The Relationship Between Spectral Bias and Saddle Points in Deep Learning Optimization

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Introduction: In deep learning, neural networks often exhibit spectral bias, prioritizing low-frequency data components over high-frequency ones. The optimization process can also encounter saddle points, causing plateaus in the learning curve. This research explores whether saddle points contribute to the stage-like drops in the loss function, linked to learning higher-frequency components.

Methods: We generated a synthetic 1D dataset of sine waves with frequencies 1 Hz, 3 Hz, 7 Hz, and 15 Hz (amplitudes 1.0, 0.8, 0.6, 0.4). To study spectral bias and saddle points, we trained four ReLU-based Multilayer Perceptrons (MLPs): a baseline (2 layers, 50 units), a wider model (2 layers, 100 units), a deeper model (5 layers, 50 units), and the baseline with SGD instead of Adam. Models were trained for 10,000 epochs (learning rate 0.01), with loss monitoring, plateau detection via gradient methods, and Fourier analysis of predictions to track frequency learning and jumps.

Results: The network displayed clear spectral bias, mastering lower frequencies first. Plateaus appeared in the loss curve, during which higher-frequency learning stalled. Intriguingly, jumps in frequency learning often occurred near plateau edges, hinting that escaping saddle points may trigger progress in capturing more complex, high-frequency features.

Conclusion: These findings suggest saddle points play a role in spectral bias. Plateaus reflect struggles with higher frequencies, and overcoming saddle points allows the network to advance. This connection could guide future efforts to enhance training efficiency, perhaps through better optimization methods or network designs.

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