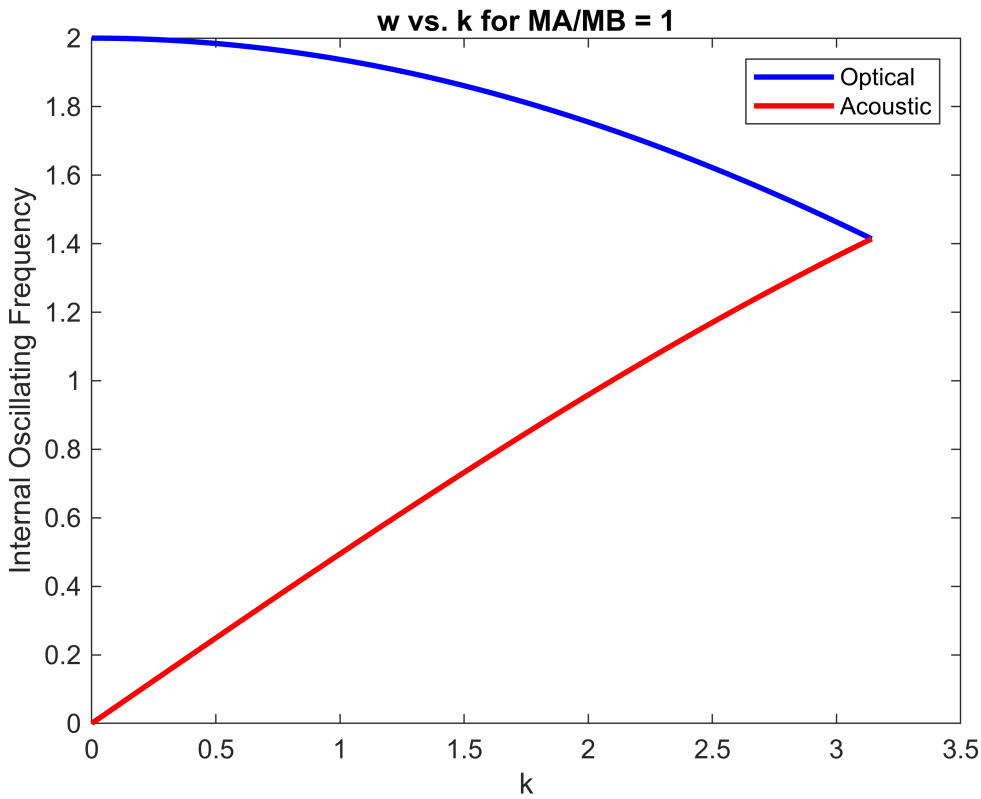
b)

$$\omega^2 = \gamma \frac{M_A + M_B}{M_A M_B} \pm \frac{\gamma}{M_A M_B} \sqrt{(M_A + M_B)^2 - 4(M_A M_B) \sin(\frac{ka}{2})^2}$$

```
clc
clear
close all

a = 1;
gamma = 1;
k = linspace(0, pi/a, 50);
w_plus = zeros(1, length(k));
w_minus = zeros(1, length(k));
```

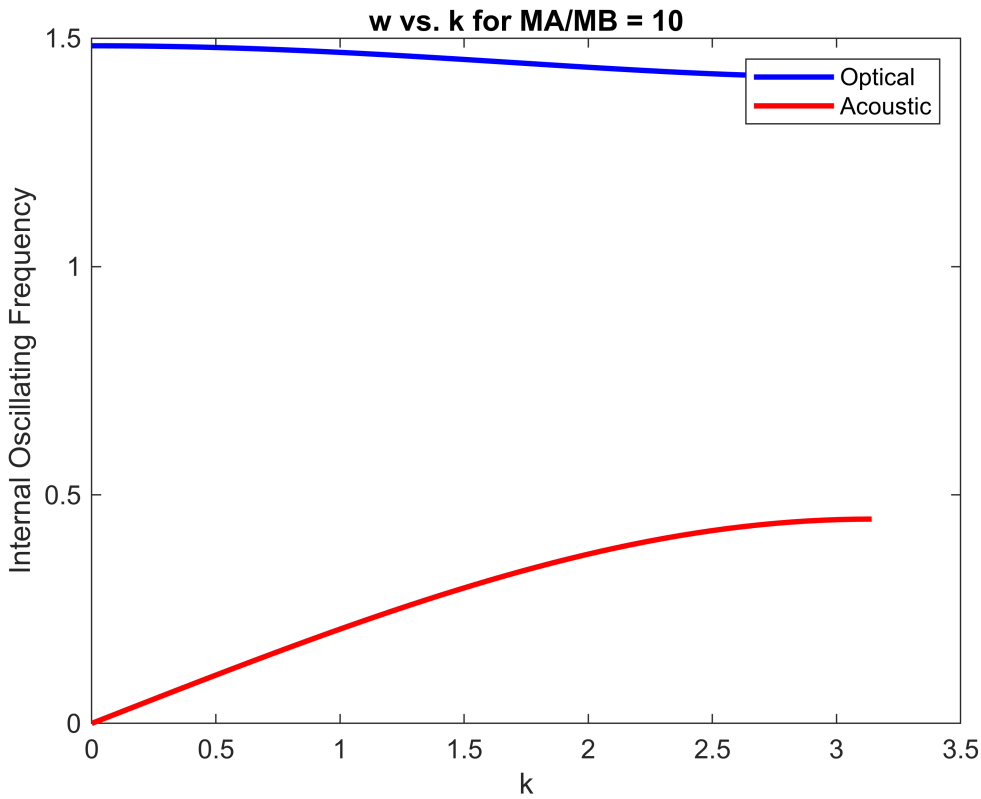
$M_A/M_B = 1$

```
MA = 1;
MB = 1;

w_plus = gamma * (MA + MB) / (MA * MB) + (gamma / (MA * MB)) * ...
    sqrt((MA + MB) ^ 2 - 4 * MA * MB * sin(k * a / 2) .^ 2);
w_minus = gamma * (MA + MB) / (MA * MB) - (gamma / (MA * MB)) * ...
    sqrt((MA + MB) ^ 2 - 4 * MA * MB * sin(k * a / 2) .^ 2);

figure;
plot(k, sqrt(w_plus), 'b-', 'LineWidth', 2);
hold on;
plot(k, sqrt(w_minus), 'r-', 'LineWidth', 2);
ylabel('Internal Oscillating Frequency');
xlabel('k');
title('w vs. k for MA/MB = 1');
legend('Optical', 'Acoustic');
```



$M_A/M_B = 10$

```
MA = 10;
MB = 1;

w_plus = gamma * (MA + MB) / (MA * MB) + (gamma / (MA * MB)) * ...
    sqrt((MA + MB) ^ 2 - 4 * MA * MB * sin(k * a / 2) .^ 2);
w_minus = gamma * (MA + MB) / (MA * MB) - (gamma / (MA * MB)) * ...
    sqrt((MA + MB) ^ 2 - 4 * MA * MB * sin(k * a / 2) .^ 2);

figure;
plot(k, sqrt(w_plus), 'b-', 'LineWidth', 2);
hold on;
plot(k, sqrt(w_minus), 'r-', 'LineWidth', 2);
ylabel('Internal Oscillating Frequency');
xlabel('k');
title('w vs. k for MA/MB = 10');
legend('Optical', 'Acoustic');
```



c)

$$\begin{bmatrix} M_A\omega^2 - 2\gamma & \gamma(1+e^{-ika}) \\ \gamma(1+e^{ika}) & M_B\omega^2 - 2\gamma \end{bmatrix} \begin{bmatrix} u_n^A \\ u_n^B \end{bmatrix} = 0$$

$$\frac{u_n^A}{u_n^B} = \frac{-\gamma(1+e^{-ika})}{M_A\omega^2 - 2\gamma}$$

For Acoustic

$k \to 0$

$\omega^2 = 0$

$$\frac{u_n^A}{u_n^B} = \frac{(1+e^0)}{2} = 1$$

$k \to \frac{\pi}{a}$

$$\frac{u_n^A}{u_n^B} = 0$$

For Optical

$k \to 0$

$$\omega^2 = \frac{2\gamma(M_A + M_B)}{M_A M_B}$$

$$\frac{u_n^A}{u_n^B} = \frac{-(1 + e^0)}{\frac{2(M_A + M_B)}{M_B} - 2} = \frac{-M_B}{M_A}$$

$k \to \frac{\pi}{a}$

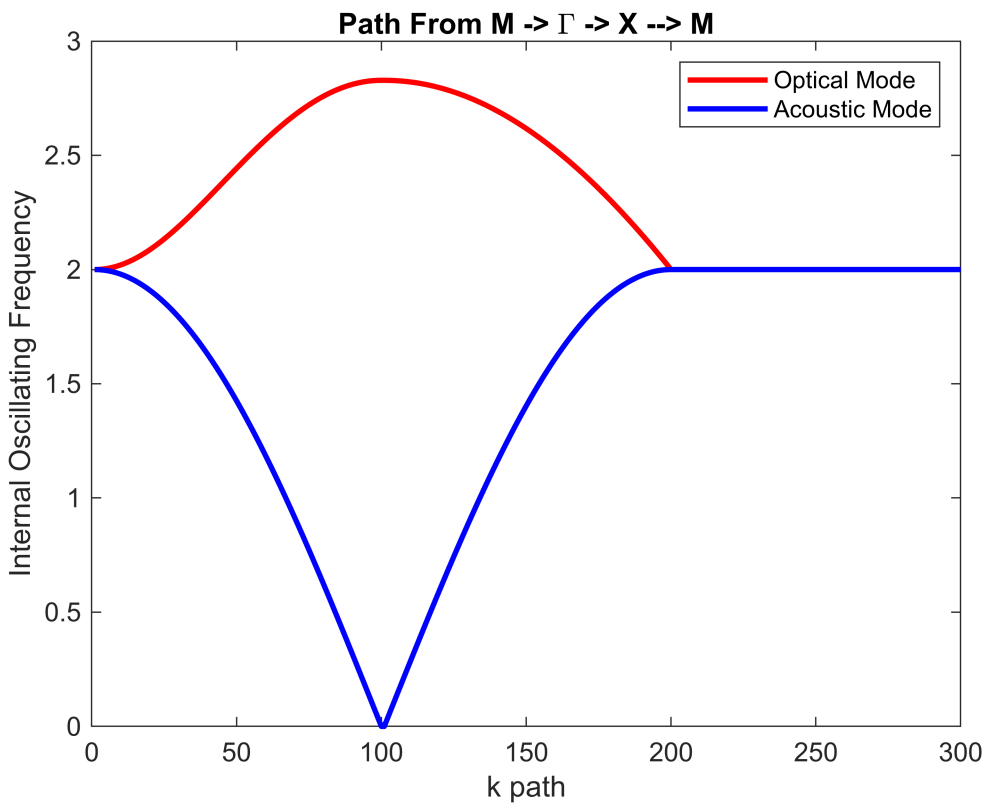
$$\frac{u_n^A}{u_n^B} = 0$$

Problem 3

$$\omega^2 = 2\gamma \frac{M_A + M_B}{M_A M_B} \pm 4\gamma \sqrt{(M_A + M_B)^2 - 4M_A M_B (1 - \cos^2(\frac{ak_x}{2}) \cos^2(\frac{ak_y}{2}))}$$

```
clc
clear
close all

a = 1;
spring = 1;
MA = 1;
MB = 1;

gamma = [0, 0];
M = [pi / a, pi / a];
X = [pi / a, 0];

k_M_GAMMA = [linspace(M(1), gamma(1), 100); linspace(M(2), gamma(2), 100)];
k_GAMMA_X = [linspace(gamma(1), X(1), 100); linspace(gamma(2), X(2), 100)];
k_X_M = [linspace(X(1), M(1), 100); linspace(X(2), M(2), 100)];

w_plus_M_GAMMA = 2 * spring * (MA + MB) / (MA * MB) + (2 * spring / (MA * MB)) * ...
    sqrt((MA + MB)^2 - 4 * MA * MB * (1 - ...
    cos(a * k_M_GAMMA(1,:) /2).^2 .* cos(a * k_M_GAMMA(2,:) / 2).^2));
w_minus_M_GAMMA = 2 * spring * (MA + MB) / (MA * MB) - (2 * spring / (MA * MB)) *
...
    sqrt((MA + MB)^2 - 4 * MA * MB * (1 - ...
    cos(a * k_M_GAMMA(1,:) /2).^2 .* cos(a * k_M_GAMMA(1,:) / 2).^2));

w_plus_GAMMA_X = 2 * spring * (MA + MB) / (MA * MB) + (2 * spring / (MA * MB)) * ...
    sqrt((MA + MB)^2 - 4 * MA * MB * (1 - ...
    cos(a * k_GAMMA_X(1,:) /2).^2 .* cos(a * k_GAMMA_X(2,:) / 2).^2));
w_minus_GAMMA_X = 2 * spring * (MA + MB) / (MA * MB) - (2 * spring / (MA * MB)) *
...
    sqrt((MA + MB)^2 - 4 * MA * MB * (1 - ...
    cos(a * k_GAMMA_X(1,:) /2).^2 .* cos(a * k_GAMMA_X(1,:) / 2).^2));

w_plus_X_M = 2 * spring * (MA + MB) / (MA * MB) + (2 * spring / (MA * MB)) * ...
    sqrt((MA + MB)^2 - 4 * MA * MB * (1 - ...
    cos(a * k_X_M(1,:) /2).^2 .* cos(a * k_X_M(2,:) / 2).^2));
w_minus_X_M = 2 * spring * (MA + MB) / (MA * MB) - (2 * spring / (MA * MB)) * ...
    sqrt((MA + MB)^2 - 4 * MA * MB * (1 - ...
    cos(a * k_X_M(1,:) /2).^2 .* cos(a * k_X_M(1,:) / 2).^2));

omega_plus = [sqrt(w_plus_M_GAMMA) sqrt(w_plus_GAMMA_X) sqrt(w_plus_X_M)];
omega_minus = [sqrt(w_minus_M_GAMMA) sqrt(w_minus_GAMMA_X) sqrt(w_minus_X_M)];

figure;
plot(omega_plus, 'r', 'LineWidth', 2);
```

```
hold on;
plot(omega_minus, 'b', 'LineWidth', 2);
legend('Optical Mode', 'Acoustic Mode');
ylabel('Internal Oscillating Frequency');
xlabel('k path')
title('Path From M -> \Gamma -> X --> M');
```



Problem 4

```
clc
clear
close all

q = 1.602e-19; % C
h = 6.626e-34; % Joule seconds
h_bar = h / (2*pi); % Joule seconds
KB_J = 1.381e-23; % Boltzmann constant in J/K
T = 300; % room temperature at degrees K
m_e = 9.109e-31; % electron rest mass in kg
m_gamma = 0.067 * m_e; % effective mass of electron in gamma valley
m_X = 0.58 * m_e; % effective mass of electron in X valley
m_L = 0.17 * m_e; % effective mass of electron in L valley
m_eff = [m_gamma m_X m_L];
E_phonon = 35.36e-3 * q; % energy of a phonon in GaAs in J
w_LO = E_phonon / h_bar; % rad/s for phonon in GaAs
e_s = 12.9; % static dielectric constant
e_inf = 10.9; % high-frequency dielectric constant
e_0 = 8.854e-12; % vacuum permittivity in F/m
N_0 = 1 / (exp(E_phonon / (KB_J * T)) - 1); % phonon occupation number
p = 5360; % mass density of GaAs, kg/m^3
```

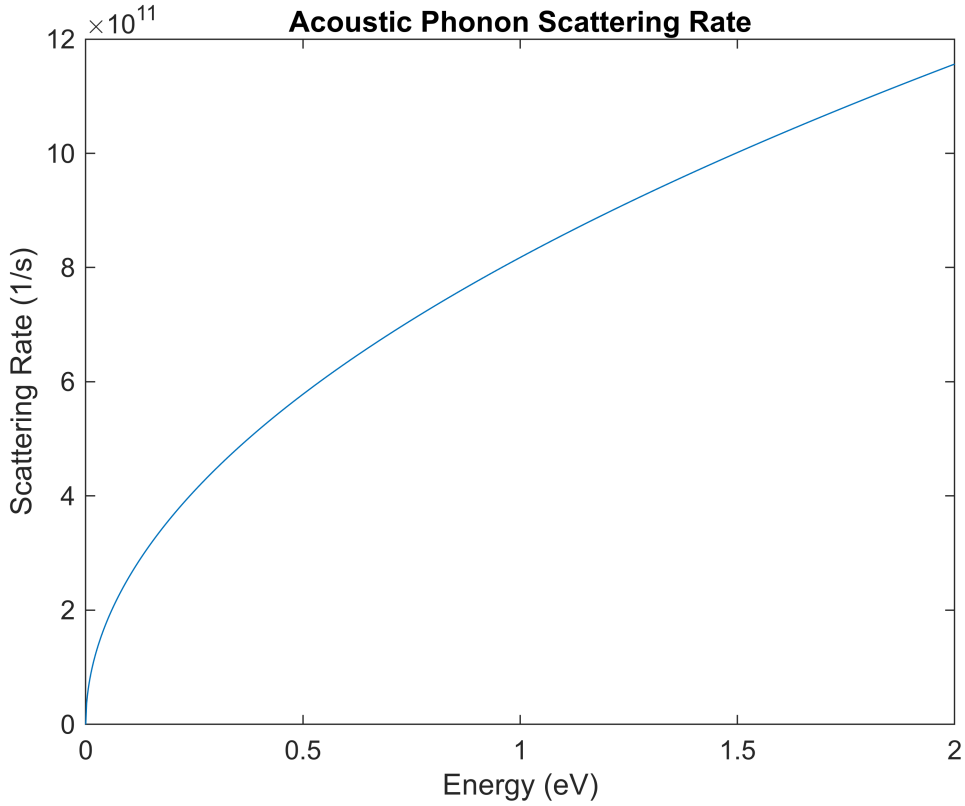
Initial Values

```
E_kin = linspace(0, 2, 1000) * q;
Z_if = [0 3 4
        1 2 4
        1 3 3]; % # of final vallies
D_if = [0 10 1.8
        10 10 1
        1.8 1 5] * (q / 1e-10); % from eV / Angstrom to J/m
E_if = [0 .48 .29
       -.49 0 -.19
       -.29 .19 0] * q;
inter_phonon = [0 .0299 .0278
              .0299 .0299 .0293
              .0278 .0293 .029] * q; % intervalley phonon energy in eV
w_if = inter_phonon / h_bar;
D_acoustic = [7 9 9.2] * q; % Deformation potential in J for gamma, X, L valley
v_s = 5110; % sound velocity in GaAs in m/s
```

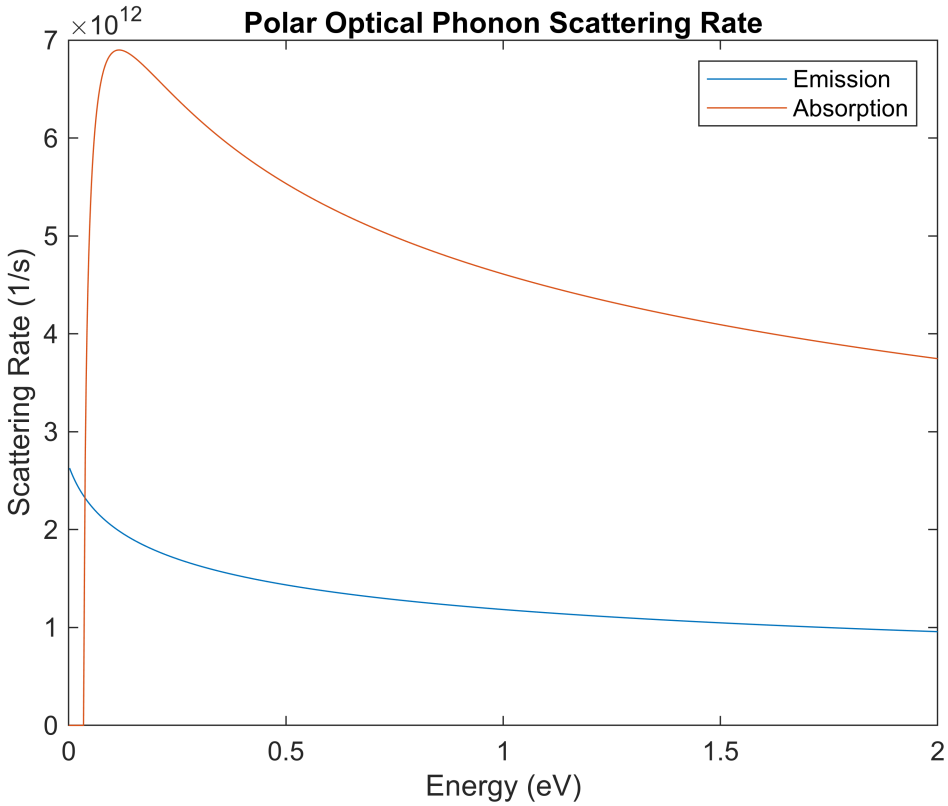
a)

```
% Acoustic scattering
x = 1;
acoustic = ac_scatter(E_kin, m_eff(x), h_bar, p, v_s, D_acoustic(x), KB_J, T);
figure;
plot(E_kin / q, acoustic);
```

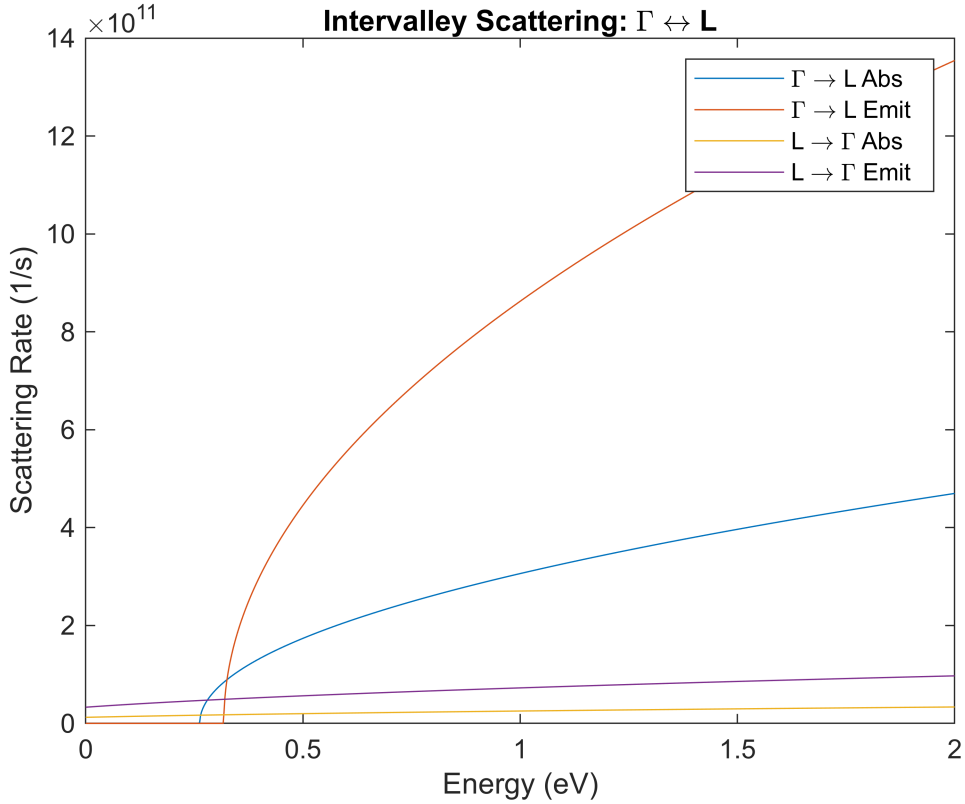
```
title('Acoustic Phonon Scattering Rate');
xlabel('Energy (eV)');
ylabel('Scattering Rate (1/s)');
```



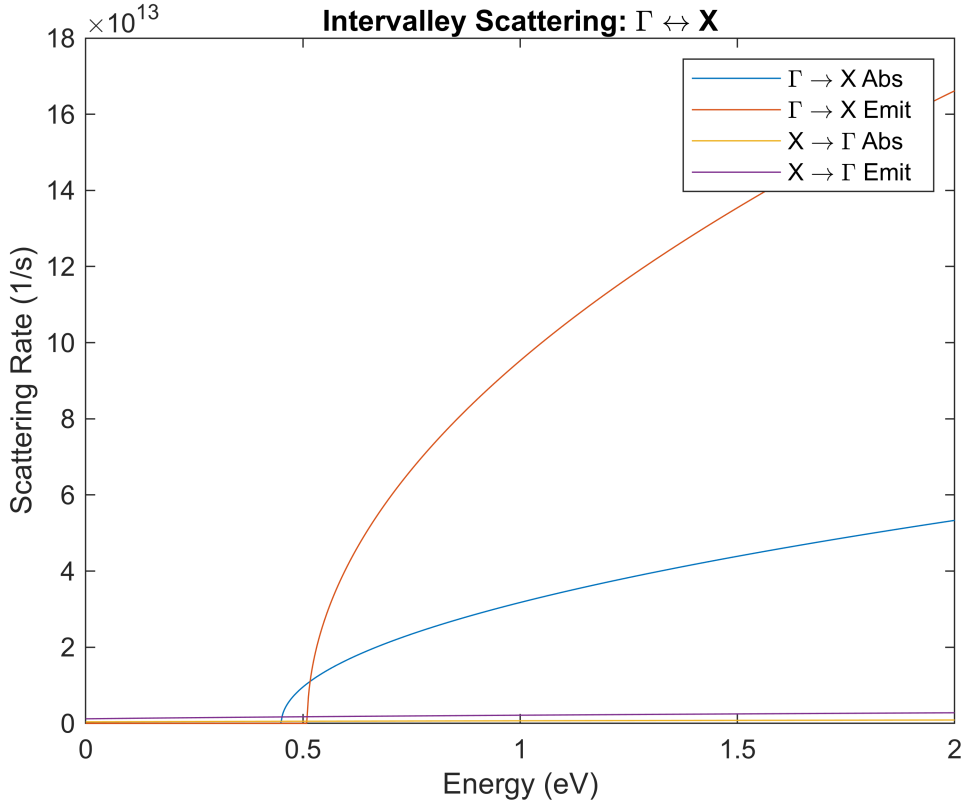

```
% Polar Optical (POP) scattering
figure;
emit_pop = pop_abs(E_kin, m_eff(x), h_bar, w_LO, E_phonon, KB_J, q, e_inf, e_s,
e_0, T);
abs_pop = pop_emit(E_kin, m_eff(x), h_bar, w_LO, E_phonon, KB_J, q, e_inf, e_s,
e_0, T);
plot(E_kin / q, emit_pop);
hold on;
plot(E_kin / q, abs_pop);
title('Polar Optical Phonon Scattering Rate');
xlabel('Energy (eV)');
ylabel('Scattering Rate (1/s)');
legend('Emission', 'Absorption');
```



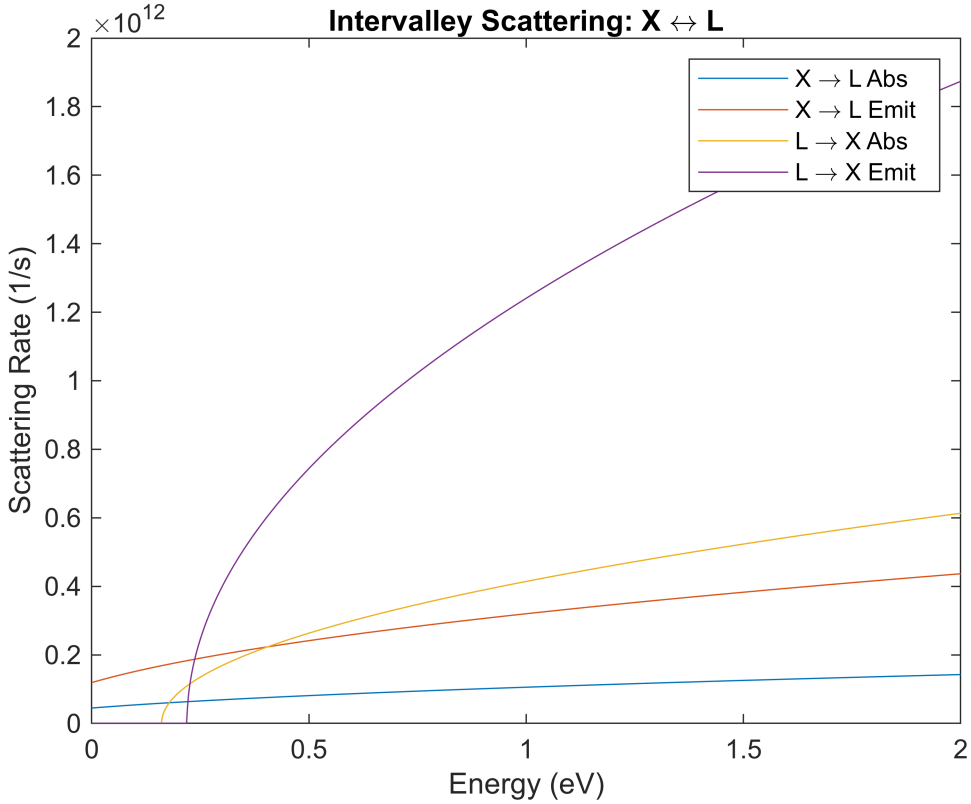

```
% Gamma <-> L
figure;
y = 3;
gamma_abs_L = inter_scatter_abs(E_kin, h_bar, KB_J, T, D_if(x, y), Z_if(x, y),
w_if(x, y), E_if(x, y), m_eff(y), p, inter_phonon(x, y));
gamma_emit_L = inter_scatter_emit(E_kin, h_bar, KB_J, T, D_if(x, y), Z_if(x, y),
w_if(x, y), E_if(x, y), m_eff(y), p, inter_phonon(x, y));
L_abs_gamma = inter_scatter_abs(E_kin, h_bar, KB_J, T, D_if(y, x), Z_if(y, x),
w_if(y, x), E_if(y, x), m_eff(x), p, inter_phonon(y, x));
L_emit_gamma = inter_scatter_emit(E_kin, h_bar, KB_J, T, D_if(y, x), Z_if(y, x),
w_if(y, x), E_if(y, x), m_eff(x), p, inter_phonon(y, x));
plot(E_kin / q, gamma_abs_L);
hold on;
plot(E_kin / q, gamma_emit_L);
plot(E_kin / q, L_abs_gamma);
plot(E_kin / q, L_emit_gamma);
title('Intervalley Scattering: \Gamma \leftrightarrow L');
xlabel('Energy (eV)');
ylabel('Scattering Rate (1/s)');
legend('\Gamma \rightarrow L Abs', '\Gamma \rightarrow L Emit', ...
       'L \rightarrow \Gamma Abs', 'L \rightarrow \Gamma Emit');
```




```
% Gamma <-> X
figure;
y = 2;
gamma_abs_X = inter_scatter_abs(E_kin, h_bar, KB_J, T, D_if(x, y), Z_if(x, y),
w_if(y, x), E_if(x, y), m_eff(y), p, inter_phonon(x, y));
gamma_emit_X = inter_scatter_emit(E_kin, h_bar, KB_J, T, D_if(x, y), Z_if(x, y),
w_if(y, x), E_if(x, y), m_eff(y), p, inter_phonon(x, y));
X_abs_gamma = inter_scatter_abs(E_kin, h_bar, KB_J, T, D_if(y, x), Z_if(y, x),
w_if(y, x), E_if(y, x), m_eff(x), p, inter_phonon(y, x));
X_emit_gamma = inter_scatter_emit(E_kin, h_bar, KB_J, T, D_if(y, x), Z_if(y, x),
w_if(y, x), E_if(y, x), m_eff(x), p, inter_phonon(y, x));
plot(E_kin / q, gamma_abs_X);
hold on;
plot(E_kin / q, gamma_emit_X);
plot(E_kin / q, X_abs_gamma);
plot(E_kin / q, X_emit_gamma);
title('Intervalley Scattering: \Gamma \leftrightarrow X');
xlabel('Energy (eV)');
ylabel('Scattering Rate (1/s)');
legend('\Gamma \rightarrow X Abs', '\Gamma \rightarrow X Emit', ...
       'X \rightarrow \Gamma Abs', 'X \rightarrow \Gamma Emit');
```

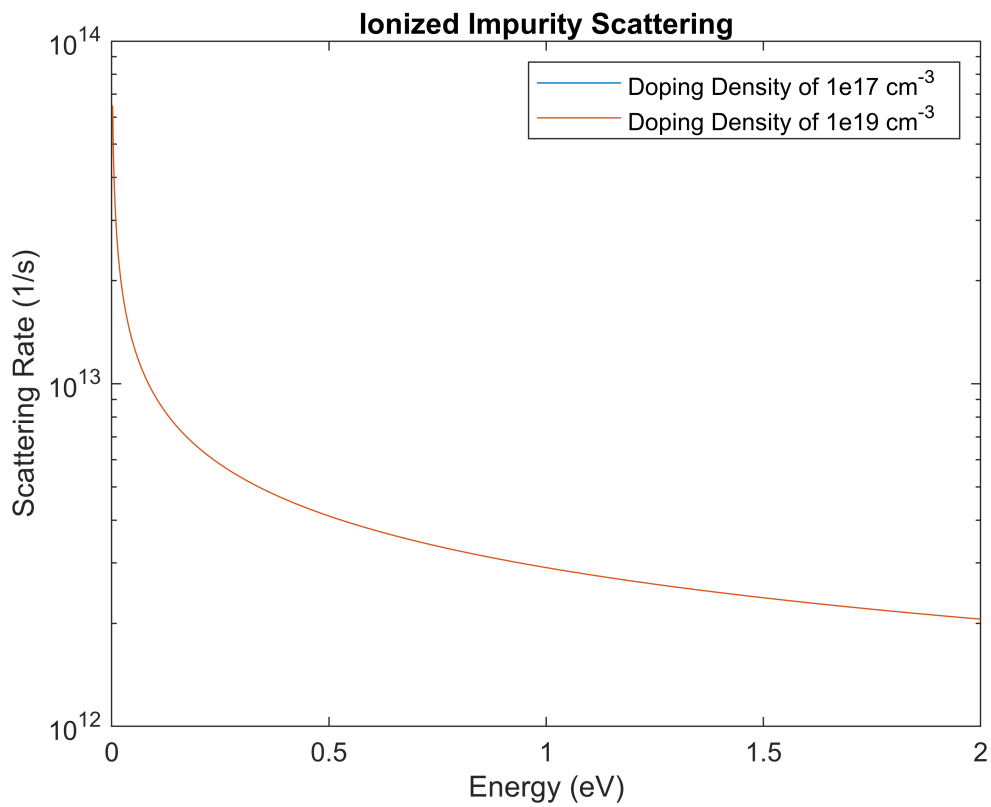



```
% X <-> L
figure;
x = 2; y = 3;
X_abs_L = inter_scatter_abs(E_kin, h_bar, KB_J, T, D_if(x, y), Z_if(x, y), w_if(y,
x), E_if(x, y), m_eff(y), p, inter_phonon(x, y));
X_emit_L = inter_scatter_emit(E_kin, h_bar, KB_J, T, D_if(x, y), Z_if(x, y),
w_if(y, x), E_if(x, y), m_eff(y), p, inter_phonon(x, y));
L_abs_X = inter_scatter_abs(E_kin, h_bar, KB_J, T, D_if(y, x), Z_if(y, x), w_if(y,
x), E_if(y, x), m_eff(x), p, inter_phonon(y, x));
L_emit_X = inter_scatter_emit(E_kin, h_bar, KB_J, T, D_if(y, x), Z_if(y, x),
w_if(y, x), E_if(y, x), m_eff(x), p, inter_phonon(y, x));
plot(E_kin / q, X_abs_L);
hold on;
plot(E_kin / q, X_emit_L);
plot(E_kin / q, L_abs_X);
plot(E_kin / q, L_emit_X);
title('Intervalley Scattering: X \leftrightarrow L');
xlabel('Energy (eV)');
ylabel('Scattering Rate (1/s)');
legend('X \rightarrow L Abs', 'X \rightarrow L Emit', ...
       'L \rightarrow X Abs', 'L \rightarrow X Emit');
```




```
% X <-> X and L <-> L
figure;
y = 3;
L_abs_L = inter_scatter_abs(E_kin, h_bar, KB_J, T, D_if(y, y), Z_if(y, y), w_if(y,
y), E_if(y, y), m_eff(y), p, inter_phonon(y, y));
L_emit_L = inter_scatter_emit(E_kin, h_bar, KB_J, T, D_if(y, y), Z_if(y, y),
w_if(y, y), E_if(y, y), m_eff(y), p, inter_phonon(y, y));
y = 2;
X_abs_X = inter_scatter_abs(E_kin, h_bar, KB_J, T, D_if(y, y), Z_if(y, y), w_if(y,
y), E_if(y, y), m_eff(y), p, inter_phonon(y, y));
X_emit_X = inter_scatter_emit(E_kin, h_bar, KB_J, T, D_if(y, y), Z_if(y, y),
w_if(y, y), E_if(y, y), m_eff(y), p, inter_phonon(y, y));
plot(E_kin / q, L_abs_L);
hold on;
plot(E_kin / q, L_emit_L);
plot(E_kin / q, X_abs_X);
plot(E_kin / q, X_emit_X);
title('Intravalley Scattering: L \leftrightarrow L and X \leftrightarrow X');
xlabel('Energy (eV)');
ylabel('Scattering Rate (1/s)');
legend('L \rightarrow L Abs', 'L \rightarrow L Emit', ...
       'X \rightarrow X Abs', 'X \rightarrow X Emit');
```

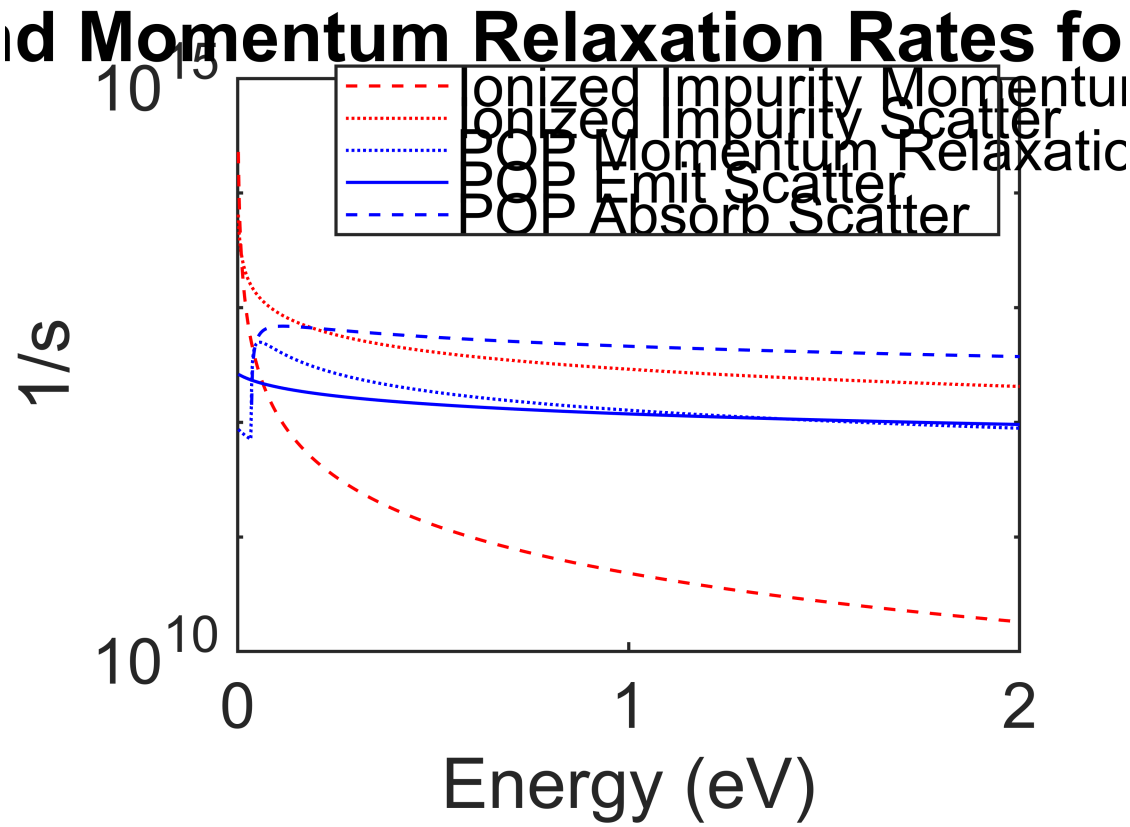
```
figure;
x = 1;
ion_imp_less_doped = ion_imp(E_kin, 1e17, 1, q, h_bar, e_0, e_s, m_eff(x), T, KB_J);
ion_imp_more_doped = ion_imp(E_kin, 1e19, 1, q, h_bar, e_0, e_s, m_eff(x), T, KB_J);
semilogy(E_kin / q, ion_imp_less_doped);
hold on;
semilogy(E_kin / q, ion_imp_more_doped);
title('Ionized Impurity Scattering');
xlabel('Energy (eV)');
ylabel('Scattering Rate (1/s)');
legend('Doping Density of 1e17 cm^{-3}', 'Doping Density of 1e19 cm^{-3}');
```



b)

```
x = 1;
% Momentum Rates
ionized_momentum = ionized_k(E_kin, 1e19, 1, q, h_bar, e_0, e_s, m_eff(x), T, KB_J);
figure;
semilogy(E_kin/ q, ionized_momentum, 'r--');
hold on;
semilogy(E_kin / q, ion_imp_more_doped, 'r:');

% Scatter Rates
emit_pop_scatter = pop_abs(E_kin, m_eff(x), h_bar, w_LO, E_phonon, KB_J, q, e_inf,
e_s, e_0, T);
```

```
abs_pop_scatter = pop_emit(E_kin, m_eff(x), h_bar, w_LO, E_phonon, KB_J, q, e_inf,
e_s, e_0, T);
pop_k = pop_momentum(E_kin, E_phonon, m_eff(x), h_bar, KB_J, T, q, e_inf, e_s, e_0,
w_LO);
semilogy(E_kin / q, pop_k, 'b:');
semilogy(E_kin / q, emit_pop_scatter, 'b-');
semilogy(E_kin / q, abs_pop_scatter, 'b--');
legend('Ionized Impurity Momentum Relaxation', 'Ionized Impurity Scatter', ...
    'POP Momentum Relaxation', 'POP Emit Scatter', 'POP Absorb Scatter');
title('Comparing Scattering and Momentum Relaxation Rates for POP and Ionzied
Impurities');
ylabel('1/s');
xlabel('Energy (eV)');
```



Ionized Impurity Momentum Relaxation

```
function ion_relax = ionized_k(E, n, Z, q, h_bar, e_0, e_s, m, T, KB_J)
    e_0 = e_0 / 100; % converting to F/cm to match cm^-3 concentration
    k = sqrt(2 * m * E / h_bar^2);
    ion_relax = zeros(1, length(k));
    Ld = sqrt((e_0 * e_s * T * KB_J) / (n * q ^ 2));

    X = (n * Z^2 * q^4 * m) / (8 * pi * h_bar^3 * e_0^2 * e_s^2);

    for j = 1:length(k)
        B = 4 * k(j)^2 * Ld^2;
        ion_relax(j) = (X / k(j)^3) * (log(1 + B) - (B / (1 + B)));
```

```
    end
end
```

POP Relaxation

```
function pop_k = pop_momentum(E_kin, E_phonon, m, h_bar, KB_J, T, q, e_inf, e_s,
e_0, w_LO)
    k = sqrt(2 * m * E_kin / h_bar^2);
    k_ph = sqrt(2 * m * E_phonon / h_bar^2);
    N0 = 1 / (exp(E_phonon / (KB_J * T)) - 1);
    pop_k = zeros(1, length(E_kin));
    absorb = pop_k; emit = pop_k;

    for j = 1:length(E_kin)
        X = (q^2 * w_LO * (e_s / e_inf - 1)) / (4 * pi * e_s * e_0 * h_bar * sqrt(2
* E_kin(j) / m));

        absorb(j) = X * (N0 * sqrt(1 + E_phonon / E_kin(j)) - ...
            (E_phonon * N0 / E_kin(j)) * asinh(sqrt(E_kin(j) / E_phonon)));
        if E_kin(j) > E_phonon
            emit(j) = X * ((N0 + 1) * sqrt(1 - E_phonon / E_kin(j)) + ...
                (E_phonon * (N0 + 1) / E_kin(j)) * asinh(sqrt(E_kin(j) / E_phonon -
1)));
        else
            emit(j) = 0;
        end
    end
    pop_k = absorb + emit;
end
```

Polar-Optical-Phonon Scattering

```
function pop_scatter = pop_emit(E, m, h_bar, w_0, E_phonon, KB_J, q, e_inf, e_s,
e_0, T)
    pop_scatter = zeros(1, length(E));
    for i = 1:length(E)
        x = h_bar * w_0 / E(i);
        C = ((q^2 * w_0) / (8 * pi * e_0)) * sqrt(2 * m / (h_bar^2)) * ...
            ((1 / e_inf) - (1 / e_s)) * (1 / sqrt(E(i)));
        N_0 = 1 / (exp(E_phonon / (KB_J * T)) - 1);

        if E(i) > (h_bar * w_0)
            pop_scatter(i) = C * ((N_0 + 1) * log(abs((1 + sqrt(1 - x)) / (1 -
sqrt(1 - x)))));
        end
    end
end

function pop_scatter = pop_abs(E, m, h_bar, w_0, E_phonon, KB_J, q, e_inf, e_s,
e_0, T)
```

```
    pop_scatter = zeros(1, length(E));
    for i = 1:length(E)
        x = h_bar * w_0 / E(i);
        C = ((q^2 * w_0) / (8 * pi * e_0)) * sqrt(2 * m / (h_bar^2)) * ...
            ((1 / e_inf) - (1 / e_s)) * (1 / sqrt(E(i)));
        N_0 = 1 / (exp(E_phonon / (KB_J * T)) - 1);

        pop_scatter(i) = C * (N_0 * log(abs((1 + sqrt(1 + x)) / (-1 + sqrt(1 +
x)))));
    end
end
```

Acoustic

```
function acoustic = ac_scatter(E, m, h_bar, p, v_s, D, KB_J, T)
    acoustic = (D^2 * KB_J * T) / (2 * pi * h_bar * p * v_s^2) * ...
        ((2 * m) / (h_bar ^ 2)) ^ 1.5 * sqrt(E);
end
```

Intervalley Scattering

```
function gamma_inter_abs = inter_scatter_abs(E, h_bar, KB_J, T, D_if, Z_if, w_if,
E_if, m_final, p, E_phonon)
    gamma_inter_abs = zeros(1, length(E));
    for j = 1:length(E)
        N_if = 1 / (exp(h_bar * w_if / (KB_J * T)) - 1);
        g_DOS = DOS(E(j) + E_phonon - E_if, m_final, h_bar);
        gamma_inter_abs(j) = N_if * (pi * D_if^2 * Z_if / (2 * p * w_if)) * g_DOS;
    end
end

function gamma_inter_emit = inter_scatter_emit(E, h_bar, KB_J, T, D_if, Z_if, w_if,
E_if, m_final, p, E_phonon)
    gamma_inter_emit = zeros(1, length(E));
    for j = 1:length(E)
        N_if = 1 / (exp(h_bar * w_if / (KB_J * T)) - 1);
        g_DOS = DOS(E(j) - E_phonon - E_if, m_final, h_bar);
        gamma_inter_emit(j) = (N_if + 1) * (pi * D_if^2 * Z_if / (2 * p * w_if)) *
g_DOS;
    end
end
```

Ionized Impurities

```
function ion_impurities = ion_imp(E, n, Z, q, h_bar, e_0, e_s, m, T, KB_J)
    k = sqrt(2 * m * E / h_bar^2);
    ion_impurities = zeros(1, length(k));
    Ld = sqrt((e_0 * e_s * T * KB_J) / (n * q ^ 2));

    X = (n * Z^2 * q^4 * Ld^4 * m) / (pi * h_bar^3 * e_0^2 * e_s^2);
```

```
    for j = 1:length(k)
        ion_impurities(j) = X * (k(j) ./ (4 * k(j)^2 * Ld^2 + 1));
    end
end
```

DOS

```
function g_DOS = DOS(E, m, h_bar)
    if E > 0
        g_DOS = (sqrt(E) / (2 * (pi^2))) * (2 * m / (h_bar ^ 2)) ^ 1.5;
    else
        g_DOS = 0;
    end
end
```

Problem 7

```
clc
clear
close all

T = 300;
q = 1.602e-19; % C
KB_J = 1.381e-23;
KB_eV = 8.617e-5;
h = 6.626e-34;
h_bar = h / (2*pi);
m_e = 9.109e-31;
m_gamma = 0.067 * m_e;
p = 5360;
v_s = 5220;
D_acoustic = 7.01 * q;
e_s = 12.9; % static dielectric constant
e_inf = 10.9; % high-frequency dielectric constant
e_0 = 8.854e-12; % vacuum permittivity in F/m
e_0_cm = e_0 / 100; % vacuum permittivity in F/cm
E_phonon = 35.36e-3 * q; % energy of a phonon in GaAs in J
w_LO = E_phonon / h_bar; % rad/s for phonon in GaAs
N = 1e17; % concentration (cm^-3)
Z = 1; % charge state of GaAs

% Energy in eV
E = linspace(0, 5 * KB_eV * T, 100);      % eV
E_J = E * q;                              % Convert to joules

% Compute everything
D = DOS(E_J, m_gamma, h_bar);            % Use E in joules
f0 = exp(-(E + 0.042) / (KB_eV * T));    % All in eV

tau0_acoustic = (2 * pi * p * v_s^2 * h_bar^4) / ...
    (D_acoustic^2 * (2 * m_gamma)^1.5 * (KB_J * T)^1.5); % seconds
S_acoustic = -0.5;
tau_f_acoustic = tau0_acoustic * (4 / (3 * sqrt(pi))) * gamma(S_acoustic + 2.5); %
seconds
mu_acoustic = tau_f_acoustic * q / m_gamma
```

```
mu_acoustic =
15.6263
```

```
fprintf('Mobility Limtied by Acoustic Phonon Scattering: %f m^2/Vs', mu_acoustic);
```

```
Mobility Limtied by Acoustic Phonon Scattering: 15.626295 m^2/Vs
```

```
tau0_ionized = 2^4.5 * (pi * e_0^2 * e_s^2 * sqrt(m_gamma)) / ...
    (N * Z^2 * q^4);
```

```
S_ionized = 1.5;
tau_f_ionized = tau0_ionized * (4 / (3 * sqrt(pi))) * gamma(S_ionized  + 2.5); %
seconds
mu_ionized = tau_f_ionized * q / m_gamma;
fprintf('Mobility Limtied by Acoustic Phonon Scattering: %f m^2/Vs', mu_ionized);
```

```
Mobility Limtied by Acoustic Phonon Scattering: 4120956897166821615693934799408752230 4.000000 m^2/Vs
```

```
gamma_pop = pop_scatter(E_J, m_gamma, h_bar, w_LO, E_phonon, KB_J, q, e_inf, e_s,
e_0, T);
tau = 1 ./ gamma_pop;
numerator = trapz(E, E .* tau .* D .* f0);
denominator = trapz(E, E .* D .* f0);
avg_tau = numerator / denominator;
mu_pop = q * avg_tau / m_gamma;
fprintf('Mobility Limtied by Acoustic Phonon Scattering: %f m^2/Vs', mu_pop);
```

```
Mobility Limtied by Acoustic Phonon Scattering: 0.551842 m^2/Vs
```

c)

```
mu_total = (1/mu_pop) + (1/mu_ionized) + (1/mu_acoustic);
mu_total = (1/mu_total) * 1e4; % in cm^2/Vs
fprintf('Total Mobility (Matthiessens Rule: %f cm^2/Vs', mu_total);
```

```
Total Mobility (Matthiessens Rule: 5330.181315 cm^2/Vs
```

```
conductivity = q * N * mu_total * 100; % S/m
fprintf('Conductivity: %f S/m', conductivity);
```

```
Conductivity: 8538.950467 S/m
```

d)

```
gamma_acoustic = ac_scatter(E_J, m_gamma, h_bar, p, v_s, D_acoustic, KB_J, T);
gamma_ionized = ion_imp(E_J, N, Z, q, h_bar, e_0, e_s, m_gamma, T, KB_J);
total_gamma = gamma_ionized + gamma_acoustic + gamma_pop;
total_gamma(1) = total_gamma(2);
tau_total = 1 ./ total_gamma;
numerator = trapz(E, E .* tau_total .* D .* f0);
denominator = trapz(E, E .* D .* f0);
avg_tau = numerator / denominator;
mu_total = 1e4 * q * avg_tau / m_gamma;
fprintf('Total Mobility (without Matthiessens Rule: %f cm^2/Vs', mu_total);
```

```
Total Mobility (without Matthiessens Rule: 1295.172484 cm^2/Vs
```

Polar-Optical-Phonon Scattering

```
function pop_scatter = pop_emit(E, m, h_bar, w_0, E_phonon, KB_J, q, e_inf, e_s,
e_0, T)
    pop_scatter = zeros(1, length(E));
    for i = 1:length(E)
```

```
        x = h_bar * w_0 / E(i);
        C = ((q^2 * w_0) / (8 * pi * e_0)) * sqrt(2 * m / (h_bar^2)) * ...
            ((1 / e_inf) - (1 / e_s)) * (1 / sqrt(E(i)));
        N_0 = 1 / (exp(E_phonon / (KB_J * T)) - 1);

        if E(i) > (h_bar * w_0)
            pop_scatter(i) = C * ((N_0 + 1) * log(abs((1 + sqrt(1 - x)) / (1 -
sqrt(1 - x)))));
        end
    end
end

function pop_scatter = pop_abs(E, m, h_bar, w_0, E_phonon, KB_J, q, e_inf, e_s,
e_0, T)
    pop_scatter = zeros(1, length(E));
    for i = 1:length(E)
        x = h_bar * w_0 / E(i);
        C = ((q^2 * w_0) / (8 * pi * e_0)) * sqrt(2 * m / (h_bar^2)) * ...
            ((1 / e_inf) - (1 / e_s)) * (1 / sqrt(E(i)));
        N_0 = 1 / (exp(E_phonon / (KB_J * T)) - 1);

        pop_scatter(i) = C * (N_0 * log(abs((1 + sqrt(1 + x)) / (-1 + sqrt(1 +
x)))));
    end
end

function total_pop = pop_scatter(E, m, h_bar, w_0, E_phonon, KB_J, q, e_inf, e_s,
e_0, T)
    abs = pop_abs(E, m, h_bar, w_0, E_phonon, KB_J, q, e_inf, e_s, e_0, T);
    emit = pop_emit(E, m, h_bar, w_0, E_phonon, KB_J, q, e_inf, e_s, e_0, T);
    total_pop = abs + emit;
    total_pop(1) = total_pop(2);
end
```

DOS

```
function g_DOS = DOS(E, m, h_bar)
    g_DOS = zeros(1, length(E));
    for i = 1:length(E)
        if E(i) > 0
            g_DOS(i) = (sqrt(E(i)) / (2 * (pi^2))) * (2 * m / (h_bar ^ 2)) ^ 1.5;
        else
            g_DOS(i) = 0;
        end
    end
end
```

Acoustic

```
function acoustic = ac_scatter(E, m, h_bar, p, v_s, D, KB_J, T)
```

```
    acoustic = (D^2 * KB_J * T) / (2 * pi * h_bar * p * v_s^2) * ...
        ((2 * m) / (h_bar ^ 2)) ^ 1.5 * sqrt(E);
end
```

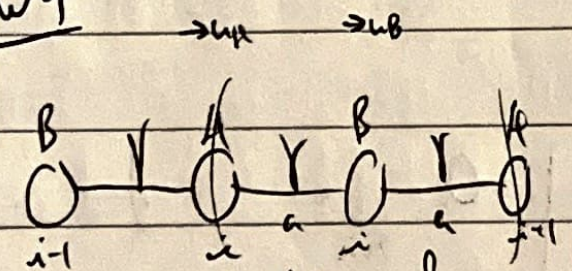
Ionized Impurities

```
function ion_impurities = ion_imp(E, n, Z, q, h_bar, e_0, e_s, m, T, KB_J)
    k = sqrt(2 * m * E / h_bar^2);
    ion_impurities = zeros(1, length(k));
    Ld = sqrt((e_0 * e_s * T * KB_J) / (n * q ^ 2));

    X = (n * Z^2 * q^4 * Ld^4 * m) / (pi * h_bar^3 * e_0^2 * e_s^2);

    for j = 1:length(k)
        ion_impurities(j) = X * (k(j) ./ (4 * k(j)^2 * Ld^2 + 1));
    end
end
```

$= -v_x B_z \partial k_y \left(k_x \tau_m(\varepsilon) \frac{\partial}{d\varepsilon}\right) + v_y B_z \partial k_x \left(k_x \tau_m(\varepsilon) d\varepsilon\right)$

HW4

1) a)



Phonons are vibrations from atoms that are oscillating, dispersion relations arise when solving for them, $\omega(k)$, called branches — acoustic / optical

acoustic: in phase (like sound waves)

optical: out phase

Should be two lattice wave branches (one acoustic / one optical)

$M_A \ddot{u}_A = \gamma(u_B - u_A) - \gamma(u_A - u_{B-1}) + \gamma(u_{B-1} - u_A)$

$M_B \ddot{u}_B = \gamma(u_{A+1} - u_B) - \gamma(u_B - u_A)$

$M_A \ddot{u}_A = \gamma(u_B + u_{B-1} - 2u_A) = -\gamma(2u_A - u_B - u_{B-1})$

$M_B \ddot{u}_B = \gamma(u_A + u_{A+1} - 2u_B) = -\gamma(2u_B - u_A - u_{A+1})$

$\ddot{u}_x = \frac{d^2}{dt^2}\left(u_0 e^{i(kna - \omega t)}\right) = -\omega^2 u_x(t)$

$*\omega^2 u_A$

$M_A \ddot{u}_A = *\gamma(2u_A - u_B - u_{B-1}) = M_A\omega^2 u_A = \gamma(2u_A - u_B - u_{B-1})$

$M_B \ddot{u}_B = -\gamma(2u_B - u_A - u_{A+1}) = M_B\omega^2 u_B = \gamma(2u_B - u_A - u_{A+1})$

want solutions $u_n = u_0 e^{+ix}$, $u_0 e^{-ix}$

$u_n^A = u_A e^{i(nka-\omega t)}$

$u_{n+1}^A = u_A e^{i((n+1)ka-\omega t)} = u_A e^{i(nka-\omega t)} e^{ika} = e^{ika} u_n^A$

$e^{i(nka+ka-\omega t)} \to e^{ika} e^{i(kan-\omega t)}$

$u_n^B = u_B e^{i(knb-\omega t)}$

$u_{B_{n+1}} = u_B e^{i((n+1)kb-\omega t)} = e^{-ibk} e^{i(knb-\omega t)} u_B = e^{-ibk} u_n^B$

$e^{i(nkb-kb-\omega t)} \to e^{-ikb} e^{i(knb-\omega t)}$

$M_A\omega^2 u_A = \gamma(2u_n^A - u_n^B - u_n^B e^{-ibk}) = \gamma(2u_n^A - u_n^B(1+e^{-ibk}))$

$M_B\omega^2 u_B = \gamma(2u_n^B - u_n^A - u_n^A e^{-ika}) = \gamma(2u_n^B - u_n^A(1+e^{-ika}))$

$$\begin{bmatrix} M_A\omega^2 & 0 \\ 0 & M_B\omega^2 \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix} = \gamma\begin{bmatrix} 2 & -(1+e^{-ika}) \\ -(1+e^{-ikb}) & 2 \end{bmatrix}\begin{bmatrix} u_A \\ u_B \end{bmatrix}$$

$M_A\omega^2 u_A - \gamma 2u_n^A = -\gamma u_n^B(1+e^{-ibk})$

$M_B\omega^2 u_B - \gamma 2u_n^B = -\gamma u_n^A(1+e^{-ika})$

$$\begin{bmatrix} M_A\omega^2 - \gamma 2 & \gamma(1+e^{-ikb}) \\ \gamma(1+e^{-ika}) & M_B\omega^2 - \gamma 2 \end{bmatrix}\begin{bmatrix} u_n^A \\ u_n^B \end{bmatrix} = 0$$

$(M_A\omega^2 - 2\gamma)(M_B\omega^2 - 2\gamma) - \gamma^2(1+e^{-ikb})(1+e^{-ika}) = 0$

$2\gamma^2(1-\cos(ka))$

$= 2\gamma^2(2\sin^2(\frac{ka}{2}))$

I forgot only one lattice constant, a, & $b \to a$

$(M_A\omega^2 - 2\gamma)(M_B\omega^2 - 2\gamma) - \gamma^2(1+e^{-ika})(1+e^{-ika}) = 0$

$M_AM_B\omega^4 - 2\gamma M_A\omega^2 - 2\gamma M_B\omega^2 + 4\gamma^2 - \gamma^2(1+e^{-ika}+e^{-ika}+e^{0}) = 0$

$M_AM_B\omega^4 - 2\gamma M_A\omega^2 - 2\gamma M_B\omega^2 + 4\gamma^2 - \gamma^2(2+2\cos(ka)) = 0$

$M_AM_B\omega^4 - 2\gamma M_A\omega^2 - 2\gamma M_B\omega^2 + 2\gamma^2 - 2\gamma^2\cos(ka) = 0$

$M_AM_B\omega^4 - 2\gamma M_A\omega^2 - 2\gamma M_B\omega^2 + 4\gamma^2\sin^2(\frac{ka}{2}) = 0$

$\omega^4(M_AM_B) + \omega^2(-2\gamma(M_A+M_B)) + 4\gamma^2\sin^2(\frac{ka}{2}) = 0$

$$\omega^2 = \frac{2\gamma(M_A+M_B) \pm \sqrt{(-2\gamma(M_A+M_B))^2 - 4(M_AM_B)(4\gamma^2\sin^2(\frac{ka}{2}))}}{2(M_AM_B)}$$

$$\left(-2\gamma(M_A+M_B)\right)\left(-2\gamma(M_A+M_B)\right)$$

$$\sqrt{4\gamma^2(M_A+M_B)^2-4(M_AM_B)\left(4\gamma^2\sin^2\left(\frac{ka}{2}\right)\right)}$$

$$\omega^2=\frac{-2\gamma(M_A+M_B)\pm\sqrt{4\gamma^2(M_A+M_B)^2-16\gamma^2(M_AM_B)\sin^2\left(\frac{ka}{2}\right)}}{2(M_AM_B)}$$

$$\omega^2=\frac{\cancel{2}\gamma(M_A+M_B)}{\cancel{2}(M_AM_B)}\pm\frac{\gamma\cancel{2}}{\cancel{2}(M_AM_B)}\sqrt{(M_A+M_B)^2-4(M_AM_B)\sin^2\left(\frac{ka}{2}\right)}$$

$$\omega^2=\frac{\gamma(M_A+M_B)}{M_AM_B}\pm\frac{\gamma}{M_AM_B}\sqrt{(M_A+M_B)^2-4(M_AM_B)\sin^2\left(\frac{ka}{2}\right)}$$

two branch

$k\to 0$ $\quad \omega^2=\frac{\gamma(M_A+M_B)}{M_AM_B}\pm\frac{\gamma(M_A+M_B)}{M_AM_B}$

optical, $\omega^2=\frac{\gamma(M_A+M_B)+\gamma(M_A+M_B)}{M_AM_B}=\frac{2\gamma(M_A+M_B)}{M_AM_B}$

acoustic, $\omega^2=\frac{\gamma(M_A+M_B)-\gamma(M_A+M_B)}{M_AM_B}=0$

$$(M_A\omega^2-2\gamma)u_n^A+\gamma u_n^B(1+e^{-ika})=0$$

$$(M_A\omega^2-2\gamma)u_n^A=-\gamma u_n^B(1+e^{-ika})$$

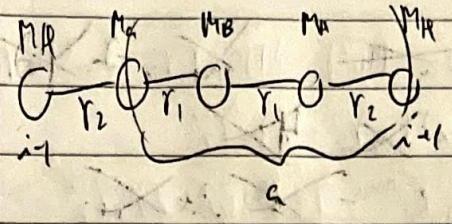
$$\frac{u_n^A}{u_n^B}=\frac{-\gamma(1+e^{-ika})}{M_A\omega^2-2\gamma}$$

$k\to\frac{\pi}{a}$, $\quad \omega^2=\frac{\gamma(M_A+M_B)}{M_AM_B}\pm\frac{\gamma}{M_AM_B}\sqrt{(M_A+M_B)^2-4(M_AM_B)}$

optical

$$\frac{-2}{\frac{2(M_A+M_B)}{M_B}-\frac{2(M_B)}{M_B}}\qquad\frac{-2}{\frac{2M_A+\cancel{2M_B}-\cancel{2M_B}}{M_B}}\qquad\frac{-2}{\frac{2M_A}{M_B}}=\frac{-\cancel{2}M_B}{\cancel{2}M_A}=-\frac{M_B}{M_A}$$

2)



$$M_A \ddot{a}_i^1 = \gamma_1(b_i - a_i^1) + \gamma_2(a_{i-1}^2 - a_i^1)$$
$$M_B \ddot{b}_i = \gamma_1(a_i^2 - b_i) + \gamma_1(a_i^1 - b_i)$$
$$M_A \ddot{a}_i^2 = \gamma_2(a_{i+1}^1 - a_i^2) + \gamma_1(b_i - a_i^2)$$
$$M_A(-\omega^2) a_i^1 = \gamma_1(b_i - a_i^1) + \gamma_2(a_i^2 e^{-jka} - a_i^1)$$
$$M_B b_i(-\omega^2) = \gamma_1(a_i^2 - b_i) + \gamma_1(a_i^1 - b_i)$$
$$M_A a_i^2(-\omega^2) = \gamma_2(a_i^1 e^{jka} - a_i^2) + \gamma_1(b_i - a_i^2)$$

$$\begin{bmatrix} M_A(-\omega^2) + \gamma_1 + \gamma_2 & -\gamma_1 & -\gamma_2 e^{-jka} \\ -\gamma_1 & M_B(-\omega^2) + 2\gamma_1 & -\gamma_1 \\ -\gamma_2 e^{jka} & -\gamma_1 & M_A(-\omega^2) + \gamma_2 + \gamma_1 \end{bmatrix} \det = 0$$

$$\omega^6(-M_A^2 M_B) + \omega^4(2M_A^2\gamma_1 + 2M_A M_B \gamma_1 + 2M_A M_B \gamma_2)$$
$$+ \omega^2(-2M_A\gamma_1^2 - 4M_A\gamma_1\gamma_2 - M_B\gamma_1^2 - 2M_B\gamma_1\gamma_2)$$
$$+ (2\gamma_1^2\gamma_2(1+\cos(ka)))$$

$$(M_A(-\omega^2) + \gamma_1 + \gamma_2)((M_B(-\omega^2) + 2\gamma_1)(M_A(-\omega^2) + \gamma_2 + \gamma_1) - \gamma_1^2)$$
$$+ \gamma_1(-\gamma_1(M_A(-\omega^2) + \gamma_2 + \gamma_1) - \gamma_1\gamma_2 e^{jka})$$
$$- \gamma_2 e^{-jka}(\gamma_1^2 + (\gamma_2 e^{jka})(M_B(-\omega^2) + 2\gamma_1))$$

$$(M_A(-\omega^2) + \gamma_1 + \gamma_2)(M_A M_B \omega^4 - \omega^2 M_B\gamma_2 - \omega^2 M_B\gamma_1 - \omega^2 M_A 2\gamma_1 + 2\gamma_1\gamma_2 + \gamma_1^2)$$
$$+ \gamma_1(\gamma_1 M_A \omega^2 - \gamma_1\gamma_2 - \gamma_1^2 - \gamma_1\gamma_2 e^{jka})$$
$$- \gamma_2 e^{-jka}(\gamma_1^2 + M_B\omega^2\gamma_2 e^{jka} - 2\gamma_1\gamma_2 e^{jka})$$

$$-\omega^6 M_A^2 M_B + 2\omega^4 M_A M_B \gamma_2 + 2\omega^4 M_A M_B \gamma_1 + \omega^4 M_A^2 2\gamma_1 - \omega^2 M_A 3\gamma_1\gamma_2 - 3\omega^2 M_A \gamma_1^2$$
$$+ \cancel{M_A M_B \omega^4 \gamma_1} - 2\omega^2 M_B \gamma_1\gamma_2 - \omega^2 M_B \gamma_1^2 - \omega^2 \cancel{M_A 2\gamma_1^2} + 2\gamma_1^2\gamma_2 + \gamma_1^3 + \cancel{M_A M_B \omega^4 \gamma_2} - \omega^2 M_B \gamma_2^2$$
$$- \omega^2 \cancel{M_B \gamma_1\gamma_2} - \omega^2 \cancel{M_A 2\gamma_1}\gamma_2 + 2\gamma_1\gamma_2^2 + \cancel{\gamma_1^2\gamma_2}$$

$$-\omega^6 M_A^2 M_B + 2\omega^4 M_A M_B \gamma_2 + 2\omega^4 M_A M_B \gamma_1 + 2\omega^4 M_A^2 \gamma_1 - 4\omega^2 M_A \gamma_1\gamma_2 - 3\omega^2 M_A \gamma_1^2$$
$$- 2\omega^2 M_B \gamma_1\gamma_2 - \omega^2 M_B \gamma_1^2 + 3\gamma_1^2\gamma_2 + \gamma_1^3 - \omega^2 M_B \gamma_2^2 + 2\gamma_1\gamma_2^2$$

$$-\omega^6 M_A^2 M_B + 2\omega^4 M_A M_B \gamma_2 + 2\omega^4 M_A M_B \gamma_1 + 2\omega^4 M_A^2 \gamma_1 - 4\omega^2 M_A \gamma_1 \gamma_2 - 2\omega^2 M_A \gamma_1^2$$

$$-2\omega^2 M_B \gamma_1 \gamma_2 - \omega^2 M_B \gamma_1^2 + 2\gamma_1^2 \gamma_2 + \gamma_1^3 - \omega^2 M_B \gamma_2^2 + 4\gamma_1 \gamma_2^2$$

$$+\gamma_1^2 M_A \omega^2 - \gamma_1 \gamma_2 - \gamma_1 - \gamma_1^2 \gamma_2 e^{jka} - \gamma_2 \gamma_1^2 e^{-jka} - \gamma_2 M_B \omega^2 + 2\gamma_1 \gamma_2^2$$

ω^6: $-M_A^2 M_B$

ω^4: $2M_A M_B \gamma_2 + 2M_A M_B \gamma_1 + 2M_A^2 \gamma_1$

ω^2: $-4M_A \gamma_1 \gamma_2 - 2M_A \gamma_1^2 - 2M_B \gamma_1 \gamma_2 - M_B \gamma_1^2 - M_B \gamma_2^2 - M_B \gamma_2$

0: $2\gamma_1^2 \gamma_2 + 4\gamma_1 \gamma_2^2 - \gamma_1^2 \gamma_2 (e^{jka} + e^{-jka})$

$$- \gamma_1^2 \gamma_2 2\cos(ka)$$

$$2\gamma_1^2 \gamma_2 (1 - \cos(ka)) + 4\gamma_1 \gamma_2^2$$

$$\frac{1}{\tau_m} \Gamma \quad \frac{D_{ac}^2 (k_B T)^{3/2}}{2\pi \hbar \rho v_s^2} \left(\frac{2m^*}{\hbar^2}\right)^{3/2} \frac{\varepsilon^{1/2}}{(k_B T)^{1/2}}$$

$$\tau_m = \frac{2\pi \hbar \rho v_s^2}{D_{ac}^2 (k_B T)^{3/2}} \left(\frac{\hbar^2}{2m^*}\right)^{3/2} \left(\frac{\varepsilon}{k_B T}\right)^{-1/2}$$

$$\tau_c = \frac{2\pi \hbar \rho v_s^2}{D_{ac}^2 (k_B T)^{3/2}} \frac{\hbar^3}{(2m^*)^{3/2}} = \frac{2\pi \hbar^4 \rho v_s^2}{D^2 (k_B T)^{1/2} (2m^*)^{3/2}}$$

$$\frac{(J \cdot s)^4 (kg/m^3)(m/s)^2}{(J)^2 (J)^{3/2} (kg)^{3/2}} \qquad \frac{J^4 s^2 kg \cdot m^{-1}}{J^{7/2} kg^{3/2}}$$

$$\frac{J^{1/2} s^2}{m\, kg^{1/2}} \qquad J^{1/2} = kg^{1/2} m/s$$

$$\frac{kg^{1/2} \cdot m \cdot s^2}{s \cdot m \cdot kg^{1/2}} = [s]$$

$$\frac{A \cdot s \cdot s}{kg} \quad \frac{A s^2}{kg} \quad \frac{C}{V} \frac{s^2}{kg} \quad \frac{Cs}{kg}$$

$$E = \left(\frac{\hbar^2 k^2}{2m^*}\right)^{3/2} \qquad E^{3/2} = \frac{\hbar^3 k^3}{(2m^*)^{3/2}}$$

$$k^3 = \frac{(2m^*)^{3/2}}{\hbar^3} \varepsilon^{3/2}$$

7) $$\Gamma = \frac{N Z^2 e^4 m^*}{8\pi\hbar^3 \epsilon_0^2 \epsilon_s^2} \left(\frac{1}{k^3}\right)$$

$$\frac{m^{3/2}}{m} = m^{1/2}$$

$$\tau_m^{(?)} = \frac{8\pi\hbar^3 \epsilon_0^2 \epsilon_s^2}{N_I Z^2 e^4 m^*} k^3 = \frac{8\pi\hbar^3 \epsilon_0^2 \epsilon_s^2}{N_I Z^2 e^4 m^*} \left(\frac{(2m^*)^{3/2}}{\hbar^3}\right) \varepsilon^{3/2}$$

$Z = 1$

$$2^3 \cdot 2^{3/2} = 2^{9/2}$$

$$\tau_m = 2^{9/2} \frac{\pi \epsilon_0^2 \epsilon_s^2 m^{1/2}}{N_I Z^2 e^4} \varepsilon^{3/2}$$

$$\tau_m = \frac{8\pi\hbar^3 \epsilon_0^2 \epsilon_s^2 k^3}{N_I Z^2 e^4 m^*}, \quad k^3 = \left(\frac{2m^*}{\hbar^2}\right)^{3/2} \varepsilon^{3/2} = (2m^* \varepsilon)^{3/2} \cdot \frac{1}{\hbar^3}$$

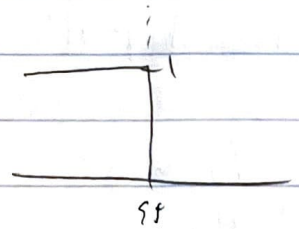
$$\tau_m = \underbrace{\frac{8\pi\hbar^3 \epsilon_0^2 \epsilon_s^2}{N_I Z^2 e^4 m^*} \left(\frac{(2m^*)^{3/2}}{\hbar^3}\right)}_{2^3 \cdot 2^{3/2} = 2^{9/2}} \cdot m^{1/2} \cdot \varepsilon^{3/2}$$

$(\varepsilon)^s, \; s = 3/2$

$$\frac{C m^2}{Vs} \cdot C \frac{1}{Cm^2} \qquad \frac{C}{Vs} \cdot \frac{1}{Cm}$$

$$\omega^6(-\mu_A^2 \gamma_B) + \omega^4(2\mu_A^2 \gamma_1 + 2\mu_A \mu_B \gamma_1 + 2\mu_A \mu_B \gamma_2)$$
$$+ \omega^2(+\gamma \mu_A \gamma^2 + 4\mu_A \gamma_1 \gamma_2 + \mu_B \gamma_1^2 + \gamma \mu_B \gamma_1 \gamma_2)$$
$$-2\gamma_1^2 \gamma_2 \cos(ak) + 2\gamma_1^2 \gamma_2$$

8) a) The fermi-level for a metal is within a conduction band. Therefore, the distribution (Fermi-Dirac) is full, until we reach the fermi-level, then it's 0, That is a step function.



$f_0 \sim 1 - u(\varepsilon - \varepsilon_f)$

$\frac{df_0}{d\varepsilon} \sim \delta(\varepsilon - \varepsilon_f)$

impulse, only a value (∞) at $\varepsilon_f = \varepsilon$

$\frac{df_0}{d\varepsilon} \sim \delta(\varepsilon - \varepsilon_f)$

$$\langle v_z \rangle = \frac{2\sum_k v_i(k) f_A(k)}{2\sum_k f_0(k)} \qquad v_i = \frac{\hbar k_i}{m^*_i}$$

$$= \frac{2\sum_k \frac{\hbar k_i}{m^*_i} \frac{\tau_m e}{\hbar} E \left(\frac{df}{d\varepsilon}\right) \frac{\hbar k_z}{m^*_z}}{2\sum_k f_A(k)} \rightarrow 0 \text{ if } z \neq i$$

$$\langle v_z \rangle = \frac{\frac{e}{m^*} \sum_k \frac{\hbar^2 k^2}{2m^*} \delta(\varepsilon - \varepsilon_f) \tau}{\sum_k f_A(k)} \rightarrow \frac{\frac{e\tau_m}{m^*} \sum_k \left(\frac{\hbar k^2}{2m^*}\right) \delta(\varepsilon - \varepsilon_f)}{\sum_k f_A(k)}$$

ε

only 1 at $\varepsilon = \varepsilon_f$

$$\langle v \rangle = \frac{e\tau_m}{m^*} E$$

$$\boxed{\mu = \frac{\langle v \rangle}{E} = \frac{e\tau_m}{m^*}}$$

6) a) $\frac{\partial f_0}{\partial t} = 0$ for equilibrium

a) rate in = rate out

$$S_0(\vec{p}', \vec{p}) f_0(\vec{p}') = S_0(\vec{p}, \vec{p}') f_0(\vec{p})$$

$$\frac{S_0(\vec{p}', \vec{p})}{S_0(\vec{p}, \vec{p}')} = \frac{f_0(\vec{p})}{f_0(\vec{p}')} = \frac{\exp\left(\frac{\varepsilon(\vec{p})}{k_B T}\right)}{\exp\left(\frac{\varepsilon(\vec{p}')}{k_B T}\right)}$$

$$\frac{S_0(\vec{p}', \vec{p})}{S_0(\vec{p}, \vec{p}')} = \exp\left(\frac{\varepsilon(\vec{p}) - \varepsilon(\vec{p}')}{k_B T}\right)$$

b) Phonons with more energy are needed, which is just not likely around room temp

7) a) $n = N \exp\left(\frac{-E_{Diff}}{k_B T}\right)$

$$\frac{n}{N} = \exp\left(-\frac{E_{Diff}}{k_B T}\right)$$

$$-k_B T \ln\left(\frac{n}{N}\right) = E_{Diff}$$

N for GaAs is $\approx 4.7e17\ cm^{-3}$

$$-(25.8\,mV) \ln\left(\frac{1}{4.7}\right) = E_{Diff} = E_c - E_F = +34.9\,meV$$ (below conduction)

b)