SNU General Physics 1 , HW#5 Due by 5/ 26/17

1. Consider a torsional oscillator as shown in Fig. 1. There is an unknown object firmly attached at the center inside the container.

(a) By solving the equation of the motion for a damped oscillator

$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + \kappa\theta = 0 \qquad \text{Eq. 1}$$

where I is the rotational inertia of the container and the object, b is the damping constant, and κ is a torsional spring constant of the wire.

Show that the solution $\theta(t)$ is given by

$$\theta(t) = \theta_m e^{-\frac{bt}{2I}}\cos(\omega' t + \phi) \qquad \text{Eq. 2}$$

where $\omega' = \sqrt{\frac{\kappa}{I} - \frac{b^2}{4I^2}}$, Eq. 3

ϕ is an arbitrary phase when $\frac{\kappa}{I} > \frac{b^2}{4I^2}$. (10 pts.)

(b) If there is a motor driving this torsional oscillator, the equation of the motion becomes

$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + \kappa\theta = F(t). \qquad \text{Eq. 4}$$

Assume that $F(t)$ is given by a sinusoidal function $F(t) = F_m\cos(\omega_d t)$. Show that the solution $\theta(t)$ is given by $\theta(t) = \theta_0(t) + \theta_{md}\cos(\omega_d t + \phi_d)$, where $\theta_0(t)$ is given by the Eq. 2 and the θ_{md} is given by

$$\theta_{md} = \frac{F_m}{[I^2\left(\omega_d^2 - \omega'^2\right)^2 + b^2\omega_d^2]^{\frac{1}{2}}}, \qquad \text{Eq. 5}$$

where ω' is given by Eq. 3.

(Hint : Set $\theta(t) = \theta_{md}e^{-i(\omega_d t + \phi_d)}$ and solve the Eq. 4. Use the relation $\cos(\omega_d t) = Real[e^{-i\omega_d t}]$.)

(10 pts.)

(c) Plot $\theta(t)$ vs t graph when ω_d is *3.1* radHz, 6.2 radHz, 9.3 radHz respectively using the following parameters of the torsional oscillator :

I = 2.5 kgm^2, κ = 100 N/rad, b = 1.5 N/(rad/s), F_m = 100 N, θ_0 = 0.5 rad, $\phi = \phi_d = 0$.

(10 pts.)

(d) Plot θ_m vs ω_d. At what value of ω_d does the θ_m become maximum? (10 pts.)

(e) It is known that the object inside the cylindrical container melts to become a liquid at a certain temperature. How can one find this transition temperature precisely? Explain. (The liquid does not make any sound when the container is moving.) (10 pts.)

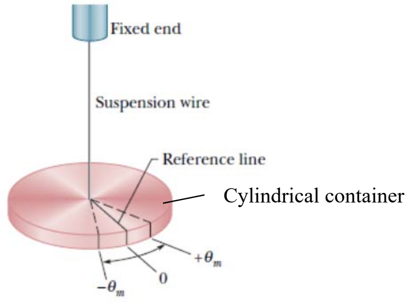

Fig. 1

2. Consider an open pipe of the area A and the length L which is filled with a gas of the density ρ and the bulk modulus B. Imagine a small volumes $A\Delta x$ of gas in the pipe which are separated by small distance a as shown in fig. 2 ($\Delta x < a << L$). The displacement about the equilibrium position of this small volume of the gas is denoted by s^l. At equilibrium, pressure $P_1 = P_2 = P = constant$. One moved the gas by blowing from a one end of the pipe and the gas started to oscillate. Bulk modulus B is defined as $\Delta P = -B\frac{\Delta V}{V}$.

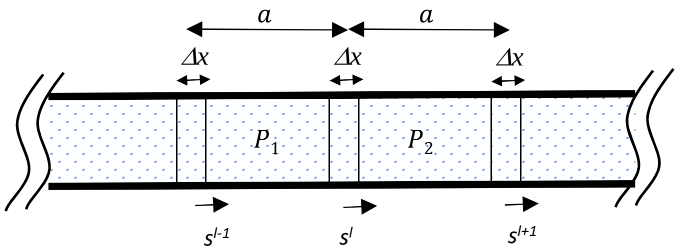

Fig. 2

Table 17-1

The Speed of Sound[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000

[a]At 0°C and 1 atm pressure, except where noted.
[b]At 20°C and 3.5% salinity.

a) Express the pressure P_1 and P_2 in terms of s^{l-1}, s^l, s^{l+1}, and P. (10 pts.)

b) Find the equation of motion for an lth small volume of the gas (hint : assume this small volume of the gas moves as if it is a rigid body. $F = ma = V\rho\frac{\partial^2 s}{\partial t^2} = PA$. Use the bulk modulus B to relate pressure and displacements s^{l-1}, s^l, s^{l+1}). (10 pts.)

c) show that the equation of motion from b) can be written as a wave equation in a form $\frac{\partial^2 s}{\partial t^2} = v^2\frac{\partial^2 s}{\partial x^2}$. What is the velocity (speed) v of the wave of the gas in this pipe? (10 pts.)

d) Find the solution to the wave equation for given k and ω. Namely, find the expression of $s(x,t)$. (10 pts.)

e) If this pipe has two open ends, L = 10 cm, and the gas is helium at room temperature, what is the frequency of the first harmonic standing wave? Compare it to the case when the gas is air and explain how one's voice will sound after he/she enhales a sizable amount of He gas. (10 pts.)