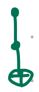
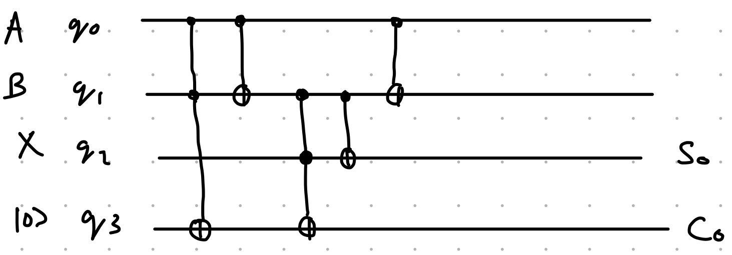
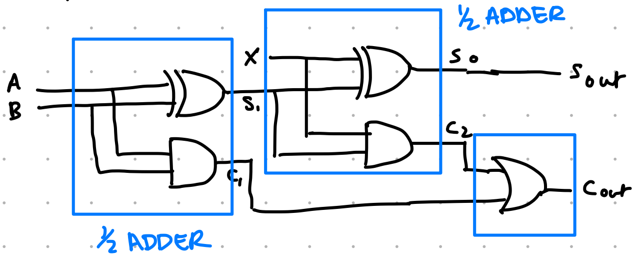
A Input	B Input	X Carry input	S Sum	C Carry out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

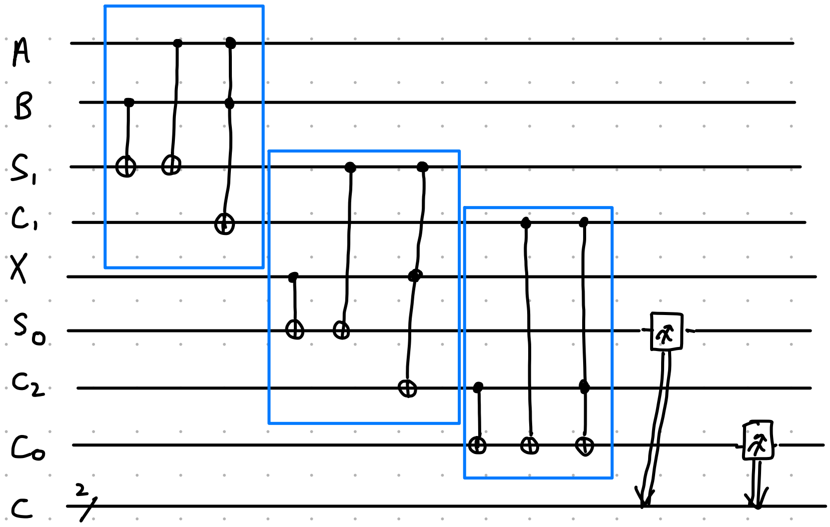

= Bit flip if 1 and 2 ≡ $|11\rangle$

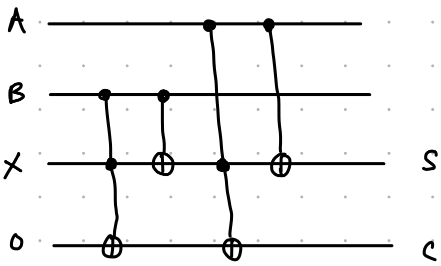

= BIT FLIP if = 1





A Input	B Input	X Carry input	S_o Sum	C_o Carry out	
0	0	0	0	0	00
0	0	1	1	0	01
0	1	0	1	0	01
0	1	1	0	1	10
1	0	0	1	0	01
1	0	1	0	1	10
1	1	0	0	1	10
1	1	1	1	1	11





A Input	B Input	X Carry input	S_o Sum	C_o Carry out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

a	b	c	$(a \oplus b)$	$(a \oplus b) \oplus c$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

	1	2	3
1	0	1	2
2	3	4	5
3	6	7	8

let $C \in \mathbb{Z}_2^{3\times3}$ ← mod 2 be a starting light configuration

$A_{i,j}$ represents a button press on i,j

$C' = C + A_{i,j}$

winning combo $C + \sum_{i,j} \beta_{i,j} A_{i,j} = 0$ ← indicates if a button must be pressed

$-1 = 1 \pmod 2$

$\Rightarrow C = \sum_{i,j} \beta_{i,j} A_{i,j}$

$0 \rightarrow 1, 3$

$1 \rightarrow 0, 4, 2$

$2 \rightarrow 1, 5$

$3 \rightarrow 0, 4, 6$

$4 \rightarrow 1, 3, 5, 7$

$5 \rightarrow 2, 4, 8$

$6 \rightarrow 3, 7$

$7 \rightarrow 4, 6, 8$

$8 \rightarrow 5, 7$

A can be represented as a 9×9 matrix where $A_{i,j}$ forms the rows

$$\begin{pmatrix} A_{1,1} \\ \vdots \\ A_{n,m} \end{pmatrix} \cdot \begin{pmatrix} \beta_{1,1} \\ \vdots \\ \beta_{n,m} \end{pmatrix} = \begin{pmatrix} C_{1,1} \\ \vdots \\ C_{i,m} \\ \vdots \\ C_{n,m} \end{pmatrix}$$

$$Z = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\beta = A^{-1} C$

$$A = \begin{bmatrix} Z & I & 0 \\ I & Z & I \\ 0 & I & Z \end{bmatrix}$$

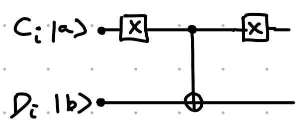
Input $= [0, 1, 1, 1, 0, 0, 1, 1, 1] = C$ (lights at start)

if $[v_0 \rightarrow v_8]$ are a solution : $f = 1$ else $f = 0$

$\uparrow$ $\vec{\beta}$

β_0 ——
$\vdots$
β_8 ——
C_0 ——
$\vdots$
C_8 ——

$$\overset{A}{\begin{pmatrix} \vdots & \vdots & \vdots \end{pmatrix}} \overset{\beta}{\begin{pmatrix} \vdots \end{pmatrix}} = \overset{C}{\begin{pmatrix} \vdots \end{pmatrix}}$$



$\|a\rangle$	$\|b\rangle$	$\|a\rangle$	$\|b\rangle$
0	0	0	1
0	1	0	0
1	0	1	0
1	1	1	1

$D = \begin{pmatrix} \vdots \end{pmatrix}$

```
for i in range(9)
    for j in range(9)
        D_i flipped if A_{i,j}, β_j = 1
for i in range(9)
```

Multi CNOT for D

qRAM

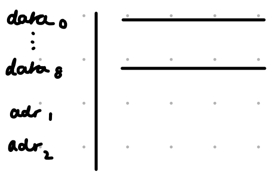
IN → lightsout4 = [[Board0], [Board1], [Board2], [Board3]]

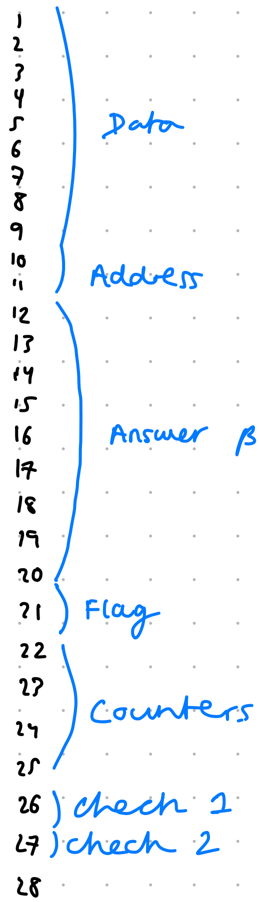
OUT ← which boards are solvable in 3 switch operations — 00, 01, 10, 11 → " 0 0 1 0 0 1 1

$n = 4$

h = lightsout 4 $\hat{x}$

m = Board which can be solved in 3 switches





for address i (00→11) store lightsout4[i] in data 0→8

qRAM

for i in lightsout 4[i]:

if i = 1

→ CNOT on data(i) with addr.

ORACLE δ

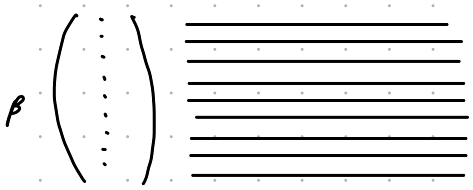
for data 1→9 δ return 1 if can be solved in 3 switches

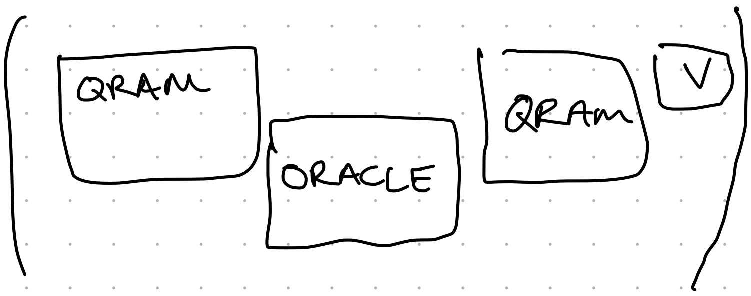
data ≡ C

$$\left(A^{-1}\right)\left(C\right) = \left(\beta\right)$$

for i, j in range 9:

flip β_i if $A^{-1}_{ij}, C_i = 1$

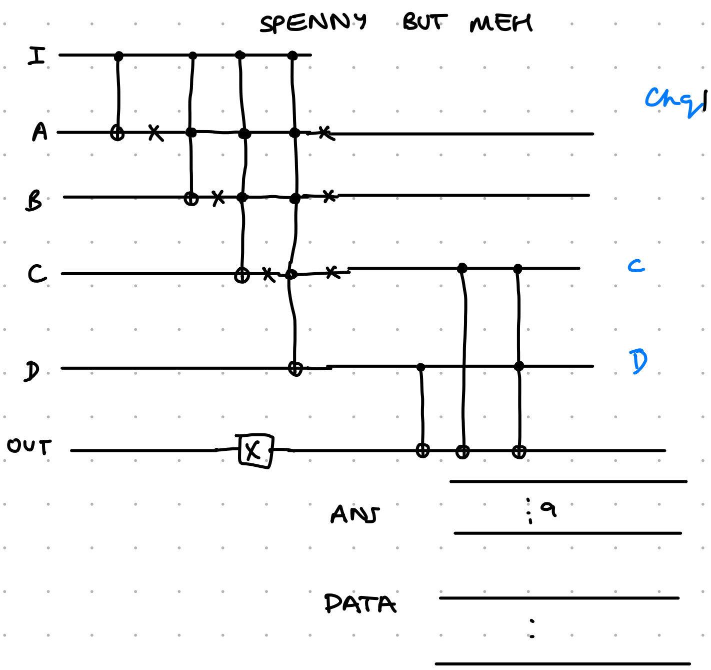


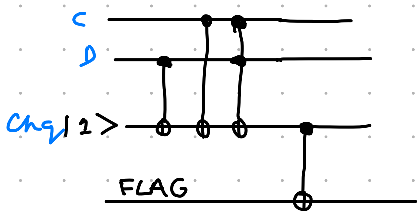


OUT = 1 if 3 ones in β

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1

DCBA

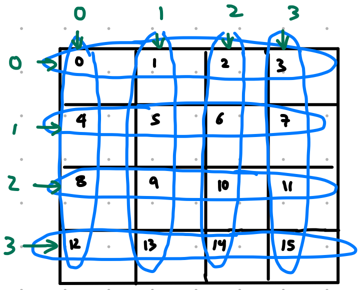




If answer[j] = 1

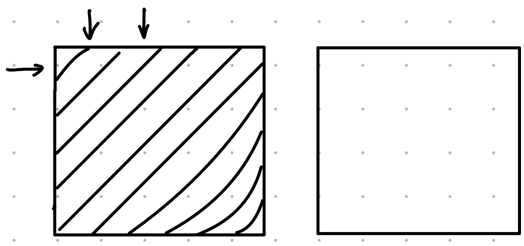
If matrix[i, j] = 1

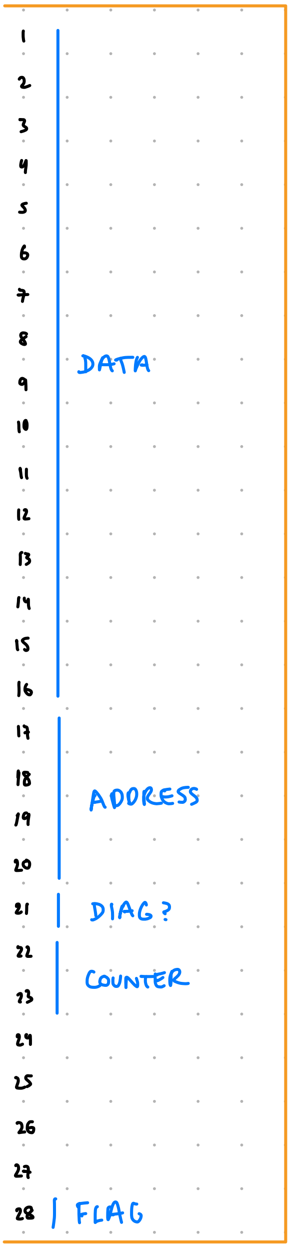
flip



PROBLEM SET [i] = [[0,2],[1,0],[1,2],[1,3],[2,0],[3,3]]

↳ $\underline{\underline{N}} \left[|0010\rangle + |0100\rangle + |0110\rangle + |0111\rangle + |1000\rangle + |1111\rangle \right]$





RETURNS 1 IFF SOLUTION REQUIRES 4 BEAMS

$(8 \times 16)(16 \times 1) = (8 \times 1)$ Solution $= \left(8 \times 1 \right)$

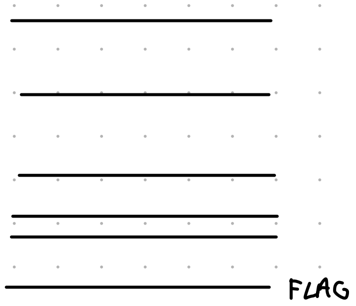
$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

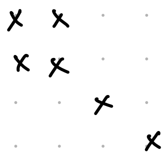
NUMBER OF LASAR BEAMS IS EQUAL TO THE <u>RANK</u>

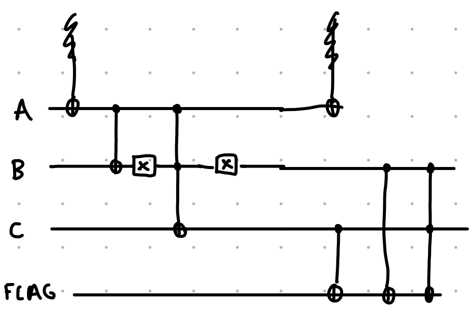
for r in range (4)

0 0 1
0 1 0
1 0 1
1 1 1

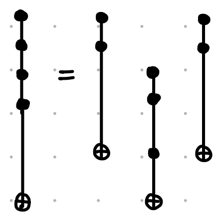
0 1 1 OR 1 1 0 OR 1 1 1

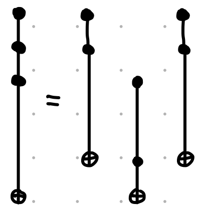






OPTIMISATION





2 QUBIT CONTROL: CCX → 33 + 10 × 18 = 213

$\boxed{\text{RCCX}}$ → 6 + 10 × 3 = 36

MCT → 9 + 10 × 6 = 69

ANCILLA MCT → 9 + 10 × 6 = 69

3 QUBIT CONTROL: RCCCX → 12 + 10 × 6 = 72

MCT → 23 + 10 × 20 = 223

ANCILLA MCT → 21 + 10 × 12 = 141

4 QUBIT CONTROL: SYNTHESIS → 24 + 12 × 10 = 144

MCT →

ANCILLA MCT →