Decarbonize Energy Systems

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Caltech

NSF Workshop April 2023

Outline

Trends and challenges (15)

Some experiences

- From EV charging (5)
- … to workplace decarbonization (10)
- … to unbalanced 3-phase power flows (15)

Why this workshop

temperature rise → crises:
- humanitarian
- economic
- refugee
- political
- national security

↑

GHG emissions

↑

economy

↑

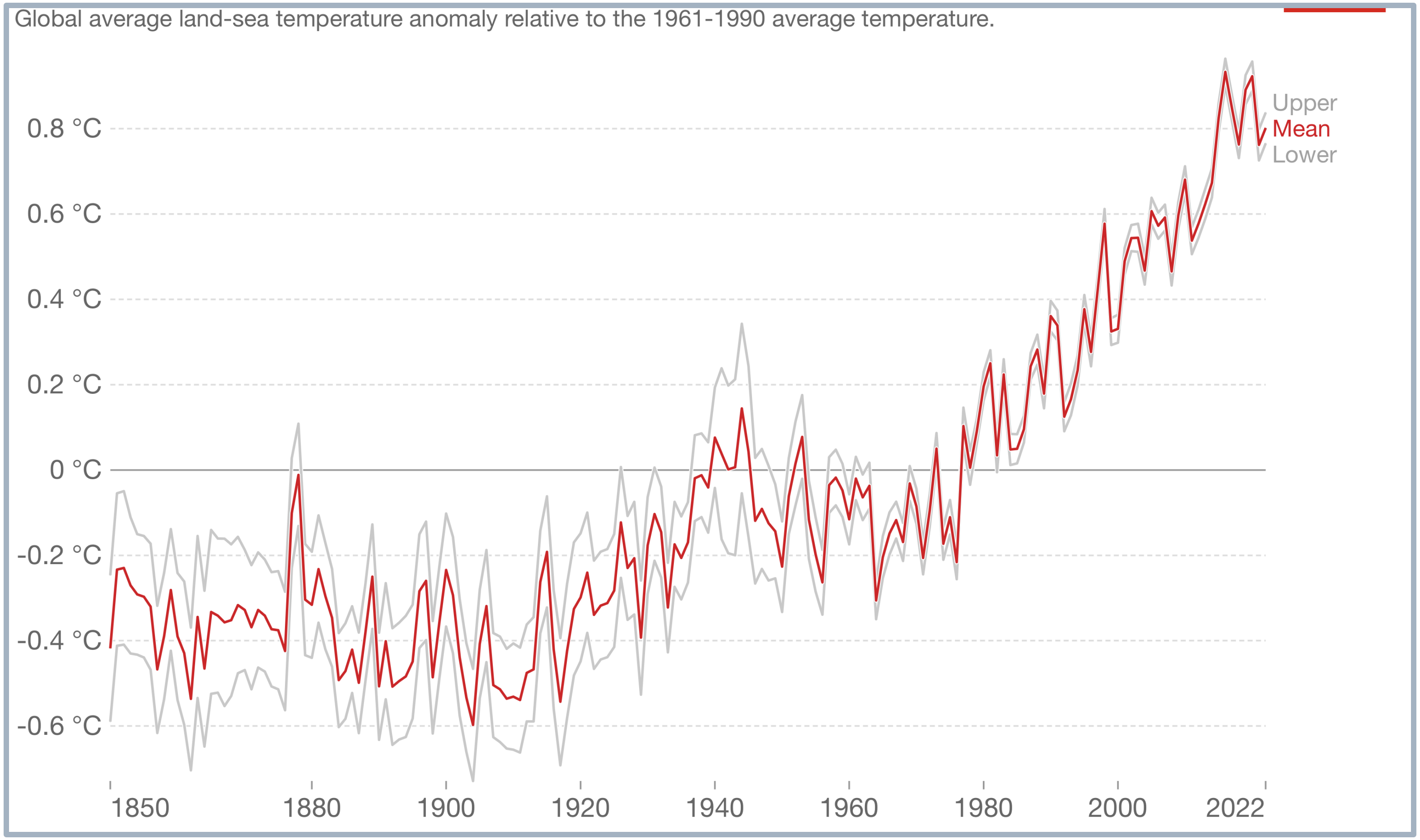
energy use

↑

electricity & transportation

this NSF Workshop

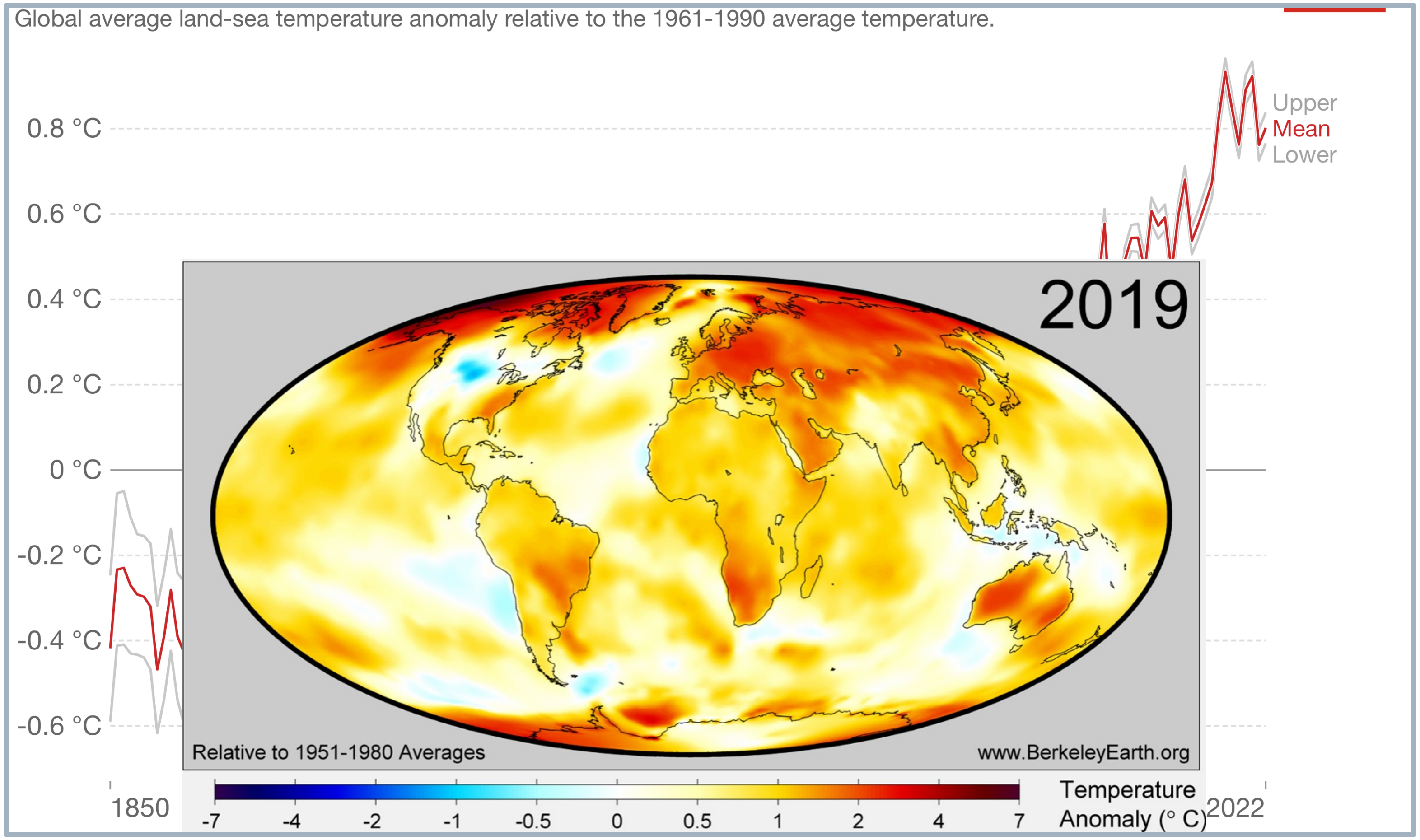
Average temperature



Global average temp has increased by >1C since pre-industrial time

https://ourworldindata.org/co2-and-greenhouse-gas-emissions

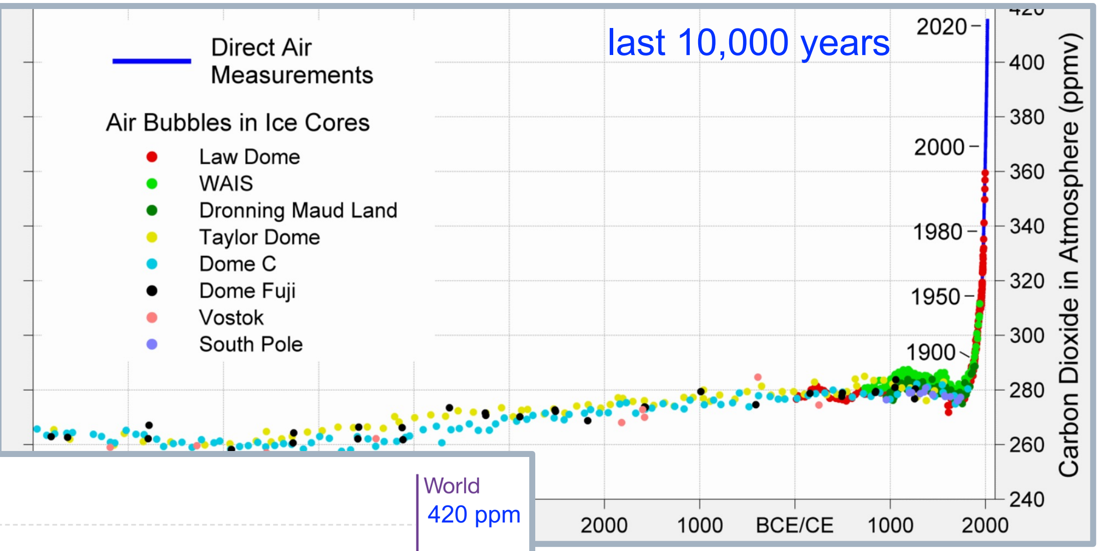
Average temperature



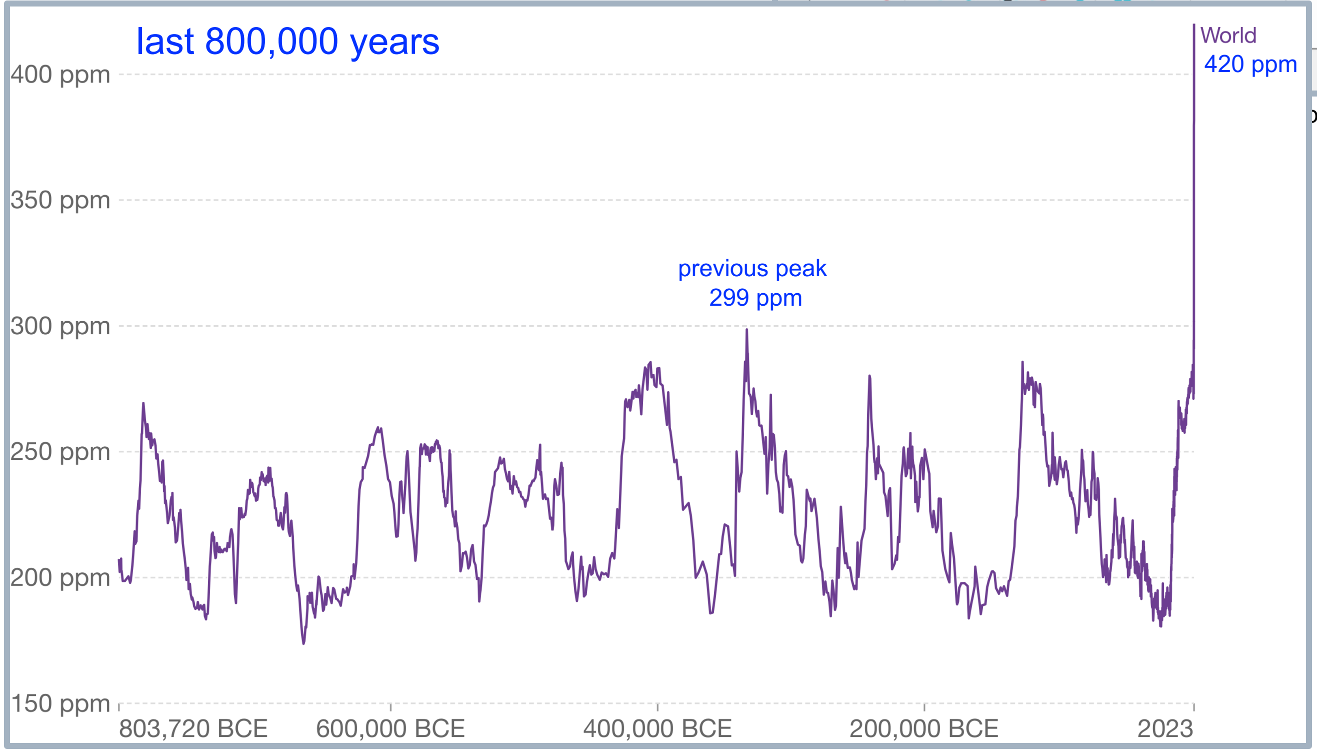
Local temperature can be much warmer than global average

https://ourworldindata.org/co2-and-greenhouse-gas-emissions

Atmospheric CO2

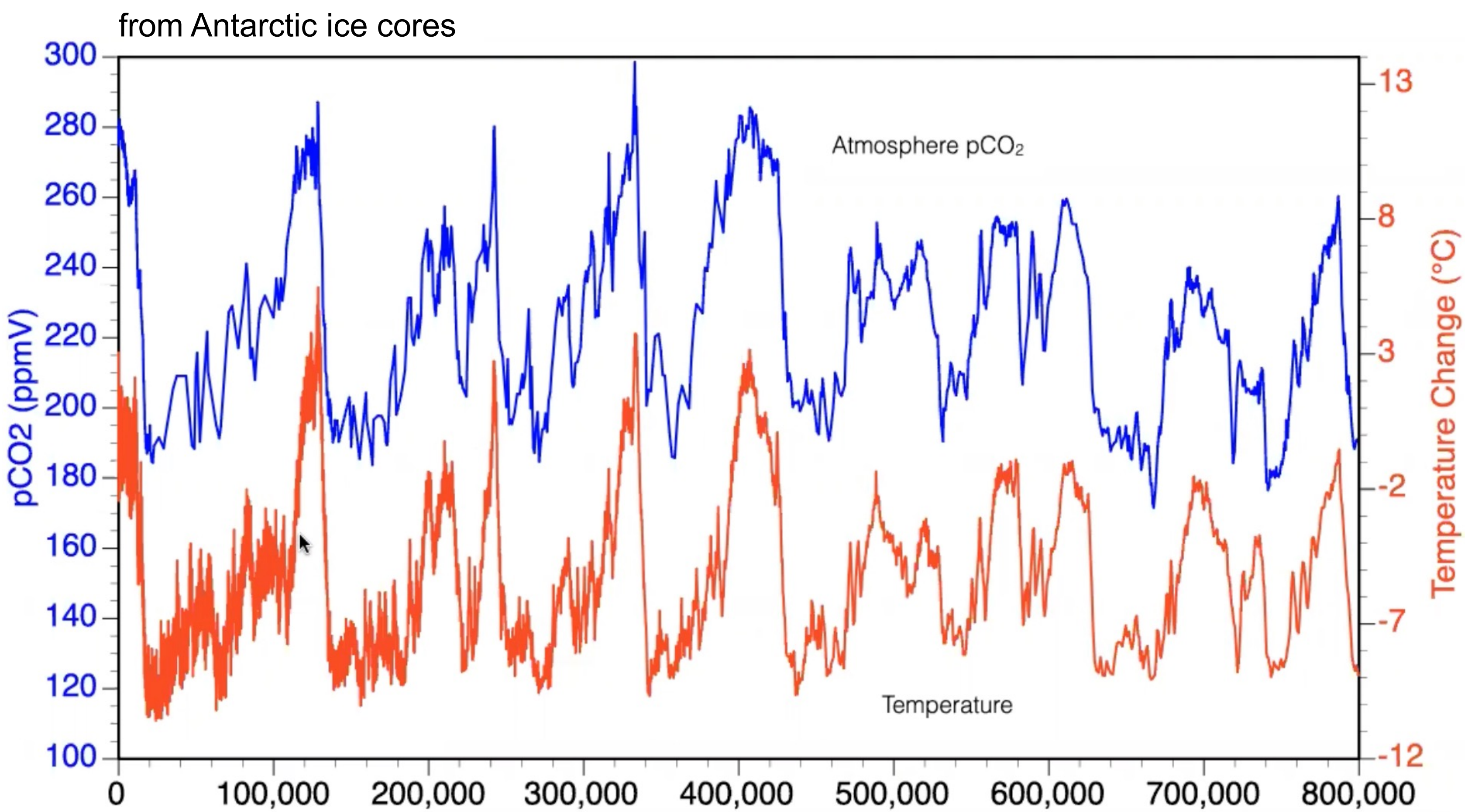


berkeleyearth.org/dv/10000-years-of-carbon-dioxide/



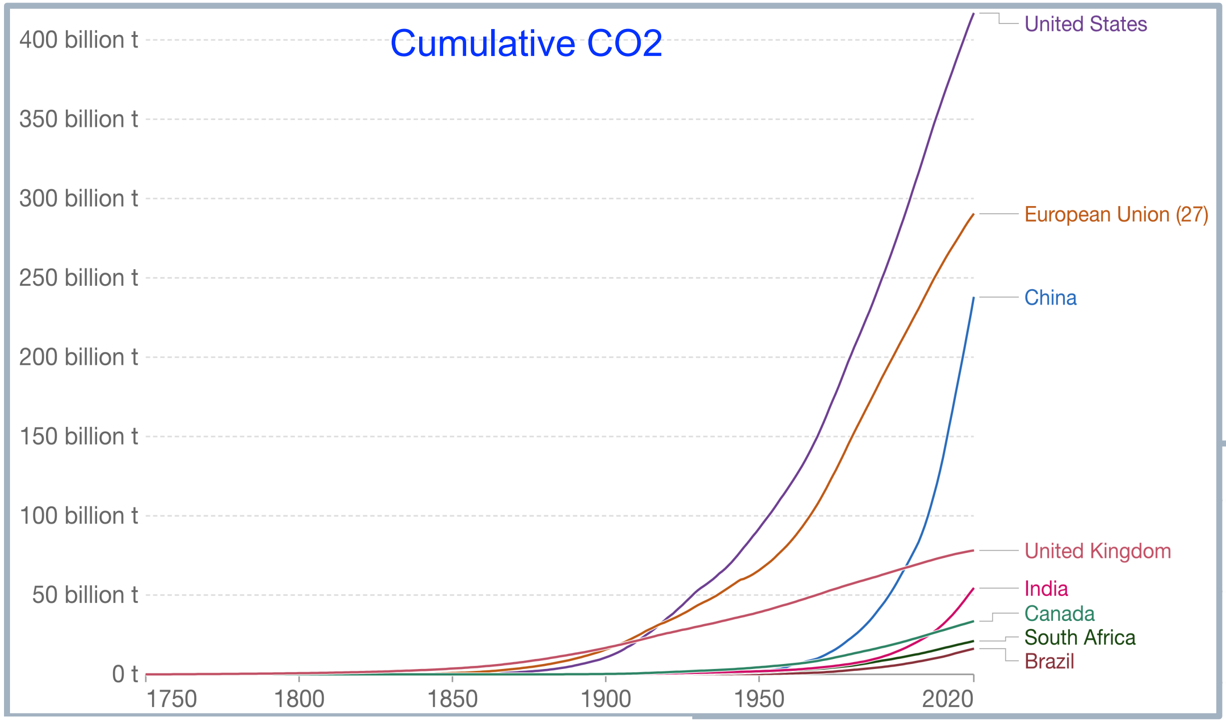
https://ourworldindata.org/co2-and-greenhouse-gas-emissions

CO2 and temperature

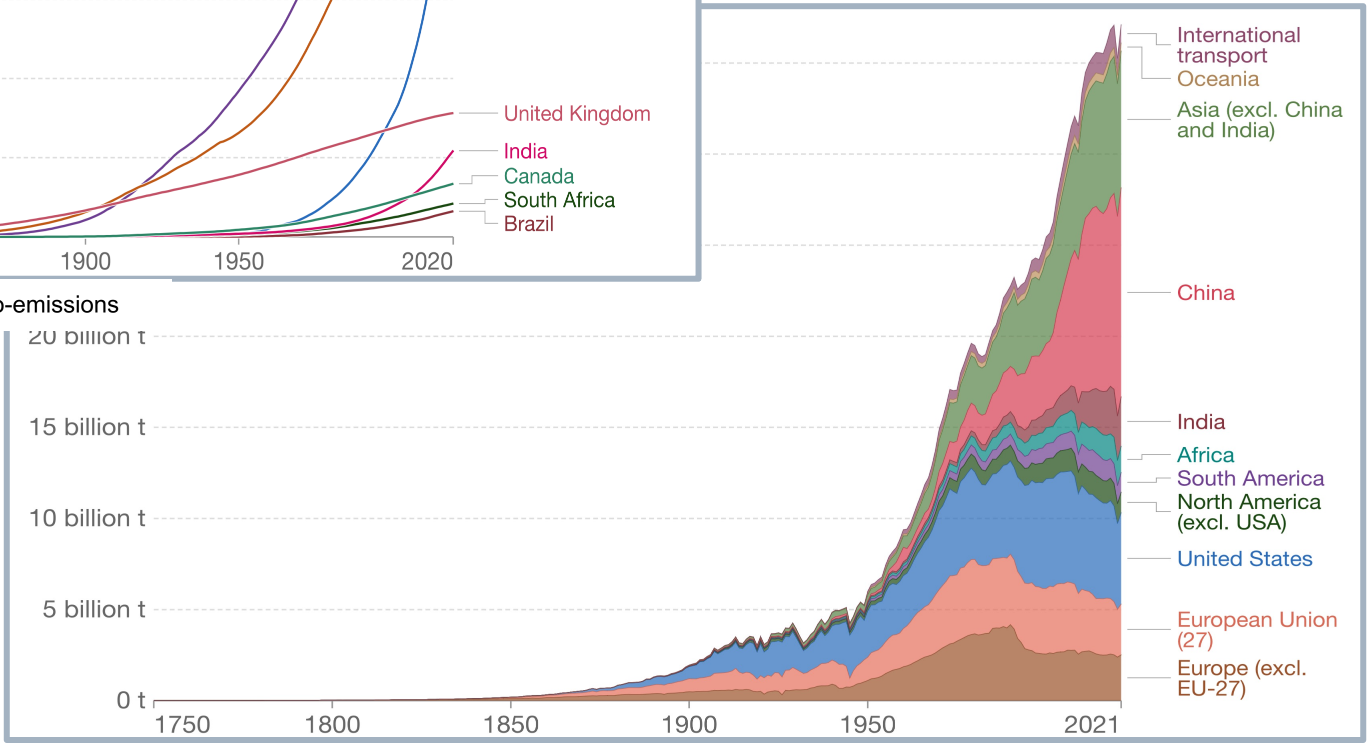


[Jess Adkins, Caltech, CS179, 2022]

CO2 emissions

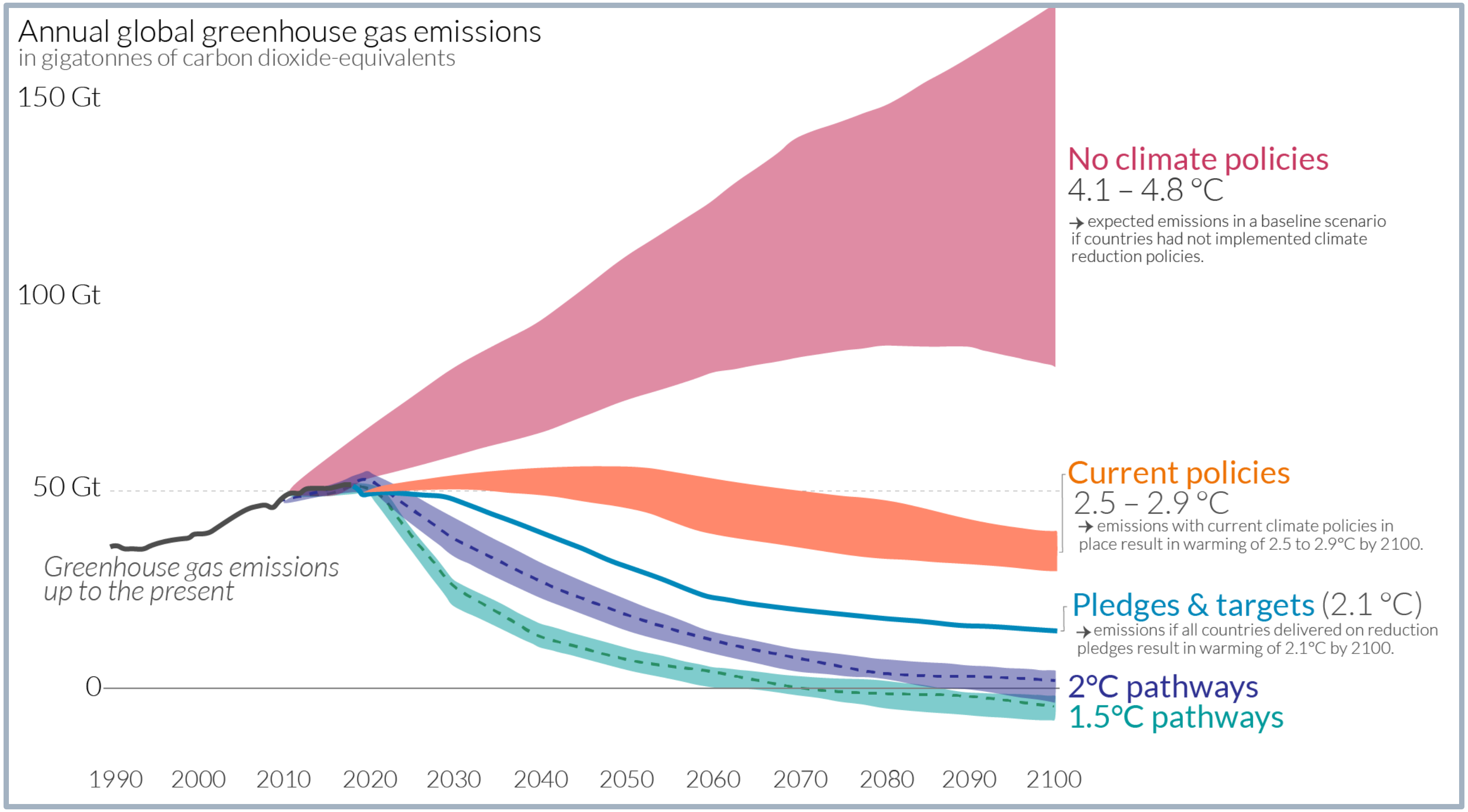


https://ourworldindata.org/grapher/cumulative-co-emissions



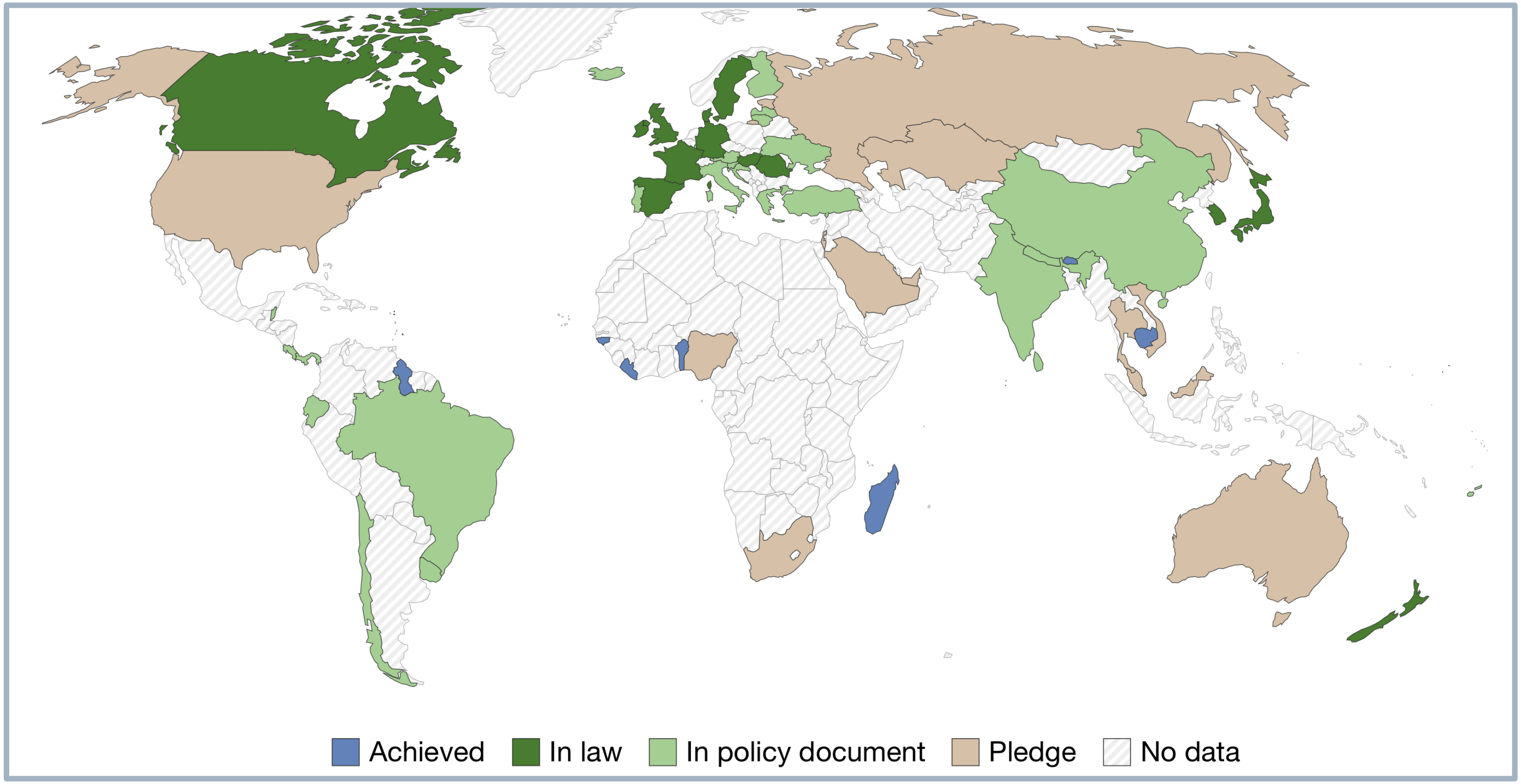
https://ourworldindata.org/co2-and-greenhouse-gas-emissions

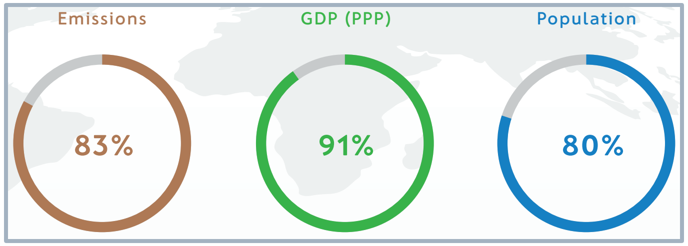
GHG pathways



https://ourworldindata.org/co2-and-greenhouse-gas-emissions

Net zero GHG pledges

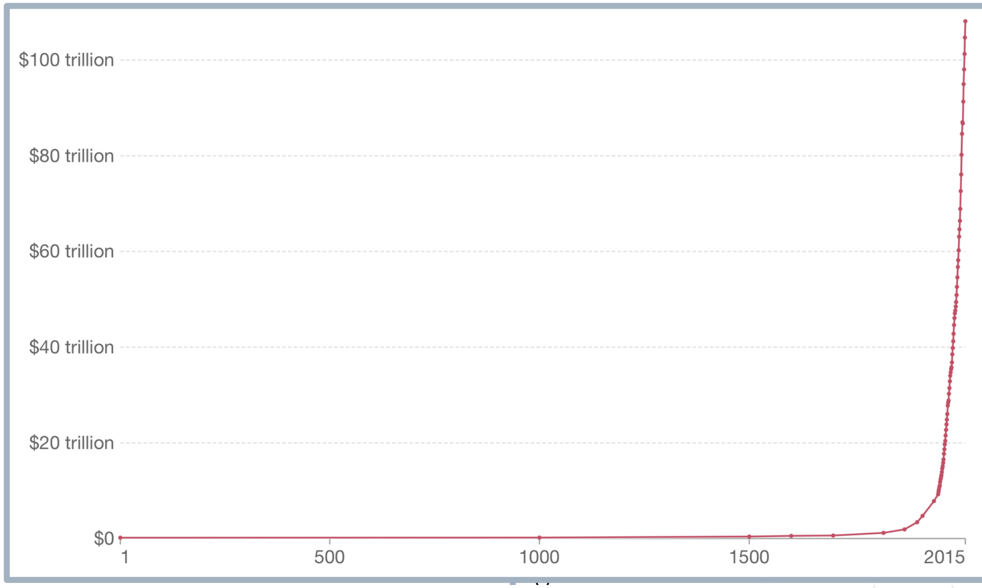




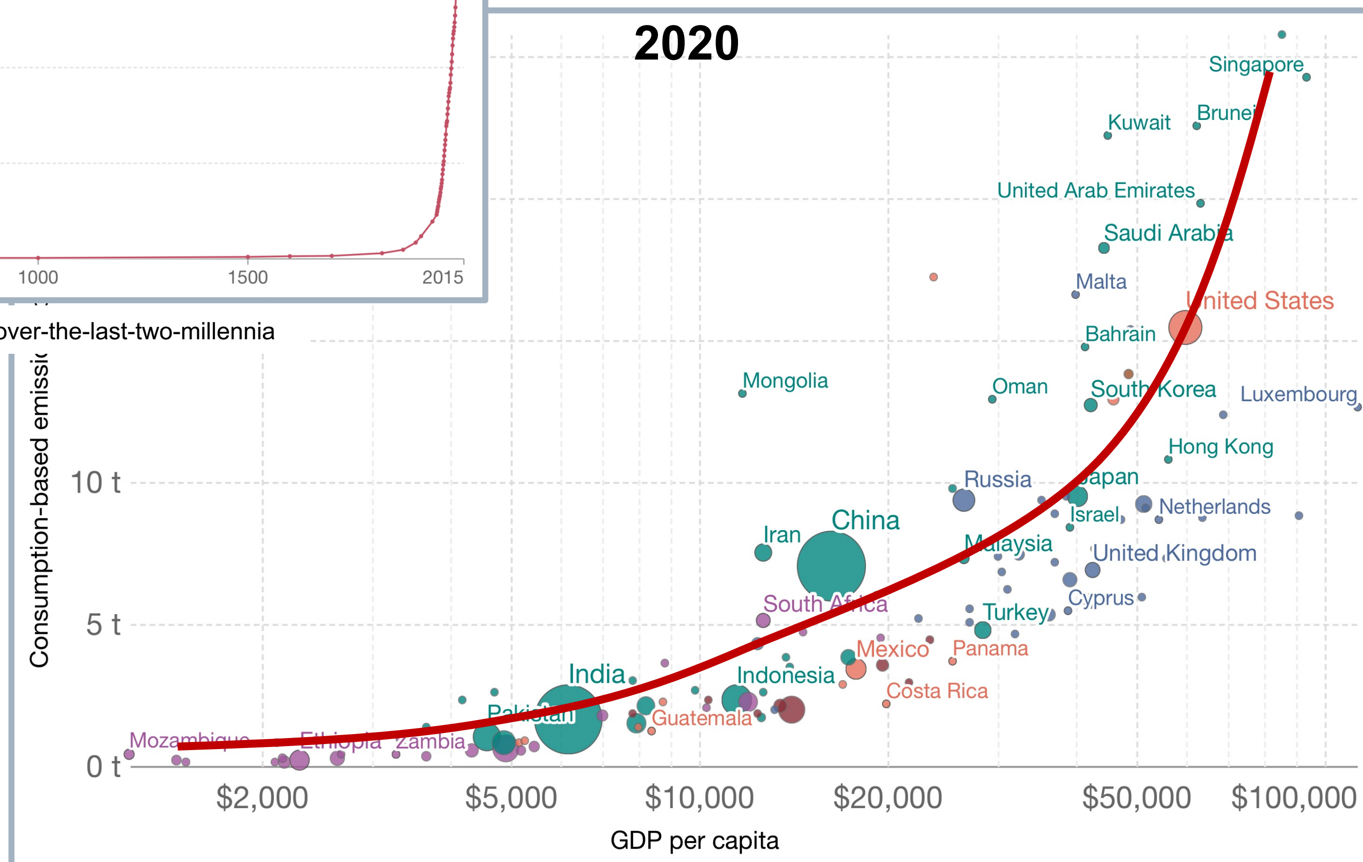
% coverage of net zero GHG pledges (Oxford 2022)
(2019: coverage = 16% GDP)

https://ourworldindata.org/co2-and-greenhouse-gas-emissions

CO2 and GDP



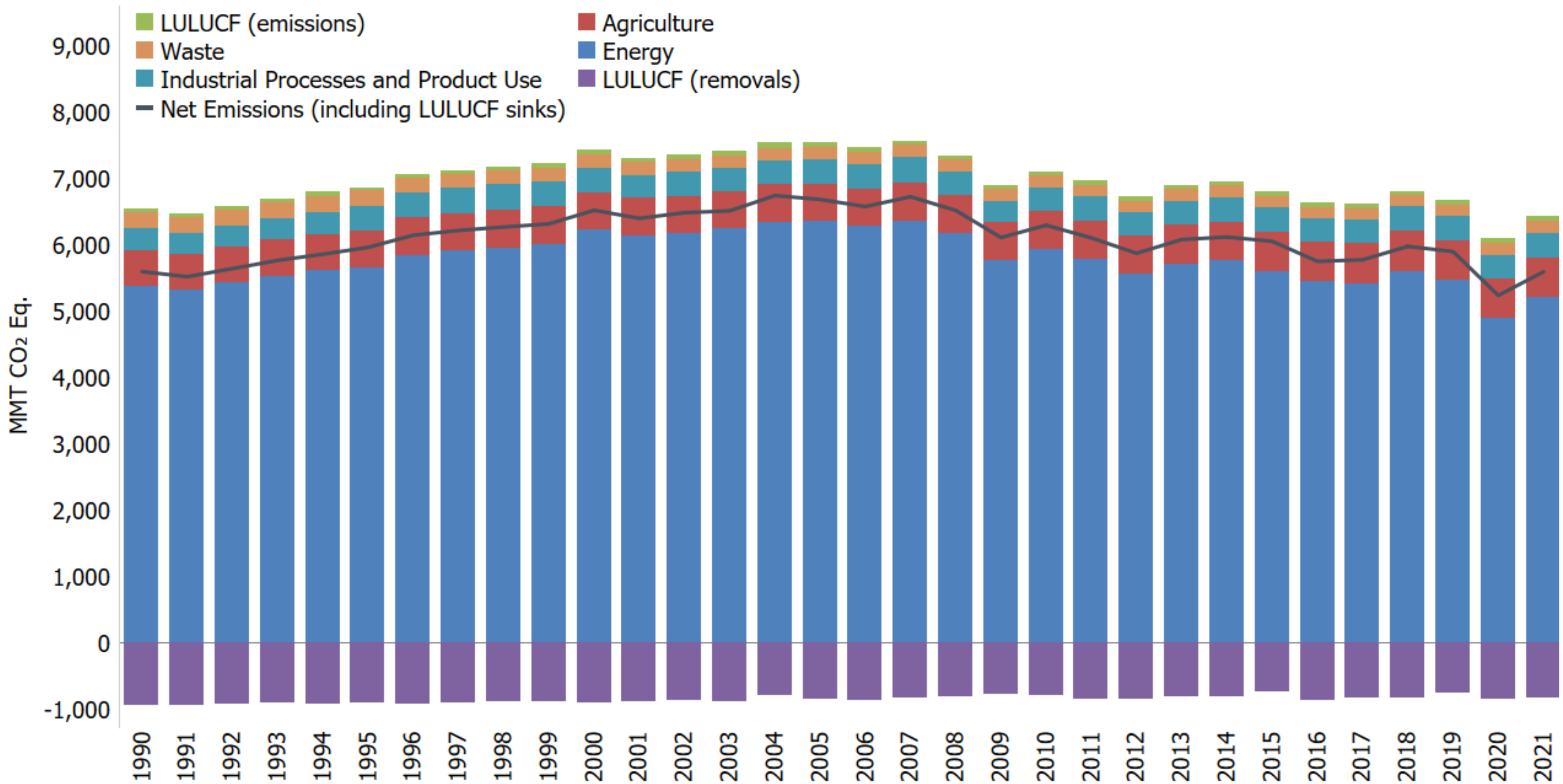
https://ourworldindata.org/grapher/world-gdp-over-the-last-two-millennia



x-axis is not time:
energy inequality !

https://ourworldindata.org/grapher/consumption-co2-per-capita-vs-gdppc

GHG and energy use

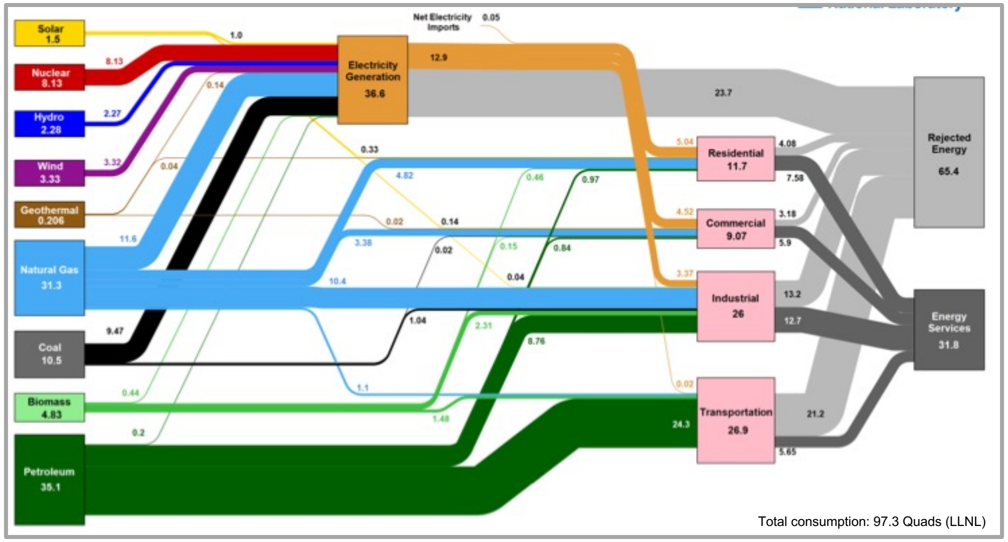


Energy use emitted 82% of total greenhouse gas emissions in US in 2021 (EPA)

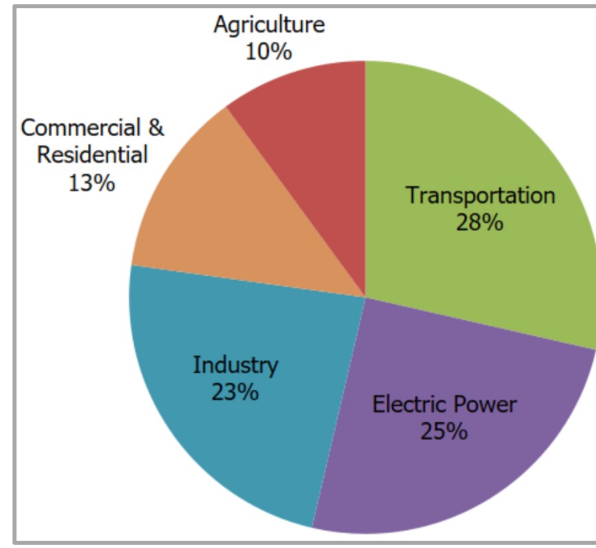
https://www.epa.gov/system/files/documents/2023-04/US-GHG-Inventory-2023-Main-Text.pdf

Electricity gen & transportation

2021 consumption: fossil 79.0%; renewables 12.5% (US EPA)



https://flowcharts.llnl.gov/sites/flowcharts/files/2022-09/Energy_2021_United-States.pdf



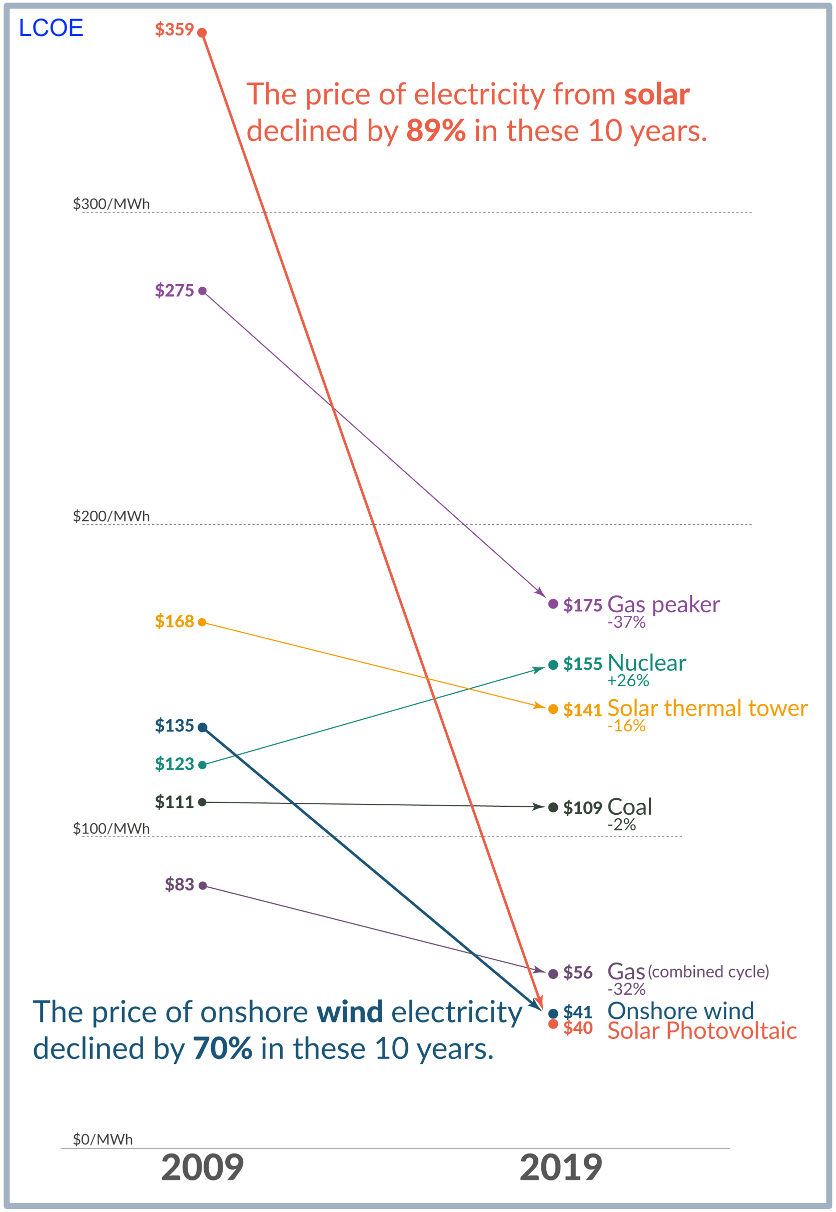
https://www.epa.gov/ghgemissions/sources-greenhouse-gas-emissions#transportation

Electricity generation & transportation in US:

- Consume 65% of all energies in 2021 (US EPA)
- Emit 53% of all greenhouse gases in 2021 (US EPA)

both numbers are lower than 2019 numbers by only ~2% !

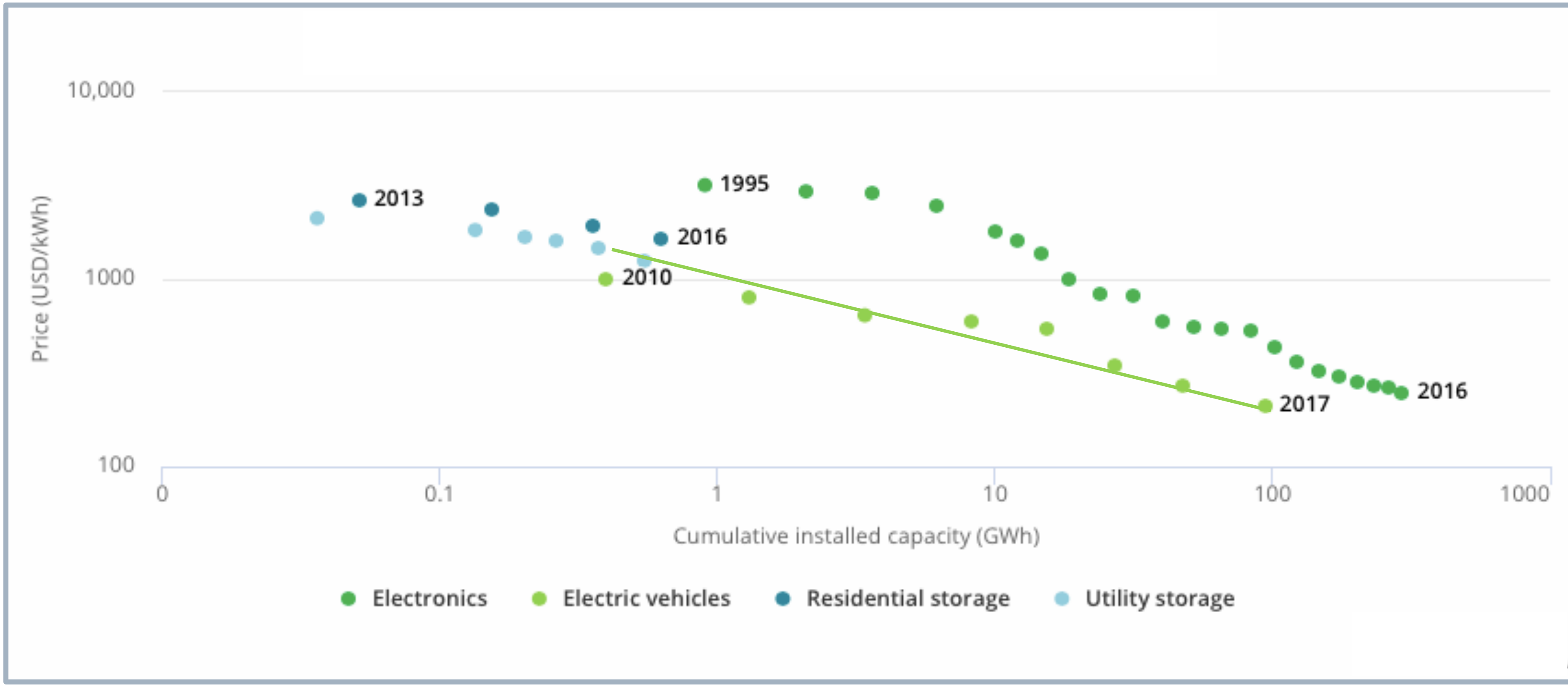
Electricity cost



PV & on-shore wind have lowest LCOE

https://ourworldindata.org/cheap-renewables-growth

Li-ion battery cost



Electric vehicle battery:

- 2010: $1,000 / kWh
- 2016: $ 275 / kWh
- 2030e: $ 73 / kWh (Bloomberg New Energy Finance 2016)

Source: https://www.iea.org/gevo2018/

Some technical challenges

Numerous research needs/opportunities

- Many experts in this NSF Workshop !

Some technical challenges

Integration of grid & mobility

Panel 1

- Technologies, economics, deployment

Data, learning, control

Panels 2, 4

- Unknown/unreliable models, uncertainty, scalability, multiple timescales, reliability

Equitable development

Panel 3

- Per capita CO_2(consumption): US(15.5t) vs Mexico(3.4t), AU(13.8t) vs Indonesia(2.3t), Switzerland(12.4t) vs Portugal(4.7t) (D. Kammen)

Inverter-based resources

- Dynamics, stability, scalability

Economics

- NEM: PV+EV charging+storage, aggregation; hosting cap. (L. Tong)

Architecture

- Layering, constraints that deconstrain, RYF [John Doyle, Caltech]

A policy challenge

1. Infrastructure challenge
   - Can necessary infrastructure be built in time for net zero by 2050?

2. Equitable development challenge
   - Can inequity be overcome while transition at the necessary speed?

Two most important determining factors

- Political system and political culture
- Economic system and economic culture
- Technology is the easy part

Political & economic systems

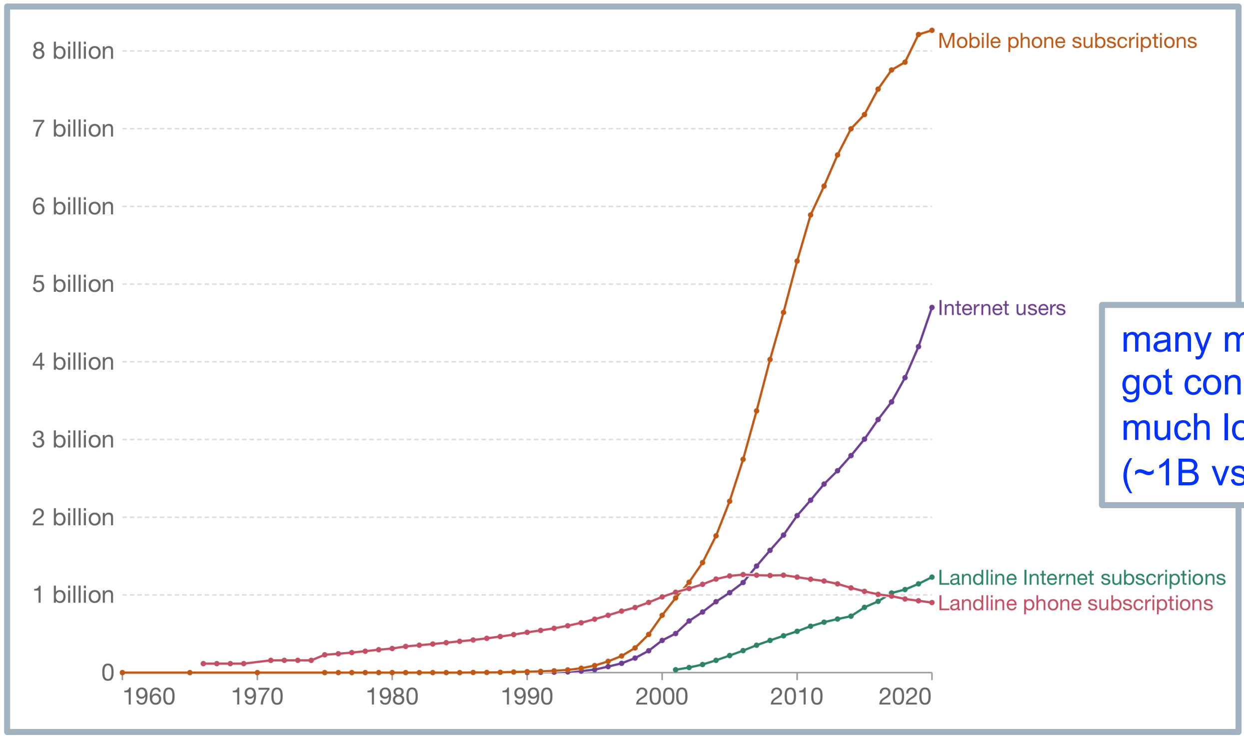
- Authoritarian, central planning; e.g. China (central planning+market)
- Democratic, market based; e.g. US
- Convex combinations; e.g. Nordic countries

Are there examples of transformation of similar scale and speed?   Yes!

Example 1: Internet

Telephony → Internet: US

- Communication network transformed from telephony to Internet over 2-3 decades from 1990s
- Centered in Silicon Valley and spread across the globe
- … with implications way beyond communications, & changing multiple industries
- Driven almost entirely by private sector



many more people got connected b'cos of much lower comm cost (~1B vs ~8B ppl)

https://ourworldindata.org/internet#internet-access

Example 1: Internet

New platform for communication (& innovation)

- Cheaper, better, faster communication

Transformation has been very profitable

- Both transfer from and growth of existing markets, e.g., telephone services, advertisements, retails
- New markets: video conferencing/sales/services/education, etc.

Challenge: for market to drive energy transition

- Are there large existing markets for transfer (besides ICE cars)?
- Will large profitable markets be created?

The five largest companies in 2006 . . .

1	Exxon Mobil	**$540** BILLION MARKET CAP
2	General Electric	**463**
3	Microsoft	**355**
4	Citigroup	**331**
5	Bank of America	**290**

. . . and by April 20, 2017

1	Apple	**$794**
2	Alphabet (Google)	**593**
3	Microsoft	**506**
4	Amazon	**429**
5	Facebook	**414**

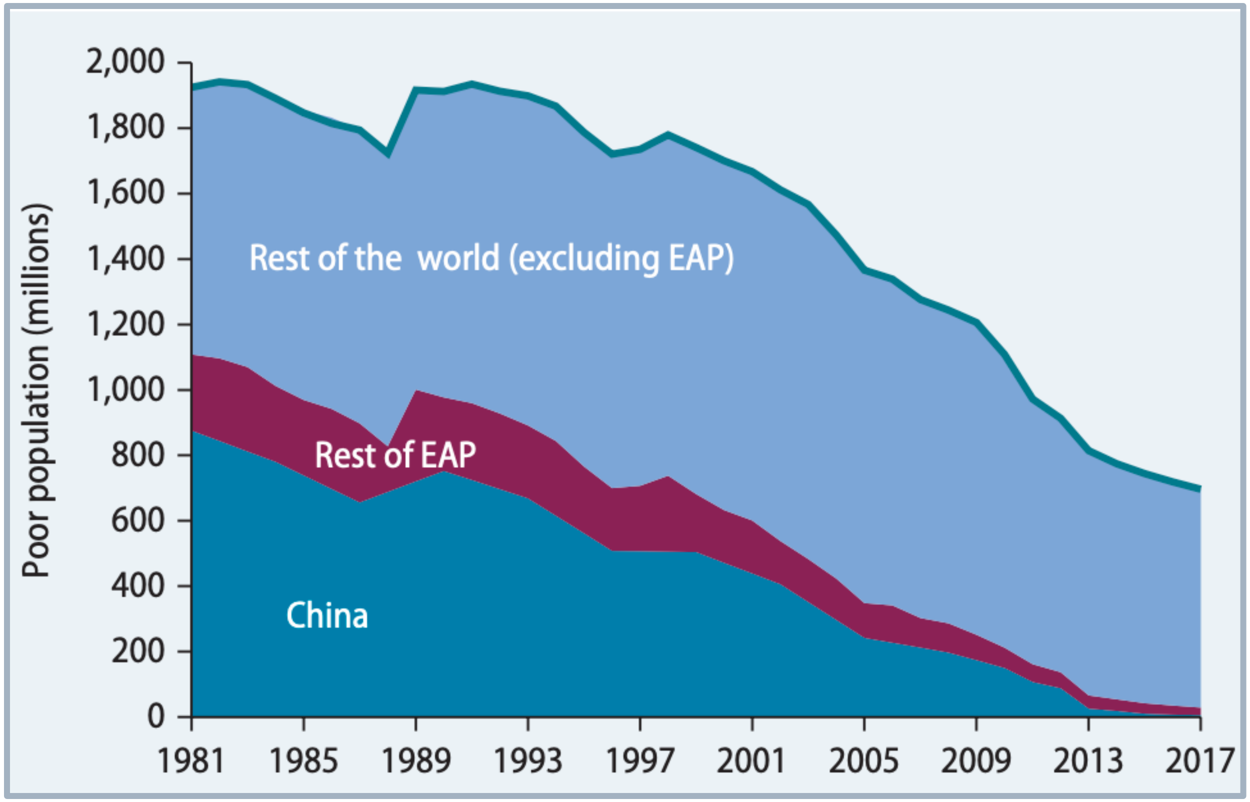
Example 2: Poverty reduction

Extreme poverty reduction: China

- Lifted ~800M people out of extreme poverty since 1981
- Account for almost ¾ of global extreme poverty reduction

Challenge: for equitable development

- Can we combine, or balance, the strengths of two extreme points?
- e.g. protection of minority vs benefits to majority



Int'l poverty line: US$1.9/person-day (2011 PPP) (EAP = East Asia & Pacific)

Lugo, Niu, and Yemtsov 2021

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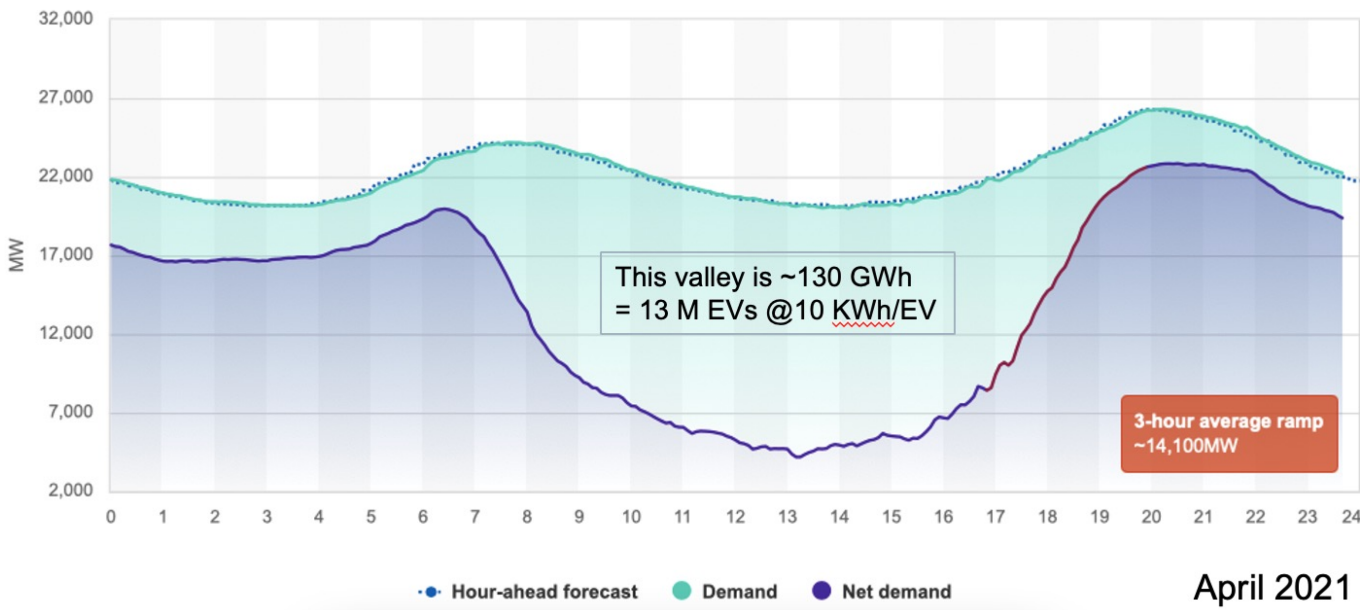
Workplace charging

CA commitment

- ~~50%~~ 60% renewables by 2030, 100% by 2045
- 1.5M ZEV by 2025, 5M by 2030 (CA has ~15M cars)



Drivers <u>twice as likely</u> to get EV when workplace charging is available
(EDF Renewables survey Feb 2018)

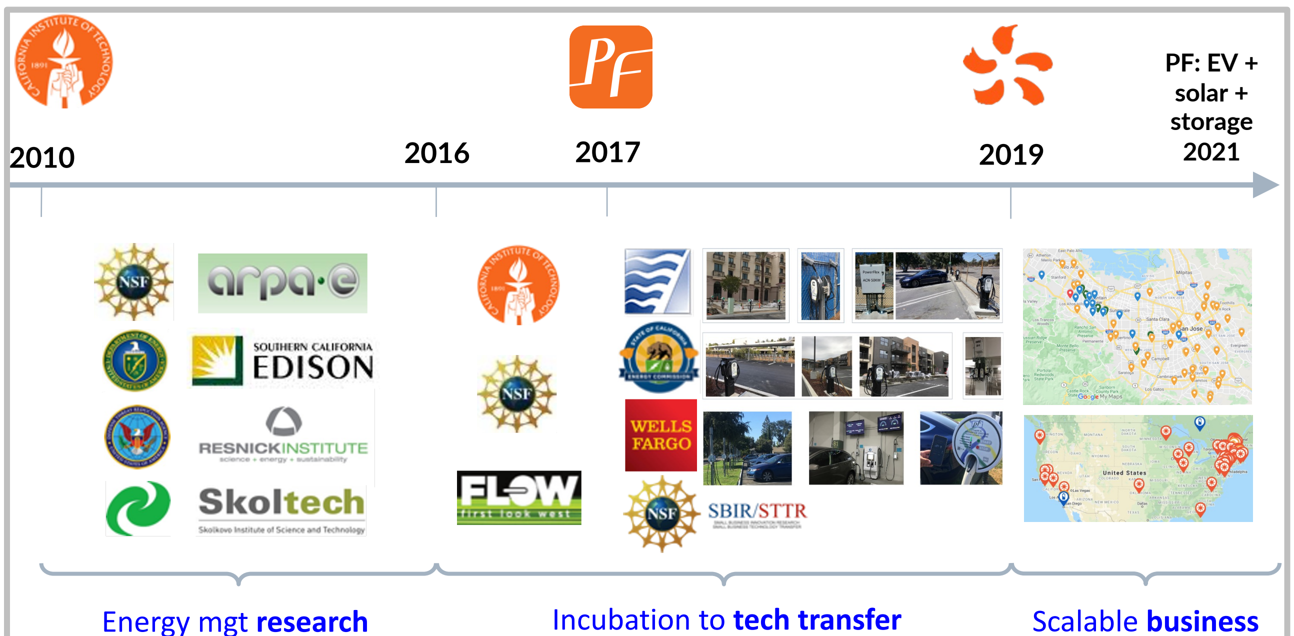
EV charging: research → impact

Theory and algorithms

1. **Broad power systems research (since 2010)**
   Nonconvex optimization, control & dynamical systems, distributed real-time algorithms

2. **Application to EV charging**
   Optimal decentralized protocol for EV charging (IEEE Trans. Power Systems, 2013)
   Theorem: Online LP attains offline optimal (IEEE PES General Meeting, 2017)

Industry	Online LP	Theoret. max
28%	53%	54%



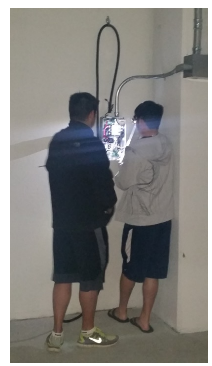
EV charging: research → impact

Testbed → deployment

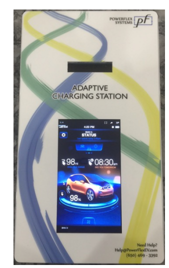
3. **First pilot: Caltech garage (2016)**
   By July 2020: delivered 3M+ electric miles, avoided 1,000 tons of CO_2e

4. **Caltech startup: PowerFlex (2017)**
   Value proposition: Enable large-scale EV charging by reducing capital & operating costs
   Acquired by EDF Renewables to scale business

debugging

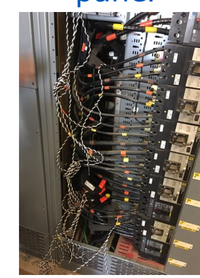


charger



transformer & subpanels

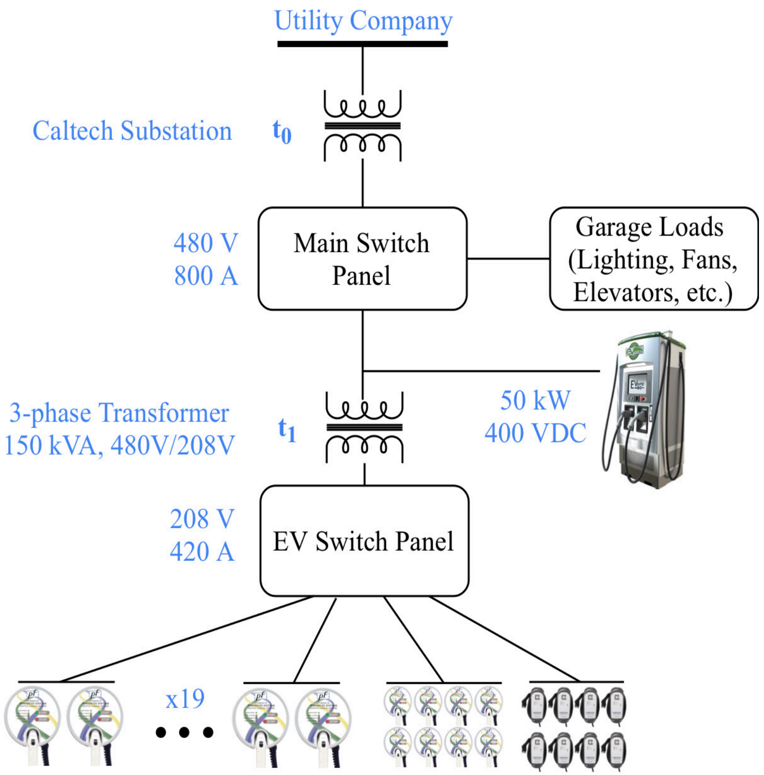
main panel



G. Lee (Co-founder)

powerflex
EDF Renewables

Caltech ACN: physical system



Caltech ACN: cyber system

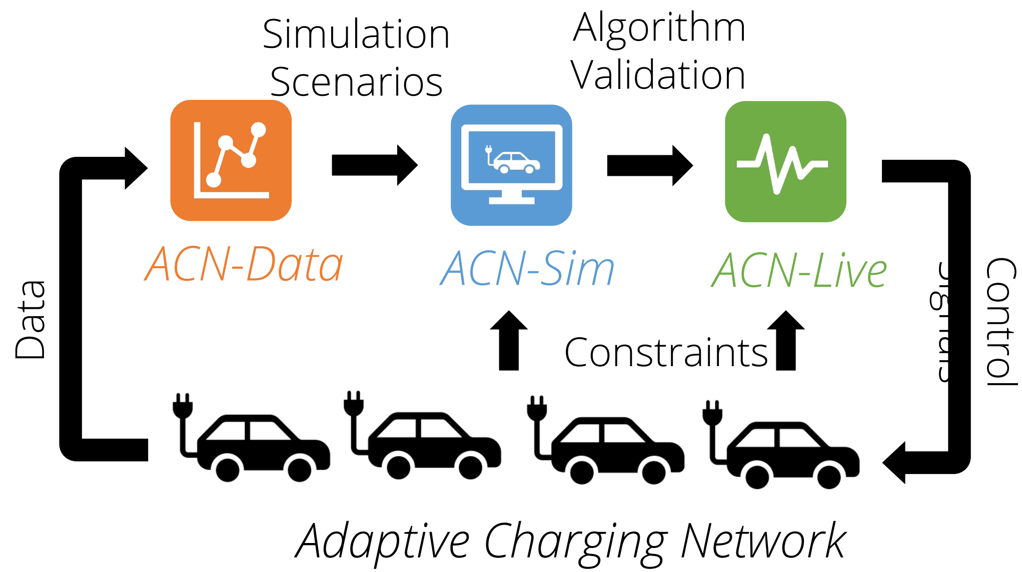
Model predictive control: QCQP

$$
\begin{aligned}
\max_{r} \quad & \sum_{v} \alpha_v u_v(r) \\
\text{s.t.} \quad & 0 \;\leq\; r_i(t) \;\leq\; \bar{r}_i(t) \\
& \sum_{t \in \mathcal{T}} r_i(t) \;\leq\; e_i \\
& \left| \sum_{i \in \mathcal{V}} A_{li} r_i(t) e^{j\phi_i} \right| \;\leq\; c_{lt}(t)
\end{aligned}
$$

Highly customizable QCQP
- objectives: cost, PV, asap, regularization
- constraints: energy, deadlines, capacities
- determine charging rates for all EVs

Caltech ACN: open research tool

- ACN-Data
- ACN-Sim
- ACN-Live (HW-in-the-loop)



open-sourced and extensible

Lee, Li, Low. ACN-Data: analysis and applications of an open EV charging Dataset
ACM e-Energy, June 2019

Lee, Johansson, Low. ACN-Sim: an open-source simulator for data-driven EV charging research
IEEE SmartGridComm, October 2019

ACN research portal

Lessons learnt

Smart EV charging

- R&D to extract untapped value intrinsic to EV charging
- Critical to maintain broad theory research
- Translation of energy R&D is hard

Workplace energy systems

- Large untapped value in current system
- Bigger & more complicated system, more expensive infrastructure, more difficult & diverse technical challenges

Caltech energy systems

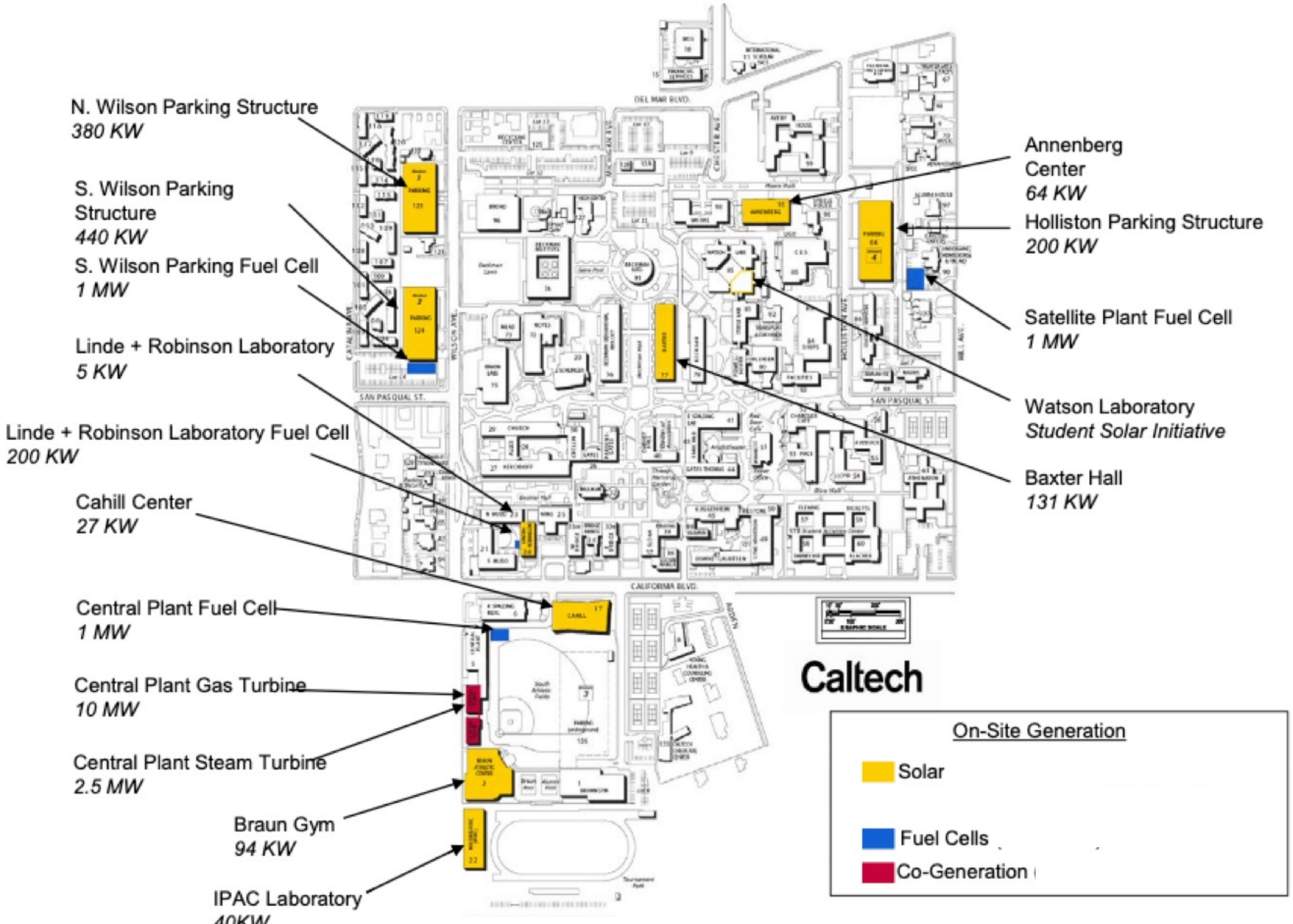
Caltech microgrid

- ~200,000-people city
- >100 commercial-size buildings

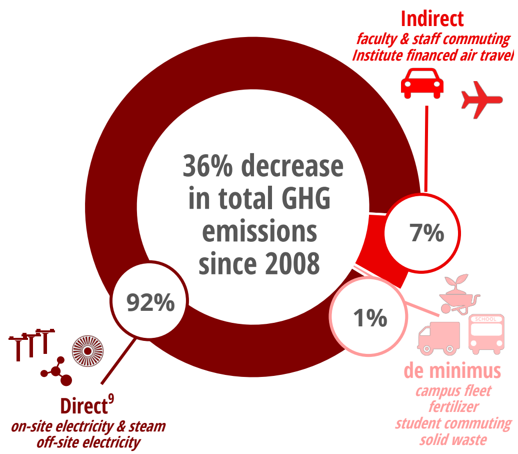
- 3 grid interconnections
- 4 substations
- 20 MW peak load

- 2.1 MW onsite solar
- 4 MW NG fuel cells
- 12.5 MW gas co-gen

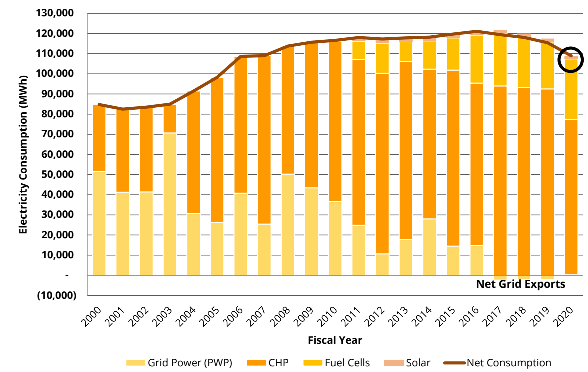
- Chilled water distribution
- Fossil-based steam and HW distribution



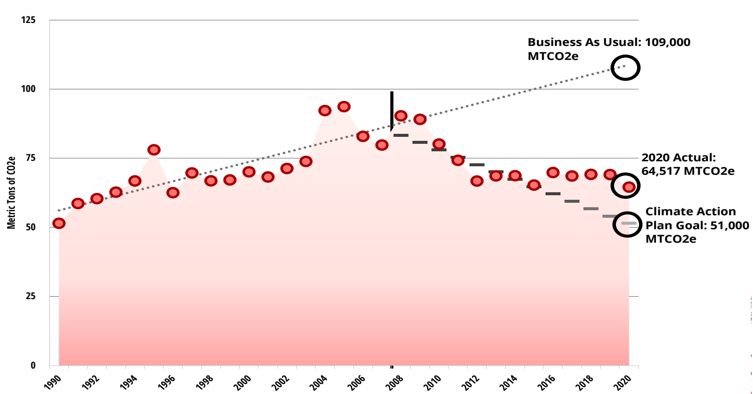
Opportunities



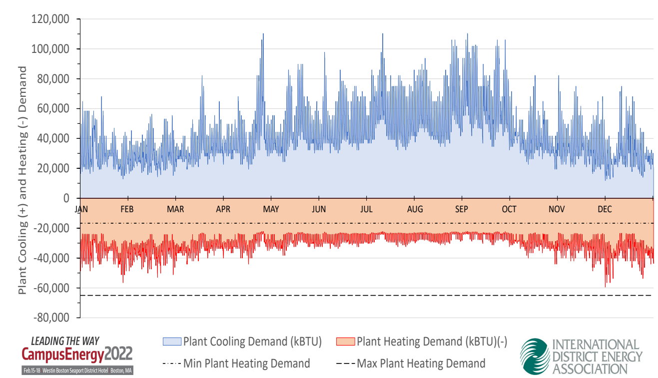
Energy is a 92%-opportunity to reduce GHG



Co-gen generated 78% of electricity consumed in 2020



Further reduction needs to retire campus co-gen



Simultaneous heating and cooling demands

Caltech Facilities, 2020 Sustainability Report
Christman & Warring 2022

Basic idea

Integrate and holistically optimize operation of electric, heating & cooling systems

- They operate independently today
- HRCs to provide net heating & cooling demand

Exploit storage (batteries & thermal) and HRCs to shape electricity demand

- To adapt to random fluctuations in demand, prices & CO_2 intensity
- Greatly reduces capital and operating costs for 24/7 CO_2 neutrality

Campus decarbonization

Infrastructure (Caltech Admin/Facilities)

- Retiring co-gen, electrify hot & chilled water, HRCs, thermal storage, batteries, tunnels & pipes

Data (Caltech DER testbed)

- Comprehensive reliable data on electric, cooling & heating systems, cost & emission data

Theory, algorithms & prototypes (focus of R&D)

- Theory & algorithms for real-time learning, control & optimization of DERs
- Software prototypes (Digital Twin)

Pilot & deployment

- Work with Caltech Facilities
- Work with industry

R&D: theory, algorithms, prototypes

Layer	R&D
Control	Optimization-based decision making for planning and operation in uncertainty
Learning (Digital Twin)	Data-driven continuous learning, identification & tracking of system models & current states
Data (Meter Caltech)	Testbed to provide real-time comprehensive & reliable data

Open problems (examples)

- Data-driven stochastic optimization
- Data-driven real-time OPF

- Network identification
- Aggregate flexibility & control

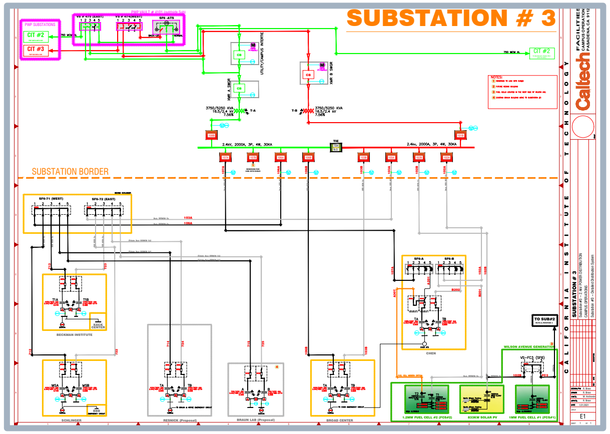
Expected outcomes:

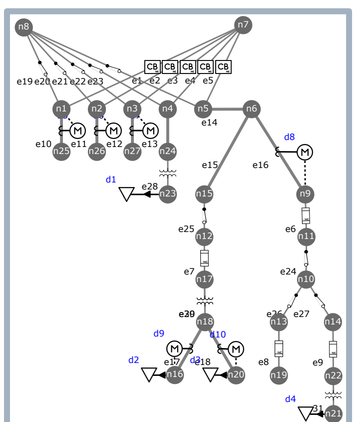
- DER live testbed: PV, building, EV, storage, monitoring system (meters & software)
- Theory & algorithms for learning, control, and optimization of networked DERs
- Software prototypes of some algorithms

DER testbed

Substation 3 (16.5kV/2.4kV/480V)

- Buildings
- Rooftop PVs
- Fuel cells
- EV chargers

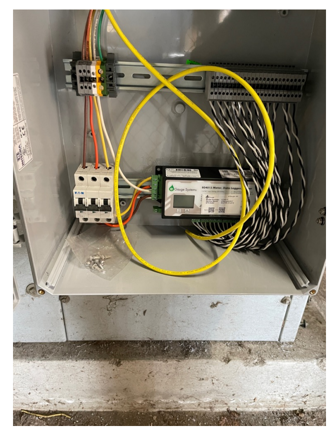




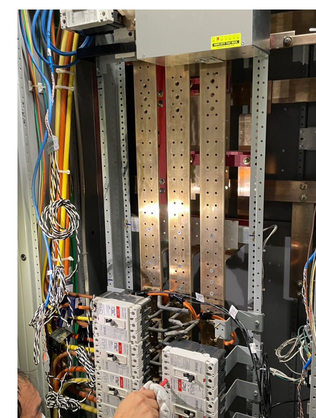
digital circuit diagram

electric room

metering cabinet



meters, CTs



3p voltage taps

Network identification

$$I \;=\; Y V \text{ where } Y_{jk} = \begin{cases} -y^s_{jk}, & j \sim k \quad (j \neq k) \\ \sum_{l: j\sim l} y^s_{jl}, & j = k \\ 0 & \text{otherwise} \end{cases}$$

Y is a complex symmetric (Laplacian) matrix with zero row sums

Learning Y from data

- Numerous control & optimization schemes assume Y is known
- But Y often unavailable or unreliable in distribution systems (e.g., Caltech does not know Y)
- Little is known about analytical properties of Y (e.g., invertibility only published in [Yuan et al 2022, Torizo & Molzahn 2022, Low 2022])

State of the art

- Full measurement: many schemes based on regressions, entropy, sparse recovery, graph processing, …
- With hidden nodes (for radial networks) ?

Network identification with hidden nodes

At each time t :

$$\begin{bmatrix} I_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix}$$

0 injection at hidden node

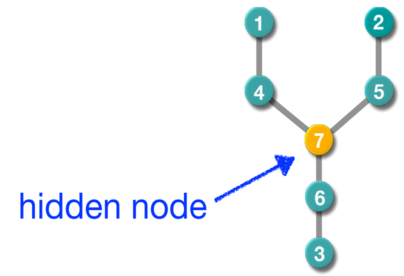
measured nodes

hidden nodes

Suppose we can exactly recover $\overline{Y}$ from $\left(V_i(t), I_i(t)\right)$ at $i \in M$

$$I_1(t) = \overline{Y} V_1(t)$$

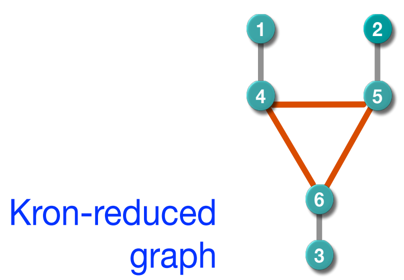
with $\overline{Y} := Y_{11} - Y_{12} Y_{22}^{-1} Y_{12}^{\mathsf{T}}$



Lemma

Kron-reduced admittance matrix $\overline{Y}$ exists, if lines are resistive & inductive

(Note that Y is complex symmetric !)



Network identification with hidden nodes

At each time t :

0 injection at hidden node

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measured nodes

hidden nodes

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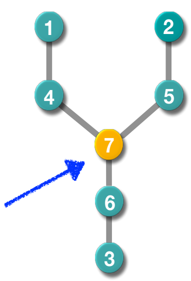
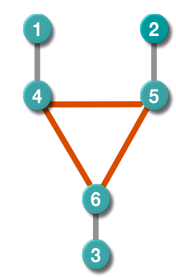
hidden node



Can we identify Y from $\bar{Y}$ for radial networks ?

Kron-reduced graph



Network identification with hidden nodes

At each time t :

0 injection
at hidden node

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measured nodes

hidden nodes

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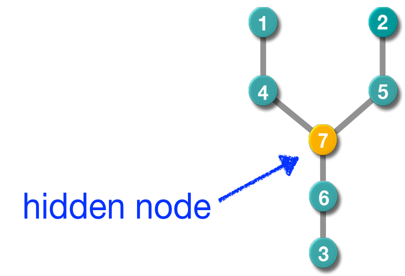
$$I_1(t) = \overline{Y} V_1(t)$$

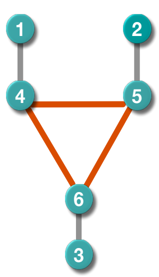
with $\overline{Y} := Y_{11} - Y_{12} Y_{22}^{-1} Y_{12}^{\mathsf{T}}$



Can we identify Y from $\overline{Y}$ for radial networks ?

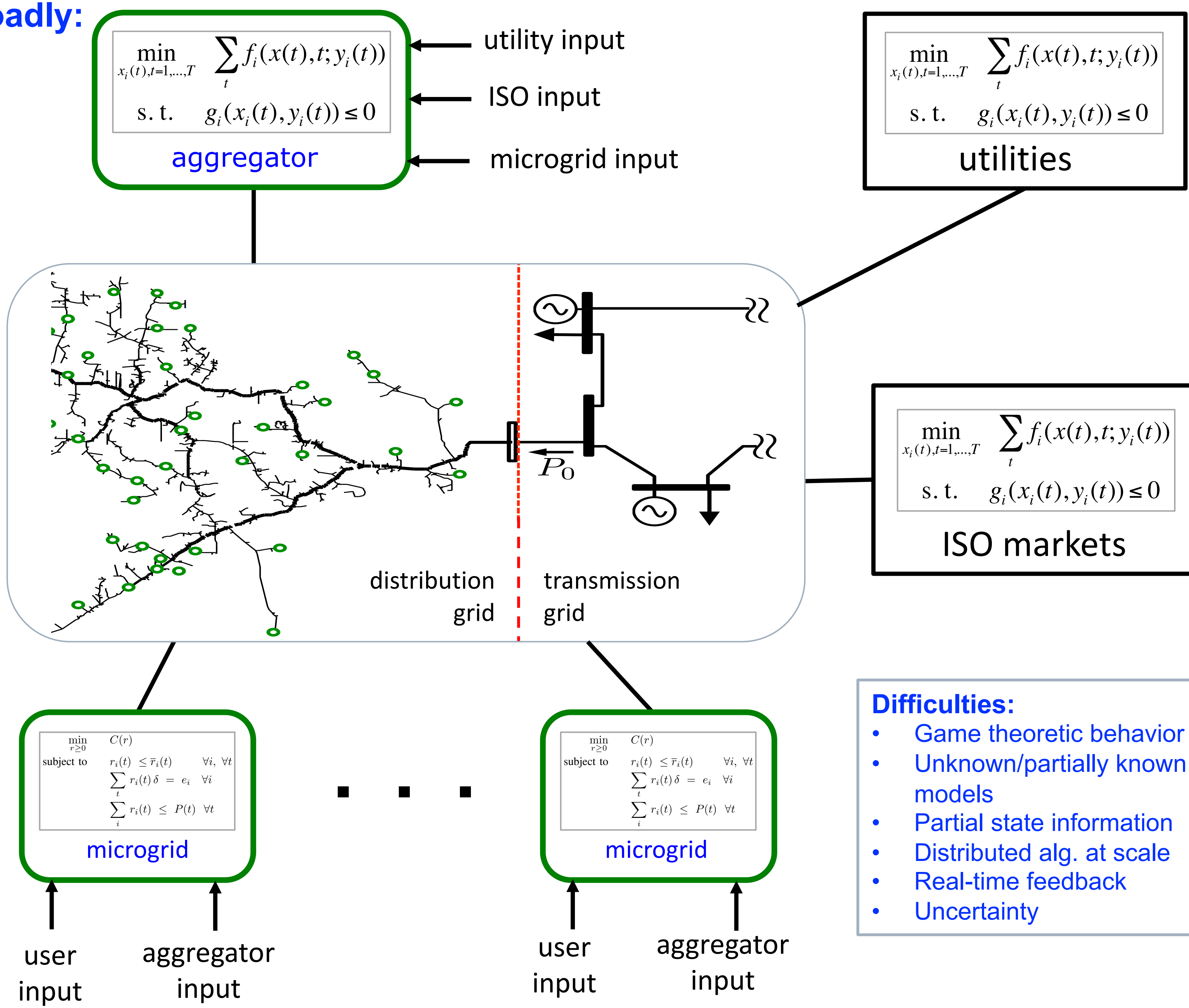
<u>Theorem</u>: Yes ! [Yuan et al 2022]

Exactly recover both topology and impedances for radial nks
Constructive proof



Kron-reduced
graph

More broadly:



Difficulties:

- Game theoretic behavior
- Unknown/partially known models
- Partial state information
- Distributed alg. at scale
- Real-time feedback
- Uncertainty

Lessons learnt

Most papers implicitly use single-phase models

- Balanced 3-phase systems have single-phase equivalents

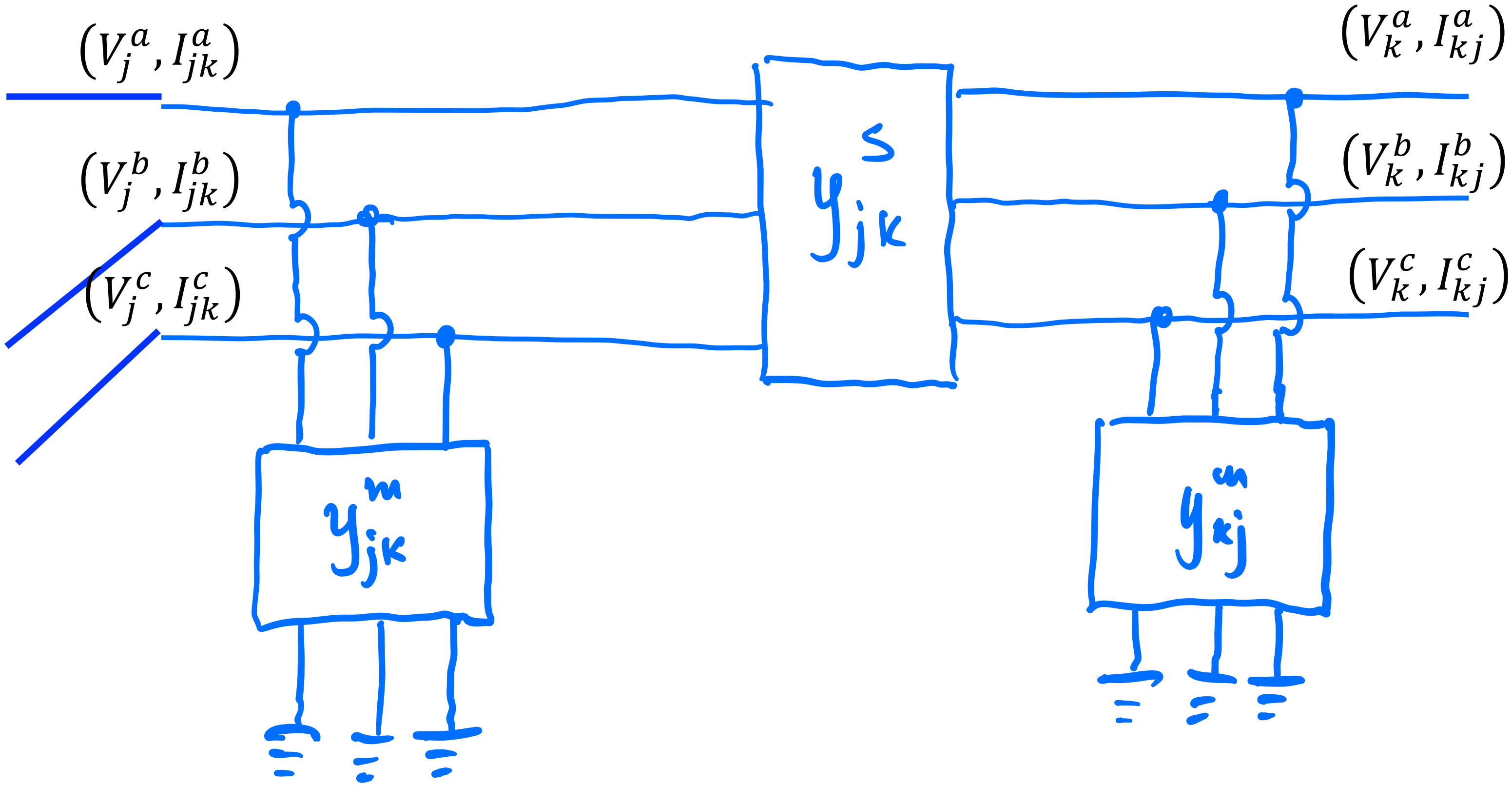
Single-phase models applicable for many purposes

- Transmission system applications
- For illustrating basic ideas and analysis of most algorithms (unbalanced 3-phase models structurally similar to 1-phase models)

Unbalanced 3-phase modeling needed

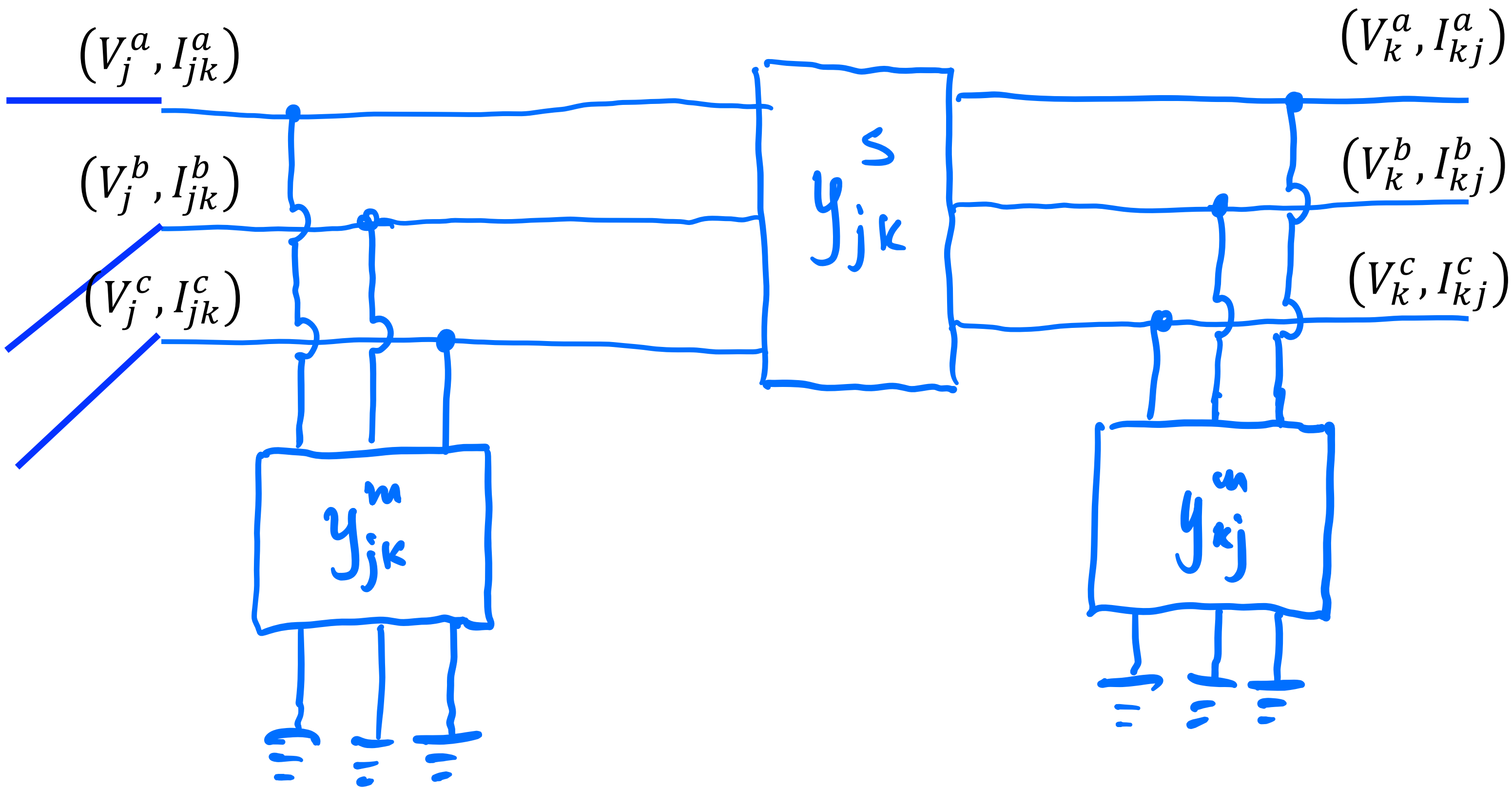
- When control & optimization are explicitly on single-phase devices making up a 3–phase device
- For implementation in real systems when phases are not balanced

Lessons learnt



- Many models assume terminal currents $\left(I_{jk}^a, I_{jk}^b, I_{jk}^c\right)$ are controllable (optimization vars)
- Extension to 3-phase setting is straightforward

Lessons learnt



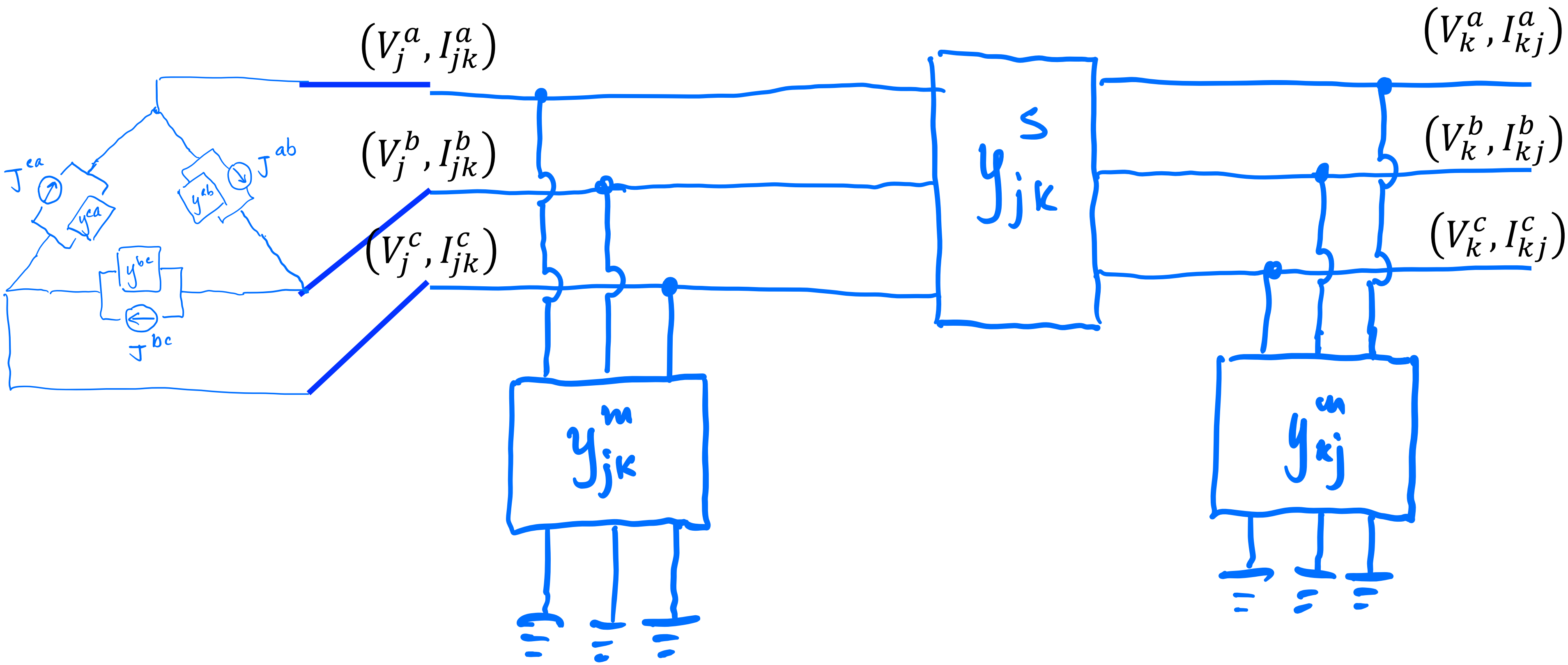
$$I_{jk} = y_{jk}^s \left(V_j - V_k \right) + y_{jk}^m V_j$$

$$I_{kj} = y_{jk}^s \left(V_k - V_j \right) + y_{kj}^m V_k$$

1-phase: $I_{jk}, V_j^a \in \mathbb{C}$. $y_{jk}^{s/m} \in \mathbb{C}$

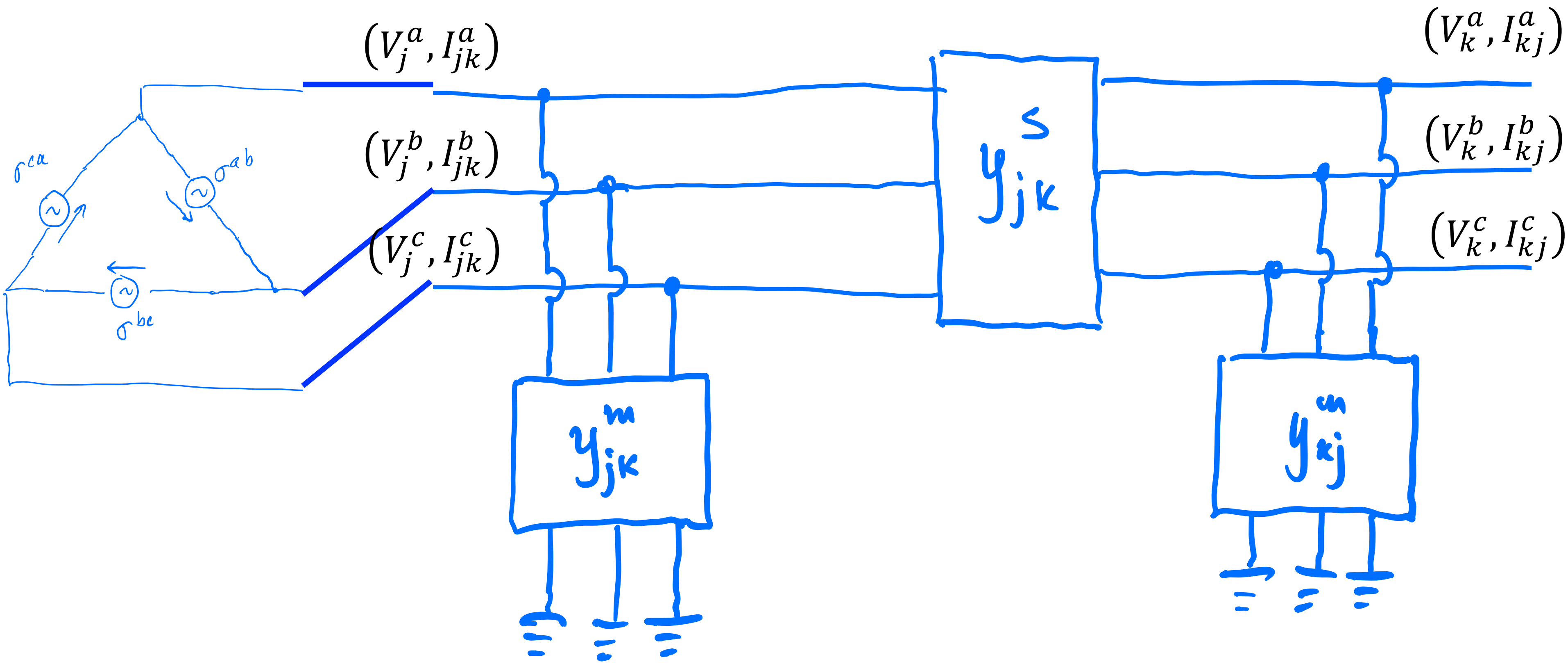
3-phase: $I_{jk}, V_j^a \in \mathbb{C}^3$. $y_{jk}^{s/m} \in \mathbb{C}^{3\times 3}$

Lessons learnt



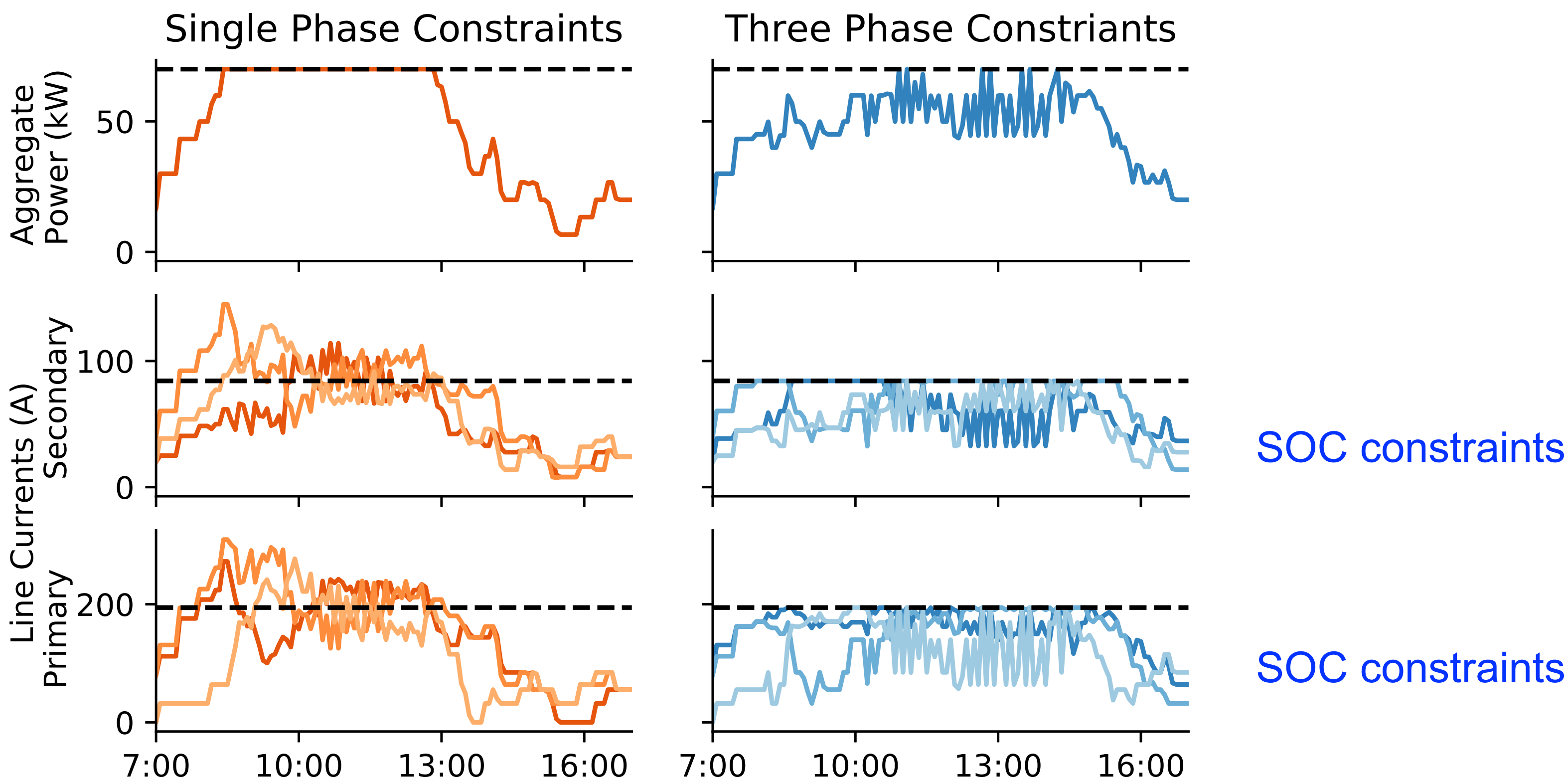
- Terminal currents I_{jk} are externally observable, but often not directly controllable
- If only internal currents $(J_j^{ab}, J_j^{bc}, J_j^{ca})$ of current sources are directly controllable, then need a 3-phase device model to convert between internal & terminal vars

Lessons learnt



Similarly for power sources or voltage sources

Lessons learnt: example

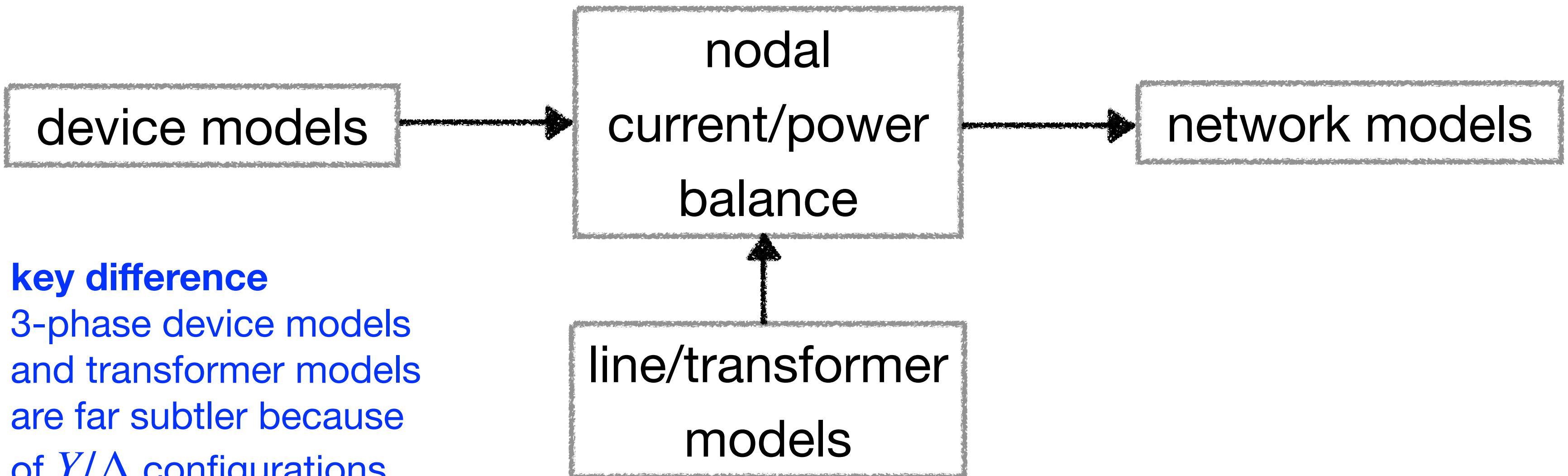


Left panel: Actual 3-phase currents violate capacity constraints if "single-phase constraints" are used (ACN-Sim based on Caltech ACN on Sept 5, 2018 data)

"single-phase constraints" : $\sum_i r_i(t) \le R$ (no phase line constraints for lack of phase info)

[Lee et al (2021), ACN-Sim, TSG]

Overview: 3-phase modeling



key difference
3-phase device models
and transformer models
are far subtler because
of Y/Δ configurations

single-phase or 3-phase

Key question

How to derive external models of 3-phase devices

1. Voltage/current/power sources, impedances (1-phase device: internal models)
2. … in Y/Δ configurations (conversion rules: int $\rightarrow$ ext)
3. … with or without neutral lines, grounded or ungrounded, zero or nonzero grounding impedances

Propose a simple and unified method to derive external models

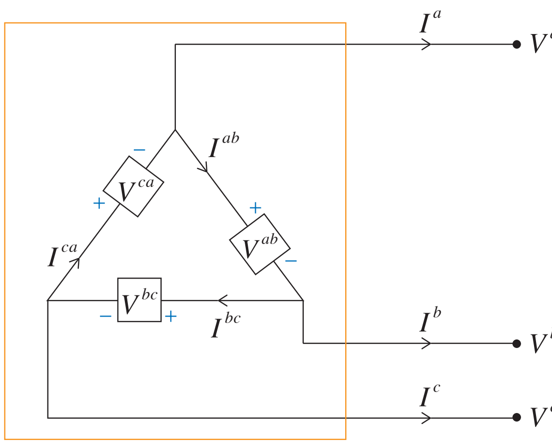
Will use 3-phase voltage source in Δ configuration to illustrate

Internal & terminal vars

Internal vars (Δ configuration)

Internal voltage, current, power across single-phase devices:

$$V^{\Delta} := \begin{bmatrix} V^{ab} \\ V^{bc} \\ V^{ca} \end{bmatrix}, \quad I^{\Delta} := \begin{bmatrix} I^{ab} \\ I^{bc} \\ I^{ca} \end{bmatrix}, \quad s^{\Delta} := \begin{bmatrix} s^{ab} \\ s^{bc} \\ s^{ca} \end{bmatrix} := \begin{bmatrix} V^{ab}\bar{I}^{ab} \\ V^{bc}\bar{I}^{bc} \\ V^{ca}\bar{I}^{ca} \end{bmatrix}$$



Terminal vars

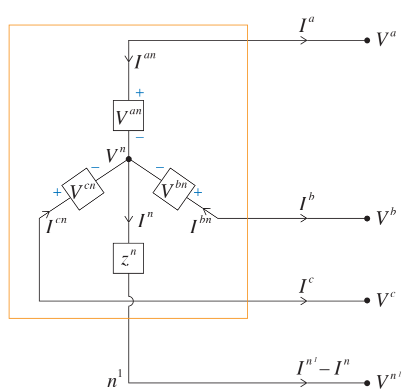
Terminal voltage, current, power (for both Y and Δ) to reference:

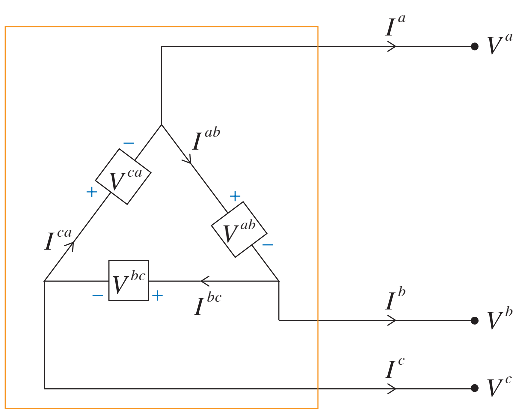
$$V := \begin{bmatrix} V^{a} \\ V^{b} \\ V^{c} \end{bmatrix}, \quad I := \begin{bmatrix} I^{a} \\ I^{b} \\ I^{c} \end{bmatrix}, \quad s := \begin{bmatrix} s^{a} \\ s^{b} \\ s^{c} \end{bmatrix} := \begin{bmatrix} V^{a}\bar{I}^{a} \\ V^{b}\bar{I}^{b} \\ V^{c}\bar{I}^{c} \end{bmatrix}$$

- V is with respect to an arbitrary common reference point, e.g. the ground
- I and s are in the direction out of the device

Internal vs external model

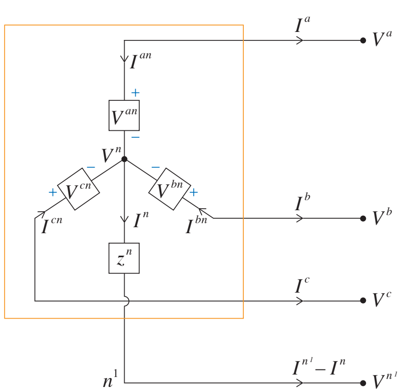
1. External model = Internal model + Conversion rule
   - External model: relation between (V, I, s)
   - Devices interact over network only through their terminal vars

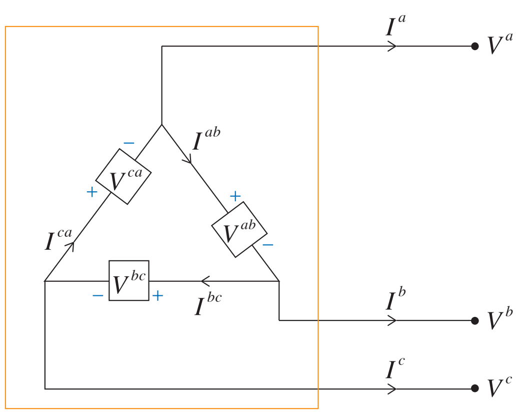




Internal vs external model

1. External model = Internal model + Conversion rule
   - External model: relation between (V, I, s)
   - Devices interact over network only through their terminal vars
2. Internal model : relation between $\left(V^{Y/\Delta}, I^{Y/\Delta}, s^{Y/\Delta}\right)$
   - Independent of Y or Δ configuration
   - Depends only on behavior of single-phase devices
   - Voltage/current/power source, impedance (voltage scr, ZIP load)





Internal vs external model

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    - External model: relation between (V, I, s)
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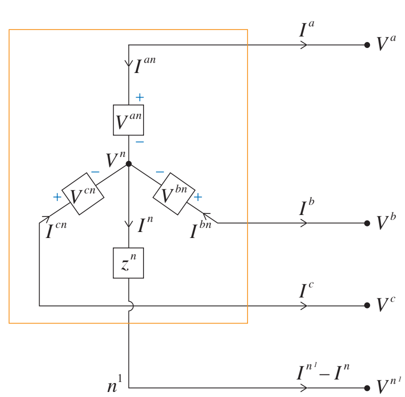
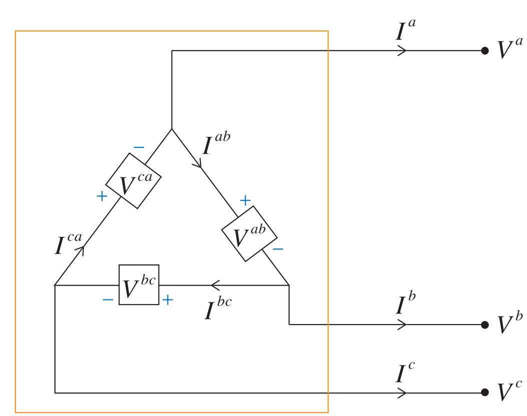
2. Internal model : relation between $\left(V^{Y/\Delta}, I^{Y/\Delta}, s^{Y/\Delta}\right)$
    - Independent of Y or Δ configuration
    - Depends only on behavior of single-phase devices
    - Voltage/current/power source, impedance (voltage scr, ZIP load)
3. Conversion rule : converts between internal and terminal vars
    - Depends only on Y or Δ configuration
    - Independent of type of single-phase devices





Conversion rule

Δ configuration

Convert between internal vars and external vars

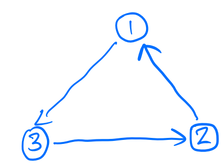
$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}}_{\Gamma} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}, \qquad \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = -\underbrace{\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{\Gamma^{\mathsf{T}}} \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

In vector form

$$\boxed{V^{\Delta} = \Gamma V, \qquad I = -\Gamma^{\mathsf{T}} I^{\Delta}}$$

internal voltage, terminal voltage, terminal current, internal current

Γ is incidence matrix of:



Conversion matrices Γ & Γ^T

Fortescue matrix F

Spectral decomposition:

$$\Gamma \;=\; F\Lambda\overline{F}, \qquad \Gamma^{\mathsf{T}} \;=\; \overline{F}\Lambda F$$

where

$$\Lambda := \begin{bmatrix} 0 & & \\ & 1-\alpha & \\ & & 1-\alpha^2 \end{bmatrix}, \qquad F := \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

eigenvectors of $\Gamma, \Gamma^{\mathsf{T}}$

positive-seq balanced vector α_+

negative-seq balanced vector α_-

and $\alpha := e^{-\mathrm{i}2\pi/3}$

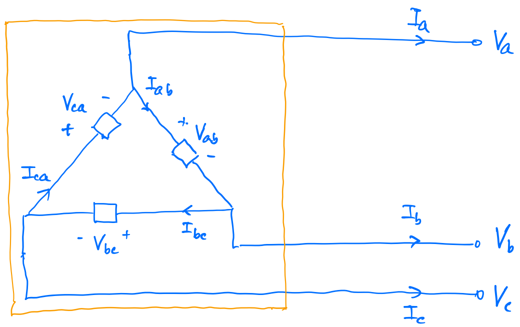
Pseudo-inverses: $\Gamma^{\dagger} \;=\; \dfrac{1}{3}\Gamma^{\mathsf{T}}, \qquad \Gamma^{\mathsf{T}\dagger} \;=\; \dfrac{1}{3}\Gamma$

Conversion rule

Δ configuration

1. Converts between internal and terminal voltages & currents

$$V^{\Delta} = \Gamma V, \qquad I = -\Gamma^{\mathsf{T}} I^{\Delta}$$



Conversion rule

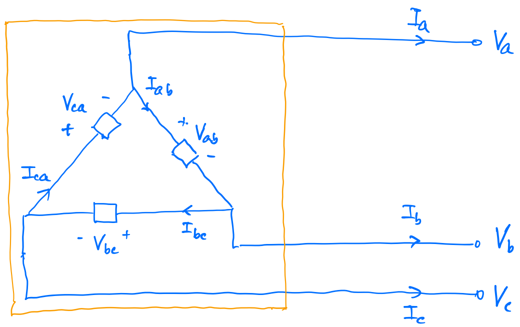
Δ configuration

1. Converts between internal and terminal voltages & currents

$$V^\Delta \;=\; \Gamma V, \qquad\qquad I \;=\; -\Gamma^{\mathsf{T}} I^\Delta$$

2. Given V^Δ: terminal voltage $V \;=\; \frac{1}{3}\Gamma^{\mathsf{T}} V^\Delta \;+\; \gamma\,1, \qquad \gamma \in \mathbb{C}$

- $\gamma := \frac{1}{3} 1^T V$: zero-sequence terminal voltage (fixed by reference voltage)



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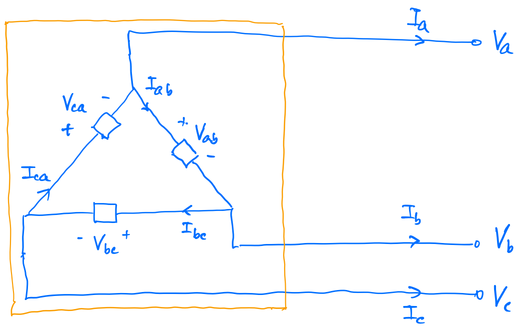

Conversion rule

Δ configuration

1. Converts between internal and terminal voltages & currents

$$V^{\Delta} = \Gamma V, \qquad I = -\Gamma^{\mathsf{T}} I^{\Delta}$$

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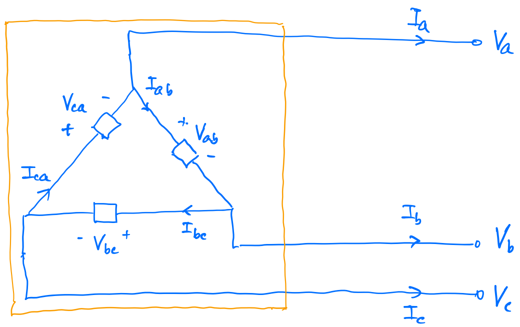


4. Relation between s and s^{Δ} through $\left(V, I^{\Delta}\right)$:

$$s = -\operatorname{diag}\left(V I^{\Delta \mathsf{H}} \Gamma\right), \qquad s^{\Delta} = \operatorname{diag}\left(\Gamma V I^{\Delta \mathsf{H}}\right)$$

(no direct relation between s and s^{Δ})

Example: transformers

Theorem 1. *The external models of three-phase transformers in YY, $\Delta\Delta$, ΔY and $Y\Delta$ configurations take the form*

$$I \;=\; D^{\mathsf{T}} Y_{YY} D\,(V-\gamma)$$

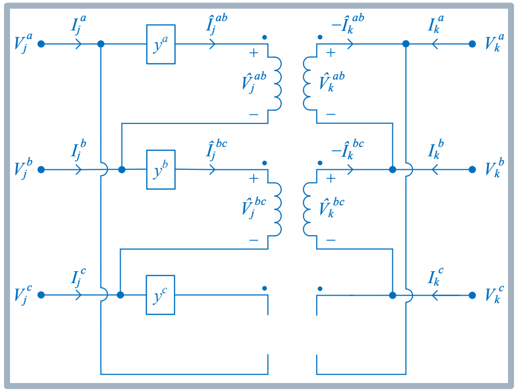
where

$$YY: \qquad D := \begin{bmatrix} \mathbb{I} & 0 \\ 0 & \mathbb{I} \end{bmatrix}$$

$$\Delta\Delta: \qquad D := \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix}$$

$$\Delta Y: \qquad D := \begin{bmatrix} \Gamma & 0 \\ 0 & \mathbb{I} \end{bmatrix}$$

$$Y\Delta: \qquad D := \begin{bmatrix} \mathbb{I} & 0 \\ 0 & \Gamma \end{bmatrix}$$



unified & modular characterization

Overall model: device + network

1. Network model relates terminal vars (V, I, s)
   - Nodal current balance (linear): $I = YV$
   - Nodal power balance (nonlinear): $s_j = \sum_{k:j\sim k} \text{diag}\left(V_j(V_j - V_k)^{\mathsf{H}} y_{jk}^{s\mathsf{H}} + V_j V_j^{\mathsf{H}} y_{jk}^{m\mathsf{H}}\right)$
   - Either can be used
2. Device model for each 3-phase device
   - Internal model $\left(V_j^{Y/\Delta}, I_j^{Y/\Delta}, s_j^{Y/\Delta}, \gamma_j, \beta_j\right)$ + conversion rules
   - External model $\left(V_j, I_j, s_j, \gamma_j, \beta_j\right)$ with internal parameters
   - Either can be used
   - Power source models are nonlinear; other devices are linear

Unbalance 3-phase modeling

Power System Analysis

A Mathematical Approach

Steven H. Low

DRAFT available at: http://netlab.caltech.edu/book/

Corrections, questions, comments appreciated!