이산수학 HW2

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1.6 #24

Logical error after step 3 and 5.
The original statement is not True in general.

1.6 #28

If $\neg R(c)$ for arbitrary c,
Universal modus tollens gives $\neg(\neg P(c) \wedge Q(c))$ from the second premise.
By De Morgan's law, $P(c) \vee \neg Q(c)$.
Universal instantiation from the first premise gives $P(c) \vee Q(c)$ for arbitrary c.
Resolution gives $P(c) \vee P(c)$ thus $P(c)$.
Universal generalization gives $\forall x(\neg R(x) \rightarrow P(x))$

1.7 #18

a) Contraposition: $\forall n \in \mathbb{N}$, if n is odd, $3n+2$ is odd.
If n is odd, $3n$ is odd and $3n+2$ doesn't change the parity of $3n$ thus $3n+2$ is odd.
b) Suppose n is odd. Then $3n+2$ is odd. Contradiction; n must be even.

1.7 #32

(i) $\Rightarrow$ (ii)
$x \in \mathbb{Q}$ then by definition, $x = \dfrac{p}{q}$ where $p, q \in \mathbb{Z}$, $q \neq 0$.
Then $\dfrac{x}{2} = \dfrac{p}{2q}$ and $\dfrac{x}{2}$ satisfies the definition of rational numbers. $\therefore \dfrac{x}{2} \in \mathbb{Q}$
(i) $\Leftrightarrow$ (iii)
$x \in \mathbb{Q}$ if and only if $3x-1$ is rational. (Trivial!)
(ii) $\Rightarrow$ (iii)
$x/2 \in \mathbb{Q}$ then $3x-1 \in \mathbb{Q}$ (Trivial)
Three statements are equivalent.

1.8 #12

Divide numbers into two groups, non-negative and negative.
There are three numbers and two groups.
Thus by pigeonhole principle two numbers must be in the same group.
Multiply those two numbers to get a non-negative product.

1.8 #36

Suppose $r \in \mathbb{Q}$, $s \in \mathbb{R} - \mathbb{Q}$.
Suppose $\dfrac{r+s}{2}$ is rational. Let this value x. Then $s = 2x - r$.

Since $\mathbb{Q}$ is closed under addition, $2x - r \in \mathbb{Q}$. Contradiction;

$\frac{r+s}{2}$ is irrational.

2.1 #10

a) T

b) T

c) F

d) T

e) T

f) T

g) F (Not a proper subset)

2.1 #28

$A \times B := \{(a, b) : a \in A \land b \in B\}$

Maps courses to professors (assignment)

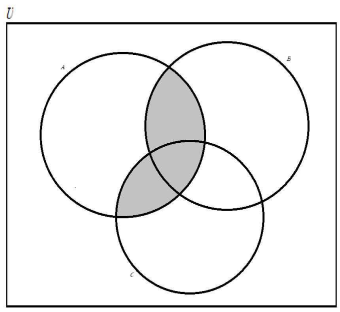
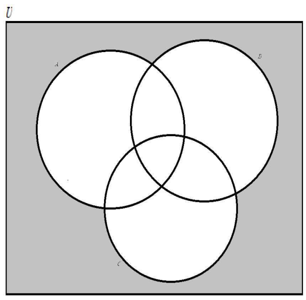
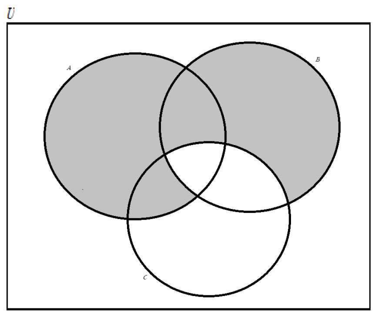
2.1 #44

a) $\{x : x \geq 1 \land x \in \mathbb{Z}\}$

b) $\varnothing$

c) $\{x : (x < 0 \lor x > 1) \land x \in \mathbb{Z}\}$

2.2 #26

a)  b)  c) 

Up-left: A, Up-right: B, Down: C

2.2 #30

a) No. $A=\{1\}$, $B=\{2\}$, $C=\{1,2\}$.

b) No. $A=\{1,3\}$, $B=\{1,4\}$, $C=\{1\}$.

c) WLOG, it suffices to show that $A\subseteq B$.

Suppose $x\in A$. $A\cup C=B\cup C$ thus $x\in B\cup C$.

$x\in B$ or $x\in C$.

If $x\in B$, we are done.

If $x\in C$, $x\in A\cap C$. Then from $A\cap C=B\cap C$, $x\in B\cap C$. Therefore $x\in B$

Thus $\forall x$, $x\in A\rightarrow x\in B$, $A\subseteq B$.

2.2 #50

a) $\bigcup_{i=1}^{\infty}A_i=\{n: n\geq 1\wedge n\in\mathbb{N}\}$, $\bigcap_{i=1}^{\infty}A_i=\varnothing$,

b) $\bigcup_{i=1}^{\infty}A_i=\mathbb{N}$ (non-negative integers), $\bigcap_{i=1}^{\infty}A_i=\{0\}$

c) $\bigcup_{i=1}^{\infty}A_i=\mathbb{R}^+$(positive real), $\bigcap_{i=1}^{\infty}A_i=\{x: 0<x<1\wedge x\in\mathbb{R}\}$

d) $\bigcup_{i=1}^{\infty}A_i=(1,\infty)$, $\bigcap_{i=1}^{\infty}A_i=\varnothing$