HW Solution 8

1. (points) Since we have

$$|f - f_n|^{\frac{3}{2}} = |f - f_n|^{\frac{1}{2}}|f - f_n|,$$

l

Hölder's inequality yields that

$$\left(\int_{\mathbb{R}} |f - f_n|^{\frac{3}{2}} dx\right) \le \left(\int_{\mathbb{R}} |f - f_n| dx\right)^{\frac{1}{2}} \left(\int_{\mathbb{R}} |f - f_n|^2 dx\right)^{\frac{1}{2}}.$$

By the assumption, we have the desired result.

- 2
- 3. (§ points) Since $||f_n||_{L^2} \le 2\pi$, $\{f_n\}$ is bounded set. Moreover, we observe that

$$||f_n - f_m||_{L^2} > \frac{1}{10000}$$
 if $n \neq m$.

It implies that any ball centered at f_n with radius $\frac{1}{10000}$ contains only f_n . Therefore, $\{f_n\}$ is closed and compact set.

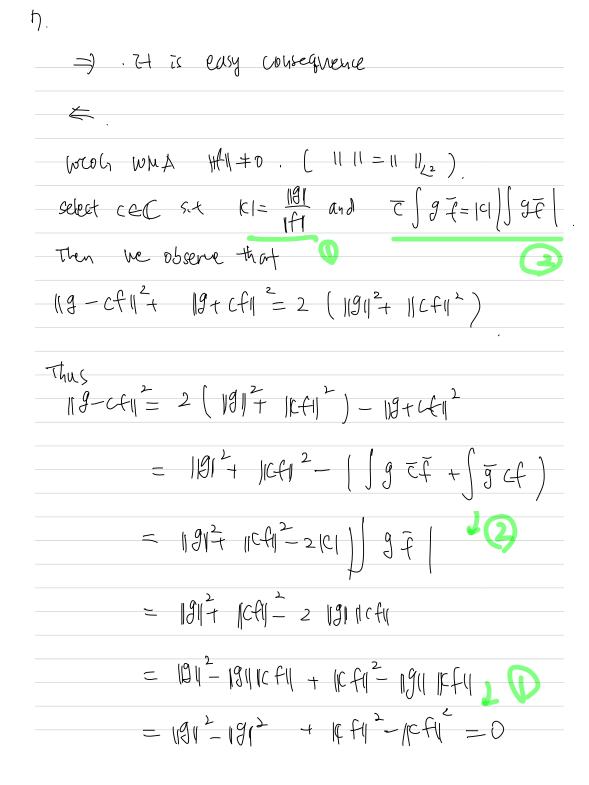
: Let frain = freinx = fre L Strie (fr-full) 2 < D = 220 n2 max(k,m) is a Cauchy sequence in 2" - be a limit of fx in Z2 he see that I fe tinx = lim for e-Inx where we have used to - I In 2 and Hölder's Tregulty Thus f= & The conx (3) Suppose 32 Pin EZ2 By Ries thm, 2 (1)2 co which is constrat

4. (6 Points) Let Enx = U \ xe [a, b], |fix|-fix) |zte \. STuce In - face, M(TEn, E) = 0. VEIN. STUR Entix (In, K and M(EI, K) CO) lim H(En. +) = M(n= En. +) = 0. Colver of and 2 20, I NEW S.T M(En,n)KJ and fica. It implies that Kon, XC Ta, J TEN, K, Ifr(x)-fm) < E.

5. (6 Points) Let $E = \{ \alpha \in (-\tau, \tau) : \lim_{n \to \infty} f_{\kappa}(n) = f_{\kappa \kappa} \}$ Since fix) is bounded, fel2(E) D PACE: JA STANKA AX -> 0 as nx ->0 o Note that me I, 0 = lim SIn NII x e TMX dx = I fix e TMX dx, where we have used Nx >> m for the first equality and LDCT for the second Thequality (sinner > fix) this implies that HATECA, =0 ~~ (+) thes I sin neady of fdx = 0 by Hölder meg 2) ACE, 2 (Sinner) -> m(A) as ne > 00, . Note that 2 JA BĨU NKX) = JA (1- COS ZUKX).

As In D. M deduce that 11 t $= \left(\sin n_{c} \alpha \right)^{2} \rightarrow 2 \left(f_{m} \right)^{2} = m \left(E \right)$ but it contradicte by, thus M(Z

b. (6 points) there is a countable set MKELL and SIN ME & 28 TREE (1) Strice thee L2, and they, =0. KE = 5 Cheins Note that Chief the CTARX MX = L Cos nxx + Csin nxx dx Et gives | Cnx | 2 5 m (E). Which Contradicts the



Ct	implies	that	g=(f	1-C.	
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