Spectrum Sensing System

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Signal Processing & Estimation Tasks

Experimental Set-up

Experimental setup should minimize variability and maximize reproducibility.

Standardized Data Acquisition

- *Standardized Data Acquisition* Requirement for controlled conditions for signal collection, ensuring consistent sampling rates and minimizing external noise sources.
- *Calibration & Normalization* Need for regular calibration of instruments and normalization techniques to maintain comparability between measurements.

Robust Signal Processing Pipeline Implement modular and reproducible Python scripts for preprocessing, filtering, and analysis, leveraging PyTorch for efficiency. PSD Verification - If measuring spectral components, verify consistency in Welch's method parameters, such as windowing and overlap, to reduce bias.

Visualization for Diagnostics Use clear comparative plots to identify anomalies in signal trends across different experimental runs.

Statistical Tests in Spectrum Sensing statistical tests help verify the consistency, accuracy, and significance of spectral components in spectrum sensing applications:

- ✓ WelchâĂŹs T-test - Compares mean spectral power across different sensing scenarios, helping detect significant changes in signal presence.

  "Optimal Threshold of Welch's Periodogram for Spectrum Sensing Under Noise Uncertainty". `P:/P01SDR/Chi-square.pdf`

- ✓ Bootstrap Resampling - Estimates confidence intervals for spectral features, improving reliability in dynamic environments.

  "Robust Spectrum Sensing Using Moving Blocks Energy Detector with Bootstrap".

- ✓ F-test for Variance Analysis - Compares the variance of spectral estimates across different sensing conditions, ensuring stability.

  "A Simple F-Test Based Multi-Antenna Spectrum Sensing Technique".

- Kolmogorov-Smirnov (KS) Test - Evaluates whether PSD distributions differ significantly between sensing periods, ensuring consistency across measurements.

  "Sequential and Parallelized FPGA Implementation of Spectrum Sensing Detector Based on Kolmogorov-Smirnov Test".

- Chi-Square Goodness-of-Fit - Assesses whether observed spectral distributions match expected noise or signal models.

  "Spectrum Sensing Method Based on Goodness of Fit Test Using Chi-Square Distribution".

- ANOVA for Multi-Scenario Analysis - Determines whether multiple sensing conditions exhibit statistically different spectral characteristics.

  "A Comparative Study of Different Entropies for Spectrum Sensing Techniques".

- Shapiro-Wilk Test - Validates normality assumptions of PSD estimates, crucial for applying parametric statistical methods.

  "Spectrum Sensing Algorithm Based on Shapiro-Wilk Test".

Power Spectral Density (PSD) Estimation

Real-Time Implementation of Multiband Spectrum Sensing Using SDR •.

Summary of key PSD estimation methods used in spectrum sensing:

1. Periodogram ✓ Given a discrete-time signal $x[n]$, the periodogram estimates the PSD as the squared magnitude of its DFT:

$$\hat{P}_{\text{per}}(f) = \mathbb{E}\left\{\left|x[n]e^{-j2\pi fn}\right|^2 : \forall n = 0, N-1\right\} \quad f \in [0, 1)$$

where N is the number of samples.

Properties

- **Resolution:** Determined by the length N of the signal. Larger N provides finer frequency resolution.
- **Variance:** The periodogram is an inconsistent estimator; it does not converge to the true PSD as $N \to \infty$, due to high variance.
- **Spectral Leakage:** Abrupt windowing of the signal causes leakage, spreading power from strong frequencies into adjacent bins.

Advantages

- Simple and fast to compute using the Fast Fourier Transform (FFT).
- Useful for preliminary spectral exploration and real-time visualization.

Disadvantages

- High variance and poor statistical consistency.
- Low detectability in low SNR conditions.
- Prone to spectral leakage, which may obscure weak signals.

Application in Spectrum Sensing

- In the context of spectrum sensing, this enables the detection of spectral peaks that correspond to active transmissions (e.g., primary users in cognitive radio).
- Identify spectral holes or vacant frequency bands.
- Estimate energy in specific subbands for energy detection.
- Serve as a baseline for comparison with more advanced PSD estimation methods (e.g., Welch, multitaper).
- useful tool for fast spectrum estimation and provides valuable insights into signal occupancy. However, its limitations in resolution and variance often motivate the use of enhanced methods in practical spectrum sensing applications.

Enhancements To mitigate its limitations, the basic periodogram is often improved via:

- **Averaging (Welch's method):** Reduces variance by averaging multiple periodograms.
- **Windowing:** Applies tapering windows (e.g., Hamming) to reduce leakage.

2. Welch's Method ✓ This method segments the input signal into overlapping windows, applies a tapering window function to each segment, computes the periodogram of each windowed segment, and then averages the resulting spectra. Step-by-Step Procedure:

Let $x[n]$ be a signal of length N:

1. Divide $x[n]$ into K overlapping segments, each of length L.
2. Apply a window function $w[n]$ (e.g., Hamming) to each segment:

$$x_k[n] = x[n + kD] \cdot w[n], \quad 0 \leq n < L$$

   where D is the shift between segments (determines overlap).
3. Compute the modified periodogram for each windowed segment:

$$\hat{P}_k(f) = \frac{1}{U} \left| \sum_{n=0}^{L-1} x_k[n] e^{-j2\pi f n} \right|^2$$

   where $U = \frac{1}{L} \sum_{n=0}^{L-1} w^2[n]$ is a normalization factor.
4. Average all K periodograms to obtain the final PSD estimate:

$$\hat{P}_{\text{Welch}}(f) = \frac{1}{K} \sum_{k=0}^{K-1} \hat{P}_k(f)$$

Advantages

- **Reduced Variance:** Averaging smooths fluctuations, leading to a more stable estimate.
- **Tunable Parameters:** Window length, overlap, and tapering window can be optimized for specific applications.
- **Spectral Leakage Control:** Use of tapering windows reduces spectral leakage.

Disadvantages

- **Reduced Frequency Resolution:** Due to windowing and segmentation.
- **Increased Computational Load:** Requires computing multiple DFTs.

Application in Spectrum Sensing

- **Detection of Spectral Occupancy:** Identifies frequency bands with significant energy.
- **Noise-Robust Detection:** Performs well in low SNR environments.
- **Real-Time Monitoring:** Efficient enough for online spectral analysis in dynamic systems.
- Welch's method offers a practical and effective compromise between spectral resolution and variance reduction. It is well-suited for real-world spectrum sensing tasks where signal detection reliability is critical under noisy and dynamic conditions.

Enhancements for Welch's PSD Estimation

- **Adaptive Windowing:** Dynamically adjusting the window length or type (e.g., Hamming, Kaiser, Blackman) based on the detected signal characteristics can improve resolution or leakage suppression.
- **Overlap Optimization:** Increasing the overlap between segments (typically 50% to 75%) enhances averaging and reduces estimation variance, at the cost of computational complexity.
- **Window Function Selection:** Choosing window functions with better side-lobe suppression (e.g., Kaiser over Hamming) can mitigate spectral leakage and improve detectability of weak signals.
- **Multiresolution Analysis:** Integrating WelchâĂŹs method with multi-resolution techniques (e.g., applying it across varying segment lengths) can provide better insights into wideband or multiband environments.

- **Noise Floor Estimation:** Combining Welch's PSD with noise floor tracking methods helps in establishing dynamic detection thresholds for energy-based sensing.
- **Parallel and GPU Implementation:** For real-time applications, WelchâĂŹs method can be parallelized across segments and implemented on GPUs to accelerate processing.
- **Pre-Filtering:** Applying bandpass filtering prior to WelchâĂŹs PSD computation can enhance the signal-to-noise ratio (SNR) for targeted frequency bands.
- **Hybrid PSD Estimators:** Combining WelchâĂŹs output with model-based or multitaper methods can provide hybrid estimates that balance resolution and variance under specific conditions.

GNU Radio for PSD – Welch's Method •.

Multitaper Method (MTM) Given a zero-mean discrete-time signal $x[n]$, $n = 0, 1, \dots, N-1$, the multitaper method estimates the power spectral density (PSD) by averaging multiple spectral estimates obtained via orthogonal taper functions.

Let $\{v_k[n]\}_{k=0}^{K-1}$ be a set of K orthogonal taper functions (typically Slepian sequences or discrete prolate spheroidal sequences, DPSS), each of length N. The tapered versions of the signal are:

$$x_k[n] = x[n] \cdot v_k[n], \quad k = 0, 1, \dots, K-1.$$

The discrete Fourier transform (DFT) of each tapered signal is:

$$X_k(f) = \sum_{n=0}^{N-1} x[n]\, v_k[n]\, e^{-j2\pi f n}, \quad f \in [0, 1).$$

The individual spectral estimates are:

$$S_k(f) = \frac{1}{N} \left| X_k(f) \right|^2 .$$

The multitaper PSD estimate is obtained by averaging over the K spectral estimates:

$$\hat{S}_{\mathrm{MT}}(f) = \frac{1}{K} \sum_{k=0}^{K-1} S_k(f) = \frac{1}{KN} \sum_{k=0}^{K-1} \left| \sum_{n=0}^{N-1} x[n] v_k[n] e^{-j2\pi f n} \right|^2 .$$

This approach reduces variance without substantially compromising frequency resolution, particularly effective in short or noisy time series.

Advantages: Reduces spectral leakage; low variance; high spectral resolution.
Disadvantages: Increased computational demand; requires taper design.
Use Case: Detection of weak signals in noisy environments.

✓ Multi-taper Method of PSD. Estimation based on multiple orthogonal tapers to reduce spectral leakage and variance.

GNU Radio for PSD – Multi-taper Method •.

4. Autoregressive (AR) Model-Based Estimation It assumes the signal is generated by an all-pole model excited by white noise.
Formula:

$$\hat{P}_{\mathrm{AR}}(f) = \frac{\sigma^2}{\left|1 + \sum_{k=1}^{p} a_k e^{-j2\pi f k}\right|^2}$$

Advantages: High resolution even with short data records.
Disadvantages: Requires accurate model order selection; less effective for broadband signals.
Use Case: Suitable for narrowband signal modeling and analysis.

5. Wavelet-Based PSD Estimation It decomposes the signal using wavelet transforms and estimates power in each frequency band from the wavelet coefficients.
Advantages: Good time-frequency localization; suitable for nonstationary signals.
Disadvantages: Depends on wavelet choice and decomposition level.
Use Case: Analysis of transient or nonstationary signals in dynamic spectral environments.

✓ Wavelet-based PSD estimation with denoising.

GNU Radio for PSD – Wavelet •.

6. Eigenvalue-Based Detection (Cyclic Spectral Analysis) It uses second-order statistics and eigenvalue distributions to detect periodicities or cyclostationarity in the signal spectrum.
Advantages: Exploits signal structure; robust to interference.
Disadvantages: Computationally intensive; requires prior knowledge of cyclic features.
Use Case: Advanced detection of modulated signals in dense spectral environments.