Engineering Mathematics I Finals

December 20, 2017

1. Solve the following integral equation.

$$\int_0^\infty f(x)\cos wxdx = e^{-|w|}$$

2. Apply the improved Euler method for the following differential equation with $h = 0.2$

$$y'' - 4y' + 4y = 0, \quad (0 \le x \le 1), \quad y(0) = 1, \quad y'(0) = -2$$

and derive the recurrence relation for $y_{1,n+1}$, and $y_{2,n+1}$ in terms of $x_n, y_{n,1}, y_{n,2}$.

3. A matrix B has the reversed columns of matrix A. Show that $A - B$ is not invertible. (Do not use determinants!)

4. (a) For a permutation matrix P, show that there exists k such that $P^k = I$. $k \in \{1, 2, 3, \cdots\}$
   (b) Find a 5×5 permutation matrix such that $P^6 = I$, and $P^k \neq I$ for $k = 1, 2, 3, 4, 5$.

5. Suppose a matrix A has eigenvalues $0, 3, 7$ and eigenvectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$, respectively. Find the least square minimum length solution for $A\mathbf{x} = \mathbf{u} + \mathbf{v} + \mathbf{w}$.

6. Perform singular value decompiosition to the following matrices.

$$\text{(a) } A_1 = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \qquad \text{(b) } A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

7. Perform singular value decomposition to the matrix.

$$A = \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix}$$