Applying Decomposition Techniques to Long-Term Investment Planning Problems

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1 Abstract

This thesis describes the development of a multistage stochastic capacity planning model applied to New Zealand's electric power generation stack. The model spans a three-decade time horizon, formulated with stages relating to blocks of five-year increments in a scenario tree structure, with uncertain demand growth and carbon prices. Discrete capacity expansions in geothermal, wind, hydro, thermal plants, battery technology, and CCS have been modelled, as has the decommissioning of Huntly's coal-powered Rankine units and the gas turbine capacity currently available.

The model is formulated as a subproblem within the Julia Decomposition-engine for General Expansion, which uses Dantzig-Wolfe decomposition to solve problems defined over a scenario tree, like this. The subproblem serves as an in-depth application of JuDGE ahead of its open-source release. Government policies have been modelled and assessed regarding their effectiveness at reaching 100% renewable electricity generation, and CO_2e emission levels under different demand growth scenarios. Carbon price variations across different demand scenarios have also been discussed.

2 Acknowledgements

I would like to thank Tony Downward and Andy Philpott for their guidance over the course of the year, the technical expertise that they imparted, and their deep involvement in the project. We could not have produced anything close to what we did without your supervision and assistance, thank you.

To my project partner, Jamie Chen, it was a pleasure working with you, and I wish you the best of luck in your future endeavours.

Table of Contents

List of Figures

List of Tables

3 Introduction

3.1 Project Scope

The project scope involves the development of a multistage stochastic capacity planning model for New Zealand's electric power generation stack, in which a scenario tree represents future uncertainty relating to demand levels and carbon prices at different states-of-the-world.

Two models have been developed, each using an industry-standard modelling methodology. The first is a load duration curve-based model [2], while the second uses a representative days approach [3]. These models have been compared and contrasted against one-another.

The primary intention of this project is to develop a framework in which modelling different government policies can be undertaken and subsequently assessed regarding the impacts on reaching 100% renewable generation, emission reductions, and the cost of implementation, in accordance with the Zero Carbon Act [1].

The JuDGE package [4] has been used to extend multistage functionality to the models and to apply Dantzig-Wolfe decomposition to the model. These two models will act as initial in-depth applications of JuDGE prior to its open-source release.

3.2 Project deliverables

The primary deliverables are the formulated and codified mixed-integer programming optimisation models. The models have also been developed in Julia [5] and integrated within JuDGE. Data visualisation capabilities have also been produced to extract meaningful interpretations of the model outputs along demand-growth trajectory subsets of a larger scenario tree.

Case studies have been conducted and included in a companion report to this, completed by Jamie Chen that contrasts the performance of each model at a single state-of-the-world. The case studies included in this thesis are multistage considerations regarding existing and proposed government emissions policies; optimal investments given carbon price variations; analysis of the effects of large-scale storage expansions; and intra-seasonal hydroelectric storage policies.

3.3 Project exclusions

This project is neither a detailed analysis of the existing generation stack, nor is it designed to consider the plausibility of individual consented projects. These are social planning models that do not account for externalities such as environmental degradation, employment considerations, and cultural concerns, for example. The emphasis has instead been placed on building an environment in which more accurate forecasting and policy analysis can be conducted.

Care has been taken to use as accurate figures as possible for the relevant case studies, however the set of potential investments provided for the multistage case studies are neither fully reflective of consented projects, nor reality.

3.4 Report structure

The report will include a review of relevant literature and existing work completed. An introduction will be provided relating to data processing and details of how New Zealand's generation stack has been modelled. The load duration curve model will be explored in depth. A brief overview will be provided regarding the representative days model and a comparison of both model's relative strengths. Detailed multistage case studies will also be explored.

3.5 Declaration of contribution

All work detailed in this report can be assumed to have been completed by the author unless otherwise referenced as an artifact used, or of that produced by the author's collaborator, Jamie Chen.

The author was primarily responsible for all data extraction and processing, the exploratory analysis of the effects of wind and solar investment on load duration curves, the development of the load duration curve model, the parallelisation and of code, the integration of this model within JuDGE as a subproblem, the generation of all multistage case studies, and data visualisation and analysis in R.

The author's collaborator was primarily responsible for the exploratory analysis of battery investments and peak load shaving effects on load duration curves, the development of a stepwise load duration curve approximation algorithm via load blocks, the development of a representative days model, the generation of two-stage case studies comparing both models, and data visualisation produced in Julia.

3.6 Original work

The authors are fortunate to be the beneficiaries of decades of research into modelling electricity network capacity investments, and directly inherited Ferris and Philpott's two-stage capacity planning model [8], in which the emphasis of this project is to build on top of by extending multistage functionality.

JuDGE [4] was not developed by the authors, although as this is a Julia package still in development, new features have been introduced, and minor bugs fixed as a result of the author's communication with the primary developer.

3.7 Notation used

$HydroR = Run\text{-}of\text{-}river\ generation.$

$Hydros = Stored\ hydro\ generation.$

$CCS = Carbon\text{-}capture\ and\ sequestration\ technology.$

$CCGT = Closed\text{-}cycle\ gas\ turbine.$

$OCGT = Open\text{-}cycle\ gas\ turbine.$

$T_I = The\ set\ of\ all\ LDC\text{-}selecting, non\text{-}dispatchable\ technologies.$

$T_B = The\ set\ of\ all\ base\ technologies.$

$T = T_I \cup T_B\ ,\ The\ set\ of\ all\ technologies.$

$W = The\ set\ of\ all\ historical\ years.$

$B_t = The\ set\ of\ all\ load\ blocks\ in\ a\ given\ season\ t.$

$\vartheta_{b,t} = The\ set\ of\ all\ load\ block\ heights\ (demand)\ in\ a\ given\ season\ t.$

$H_{b,t} = The\ set\ of\ all\ load\ block\ widths(hours)\ in\ a\ given\ season\ t.$

$P(T_I) = The\ power\ set\ of\ all\ combinations\ of\ investment\ technologies.$

$S = The\ set\ of\ all\ seasons, \{0, 1, 2, 3\}, where\ 0\ refers\ to\ summer, 1\ to\ Autumn,$
$2\ to\ Winter\ and\ 3\ to\ Spring.$

$S' = The\ set\ of\ all\ seasons, \{-1, 0, 1, 2, 3\}, where\ S_{-1} is\ equivalent\ to\ the$
$previous\ year's\ S_3.$

$\varphi = Half\ the\ distance\ between\ the\ extremities\ of\ the\ hydro\ gates.$

$U_k = The\ existing\ MW\ capacity\ of\ technology\ k.$

$u_k = Proposed\ MW\ capacity\ investment\ for\ technology\ k.$

$\eta =\ Round\ trip\ battery\ efficiency.$

$\beta_k =\ Dimensionless\ battery\ charge\text{-}rate\ scaling\ factor.$

$e_k =\ tonnes\ CO_2e\ /\ MWh\ emitted.$

$E =\ Total\ permissible\ annual\ CO_2e\ emissions.$

$\zeta =\ The\ cost\ of\ emitting\ one\ tonne\ of\ CO_2e$

$\Delta =\ Number\ of\ historical\ years\ in\ which\ CO_2e\ emissions\ are\ permitted$

$x_k \in [0, 1], x_k = 1, if\ the\ model\ expands\ technology\ k's\ generation\ capacity$

$d_j \in [0, 1], d_j = 1, if\ the\ j'th\ wind\ site\ is\ invested\ in$

$\lambda_i \in [0, 1], if\ \lambda_i = 1, then\ the\ i'th\ corresponding\ portfolio\ is\ selected$

$z_{k,b,t} \geq 0, The\ MWh\ maximum\ potential\ generation\ in\ each\ block\ for\ each$

$techhnology\ k$

$\pi_{b,w,t} \in free, the\ selected\ and\ modified\ set\ of\ load\ blocks\ for\ a\ given\ portfolio$

$r_{b,w,t} \geq 0, models\ the\ load\ shed\ in\ each\ load\ block$

$y_{k,b,w,t} \geq 0, models\ the\ MWh\ generation\ in\ each\ block\ by\ each\ technology\ k$

$g_t \geq 0, models\ the\ centres\ of\ the\ seasonal\ hydro\ gates$

$m_{t,w} \geq 0, models\ the\ intra\text{-}seasonal\ stored\ hydro\ energy$

$c_{k,b,w,t} \geq 0, models\ the\ MWh\ charged\ by\ the\ batteries\ in\ a\ given\ block$

$c'_{k,b,w,t} \geq 0, models\ the\ MWh\ discharged\ by\ the\ batteries\ in\ a\ given\ block$

$V = Value\ of\ lost\ load\ (VOLL).$

$L_k = The\ annual\ maintenance\ cost\ of\ having\ z_{k,b,t}\ MWh\ capacity\ available.$

$C_k = The\ operational\ costs\ of\ producing\ y_{k,b,w,t}\ MWh\ with\ technology\ type\ k.$

$\rho_k = The\ discounted\ capital\ cost\ of\ investing\ in\ new\ MW\ capacity\ in\ technology\ k.$

$K_k = The\ capital\ cost\ of\ investing\ in\ new\ MW\ capacity\ in\ technology\ k.$

$\delta_w \in [0, 1], Models\ the\ normal\ hydrological\ years\ in\ which\ thermal\ generation$

$can\ be\ used$

Any variable with a * superscript refers to generation technologies that are modelled for decommissioning decisions as opposed to expansions.

3.8 Project background

This project has arisen out of New Zealand, and the wider world's, collective ambition to halt the onset of climate change. In 2019, the Sixth Labour Government of New Zealand signed the Climate Change Response (Zero Carbon) Amendment Act [1] in accordance with the Paris Agreement, ratified in 2016 [6]. The Zero Carbon Act established the Climate Change Commission, tasked with advising the government about feasible actions that can be undertaken to reduce emissions. The Zero Carbon Act outlines the requirement for an emissions budget to be set at five-year intervals of time leading to 2050. The Commission must account for the, *"risks and uncertainties associated with emissions reductions and removals,"* and to consider scientific advice, both domestic and international [1].

The Paris Agreement is a United Nations accord signed by 195 nations. The agreement requires each signatory to work towards and to review its own nationally determined contribution to climate change and greenhouse gas emissions. The intended goal of this agreement is to limit the increase in average global temperatures to under 2°C above pre-industrial levels [6].

The recently re-elected Sixth Labour Government of New Zealand has announced a new policy of reaching 100% renewable generation by 2030 [7]. This will replace the previous policy of reaching 100% renewable generation by 2035 in a *normal hydrological year*, which hedged against dry-year risk. These policies have been assessed and modelled in the multistage case studies. At the time of writing, 2030 is nine years away. The Climate Change Commission will be required to work out a proposal regarding the feasibility of such policy changes, and the intention with this work is to develop the framework in which this question can be posed and answered, from a social planning perspective.

4 Literature review

4.1 On two-stage stochastic capacity planning problems

In 2019, Ferris and Philpott [8] developed a novel stochastic, two-stage capacity planning model for a hydro-dominated electric power generation stack such as New Zealand's.

This paper was written as a response to the temporary formation of the Interim Climate Change Committee and provided an analysis of different forms of emission constraints that New Zealand could introduce to facilitate the move to 100% renewable generation by 2035.

This publication has developed powerful models incorporating uncertainty and risk that can be used to plan electricity generation investments across multiple technologies for the New Zealand market, and contains the theoretical framework for our electricity model which will be expanded upon and developed in JuDGE.

The structure of the models we develop will follow closely behind the structure presented in this paper, as the authors discuss likely scenarios for changes in New Zealand's electricity demand, how New Zealand can incorporate short- and long-term energy storage, and considerations for modelling different generation technologies available to the New Zealand market.

This publication defines a two-stage stochastic programming model capable of modelling CO_2e emissions while optimising for capacity investments in new generation technology. The model includes capital costs of investments, annual maintenance costs of assets, operational costs of producing generation, and the costs associated with carbon emissions. The paper considers eleven different generation technologies, which include large-scale investments in batteries, solar, and wind. Demand is modelled as a stepwise approximation of load duration curves, which has been adopted for the purposes of our models.

The two stages refer to a build and then operate-under-uncertainty approach. The first stage governs the investment decisions in new generation capacity, while the stochastic second stage requires the model to meet demand in the most cost-effective manner in each historical year scenario.

The following quote from this paper is the primary justification for this project:

> "Since the models we are considering have long time horizons over which uncertain effects will become realized gradually, there is considerable value in developing a multistage version of our model." [8]

The intention of this project is to logically pursue the continuation of this model's development by introducing multistage functionality to more accurately model a longer time-horizon with more facets of uncertainty.

While there has been extensive research undertaken regarding transitioning to 100% renewable electricity generation in other countries [18], this publication presents the first stochastic optimisation model directly relating to New Zealand. This is important as New Zealand's grid differs from that of other countries with connected grids. New Zealand must generate adequate supply to meet all of the required demand or risk either shedding load or disconnecting parts of the grid. As this publication contains the capacity planning model that we intend to further enhance, this publication is pivotal for our project.

Wind is modelled by scaling down the maximum generation based on varying amounts of wind, by assigning probabilities that no wind is present during peak periods, and by assuming that the wind distribution is independent of the load block that it is in. Solar power is modelled a function of the time of day, region, and season. It does not accommodate randomness. Thermal plants can be stochastically modelled with random fuel supply levels and will incur carbon taxes relating to their emissions. Huntly's coal-fired Rankine units are assumed to decommission at some stage prior to 2035. Hydroelectricity can be modelled both via run-of-river flow capacity and long-term hydroelectric storage, where the latter occurs between seasons depending on the inflows of each historical year experienced. Thirteen years of historical inflow data are considered as part of the analysis, some of which include inflow data for *dry-winter* years.

The basic structure of the deterministic two-stage sub-problem provided by Ferris and Philpott [8, page 8] is reproduced with permission from one of the authors, and adapted from a MW formulation into a MWh formulation:

$$Min \qquad \sum_{k\in T}\left(K_k u_k x_k \ + \ L_k(U_k + u_k x_k)\right) + \sum_{t\in S}\left(\sum_{b\in B_t}\left(\sum_{k\in T}\left(C_k\, y_{k,b,t}\right) + V r_{b,t}\right)\right)$$

$$s.t. \qquad 0 \le z_{k,b,t} \le (U_k + u_k x_k)H_{b,t}\,, \qquad \forall k \in T, b \in B_t, t \in S \qquad (1)$$

$$0 \le y_{k,b,t} \le z_{k,b,t}\,, \qquad \forall k \in T, b \in B_t, t \in S \qquad (2)$$

$$\textstyle\sum_{k\in T}\left(y_{k,b,t}\right) \ge \vartheta_{b,t}H_{b,t} - r_{b,t}\,, \qquad \forall b \in B, t \in S \qquad (3)$$

$$0 \le r_{b,t} \le \vartheta_{b,t}H_{b,t}\,, \qquad \forall b \in B_t, t \in S \qquad (4)$$

$z_{k,b,t}$ refers to the maximum potential MWh generation available in each load block for technology k. The upper bound $(U_k + u_k x_k)H_{b,t}$ refers to the existing MW capacity and potential MW expansion $U_k + u_k x_k$, multiplied by the hours in each load block, $H_{b,t}$. $y_{k,b,t}$ refers to the MWh generated in each load block for technology k. The block generation must not exceed each technology's $z_{k,b,t}$. In each block, $\sum_{k\in T}\left(y_{k,b,t}\right)$ must exceed the MW load in each block, $\vartheta_{b,t}$ multiplied by the hours in each block minus the amount of shed load. The objective function includes the cost of new capital investment, K_k, the cost of maintenance, L_k, the cost of generating supply, C_k, and the value of lost load (VOLL), V which we seek to minimise over.

This basic model serves as a platform to build on top of in section 5.5, in which the goal is to meet demand at the lowest cost by using existing capacity, and potentially investing in new capacity. Load can be shed at a penalty equal to VOLL instead of investing in new capacity. This formulation deviates slightly from that of Ferris and Philpott [8] as the primary variables $y_{k,b,t}$ and $z_{k,b,t}$ have MWh units, and x_k is a Boolean investment decision, whereas in their model, $y_{k,b,t}$ and $z_{k,b,t}$ have MW units, and x_k is the potential capacity expansion, relating to $u_k x_k$ in our formulation. This deterministic model leaves out historical year scenarios, storage, transmission constraints, and carbon emission pricing.

4.2 Julia, version control, and supplementary programming languages

The Julia Programming Language (Julia) [5] was chosen as the platform in which to develop the capacity planning models. Julia is a high-performance, open-source language gaining traction within the Operations Research community. The use of Julia was specified as part of the project brief due to the ability to leverage JuMP [9] syntax to interface with the Gurobi Optimizer solver [10] within JuDGE.

GitHub [11] was used for version control and efficient collaboration while working from home over the course of 2020. R [12] was the primary tool used for data visualisation, exploratory data analysis, and interfacing with the National Institute of Water and Atmospheric Research's (NIWA) National Climate Database (CliFlo) [13]. A Python [14] web crawler script was written to extract decades of data from the Electricity Authority's (EA) Electricity market Information (EMI) wholesale datasets [15].

4.3 Dantzig-Wolfe decomposition in a scenario tree

Singh et al [16] develops a multistage stochastic capacity planning model that uses Dantzig-Wolfe decomposition [17] and column generation over a scenario tree. Discrete investments are modelled with at most one expansion per facility. The decomposition approach forms a Dantzig-Wolfe master problem reformulation that approximates a *deterministic equivalent* formulation, while at each node of the scenario tree, a subproblem is defined. The paper solves an electricity grid example where at each node of a scenario tree, the level of demand is realised, and additional investments can be made to increase supply. This decomposition approach has been codified into the JuDGE package, detailed below.

4.4 Julia Decomposition-Engine for General Expansion (JuDGE)

JuDGE [25] is a Julia package nearing open-source release [4] and has been in active development over the course of this project. Julia was developed by Regan Baucke and Anthony Downward, and is based on Singh et al. [16]. JuDGE is designed to optimally solve scenario-tree based stochastic capacity planning problems by leveraging JuMP syntax [9]. JuDGE builds a master problem over a user-specified scenario-tree and uses Dantzig-Wolfe decomposition [17]. The form of the problems that JuDGE is designed to solve can be found on page 4 of [25], and builds a restricted master problem during the decomposition in the form of what can be found on page 5 of [25].

Decomposition occurs both along scenarios and at each node of the tree. The single expansion per facility is important for strong problem formulations [16], however we overcome this by providing the same facility to JuDGE repeatedly, with the first instance containing the existing capacity and a single discrete investment, the remaining have no existing capacity yet alternate expansion sizes made available for utilisation.

A scenario tree such as figure 1 is defined as starting with the current state of the world as a single root node, each stage corresponds to a five-year block of time. Uncertainty exists regarding future states-of-the-world which are each represented by an individual node in the next stage. Probabilities can be specified regarding the likelihood of a state-of-the-world, however all child nodes of a parent in a given stage have been assumed to have equal probability of occurring.

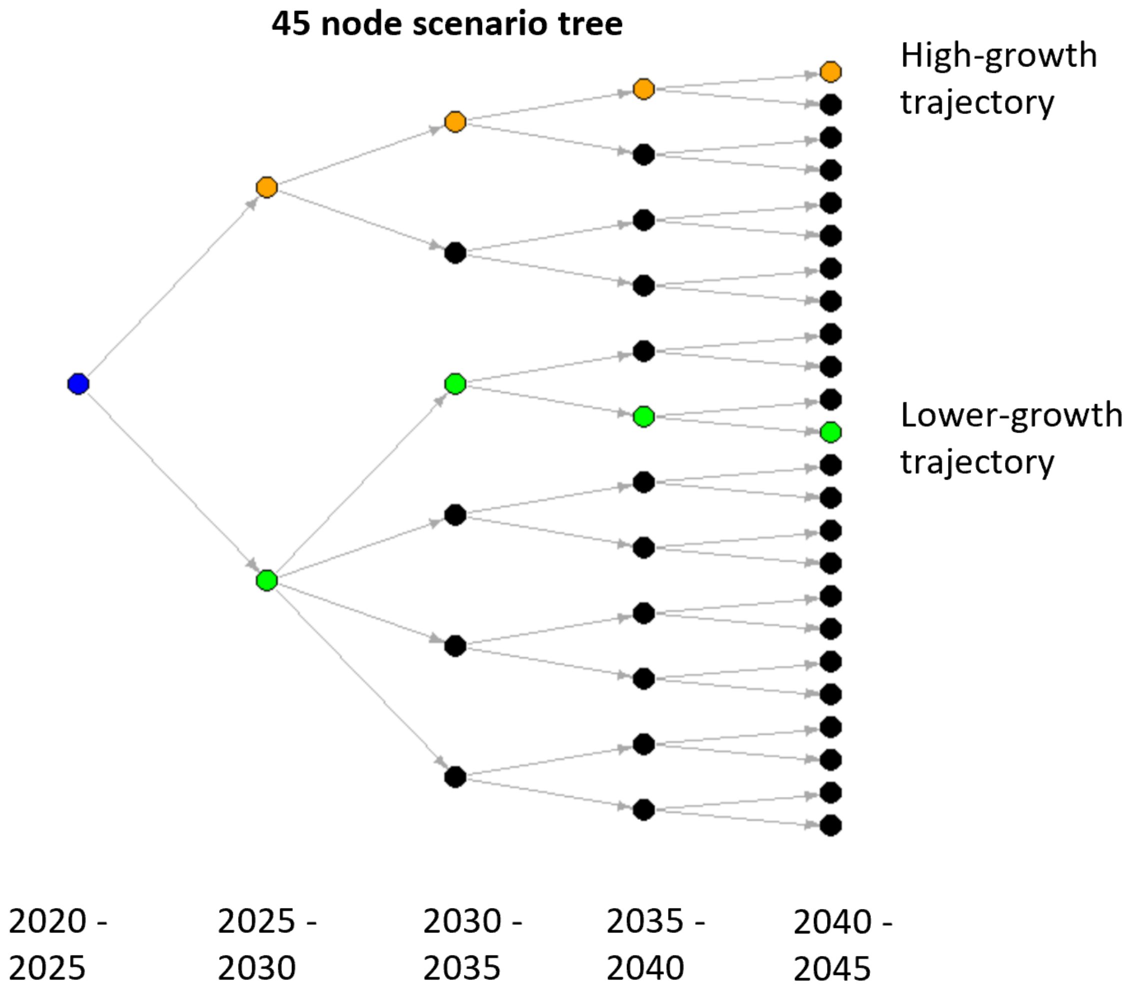

Figure 1: Primary scenario tree used in this report

Each node must have a single parent, excluding the root. JuDGE can build a scenario tree based on a text file specifying each node, the parent of the node, the demand levels at that node, and features of a given state-of-the-world such as a carbon price or the level of distributed “rooftop” solar capacity available.

The master problem optimises for investments in new generation capacity. If an investment is made at a given node of the tree, all successive nodes will be able to utilise this new capacity and will be responsible for annualised investment and maintenance costs.

The operational subproblems at each node optimise for capacity dispatch given the demand realised at each node. The model aims to meet demand at least cost. The subproblem at each node has two stages. The first stage is a building decision, where the model can choose to invest in new generation assets, while the second stage is the stochastic operational policy, where the model must be able to generate supply to meet demand across all nodal scenarios.

This problem can be solved deterministically and is feasible to run for a small number of nodes in a tree, however decomposition is required for tractability when considering a large number of potential investments or nodes within the scenario tree.

A trajectory is referred to as a path between two nodes. This will primarily refer to paths connecting the root node and states of the world in the end stage. The case studies offer comparisons of different growth and carbon price trajectories spanning the first and final stages of each scenario tree.

For clarification, at each node, the operational model is a two-stage subproblem, however the term *stage* is also used to refer to depth bands of the scenario tree, hence the tree pictured below can be thought of a five-stage tree. Care will be taken to ensure that any use of the word *stage* can be intuitively understood in the correct context.

5 Mathematical Programming Formulation

5.1 Modelling approach

The modelling rationale was to develop two multistage models using industry standard methodology, namely a load duration curve (LDC) model and a representative days model. This report with detail the LDC model, while the companion report of Jamie Chen will contain details of the representative days model's formulation and implementation. The intention with developing two models was to be able to compare and contrast them, and to see if they find the same optimal set of investments under the same state-of-the-world scenarios.

The LDC model is long-term and coarse and is capable of modelling seasonal storage well. The representative days approach models short-term, intra-day variations well, such as solar, battery, and wind scenarios over the course of a day.

5.2 Raw data processing

The EA EMI database provided electricity prices, wind generation, and grid export demand data in half-hourly increments referred to as *trading periods*. A python web crawler was written to extract almost four-hundred monthly datasets for processing, collation, and analysis.

NIWA's CliFlo database provided the solar irradiance data that was used. Geospatial information was gathered about five locations assumed to be representative of New Zealand, and solar irradiance data was extracted from each site.

The hydrological generation technologies have been modelled in a stochastic manner. Thirteen years of historical data about reservoir inflows and proportional generation capacity reductions are used. The assumption is made that these thirteen years are an acceptable distribution of typical hydrological years that may occur. These years, for example, may include a *dry winter* with reduced seasonal inflows. The JuDGE model has an additional dimension for these years, which allows the model to determine an optimal operating policy for all historical year types.

A Julia pipeline was used to extensively process the data into seasonal datasets. Wind and solar *factors* were computed, which equates to dividing each trading period's wind and solar generation by the maximum wind and solar generation, respectively. An assumption was made, in-keeping with Ferris and Philpott [8] that the maximum generation exhibited over the course of a year generates the rated capacity of a generation asset, and the remaining trading periods could produce a scaled level of generation based on the wind and solar factors experienced given the MW capacity available.

5.3 Load duration curves

The pipeline produced a set of seasonal datasets ordered by decreasing load which represent load duration curves. Murphy et al [2] states that the interpretation of a load duration curve is that for any load, the horizontal axis determines the number of hours in which load is greater than or equal to such load.

Load duration curves (LDCs) are an industry-standard modelling methodology used to represent periods of constant demand. LDCs are formatted as a non-increasing curve of load plotted against a time index. LDCs (magenta) can be approximated by load blocks (blue), as illustrated in figure 2. Murphy et al. [2] discusses the stepwise approximation of an LDC. Seasonal LDCs are formed, each approximated by ten blocks, hence a single year is effectively represented by forty blocks specifying the MW load of each block, and the hourly duration of each block at that given load.

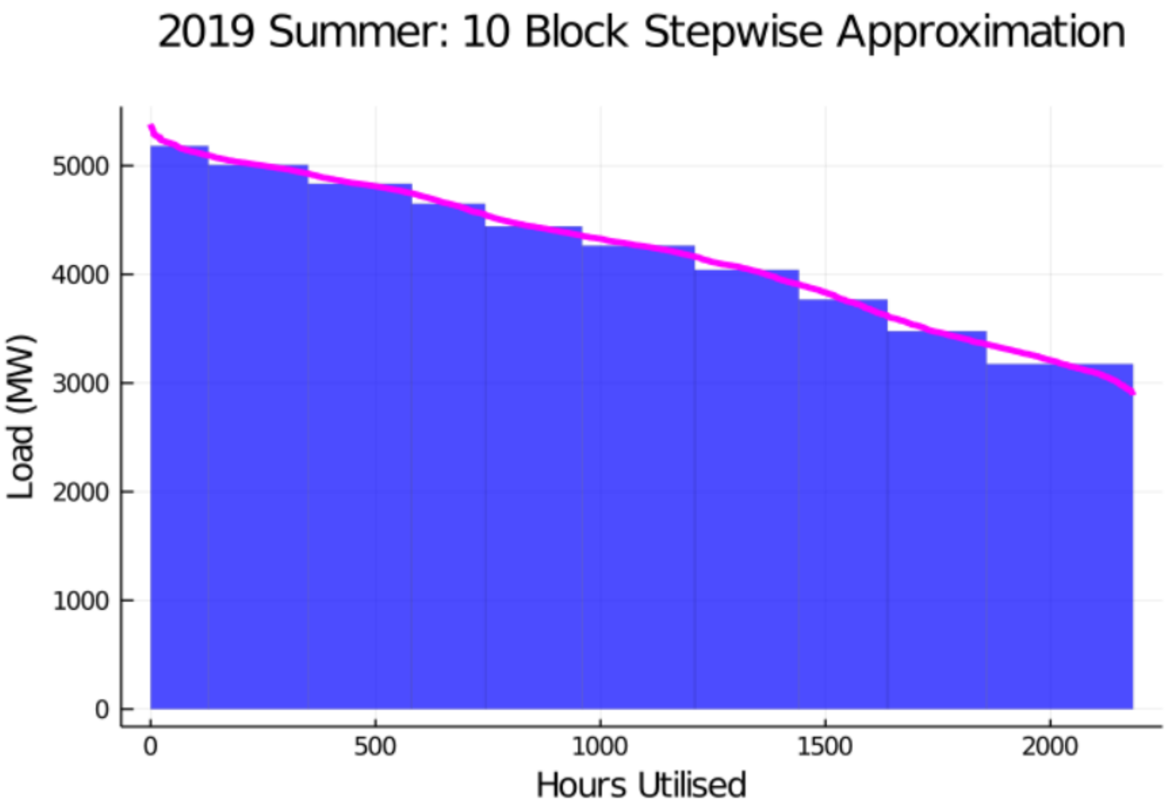

Figure 2: Stepwise approximation of a load duration curve

Load block approximations of LDCs have been calculated by minimising the price-weighted error of the LDC curve using a Bellman Ford algorithm. Refer to Jamie Chen's companion report for a detailed explanation of this process. Peak blocks represent a small interval of high demand, while the off-peak blocks represent longer periods of low demand. Load blocks are piecewise constant and are primarily used for convenience.

5.4 Generation technology, demand growth, and governmental policy

New Zealand's current electric power generation stack has a capacity of 9.3GW [19]. After accounting for cogeneration, partially embedded generation, and plants operating at lower capacity than what is installed, lowers this capacity to 8.2GW.

In 2018, 4.24Mt CO_2e were produced, 82% of which arose from the use of coal- and gas-fired plants [20]. This is a 45.4% reduction from peak emissions in 2005, where 9.35Mt CO_2e were produced. National annual demand is currently just below 40,000 GWh, with the expected 2050 demand ranging from 85,000 GWh to 56,000 GWh [21].

The scenario tree used has multiple scenarios ranging from a high growth trajectory that represents a 2% annual demand growth compounding from 2020 until 2050. The lowest growth trajectory represents a 1% annual compounded growth.

Major events that may alter demand include the closure of Rio Tinto's Tiwai-Point Aluminium Smelter which currently accounts for 13% of national electricity demand, or the uptake of electric vehicles given the recently-elected New Zealand Labour Party's electrification policy [7]. Considerations that may impact generation involve the decommissioning of Huntly's coal-powered Rankine units, with a capacity of 500MW, or potentially governmental climate policies such as reaching 100% renewable generation by 2030.

Investment in new generation technology is an active area of research, with generation stack updates and project proposals for new wind, hydro, geothermal, and solar sites completed in 2020 ahead of approaching major investment decisions. The recently re-elected Sixth Labour Government has committed to reaching 100% renewable generation by 2030. This leaves nine years for a large transformation of the generation stack to occur.

As stated, prior, the purpose of the multistage case studies are not to model consented projects, and instead to consider a wide range of large-scale investments in different generation technology types and to facilitate further work at a later stage. The two-stage case studies use more accurate expansion capacities and can be found in the companion report produced by Jamie Chen.

Twelve different generation technologies have been modelled as shown in table 1 below, each with a unique cost structure. We have not considered green hydrogen or the Lake Onslow dam proposal, as promoted by the Sixth Labour Government.

Generation technology	Existing capacity (MW)	Total potential investments (MW)	Number of sub-investments	CO_2e Emissions (tonnes / MWh)	Capital investment costs ($ NZD / MW / Year)	Operational costs ($ NZD / MWh)	Maintenance costs ($ NZD / MW / Year)
CCGT	785	1,000	1	0.38	138,000	70	45,000
OCGT	350	1,000	1	0.52	110,400	93	15,000
Diesel	155	0	NA	0.8	NA	232	15,000
CCS	0	4,200	4	0.044	242,717	75	45,000
Coal	500	0	NA	0.921	NA	48	60,000
Geothermal	892.4	320	4	0.099	430,000	1	150,000
HydroR	1,527	5,100	4	0	430,000	6	0
HydroS	3,524	4,800	4	0	516,000	6	0
Battery (Fast)	0	60	2	0	314,788	0	7,000
Battery (Med)	0	60	2	0	163,879	0	6,000
Battery (Slow)	0	60	2	0	48,364	0	5,000
Wind	375.25	4,000	5	0	178,000	12	20,000

Table 1: Technology types considered with capacities, emissions levels, and costs. Note that battery emissions of 0 tonnes / MWh have been assumed but may not be accurate.

5.5 Load Duration Curve (LDC) Model

This optimisation model follows closely behind that of Ferris and Philpott [8] and adds multistage functionality to their two-stage stochastic program. The stochasticity arises from the modelling of multiple historical years in which first-stage building decisions must be able to meet second-stage demand requirements posed in each historical year given the variation in available hydro generation. As this model has been developed for a hydro-dominated electric power generation stack, it was fitting to use this modelling methodology in the multistage version. This section of the report will challenge some of the assumptions put forward by Ferris and Philpott, and we propose some novel methods of further developing the assumptions.

The two-stage operational subproblem has also been integrated within JuDGE to provide the multistage computational tractability via decomposition required to optimally solve such a complex problem, given that deterministic equivalents fail to solve in an acceptable time.

This model needs to efficiently solve a decomposed operational model at each node quickly, hence the majority of assumptions and modelling methods used were in service of this computational efficiency goal. The effects of large-scale investments in non-dispatchable technologies such as wind and solar have been analysed regarding how load block approximations of LDCs are modified by their introduction.

The avoidance of modelling correlated scenarios like the representative days model was taken for efficiency reasons. We explored the feasibility of directly accounting for wind and solar generation at different stages of the data preparation and modelling phase.

Regarding solar investments, the interest lay in the effects of accounting for load generation before and after calculating load blocks. For large-scale investments, it was found that large solar investments would result in a non-monotonically decreasing LDC, given that solar energy's contribution only arises during daylight hours, which typically coincide with the shoulder period of an LDC.

Figure 3 below illustrates a stepwise load block approximation of a load duration curve comprised of seasonal demand data with a series of solar investments' generation subtracted from the load **prior** to the load block approximation. The investments sizes considered are 0 MW, which just leaves the existing embedded capacity, 2000 MW and 4000 MW. The stepwise functions are monotonically decreasing. Figure 4 however illustrates the same data with the contribution of the solar investments accounted for **after** producing the load blocks. We see that the approximated blocks change dramatically, and a redistribution of trading periods in each block varies. The apparent peak and shoulder blocks drop dramatically while the off-peak increases.

What is happening is that in figure 3, for the 4000 MW case, ranging on the x-axis from 2000 to 3000 correspond to trading periods with no solar generation, such as at night. These periods become the off-peak sections in figure 4. Similarly, the large solar contributions in figure 3 that significantly reduces the off-peak blocks instead contributes to substantially reducing the peaks given that these are likely periods before 10am and after 4pm with sufficient sunlight in Summer.

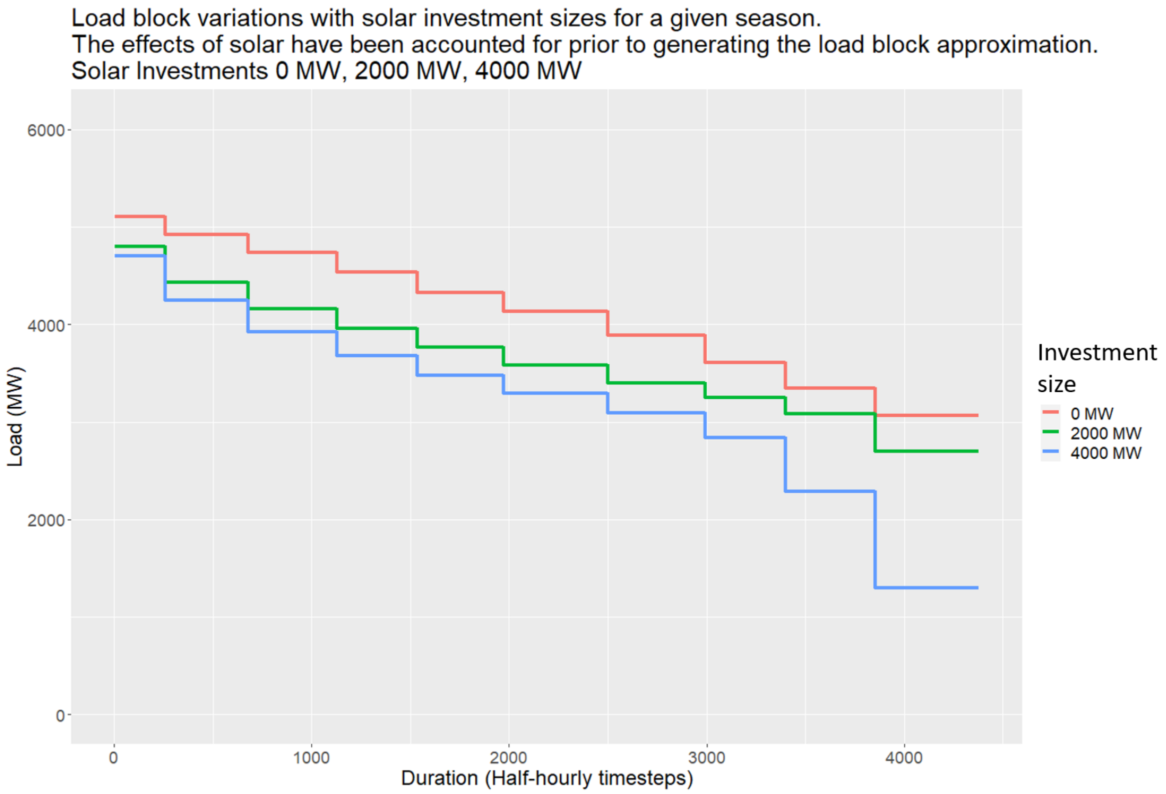

Figure 3: Load blocks with large solar generation accounted for prior to calculation

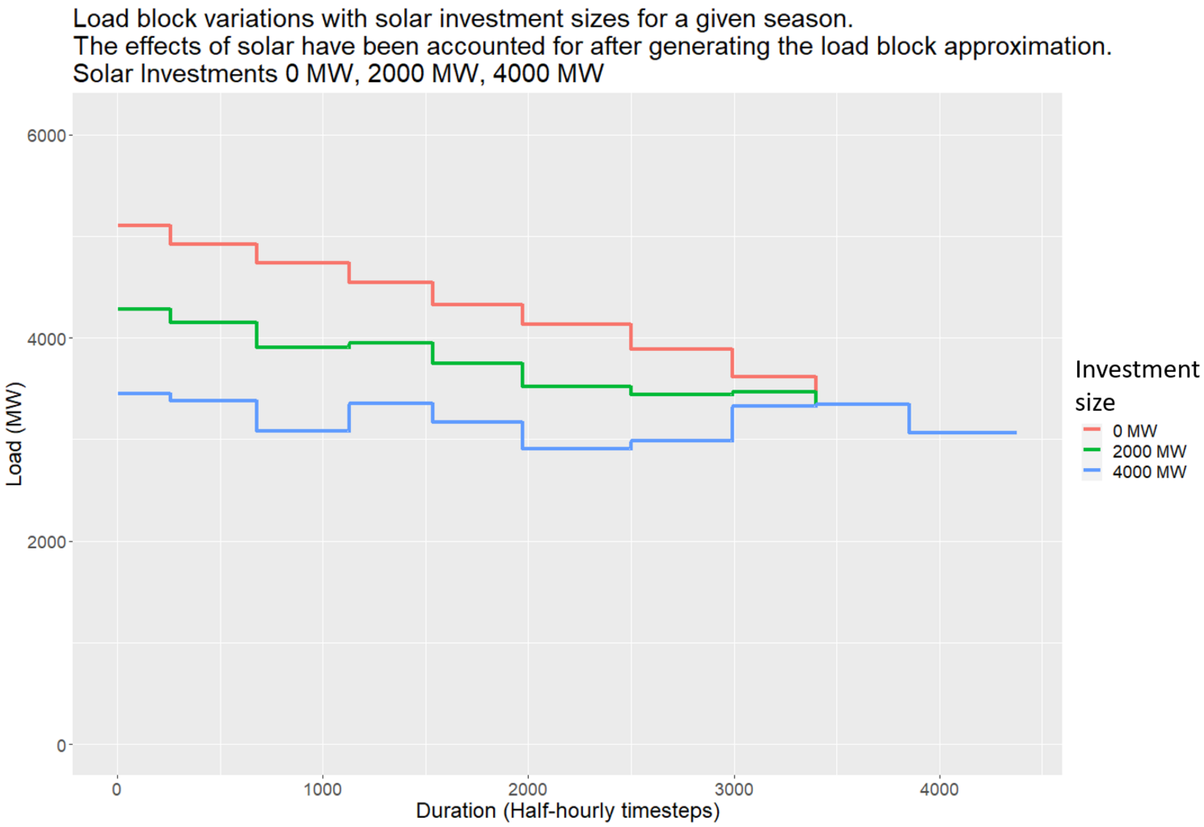

Figure 4: Load blocks with large solar generation accounted for after calculation

As part of this model, solar and wind generation are accounted for prior to calculating load blocks. Solar generation is treated as a facet of the scenario tree, in which each node has a defined solar MW capacity. As discussed before, this is distributed solar generation, referred to as *"rooftop solar"*. The uptake level of rooftop solar is assumed to be correlated to the carbon price. Transpower's *Advanced Electrification* [23] predicts the uptake levels of installed rooftop solar capacity to be 4,900 MW in 2050.

Wind generation is modelled with a portfolio selection approach. Given N different wind investments, there is a portfolio of 2^N combinations of such investments, ranging from investing in none, to investing in all N. For each element of the portfolio, seasonal LDCs are calculated and then approximated by load blocks. A matrix is formed, with each row corresponding to an element of the portfolio's forty load blocks over a year. The notation used in the following formulation is $P(T_I)$ to represent the *power set* of LDC-altering technologies. An additional matrix stores the block widths in a similar manner. Had a MW capacity-based formulation of this model been used, the blocks widths across all portfolios would be held constant to avoid a non-linear objective function.

This occurs at the root node for each portfolio and season, where load blocks are *built* from the LDC data. Given that 2020 electricity prices are used to build the load blocks given the price-weighted error minimisation algorithm mentioned prior, the block widths of each individual portfolio are held constant in each node of the scenario tree. Hence in all non-root nodes, the load blocks are *fit* to a given set of widths that were calculated at the root. For example, the "no wind investment" option has the same block widths in node 1, 11, 12, 111, et cetera, however these block widths are different to those that are constant for the "all wind investments" option at the same nodes. This avoided having to add an additional dimension to the scenario tree to estimate how future electricity prices would vary.

JuDGE extracts the correct set of load blocks based on which combination of investments have been made with the following constraints. A binary dummy variable d_j is used in the first inequality to model the LDC-altering investment portfolio, as x_j cannot be used in the third inequality given the Dantzig-Wolfe decomposition approach used to solve this problem. The second constraint states that only one row of the load block portfolio matrix may be extracted referring to an annual set of load blocks, while the complex third constraint uses a binary representation, i_2 of each portfolio in $P(T_I)$ to model this. A detailed explanation of this selection process and an example are provided in Appendix 1.

$$d_j \leq x_j, \qquad \forall j \in T_I$$

$$\sum_{i \in P(T_I)} \lambda_i = 1$$

$$\lambda_i \geq \sum_{(a,j) \in i_2} \big(j(d_a) + (1-j)(1-d_a)\big) - (\,\|T_I\| - 1)\,, \qquad \forall i_2 \in P(T_I)$$

The selected set of load blocks are extracted from the MW load matrix $\vartheta_{b,t}$ and the block hours matrix $H_{b,t}$ with λ_i.

$$\pi_{b,w,t} = \sum_{i \in P(T_I)} (\lambda_i \, \vartheta_{b,t} H_{b,t})$$

New capacity investments as well as the decommissioning of thermal plants have been modelled. Expansions for each technology type k are modelled with a binary variable x_k. Decommissioned plants are modelled similarly with x_k^*. Closed- and open-cycle gas turbines can be decommissioned for a salvage value, while the coal-powered Huntly Rankine units will be forcibly decommissioned at some stage and will hence be modelled explicitly, without the use of x_k^*.

If the model choses to invest in new generation technology, it pays an annualised capital cost of $\sum_{k \in T} \rho_k u_k x_k$ which is the discounted annualised capital cost of investment of a certain MW capacity specified by u_k, where u_k is a discrete investment size parameter provided to the model. If the model choses to decommission a thermal plant, it receives a once-off scrap value of $\sum_{k \in T^*} \rho_k^* x_k^*$ which is a discounted salvage value of a plant. These costs are only inflicted should x_k or x_k^* become active.

The potential megawatt-hour generation capacity available of each technology type k, in each load block b, of season t, is modelled with $z_{k,b,t}$, deviating from that of Ferris and Philpott [8]. This model is MWh generation-based as opposed to being MW capacity-based. In the constraint below, U_k models the existing MW capacity installed for technology type k. Selection of the appropriate set of load block widths from $H_{b,t}$ is controlled by $\sum_{i \in P(T_I)} (\lambda_i H_{b,t})$, corresponding to the block hours given the investments made in wind technology. $z'_{k,b,t}$ models any potential expansion in this potential generation arising from new capacity investments.

$$z_{k,b,t} \leq U_k \sum_{i \in P(T_I)} (\lambda_i H_{b,t}) + z'_{k,b,t}$$

A *Big M* approach is taken to model the expanded potential generation capacity. The first in the set below operates as a master-slave constraint where the binary x_k controls the expansion. u_k contains a potential expansion for generation type k, and as above, the correct set of block widths are extracted from $H_{b,t}$.

$$z'_{k,b,t} \leq BigM \; x_k$$

$$z'_{k,b,t} \leq u_k \sum_{i \in P(T_I)} (\lambda_i H_{b,t})$$

The potential generation capacity of shutdown variables is given by the constraints below. The first constraint is as above, while the second also uses a *Big M* approach to turn off the generation capacity of the decommissioned plants, should they cease operation.

$$z^*_{k,b,t} \le U^*_k \sum_{i \in P(T_I)} (\lambda_i H_{b,t})$$
$$z^*_{k,b,t} \le BigM \ (1 - x^*_k)$$

Each technology type can generate energy based on the capacity available, modelled with $y_{k,b,w,t}$ which refers to the megawatt-hours generated by technology type k in the b'th load block, of the w'th historical year, in season t. The total amount generated must be less than the installed potential generation capacity.

$$y_{k,b,w,t} \le z_{k,b,t}$$
$$y^*_{k,b,w,t} \le z^*_{k,b,t}$$

The first constraint above is not used for hydro generation. For run-of-river generation this is seasonal scaled nominally by $\mu_{t,w}$ which varies over each historical year and $\alpha_{b,t}$ is a seasonal intra-day scaling factor that proportionally scales generation in the first three load blocks and reduces the generation in the last three load blocks of each season. This is dynamically allocated to the off-peak blocks based on the load block widths selected using the peak block scaling factors of Ferris and Philpott [8].

$$y_{k,b,w,t} \le z_{k,b,t} \ \alpha_{b,t} \ \mu_{t,w}$$

Stored hydro generation uses the same seasonal gate methodology as Ferris and Philpott [8]. Seasonal gates are decided by the model, and $m_{t,w}$ which is the level of stored energy under each historical scenario in transferred from season $t \in S'$ to season $t + 1$ which must pass through the seasonal gates $g_t \pm \ \varphi$. Where φ is this width from the centre of the gate to the upper and lower bounds.

$$m_{t,w} \le g_t + \ \varphi$$
$$m_{t,w} \ge g_t - \ \varphi$$

To ensure inter-year continuity, we constrain $g_{-1} = g_3$ where g_{-1} refers to the same season as g_3 but in the preceding year.

Stored hydro generation is constrained by the following constraint. The maximum potential generation over a season $\sum_{b \in B_t} (z_{k,b,t})$ is scaled by $\upsilon_{k,w,t}$ which models the seasonal inflows, ranging from 0.2 to 0.42 in Summer, and from 0.42 to 0.75 in Winter. New Zealand is susceptible to *dry-year risk*, which results in lower than expected inflows, which is a primary consideration for modelling stored hydro in this method. The seasonal energy transfers of stored hydro are also included. The stored energy at the end of season t are subtracted, while the stored energy at the start of season $t \in S$ left over from the previous season are added to the constraint.

$$\sum_{b \in B_t} (y_{k,b,w,t}) \leq \ v_{k,w,t} \sum_{b \in B_t} (z_{k,b,t}) - m_{t,w} + m_{t-1,w}$$

The core tenet of the model is that generation must exceed demand or suffer the penalty of shedding part of the load. We can sum over all of the generation in a given block and ensure that the total generation plus any load shed exceeds or meets the demand in the same block. We also constrain the shed such that it cannot exceed the total demand in any block.

$$\sum_{k \in T_B} (y_{k,b,w,t}) + \sum_{k \in T^*} (y^*_{k,b,w,t}) \geq \pi_{b,w,t} - r_{b,w,t}$$

$$r_{b,w,t} \leq \pi_{b,w,t}$$

Batteries are modelled differently than other technology types. The non-negative vectors $c_{k,b,w,t}$ and $c'_{k,b,w,t}$ are used respectively to represent the charge and discharge amounts in a given load block for each individual battery. A round-trip efficiency value η is included in the first constraint, signifying electrical loss between the seasonal charge and discharge amounts. An assumption is made that the batteries can charge and discharge fully, at most once per day, hence the second constraint acts as a bound on the total permissible charge over a season. The total permissible discharge is implied via the first constraint also. β_k models the charge-rate of each battery type. Fast, medium, and slow batteries are considered. Fast batteries are more expensive to purchase and to maintain but have a more rapid rate of charge. Beta is used to constrain the total permissible charge via the third constraint below.

$$\eta \sum_{b \in B_t} (c_{k,b,w,t}) = \ \sum_{b \in B_t} (c'_{k,b,w,t})$$

$$\sum_{b \in B_t} (c_{k,b,w,t}) \leq \frac{1}{24} \sum_{b \in B_t} (z_{k,b,t})$$

$$c_{k,b,w,t} \leq \beta_k z_{k,b,t}$$

The effects charge and discharge vectors are added to the selected set of load blocks; hence we redefine the load block selection constraint to include these effects:

$$\pi_{b,w,t} = \sum\nolimits_{i \in P(T_I)} (\lambda_i \, \vartheta_{b,t} H_{b,t}) + \sum_{\forall k = Battery} (c_{k,b,w,t} - c'_{k,b,w,t})$$

Emissions constraints have been included to model policy considerations of the Sixth Labour Government of New Zealand. Two considerations have been included, the first involves reaching 100% renewable generation in a *normal hydrological year* by 2035. What a *normal hydrological year* exactly corresponds to is open to interpretation, however this has been interpreted as allowing emissions to be produced from thermal generation in dry years from 2035 onwards. This constraint is applied to all nodes in stage four or five of a scenario tree when used. This is modelled with the following two constraints. δ_w is a binary vector with each element active when thermal generation is permitted. Out of the thirteen historical years considered, thermal generation is permitted in at most Δ of these years. The first constraint sums over the total annual emissions of all technology types, excluding geothermal, which is dominated by δ_w over each historical year, capable of turning off emissions entirely. Geothermal energy is excluded given that even though emissions are produced, and some geothermal generators such as Ohaaki and Ngawha have a carbon emissions intensity nearing that of CCGT plants [22], they are still favourably considered renewable generation within New Zealand.

$$\sum_{k\in T^*}\left(e_k^*\sum_{t\in S}\left(\sum_{b\in B_t} y_{k,b,w,t}^*\right)\right)+\sum_{k\in T}\left(e_k\sum_{t\in S}\left(\sum_{b\in B_t} y_{k,b,w,t}\right)\right)\leq E\ \delta_w$$

$$\sum_{w\in W}(\delta_w)\leq \Delta$$

The second form of emission policy modelled is that announced in 2020 by the recently elected Labour government. It is a simplification of that above, with the intention of reaching 100% renewable generation by 2030 in any form of year. This is modelled with the constraint below, where E represents the maximum tonnes of CO_2e permitted for release in a given year.

$$\sum_{k\in T^*}\left(e_k^*\sum_{t\in S}\left(\sum_{b\in B_t} y_{k,b,w,t}^*\right)\right)+\sum_{k\in T}\left(e_k\sum_{t\in S}\left(\sum_{b\in B_t} y_{k,b,w,t}\right)\right)\leq E$$

We seek to minimise the cost of meeting demand at each node. The objective function of each subproblem consists of the operational, emission, value of lost load, and maintenance costs. Maintenance L_k is paid on any existing or newly invested capacity.

$$\sum_{k\in T}\big(L_k(U_k+u_k x_k)\big)$$

The maintenance costs L_k^* are recovered from any thermal plants that choose to decommission or are otherwise paid on the existing capacity.

$$\sum_{k\in T^*}\Big(L_k^*\big(U_k^*\,(1-\ x_k^*)\big)\Big)$$

The operational costs C_k and the carbon cost ζe_k per generated MWh are also included. These constraints are repeated for the shutdown technologies. $Vr_{b,w,t}$ models the VOLL and amount of load shed.

$$\sum_{w \in W} \left(\sum_{t \in S} \left(\sum_{b \in B_t} \left(\sum_{k \in T} \left((C_k + \zeta e_k)\, y_{k,b,w,t} \right) + Vr_{b,w,t} \right) \right) \right) + \sum_{w \in W} \left(\sum_{t \in S} \left(\sum_{b \in B_t} \left(\sum_{k \in T^*} \left((C_k^* + \zeta e_k^*)\, y_{k,b,w,t}^* \right) \right) \right) \right)$$

The cost of investing in new capacity is treated as an ongoing cost within JuDGE that is to be paid in at any node of investment as well as any descendant nodes that benefit from this capacity expansion. As previously mentioned, this is modelled as:

$$\sum_{k \in T} \rho_k u_k x_k$$

The salvage value returned by decommissioning a plant is a once-off payment, modelled with:

$$\sum_{k \in T^*} \rho_k^* x_k^*$$

There are no explicit costs defined with operating a battery, however due to a round-trip efficiency loss via peak load shaving, the cost is implied via the required additional generation to account for the efficiency loss. The use of batteries can be deduced at a given node under different scenarios and historical years given the variation in total MWh generation produced.

5.6 LDC Model Formulation

$$
\begin{aligned}
Min\ & \sum_{k\in T}\big(L_k(U_k+u_kx_k)\big) + \sum_{k\in T^*}\Big(L_k^*\big(U_k^*\,(1-\ x_k^*)\big)\Big) + \sum_{k\in T}\rho_k u_k x_k - \sum_{k\in T^*}\rho_k^* x_k^* \\
& + \sum_{w\in W}\left(\sum_{t\in S}\left(\sum_{b\in B_t}\left(\sum_{k\in T}\Big((C_k+\zeta e_k)\,y_{k,b,w,t}\Big) + Vr_{b,w,t}\right)\right)\right) \\
& + \sum_{w\in W}\left(\sum_{t\in S}\left(\sum_{b\in B_t}\left(\sum_{k\in T^*}\Big((C_k^*+\zeta e_k^*)\,y_{k,b,w,t}^*\Big)\right)\right)\right)
\end{aligned}
$$

$$s.t. \qquad d_j \le x_j\,, \quad \forall j\in T_I \tag{1}$$

$$\textstyle\sum_{i\in P(T_I)}\lambda_i = 1 \tag{2}$$

$$\lambda_i \ge \textstyle\sum_{(a,j)\in i_2}\big(j(d_a)+(1-j)(1-d_a)\big) - (\,\|T_I\|-1)\,, \quad \forall i_2\in P(T_I) \tag{3}$$

$$\pi_{b,w,t} = \textstyle\sum_{i\in P(T_I)}\big(\lambda_i\,\vartheta_{b,t}H_{b,t}\big) + \sum_{\forall k=Battery}\big(c_{k,b,w,t}-c'_{k,b,w,t}\big), \quad \forall b\in B_t, w\in W, t\in S \tag{4}$$

$$z_{k,b,t} \le U_k \textstyle\sum_{i\in P(T_I)}\big(\lambda_i H_{b,t}\big) + z'_{k,b,t}\,, \quad \forall k\in T, b\in B_t, t\in S \tag{5}$$

$$z'_{k,b,t} \le BigM\ x_k\,, \quad \forall k\in T, b\in B_t, t\in S \tag{6}$$

$$z'_{k,b,t} \le u_k \textstyle\sum_{i\in P(T_I)}\big(\lambda_i H_{b,t}\big), \quad \forall k\in T, b\in B_t, t\in S \tag{7}$$

$$z^*_{k,b,t} \le U_k^* \textstyle\sum_{i\in P(T_I)}\big(\lambda_i H_{b,t}\big), \quad \forall k\in T^*, b\in B_t, t\in S \tag{8}$$

$$z^*_{k,b,t} \le BigM\ (1-x_k^*)\,, \quad \forall k\in T^*, b\in B_t, t\in S \tag{9}$$

$$y_{k,b,w,t} \le z_{k,b,t}\,\alpha_{b,t}\,\mu_{t,w}\,, \quad \forall k=HydroR, b\in B_t, w\in W, t\in S \tag{10}$$

$$y_{k,b,w,t} \le z_{k,b,t}\,, \quad \forall k\ne Hydro, b\in B_t, w\in W, t\in S \tag{11}$$

$$y^*_{k,b,w,t} \le z^*_{k,b,t}\,, \quad \forall k\in T^*, b\in B_t, w\in W, t\in S \tag{12}$$

$$m_{t,w} \le g_t + \varphi\,, \quad \forall t\in S', w\in W \tag{13}$$

$$m_{t,w} \ge g_t - \varphi\,, \quad \forall t\in S', w\in W \tag{14}$$

$$g_{-1} = g_3 \tag{15}$$

$$\textstyle\sum_{b\in B_t}\big(y_{k,b,w,t}\big) \le v_{k,w,t}\sum_{b\in B_t}\big(z_{k,b,t}\big) - m_{t,w} + m_{t-1,w}\,, \quad \forall k=HydroS, t\in S, w\in W \tag{16}$$

$$\sum_{k\in T_B}\left(y_{k,b,w,t}\right) + \sum_{k\in T^*}\left(y^*_{k,b,w,t}\right) \geq \pi_{b,w,t} - r_{b,w,t}\,, \qquad \forall b\in B, w\in W, t\in S \tag{17}$$

$$r_{b,w,t} \leq \pi_{b,w,t}\,, \qquad \forall b\in B_t, w\in W, t\in S \tag{18}$$

$$\eta \sum_{b\in B_t}\left(c_{k,b,w,t}\right) = \sum_{b\in B_t}\left(c'_{k,b,w,t}\right), \qquad \forall k = Battery, w\in W, t\in S \tag{19}$$

$$\sum_{b\in B_t}\left(c_{k,b,w,t}\right) \leq \frac{1}{24}\sum_{b\in B_t}\left(z_{k,b,t}\right), \qquad \forall k = Battery, w\in W, t\in S \tag{20}$$

$$c_{k,b,w,t} \leq \beta_k z_{k,b,t}\,, \qquad \forall k = Battery, b\in B_t, w\in W, t\in S \tag{21}$$

$$\sum_{k\in T^*}\left(e^*_k \sum_{t\in S}\left(\sum_{b\in B_t} y^*_{k,b,w,t}\right)\right) + \sum_{k\in T}\left(e_k \sum_{t\in S}\left(\sum_{b\in B_t} y_{k,b,w,t}\right)\right) \leq E\,\delta_w\,, \qquad \forall k \neq Geothermal, w\in W \tag{22}$$

$$\sum_{w\in W}(\delta_w) \leq \Delta \tag{23}$$

$$\sum_{k\in T^*}\left(e^*_k \sum_{t\in S}\left(\sum_{b\in B_t} y^*_{k,b,w,t}\right)\right) + \sum_{k\in T}\left(e_k \sum_{t\in S}\left(\sum_{b\in B_t} y_{k,b,w,t}\right)\right) \leq E\,, \qquad \forall k \neq Geothermal, w\in W \tag{24}$$

$$x_k \in [0,1]\,, \qquad \forall k\in T$$

$$d_j \in [0,1], \qquad \forall j\in T_I$$

$$\lambda_i \in [0,1]\,, \qquad \forall i\in P(T_I)$$

$$z_{k,b,t} \geq 0\,, \qquad \forall k\in T, \forall b\in B, \forall t\in S$$

$$\pi_{b,w,t} \in free\,, \qquad \forall b\in B, \forall w\in W, \forall t\in S$$

$$r_{b,w,t} \geq 0\,, \qquad \forall b\in B, \forall w\in W, \forall t\in S$$

$$y_{k,b,w,t} \geq 0\,, \qquad \forall k\in T, \forall b\in B, \forall w\in W, \forall t\in S$$

$$g_t \geq 0\,, \qquad \forall t\in S$$

$$m_{t,w} \geq 0\,, \qquad \forall t\in S, \forall w\in W$$

$$c_{k,b,w,t} \geq 0\,, \qquad \forall k = Battery, \forall b\in B, \forall w\in W, \forall t\in S$$

$$c'_{k,b,w,t} \geq 0\,, \qquad \forall k = Battery, \forall b\in B, \forall w\in W, \forall t\in S$$

$$\delta_w \in [0,1]\,, \qquad \forall w\in W$$

5.7 Information realisation

Information realisation can be modelled by two different methods. The first method is that at any node in stage n, the demand growth for all nodes in stage $n+1$ becomes known. This facilitates the planning, consenting, and construction of new technology types for immediate use once stage $n+1$ is reached. Any node in stage n can still have multiple child nodes, however we are able to infer the known demand growth with certainty for each node in stage $n+1$, by assessing the current growth trajectory, for example.

The second method states that at any node in stage n, adequate capacity must be pre-emptively paid for and installed to meet the maximum level of demand in stage $n+1$, without regard for any child nodes with a lower-demand growth. The capacity has a lagged introduction of one stage, such that any capacity bought cannot at a stage be used until the next stage. This is a slightly more realistic approach, however, adds additional computational expensive.

5.8 Software development

This model has been coded as a subproblem in JuDGE, the Julia package. Gurobi is used for solving the problems and JuMP interfaces between JUDGE and Gurobi. The LDC approximation code is also coded as front matter within JuDGE, such that any load block optimisation is handled automatically by the model and are built according to the scenario tree structure. The code and data will be fully included in the research compendium submitted in partial fulfilment of the requirements for ENGSCI 700 A/B: Research Project.

The models formulated can be found on GitHub at [11], which contains the raw data extraction, data processing scripts, model formulation in JuDGE, and data visualisation code.

The LDC model requires a directory containing load duration curves for each element in the investment portfolio for each season. The model automatically generates load block approximations and collates the load blocks into a scenario tree directory structure.

Emissions policies, the option to decommission thermal technologies, and information realisation options are controlled by Booleans at the beginning of the script that automatically adapts the problem formulation, adding additional variables and constraints to the problem, as required.

6 Discussion

6.1 Modelling methodology comparisons

This report has discussed the LDC model in significant detail. In the companion report produced by Jamie Chen, a similar analysis of the representative days model has been offered.

The representative days model was developed out of concern for the coarse assumptions made during the production of the LDC model. It considers hourly variations in generation and demand levels over the course of daily scenarios. Different daily wind, solar, and demand profiles are modelled also. This model has more accurate peak load shaving assumptions, and models non-dispatchable technology like wind and solar within the subproblem. The model also uses an order of magnitude more data regarding different load and generation scenarios for each technology type, compared to the LDC model.

The representative days model is complex and including more than a single state-of-the-world is intractable, hence integrating the model within JuDGE became unnecessary, due to the single-stage nature. The conclusion drawn from the representative days model was that it was unnecessarily detailed. The decision was made to use the representative days model to assess the LDC model's optimal set of expansions returned at a given node in the scenario tree under the same demand growth conditions.

While the intention of this model was to integrate it into JuDGE and to have multistage functionality, the failure to do this was not an issue in itself, given that it strengthened our resolve that a load duration curve model is a more valid approach when considering a long time horizon.

6.2 Validity of the multistage formulation

Relative to Ferris and Philpott's two-stage model, we cannot make direct comparisons due to the successive nature of theirs and the LDC model. The extended functionality has improved the accuracy of the model, via the more accurate portfolio selection of load blocks when considering large scale investments in wind, battery, and solar technology. The run-of-river generation flow-scaling factors are no longer constant and are now dynamically allocated based on the block widths chosen. Using a MWh-generation based model was a requirement of this, however. We can model further than Ferris and Philpott's 2035 horizon and have introduced multiple intermediary states of the world via a scenario tree approach. We are also able to model both the forced, and optional, decommissioning of thermal plants under an expanded set of renewable constraint considerations.

New Zealand is however modelled as a single "node" by the LDC model while Ferris and Philpott split the country into three distinct regions, with transmission constraints linking each of them. This is a weakness of the LDC model, however as the intention of this model is to be coarse, fast, and facilitate multistage optimisation, we chose to forgo this aspect of the model.

6.3 Case study introduction

The underlying theme of all case studies is to assess the effects of the generation stack's emission levels produced over each stage along high and low demand-growth trajectories.

The first set of studies consists of government policy considerations, intended to facilitate a transition to 100% renewable generation, while also reducing emissions produced during generation. The scenario tree used has five stages and a deterministic carbon price and correlated rooftop solar uptake included in the structure.

The second set of case studies considers a scenario tree with carbon price as the primary facet, with an assumption that demand growth is correlated to the carbon price. Two trajectories will be considered along a three-stage tree. The carbon price determines the stage at which decommissioning of a significant electricity consumer occurs, and also coal generation assets that currently exist.

A portfolio of graphs and files will be included as part of the research compendium to provide further clarity to the case studies offered here.

6.4 Government policies aimed at reaching 100% renewable generation

Note that all graphs related to this case study can be found in Appendix 2.

Three potential government policies have been compared and contrasted regarding reducing emissions. The first policy considers setting a total emissions limit, of 1 MT CO_2e annually, which would reduce emissions by almost 75%. This has been considered as the baseline policy from which to assess the remaining two. The next two policies concern reaching 100% renewable generation; the second policy aims to reach this by 2030 and the third policy by 2035 in a normal hydrological year.

Policy three was the Labour Government's previous commitment, however during the latest election this has been modified into policy two. Policy three introduces constraints (22) and (23) into the formulation in stages four and five. $\Delta = 5$ was used to test whether thermal generation was required in five out of the thirteen historical years available. Policy two uses constraint (24), where $E = 0$ in stage three, four, and five, while policy one uses a variation of constraint (24), where $E = 1e6$ across all stages.

A forty-five node, five-stage scenario tree has been considered with a deterministic level of solar for tractability. This tree is equivalent in structure to figure 1 found in section 4.4. Two trajectories have been considered for analysis, a high growth trajectory ending at node 11111, and a lower-growth trajectory finishing at node 12422. The former has an annual MWh demand of 70.3 TWh in the final stage, while the latter trajectory has an annual MWh demand of 63.2 TWh.

The generation stacks do not differ dramatically across each policy considered, as seen in figures 6 and 7 in Appendix 2. Wind generation is used extensively, while stored hydro is gradually invested in across each stage. Geothermal is also used quite significantly, investing in all potential capacity expansions. The generation stacks are also quite constant across historical years, with the only minor variance arising from hydrological inflow variation in each historical year. Policy one invests minorly in CCS technology to meet demand in the final stage. Batteries were not invested in at any stage across all three policies considered. Interestingly, policy two chose to forgo a battery investment to redistribute energy across the blocks and instead opted to shed part of the load in peak blocks in two few historical years.

Regarding emissions, geothermal generation contributes significantly to total emissions in each policy, emitting roughly 990 kT CO_2e annually. Policy one has slightly higher emissions in all stages other than the second stage compared to the other two policies. In stage two, both policy two and three make significant use of either coal or CCGT generation, contributing an additional 450 kT CO_2e.

100% renewability is achieved by policy two, across all historical years, as is to be expected, seen in figures 8 and 9. Five historical years have thermal generation under policy three in both growth trajectories, corresponding to years with reduced hydro inflows. Interestingly policy one results in less historical years with thermal generation than policy three, given that geothermal generation is prioritised due to the larger amount of generation made possible for the same emission levels as thermal generation.

Policy three is the cheapest option, with a cost of NZ$10.9 billion for the high-growth trajectory, and a cost of NZ$8 billion for the low-growth trajectory. Policy one has similar pricing at NZ$11.2 billion and NZ$8.15 billion. Policy two is however far more expensive in the high-growth trajectory, pricing at NZ$24 billion and NZ$8.2 billion respectively.

Policy one appears to be optimal course of action. Emissions are maintained at 1 MT CO_2e annually, which is lower than both other policies considered, shown in figure 10. Policy two and three see an emission increase between stages one and five of 4%, and significant usage of coal until it is decommissioned. Policy two is however the optimal course of action regarding reducing emissions for the low-growth trajectory, with 35% less emissions than policy one and 49% less emissions than policy three in a dry-year, demonstrated in figure 11.

While policy three has the least cost, this arises due to only constraining stages four and five. Overall, it is the worst policy choice available, given that policy one has a similar cost yet constrains emissions in all stages. The emissions produced by each policy do not vary dramatically, even though thermal technology is used minorly for peak generation in policy one, this is just comprised of geothermal generation in the others. In policy two, thermal technology is decommissioned in stage three, but used in the first two stages. In policy one, OCGT decommissions in the second stage of the low-growth trajectory, while no thermal decommissioning other than coal occurs in the high-growth trajectory. No gas technology is decommissioned in policy three in either trajectory.

Given that policy one has been identified as the optimal strategy out of the tree presented, an interesting addition to this study is the effects of introducing a large battery storage option in stage one. 1200 MW of battery capacity is provided to the model, split evenly between each of the three charging-rate technologies modelled. The batteries are used in each stage along both trajectories to shave peak load blocks with the highest marginal cost of meeting demand. Batteries arbitrage between load blocks with differing costs of meeting demand, for instance, peak blocks are more costly to meet than off-peak, so batteries discharge in the peak and charge in blocks with a lower marginal cost of generation, shown in figure 12. Additional battery capacity is not invested in and is solely used given that we provide existing battery capacity to the model. Usage is highest in historical years with reduced hydro inflows, however, does not significantly contribute to modifying the load blocks.

6.5 Carbon price variations effects on emissions

Note that all graphs related to this case study can be found in Appendix 3.

This case study varies carbon prices dramatically across the trajectories of a scenario tree. The intention of this case study is to assess the impact of introducing a market incentive as another mechanism of moving to 100% renewable generation by 2030 without the need for explicit emissions policies and limits.

We consider a three-stage, scenario tree for this case study, given that our interest only lies in being able to operate with fully renewable generation in stage three. The third stage ranging from 2030 until 2035 will facilitate an assessment of the composition of the generation stack, given the first two stages allow different forms of preparatory expansions and thermal shutdowns to be made.

A visualisation of the scenario tree can be found in figure 5. The green route features a lower carbon price and a high demand-growth scenario. The orange route contains a high carbon price. A carbon price reaching NZ$50 / tonne CO_2e is assumed to trigger an immediate shutdown of the Tiwai-Point Aluminium Smelter and Huntly's coal units, resulting in a low demand-growth scenario. In all other scenarios, these are assumed to decommission in the third stage.

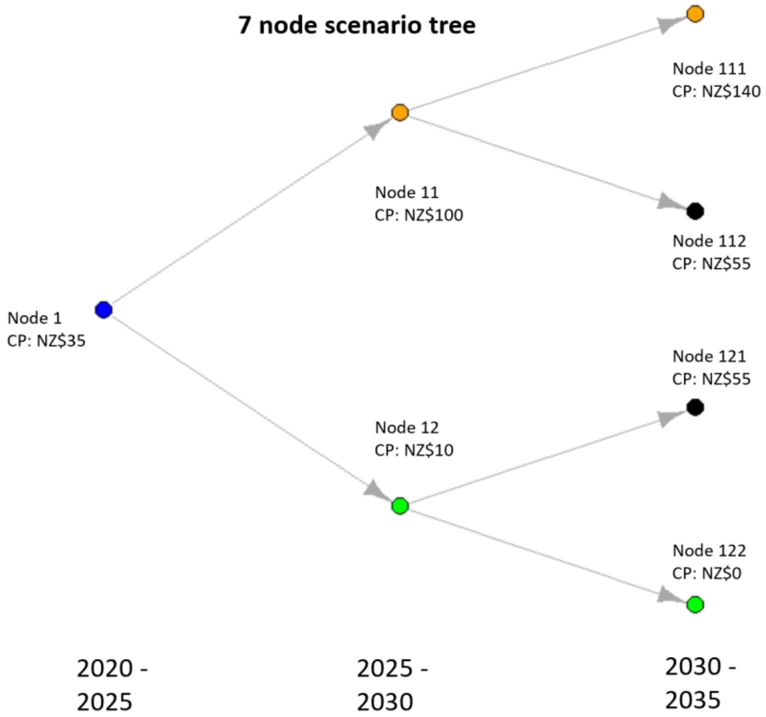

Figure 5: Seven node, carbon price-dominant scenario tree

Along the trajectory with an increasing carbon price, geothermal generation is effectively priced out of the generation stack and replaced with additional run-of-river and stored hydro generation, when the carbon price rises to NZ$140 / tonne CO_2e in stage three. This is shown in figure 13 in Appendix 3. Geothermal is however used in years with reduced inflows, while wind expansions are made and used extensively, and no battery investments exist. Emissions are 73 kT CO_2e and are entirely produced by the remaining geothermal generation, demonstrated in figure 15. This is fifteen times lower than the emissions produced in 2020 with a carbon price of NZ$35 / tonne CO_2e. All historical year scenarios reach full renewability by 2030, and half of these years do not release emissions. The other years, which are drier, emissions are solely produced by geothermal generation.

Along the trajectory with a decreasing carbon price, geothermal technology is used significantly, which can be seen in figure 14. Interestingly, OCGT and CCGT generation increases over the stages as the carbon price drops. This makes sense, as high carbon prices are an inhibiting factor against their use. Emissions peak in stage two, however coal generation is decommissioned at this stage, hence less pollutant gas generation makes up the shortfall shown in figure 16. Between stages one and three, annual emissions rise by 7% to 1.12 MT CO_2e when the carbon price falls to NZ$10 / tonne CO_2e.

In conclusion, raising carbon prices may be unpopular, but is a valid and effective method of reducing emissions and simultaneously incentivising the generation stack to become fully renewable by 2030.

7 Conclusion

Thankfully, this project has not suffered or changed drastically as a result of COVID-19 restriction measures, however this pandemic has had similarities drawn between its effects and that of climate change. The global state that we are facing in regard to COVID-19 sheds light on dire consequences arising from poor preparation for large-scale challenges that inevitably occur.

The events of 2020 have further reinforced the requirement for work of this sort to be undertaken, and for its continued development.

> “Addressing pandemics and climate risk requires the same fundamental shift, from optimizing largely for the *shorter-term performance* of systems to ensuring equally their *longer-term resiliency*. Healthcare systems, physical assets, infrastructure services, supply chains, and cities have all been largely designed to function within a very narrow band of conditions. In many cases, they are already struggling to function within this band, let alone beyond it. The coronavirus pandemic and the responses that are being implemented (to the tune of several trillion dollars of government stimulus as of this writing) illustrate how expensive the failure to build resiliency can ultimately prove. In climate change as in pandemics, the costs of a global crisis are bound to vastly exceed those of its prevention.” [24]

The Government of New Zealand has committed to reaching fully renewable generation by 2030, a date nine years from today. Two models have been built to assist with developing a plan for this transition. The load duration curve model is capable of modelling scenarios out beyond 2030, while the representative days model can be run at each node of such a scenario tree ensuring that the generation capacity is sufficient to meet hourly demand over the course of more intricate scenarios.

The load duration curve can operate as a long time-horizon, multistage model, and the case studies discussed offer an example of how introducing different scenario tree structures and model aspects of government policies. It can model a stochastic scenario tree with varying levels of demand-growth and carbon prices correlated to distributed solar uptake. Dry-year risk is accounted for via the historical-year scenario aspect of the model, which is crucial for a hydro-dominated generation stack.

The model has been integrated into the Julia Decomposition-engine for General Expansion. JuDGE provided a level of computational power via a version of Dantzig-Wolfe decomposition that facilitated the solving of multistage problems that a deterministic equivalent would not have managed to solve tractably. The LDC model serves as a functional application of JuDGE’s potential for modelling discrete capacity investments JuDGE worked well when we either had a large tree (45 to 341 nodes) or a large number of investments (35 different expansions), but not both simultaneously due to an intractably large resultant problem. Singh et al. [16] discusses the tendency to yield naturally integer solutions given this decomposition formulation, which materialises for the smaller problems considered, however larger problems often require a MIP solve due to the fractionality still present in the optimal solution once Gurobi terminated. Modelling the decommissioning of thermal plants and utilising the lag option between expansion decisions increased the solve time drastically.

The modelling methodology has facilitated case studies to be completed assessing past and present government policies, and the effects of carbon price variations on emission levels under different demand-growth scenarios.

The case studies conclude that increasing carbon prices is more effective than implementing a 100% renewable policy. Emission levels were significantly reduced by raising the carbon price and achieved fully renewable generation as a by-product.

8 Further work

We are under the impression that there is significant further work that can be carried out to further develop the models put forward.

Regarding raw data, utilising more than one year of wind, solar, and data scenarios would be useful, such as to give a representation of different seasonal effects. The wind- and solar-factor approach is flawed. We recommend research to be undertaken such as to calculate a baseline for wind speed and solar irradiance, such that if this baseline is exceed, the non-dispatchable generation is then generating at “full capacity”.

Modelling New Zealand as three nodes with transmission constraints and losses between them would add additional realism to the model. The model currently just uses Auckland’s Penrose grid exit point to price all national generation, however a more representative sample of the entire country would improve the accuracy of the price-weighted approximations of the load duration curves. A more accurate stepwise load block approximation function could be developed, that does not rely on trading period price data to minimise the error. If the price-weighting was removed, then block widths would not have to be fit to the root node’s prices.

Asset lifespans are not modelled for simplicity; however, JuDGE does provide the required functionality to specify the number of stages that a new expansion will be able to generate supply. The current model does not accurately model this lifespan, as assets last for the entire duration modelled.

The scenario tree should have probabilities assigned to weight the likelihood of certain occurrences, which is inaccurate. Along a certain trajectory, it would be more interesting to realise a more-highly weighted likelihood of and existing trends continuing. An example would be that if a trend of a high level of distributed solar uptake was present, then the following states-of-the-world would reflect this trend via a more heavily weighted probability of it occurring.

Larger, stochastic scenario trees containing at most 341 nodes have been modelled, but not included in this thesis. The reasoning behind this decision is that the JuDGE model solves in an acceptable timeframe, on the order of one or two hours for problems that either have a large number of investments, or a larger scenario tree, but not both simultaneously. The decision was made to focus on a detailed set of potential expansions that could be provided to JuDGE, as opposed to coarsely modelling the investments for a slightly more representative scenario tree. An interesting approach would be to combine both, by running a smaller tree with detailed investments to determine what types and sizes of investments are made, and then sub-setting this set of investments which are provided to a more detailed tree.

Introducing different load-shedding bands would be an improvement for the model, such as a low VOLL for industrial manufacturers, a medium VOLL for commercial consumers, and then the existing high-VOLL for residential consumers. This would facilitate different groups to shed load at earlier instances than the entire network as is currently modelled.

The emissions constraints add significant computational expense to the model, hence a Lagrangian relaxation could remove this constraint, and also to find a marginal abatement cost for the carbon price.

A better method of providing potential investment capacities to the model should be formed, and stronger constraint formulations applied to turn off any variables relating to thermal generation when including renewability constraints to speed up the solve time

The historical year methodology could be rethought given that the current model operates on the basis that a central agent “observes” the type of year to be experienced ahead of time, and can choose to operate accordingly for each historical year scenario. This is inaccurate when compared to the real world for obvious reasons. Variance regarding the inflow data corresponding to climate effects would be interesting to model in the future.

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10 Appendix 1: Logical constraints for demand curve selection

This section was written to provide a more detailed description of how constraint (3) is used to select the appropriate LDCs within JuDGE. This constraint is provided as:

$$\lambda_i \geq \sum_{(a,j)\in i_2}\big(j(d_a) + (1-j)(1-d_a)\big) - (\,\|T_I\| - 1)\,,\ \forall i_2 \in P(T_I)$$

We will use an example of having the option to invest in three "LDC-altering" technologies, labelled as A, B, and C. In this instance, $T_I = \{A, B, C\}$. We define $P(T_I)$ to be the portfolio set of a combination of investments that can be made. $P(T_I) = \{$ ∅, {A}, {B}, {C}, {A,B}, …, {A,B,C} }, however instead, we will represent this in a binary form such as, $P(T_I)_2 = \{\,000, 001, 010, 100, 011, \dots\,, 111\}$. Where 001 refers to investing in A, and not in B or C. Given that we have $n = 3$ investments, we will have $2^n = 8$ combinations of investments in our portfolio set. This explanation is provided with a binary representation example as this makes the coded version easier to interpret.

The summation symbol $\sum_{(a,j)\in i_2}(\dots)$ refers to enumerating the binary representation of $i \in P(T_I)$, denoted as i_2. The pair *(a, j)* refers to the index and value of each element in i_2 respectively. For example, given $i_2 = 001$, if we iterate over it from *least significant bit* to *most significant bit*, we will return three combinations of *(a, j)*: *(1,1) , (2,0) ,* and *(3,0)*.

Working towards the inner section of the constraint, $\|T_I\| - 1$ refers to the cardinal size of the investment set. In our example, this is $3 - 1 = 2$. The final section depends on whether $j \in [0, 1]$. If $j = 1$: The inner section of the constraint becomes d_a, and if $j = 0$: it becomes $1 - d_a$.

If we were to sum over $i_2 = 001$, the constraint would resemble:

$$\lambda_i \geq (d_1) + (1 - d_2) + (1 - d_3) - (3 - 1)$$

Further refined to: $\lambda_i \geq d_1 - d_2 - d_3$

Where the first three brackets resemble the elements "1", "0", and "0" respectively, remembering that we start at the *least significant bit* of $i_2 = 001$.

We define one λ for each different portfolio option, hence we have $2^n = 8$ in this example. Each λ has an associated constraint. The table below provides an example of all 2^n constraints as would be generated by the general form of the constraint given at the top of this section.

$\boldsymbol{i}$	$\boldsymbol{i_2}$	**Associated constraint**
1	**000**	$\lambda_1 \geq 1 - d_1 - d_2 - d_3$
2	**001**	$\lambda_2 \geq d_1 - d_2 - d_3$
3	**010**	$\lambda_3 \geq -d_1 + d_2 - d_3$
4	**100**	$\lambda_4 \geq -d_1 - d_2 + d_3$
5	**011**	$\lambda_5 \geq d_1 + d_2 - d_3 - 1$
6	**101**	$\lambda_6 \geq d_1 - d_2 + d_3 - 1$
7	**110**	$\lambda_7 \geq -d_1 + d_2 + d_3 - 1$
8	**111**	$\lambda_8 \geq d_1 + d_2 + d_3 - 2$

The purpose of this set of constraints, alongside constraint (2), is to ensure that only one λ_i can be active at any one time, based on the dummy values d_i. This constraint works for any cardinality of T_I, and avoids laborious constraint definitions when dealing with a large number of potential investments.

11 Appendix 2: Government policy case study graphs

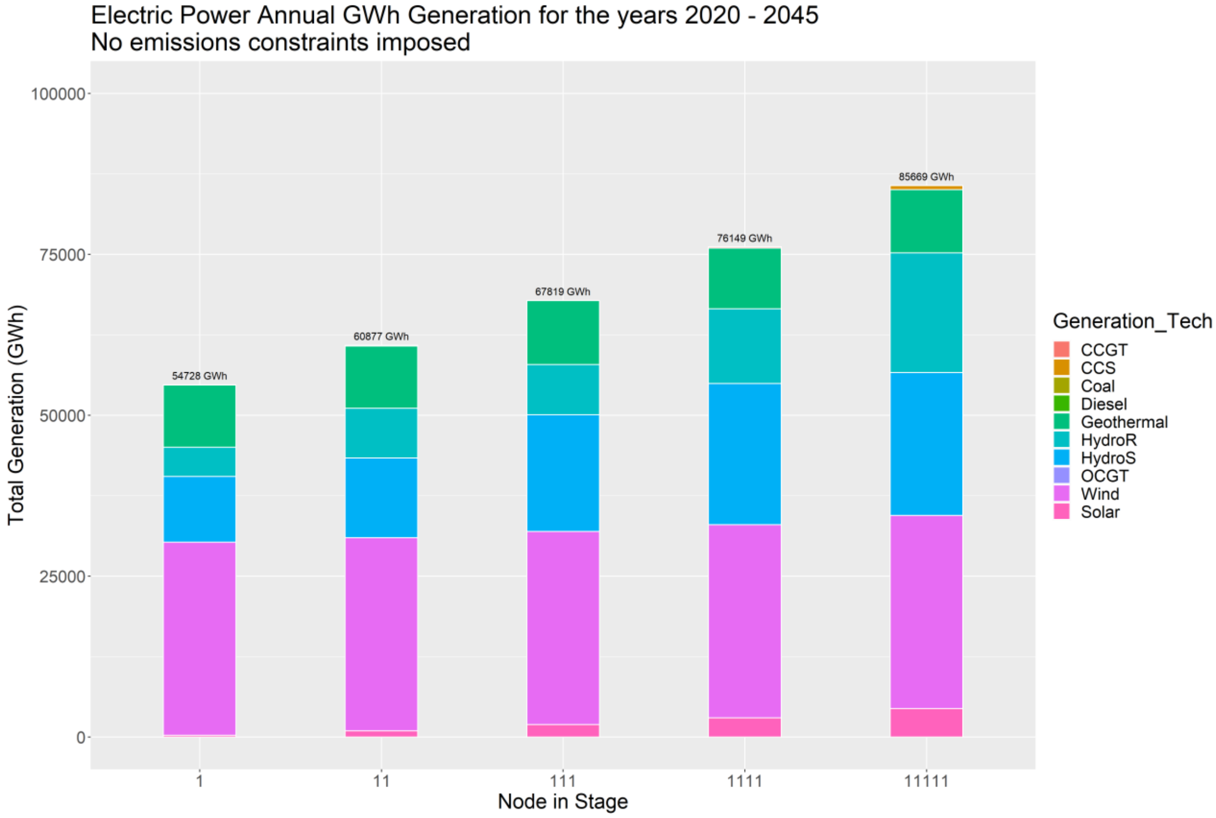

Figure 6: Generation stacks for policy one in a high-growth scenario

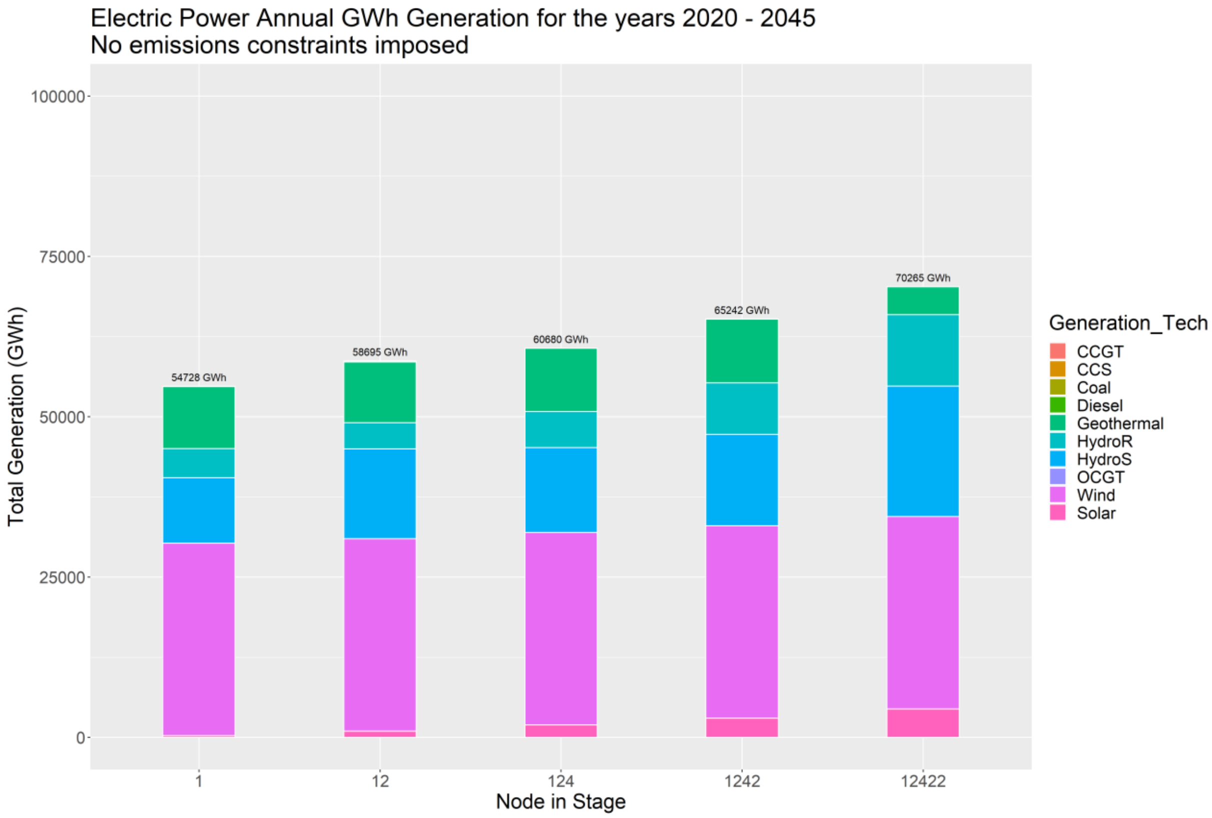

Figure 7: Generation stacks for policy one in a low-growth scenario

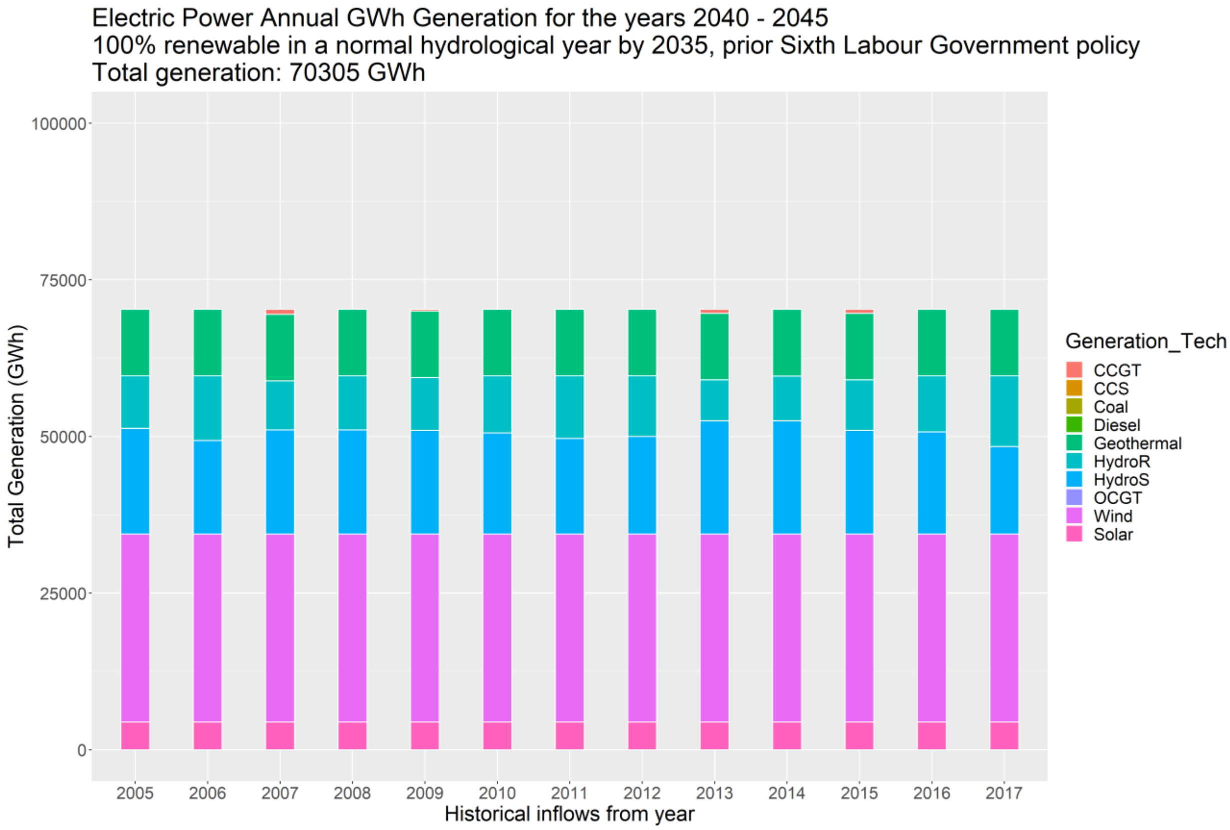

Figure 8: Policy three normal hydrological year generation stacks at the final stage of the high-growth trajectory

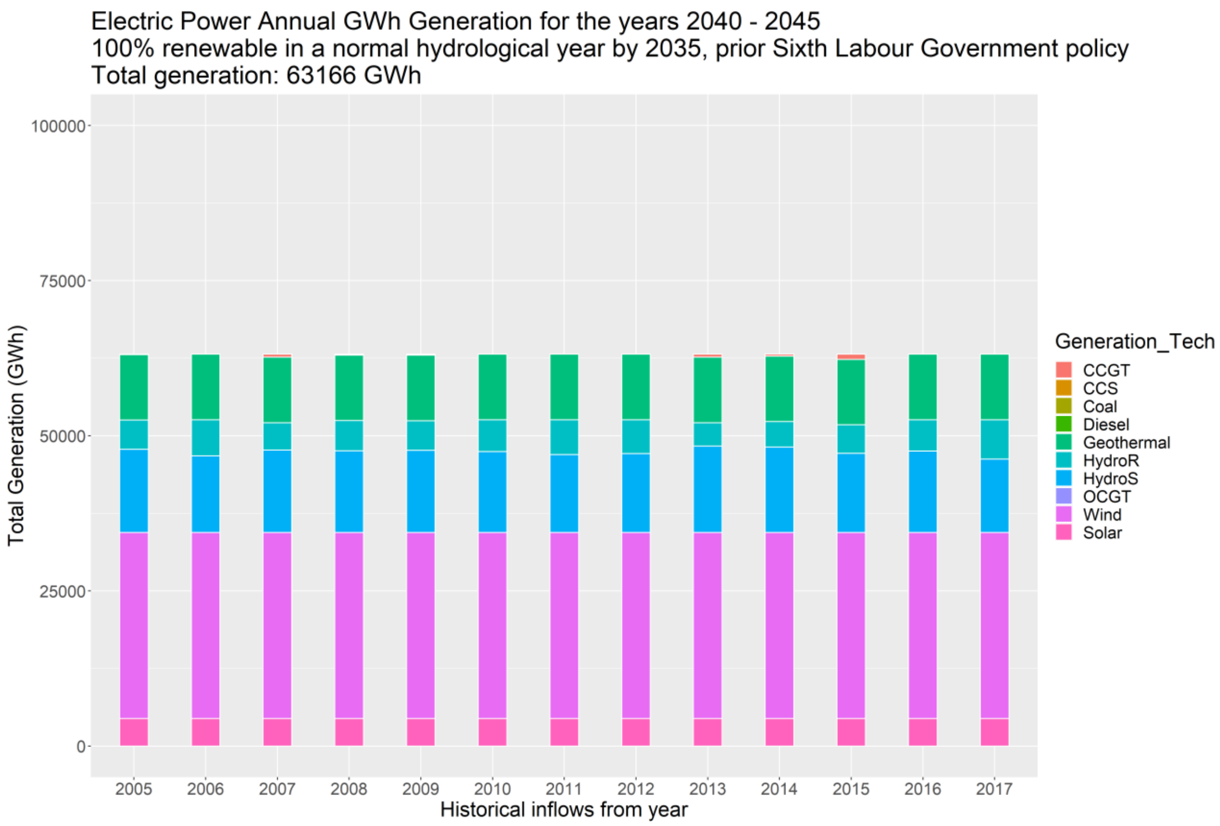

Figure 9: Policy three normal hydrological year generation stacks at the final stage of the low-growth trajectory

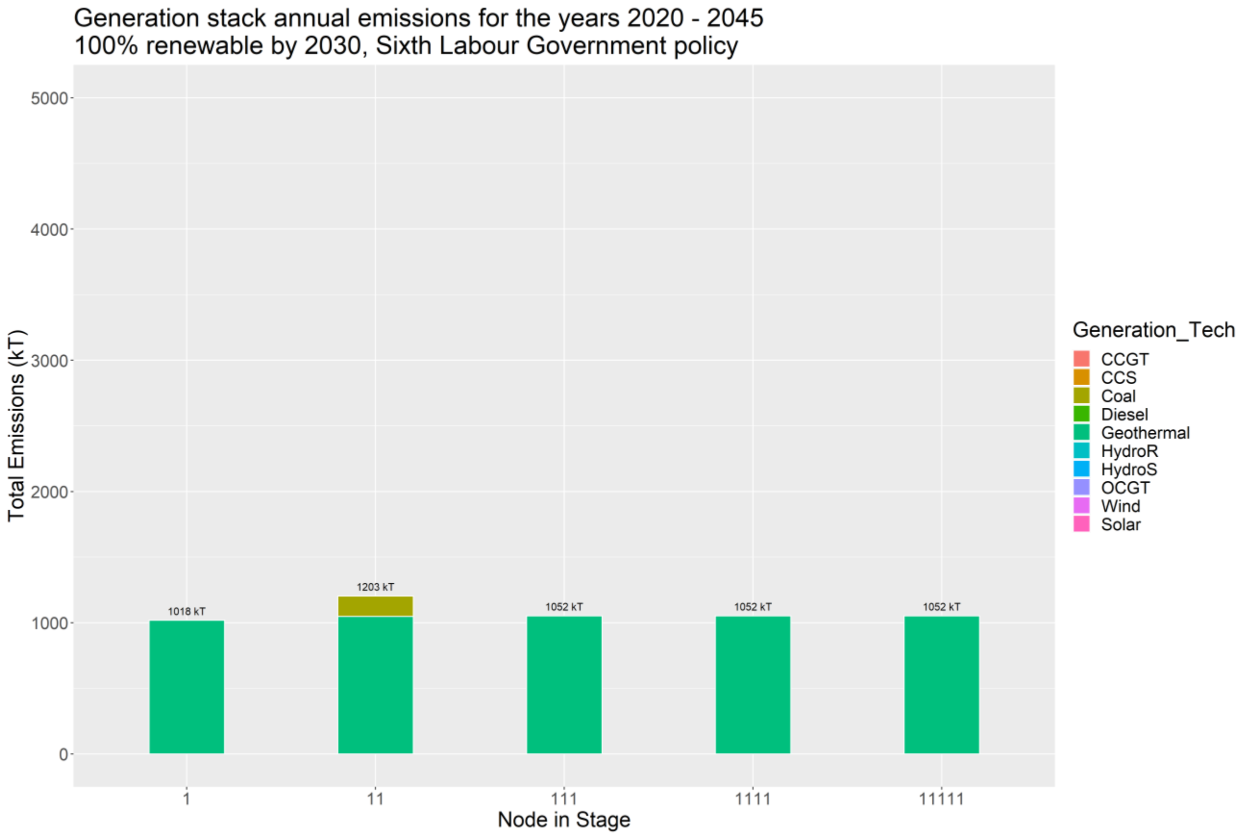

Figure 10: Emission stacks for policy two under the high-growth trajectory

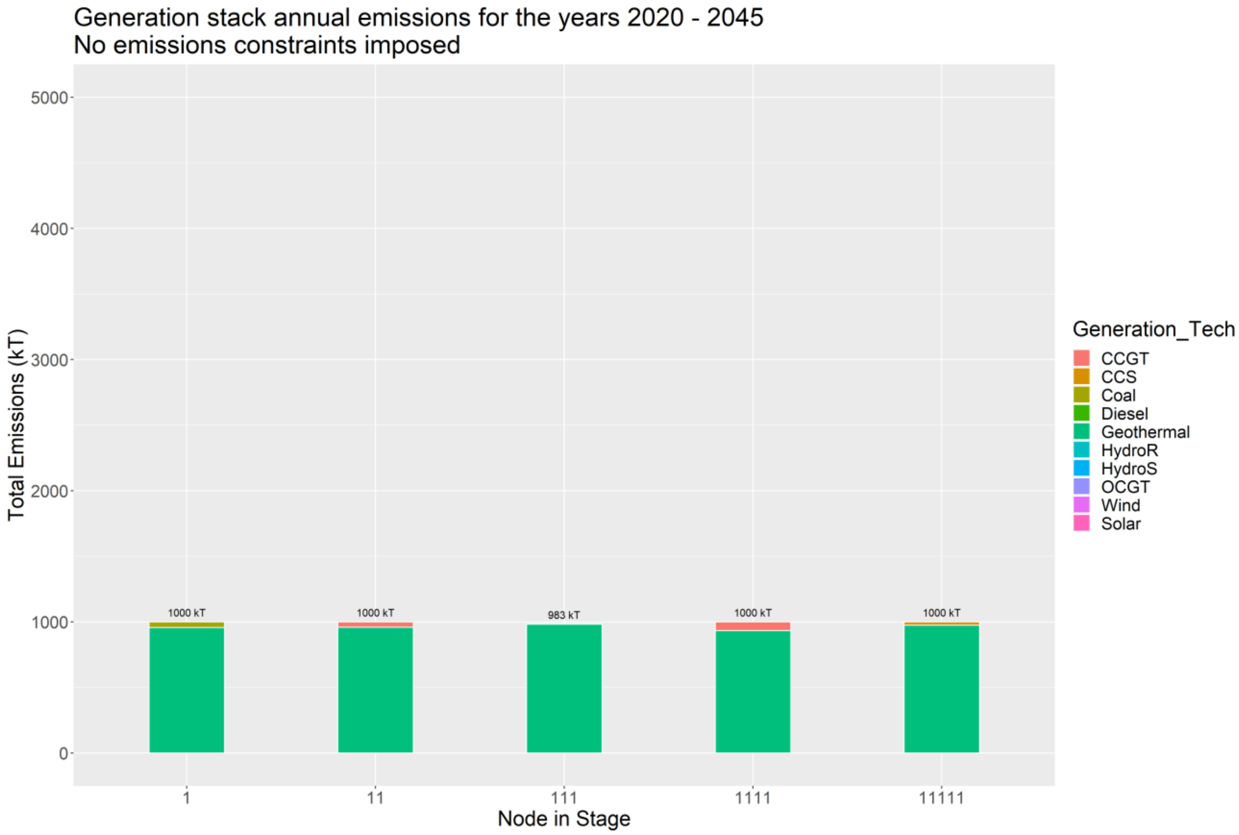

Figure 11: Emission stacks for policy one under the high-growth trajectory

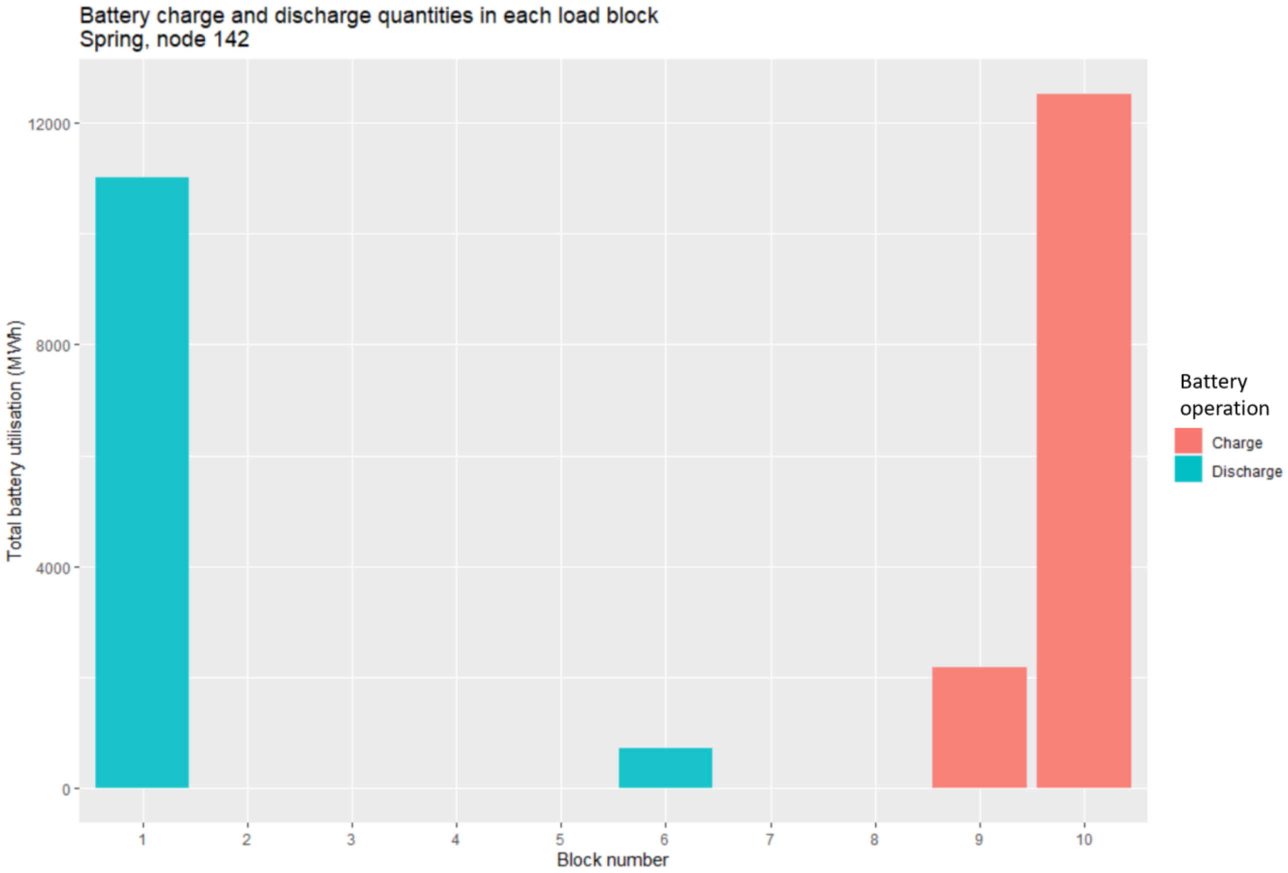

Figure 12: Example of battery charge and discharge quantities in each load block for a given season

12 Appendix 3: Carbon pricing policies

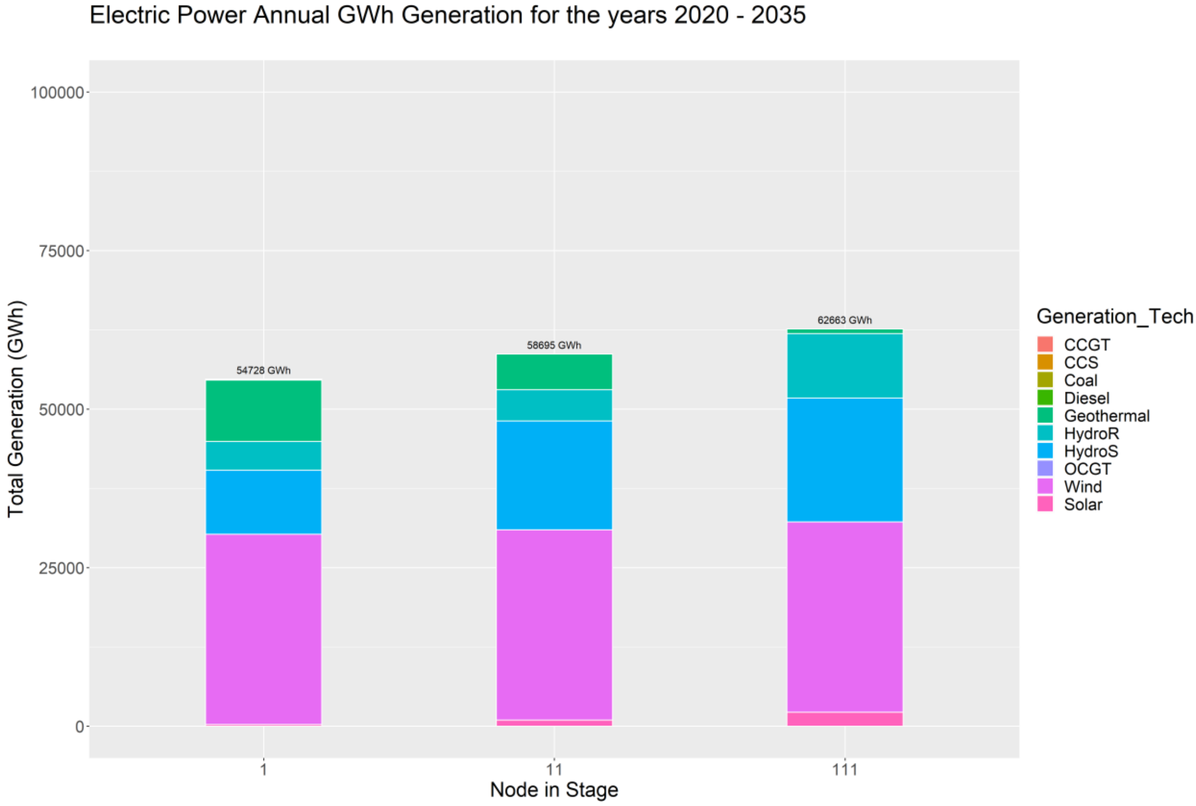

Figure 13: Generation stack along the high carbon price trajectory

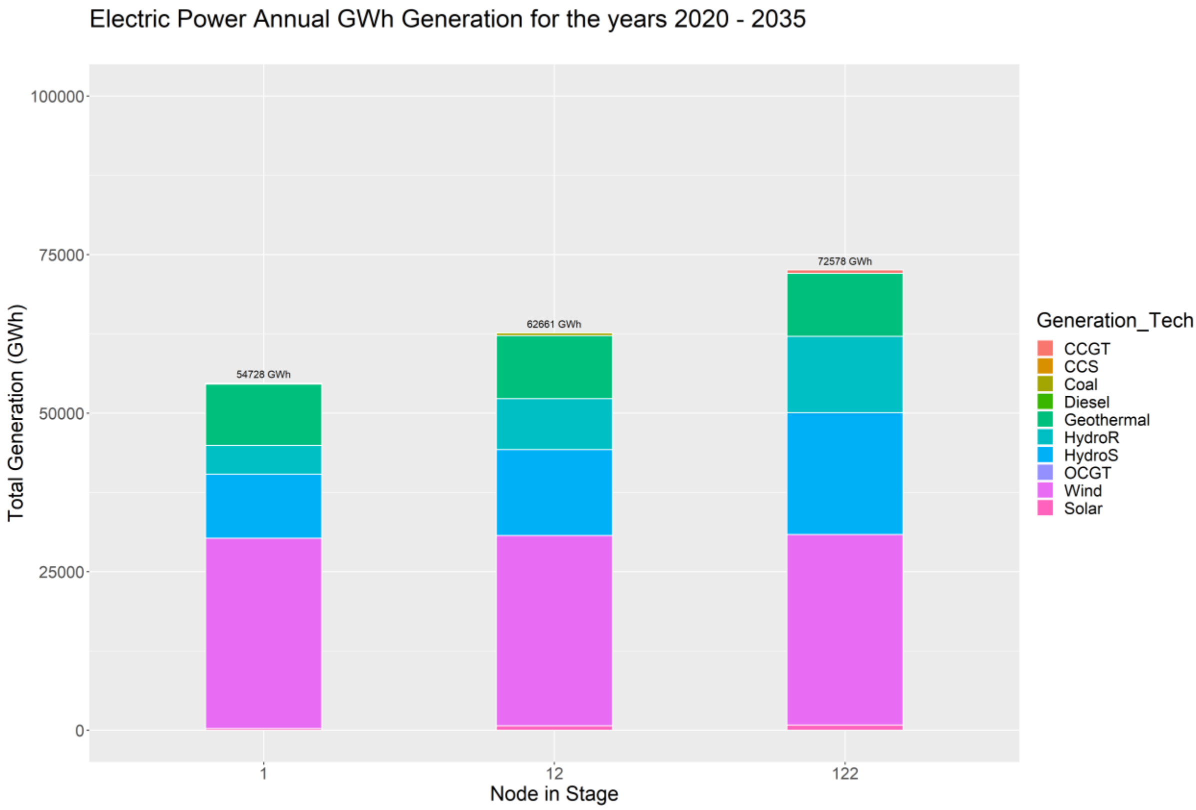

Figure 14: Generation stack along the low carbon price trajectory

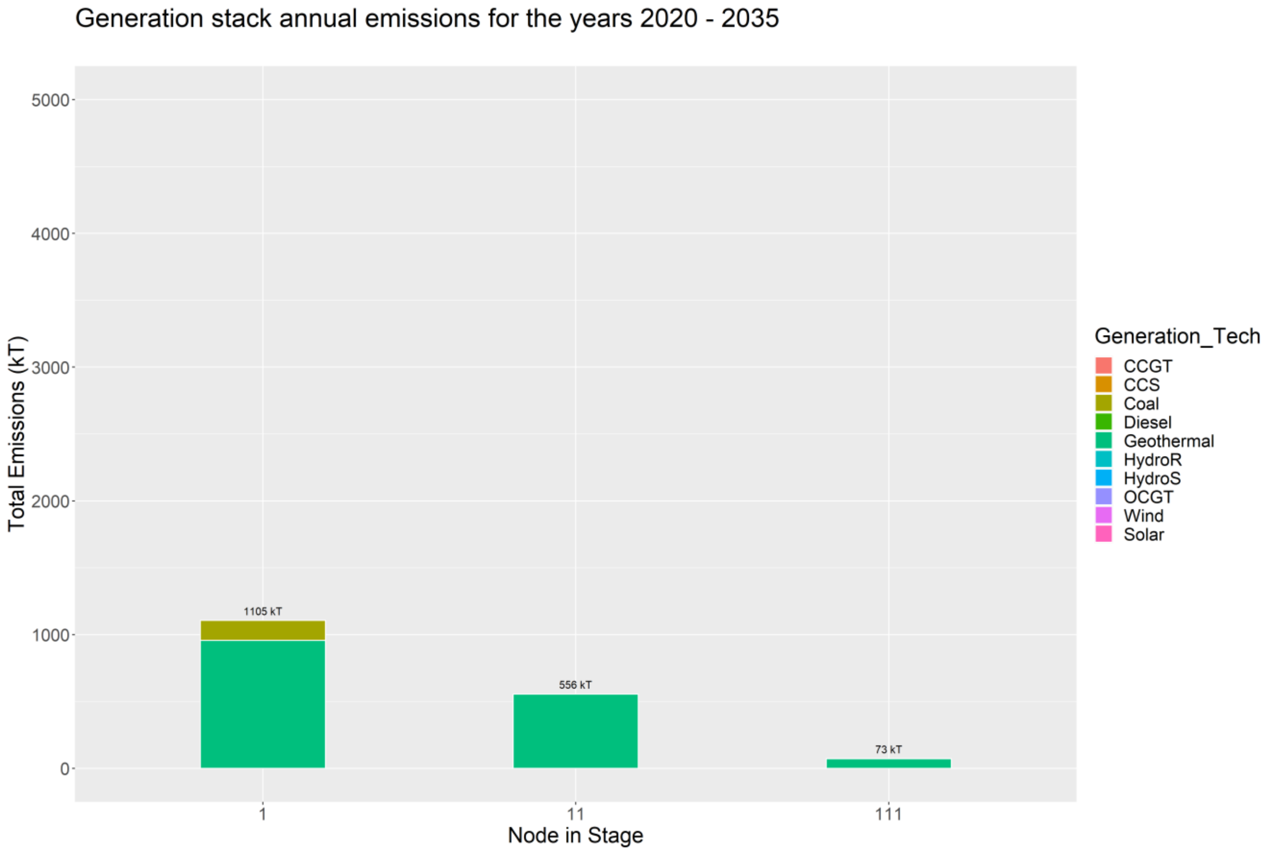

Figure 15: Emission stacks along the high carbon price trajectory

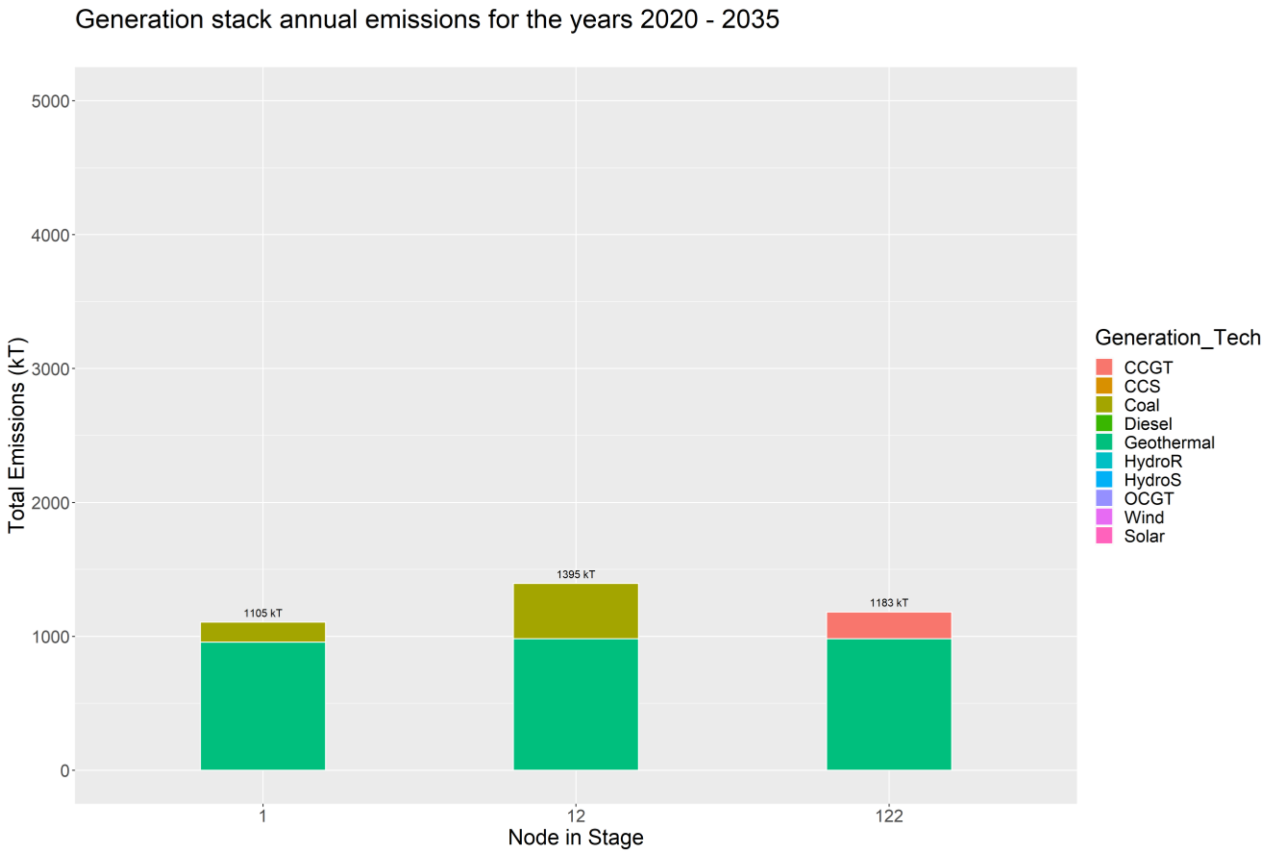

Figure 16: Emission stacks along the low carbon price trajectory