3D Rendering with Ray Tracing in Python

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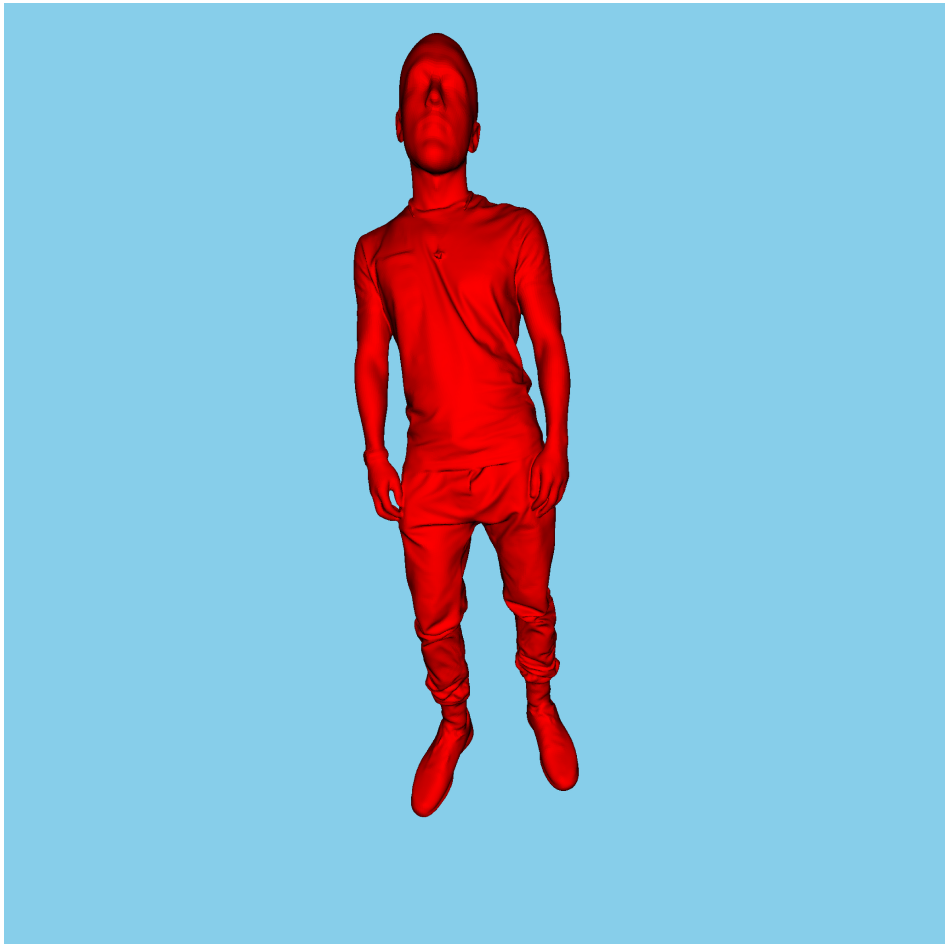
About Me

- Junior at Virginia Tech
- Began ray tracing in late April



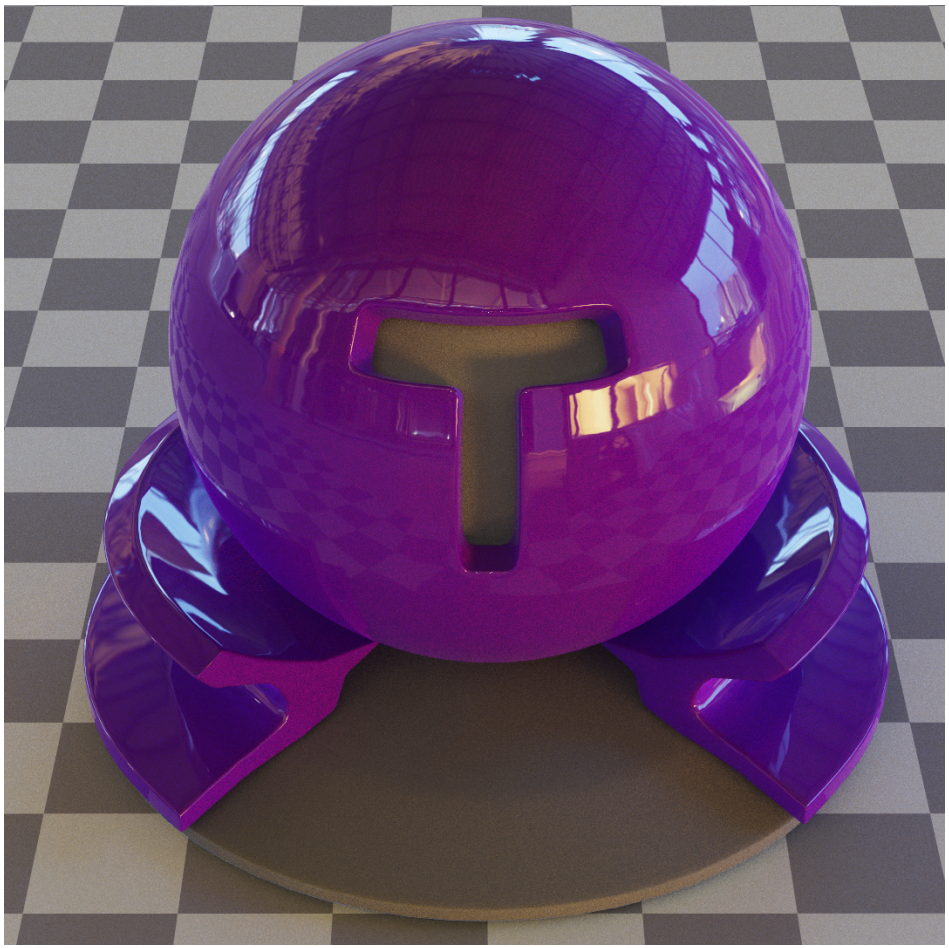
Rendered in June; 11 hours

Rendered in June; 8.5 hours

About Me

- Junior at Virginia Tech
- Began ray tracing in late April

Rendered in December; 2.5 hours



Rendered in January; 3.5 hours

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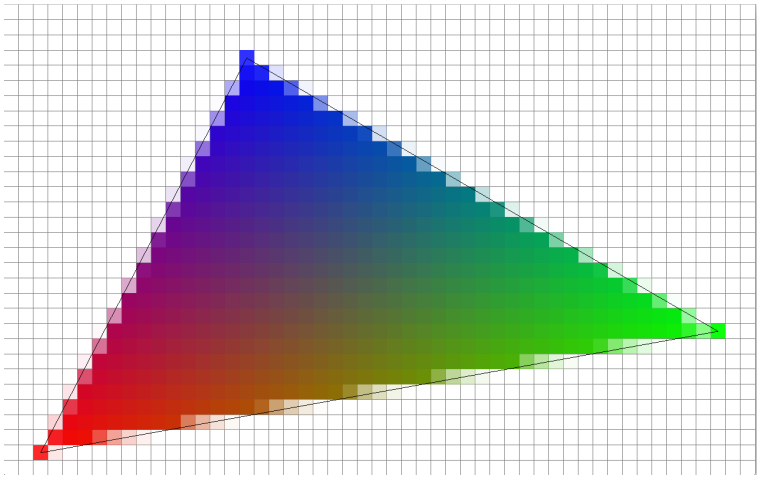
Computer Graphics

- Software Rasterization

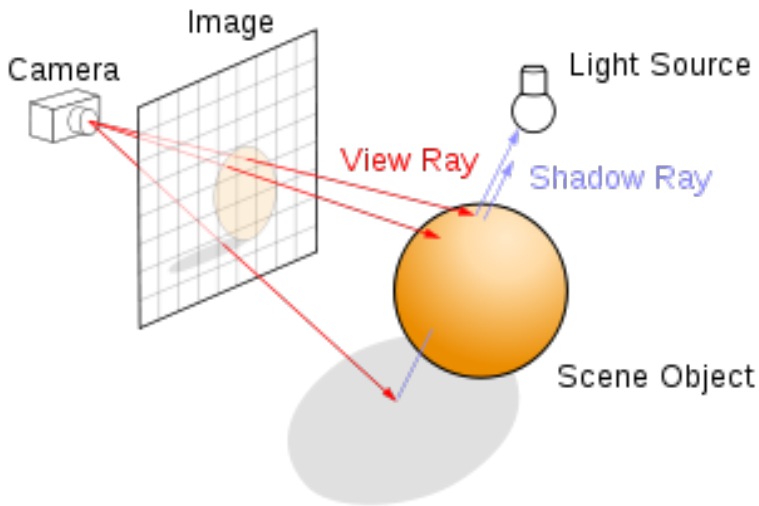


  - "Given *shape*, what pixels should show this?"
  - Faster, commonly used in cartoon-styled games
  - Not the focus of this workshop

- Ray Tracing

Computer Graphics

- Software Rasterization
- Ray Tracing



- "Given *pixel*, what object should this show?"
- Slower, but more realistic
- Involves the study of physics (light interactions), math (sampling techniques) and computer science (acceleration structures)

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Vector

```
class Vector3:
    def __init__(self, x, y, z):
        self.x, self.y, self.z = x, y, z
```

- Standard vector operations; addition and scalar multiplication
- Also contains utility functions for dot/cross product, length, normalize, etc.
- Used to represent points, vectors, and colors

Ray

```
class Ray:
    def __init__(self, origin, direction):
        self.origin = origin
        self.direction = direction

    def at(self, t):
        return self.orgin + t * self.direction
```

- Contains origin of ray and direction (both `Vector3`)
- Includes method to find point at given distance on ray

ImageWrapper

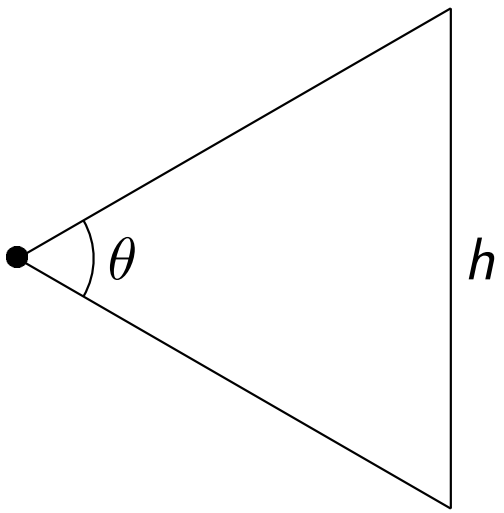
```
class ImageWrapper:
    def __init__(self, name, width, height):
        # ...

    def save(self):
        # ...

    def write_pixel(self, x, y, color):
        # Assumes color is Vector3 and
        # 0 <= x < width, 0 <= y < height
        # ...
```

- Constructor takes file name and file size
- Provides method to save file and write pixel at specified coordinate

Camera



- Requires location, view direction (target), and orientation ('up' direction), field of view, and aspect ratio
- Field of view (θ) controls 'zoom' of image; higher field of view is more zoomed out

Camera

- Constructor takes all that information and stores information about the projected plane
- Contains an additional ray generation method that will be useful later

```
class Camera:
    def __init__(self, loc, to, up, aspect_ratio, vfov):
        """Parameters:
        location     -- Vector3
        direction    -- Vector3 target of camera
        up           -- 'up' direction relative to the camera.
        aspect_ratio -- width / height of plane
        vfov         -- vertical field of view, in degrees"""
        # implementation not included

    def generate_ray(self, x, y):
        """Takes an (x, y) pair (in [0, 1]), returns
        ray that starts at the camera that goes
        through that point on the plane."""
        # implementation not included
```

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Main Rendering Loop

- First, define constants, Camera, and ImageWrapper

```
def main():
    # Define constants, ImageWrapper, and Camera
    aspect_ratio = 1
    height = 256
    width = int(height * aspect_ratio) # force to be integer
    name = "test.png"
    wrapper = ImageWrapper(name, width, height)
    # Camera pointing from origin to (0, 1, 0) with z as up
    camera = Camera(Vector3(0, 0, 0), Vector3(0, 1, 0),
                    Vector3(0, 0, 1), aspect_ratio, 90)
    # ...
    #
    #
    #
    #
    #
```

Main Rendering Loop

- Then iterate over each pixel and calculate color

```
def main():
    # Define constants, ImageWrapper, and Camera
    aspect_ratio = 1
    height = 256
    width = int(height * aspect_ratio) # force to be integer
    name = "test.png"
    wrapper = ImageWrapper(name, width, height)
    # Camera pointing from origin to (0, 1, 0) with z as up
    camera = Camera(Vector3(0, 0, 0), Vector3(0, 1, 0),
                    Vector3(0, 0, 1), aspect_ratio, 90)
    for y in range(height):
        for x in range(width):
            ray = camera.generate_ray(x / width, y / height)
            color = # ... some calculation to generate color
            wrapper.write_pixel(x, y, color)
    wrapper.save()
```

First Render Test

- Set color of pixel depending on z component of ray's direction

```
# ...
white = Vector3(1, 1, 1)
blue = Vector3(0.5, 0.7, 1)
for y in range(height):
    for x in range(width):
        ray = camera.generate_ray(x / width, y / height)
        # map z in [-1, 1] to [0, 1]
        t = (ray.direction.normalize().z + 1) / 2
        # Linearly interpolate between white and blue
        color = t * blue + (1 - t) * white
        wrapper.write_pixel(x, y, color)
wrapper.save()
```

First Render Test

- Set color of pixel depending on z component of ray's direction

Resulting image:

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Sphere-Ray Intersection

- Given ray with origin O and direction D, where does it first intersect a sphere with center C and radius R?
- Equivalent question is intersection between ray at $O - C$ and direction D and sphere centered at origin with radius R
  - Parameterize ray as $(O - C) + tD$; each value of t gives new point P on ray
  - P is on sphere if $|P|^2 = R^2$, or $|P|^2 - R^2 = 0$.
  - But $P = (O - C) + tD$, giving the following equation:

$$|(O - C) + tD|^2 - R^2 = 0$$

Sphere-Ray Intersection

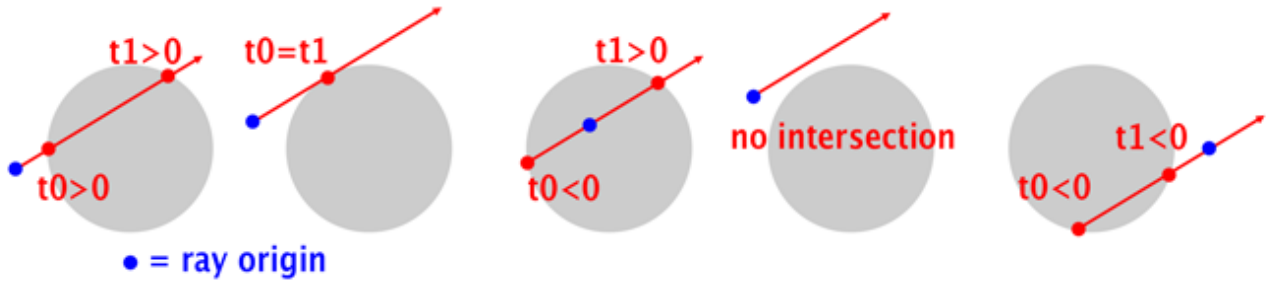
$$|(O - C) + tD|^2 = R^2 \iff$$
$$((O - C) + tD) \cdot ((O - C) + tD) = R^2 \iff$$
$$O \cdot O - 2O \cdot C + 2t(O \cdot D) + C \cdot C - 2t(C \cdot D) + t^2(D \cdot D) = R^2 \iff$$
$$(D \cdot D)t^2 + (2O \cdot D - 2C \cdot D)t + O \cdot O + C \cdot C - 2O \cdot C = R^2 \iff$$
$$(D \cdot D)t^2 + 2D \cdot (O - C)t + (O - C) \cdot (O - C) - R^2 = 0$$

where $\cdot$ indicates the dot product between two vectors

- This is a quadratic equation in t with $a = D \cdot D$, $b = 2D \cdot (O - C)$, and $c = (O - C) \cdot (O - C) - R^2$
- Two solutions to quadratic formula give two (possible) intersection points

Geometric Interpretation of Ray-Sphere Formula

- Two real solutions: Ray intersects with sphere twice
  - Two positive solutions: Two intersections "in front" of ray (example 1 below)
  - One positive, one negative solution: Ray starts within sphere (example 3)
  - Two negative solutions: Sphere is "behind" ray (example 5)
- One solution: Ray "grazes" sphere at single point (example 2)
- No real solution: Ray misses sphere (example 4)



Quadratic Formula Solver

```python
import math
def solve_quadratic(a, b, c):
    desc = b * b - 4 * a * c
    if desc < 0: # No real solution
        return []
    if desc == 0: # Single solution
        return [-b / (2 * a)]
    else:
        sqrt_desc = math.sqrt(desc)
        b_minus = (-b - sqrt_desc) / (2 * a)
        b_plus = (-b + sqrt_desc) / (2 * a)
        # Always return smaller solution first
        return [min(b_minus, b_plus), max(b_minus, b_plus)]
```

Sphere-Ray Intersection

```
def sphere_intersection(center, radius, ray):
    diff = ray.origin - center # O - C
    a = ray.direction.dot(ray.direction) # D . D
    b = diff.dot(2 * ray.direction) # 2D . (O - C)
    c = diff.dot(diff) - radius * radius
    solutions = solve_quadratic(a, b, c)
    # Because solutions are in increasing order,
    # we can iterate until we find a positive solution
    for solution in solutions:
        if solution > 0:
            return True
    return False
```

Rendering a Sphere

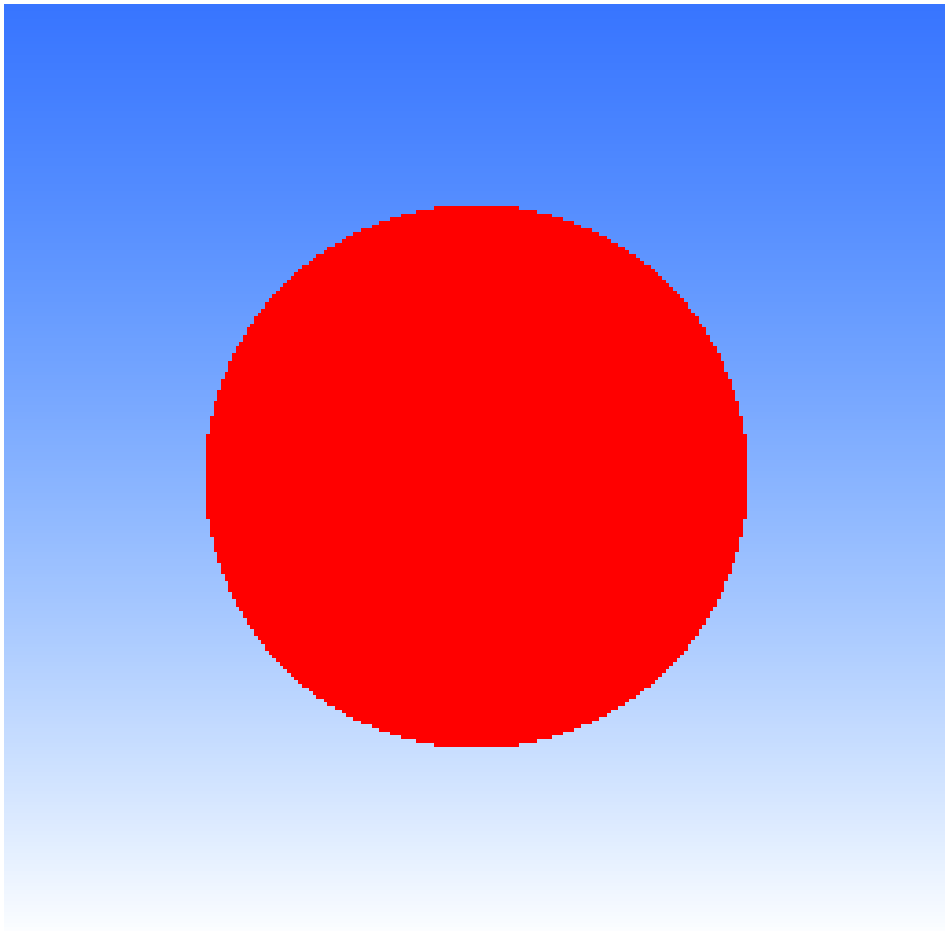
- Now update the render loop from earlier to use this

```
# ...
white = Vector3(1, 1, 1)
blue = Vector3(0.5, 0.7, 1)
for y in range(height):
    for x in range(width):
        ray = camera.generate_ray(x / width, y / height)
        center = Vector3(0, 10, 0)
        radius = 5
        if sphere_intersection(center, radius, ray):
            color = Vector3(1, 0, 0) # red
        else: # background color
            t = (ray.direction.normalize().z + 1) / 2
            color = t * blue + (1 - t) * white
        wrapper.write_pixel(x, y, color)
wrapper.save()
```

Rendering a Sphere

- Now update the render loop from earlier to use this

And the new result:



Making a Sphere Class

- Sphere contains center, radius, color, and intersection method

```
class Sphere:
    def __init__(self, center, radius, color):
        self.center = center
        self.radius = radius
        self.color = color

    def intersection(self, ray):
        diff = ray.origin - self.center
        a = ray.direction.dot(ray.direction)
        b = diff.dot(2 * ray.direction)
        c = diff.dot(diff) - self.radius * self.radius
        solutions = solve_quadratic(a, b, c)
        for solution in solutions:
            if solution > 0:
                return (True, self.color)
        return (False, self.color)
```

Storing Intersection Information

- Often require knowing a lot of information about intersection; for now we just have location and color, but in more robust renderers, it can store texture coordinates, information about the material, etc.
- Store the information in a `HitRecord` object

```
class HitRecord:
    def __init__(self, point, normal, color, time):
        self.point = point
        self.color = color
        self.time = time # time along ray
```

Updated Sphere Intersection

- We can use the new `HitRecord` in the `Sphere` intersection method

```
class Sphere:
    # ...
    def intersection(self, ray):
        diff = ray.origin - self.center
        a = ray.direction.dot(ray.direction)
        b = diff.dot(2 * ray.direction)
        c = diff.dot(diff) - self.radius * self.radius
        solutions = solve_quadratic(a, b, c)
        for solution in solutions:
            if solution > 0: # new code to make HitRecord
                p = ray.at(solution)
                return HitRecord(p, self.color, solution)
        return None
```

Rendering Multiple Spheres

- Go through all spheres, find the one with smallest positive intersection time
- Return color associated with it

```
# takes ray, list of spheres; returns color
def get_intersection(ray, spheres):
    nearest = 1e100 # big number
    found_record = None
    for sphere in spheres:
        record = sphere.intersection(ray)
        if record is None:
            continue
    # ...
    #
    #
    #
    #
    #
    #
```

Rendering Multiple Spheres

- Go through all spheres, find the one with smallest positive intersection time
- Return color associated with it

```
# takes ray, list of spheres; returns color
def get_intersection(ray, spheres):
    nearest = 1e100 # big number
    found_record = None
    for sphere in spheres:
        record = sphere.intersection(ray)
        if record is None:
            continue
        if record.time < nearest:
            found_record = record
            nearest = record.time
    if found_record == None:
        # This does simple gradient from earlier
        return get_background(ray)
    return found_record.color
```

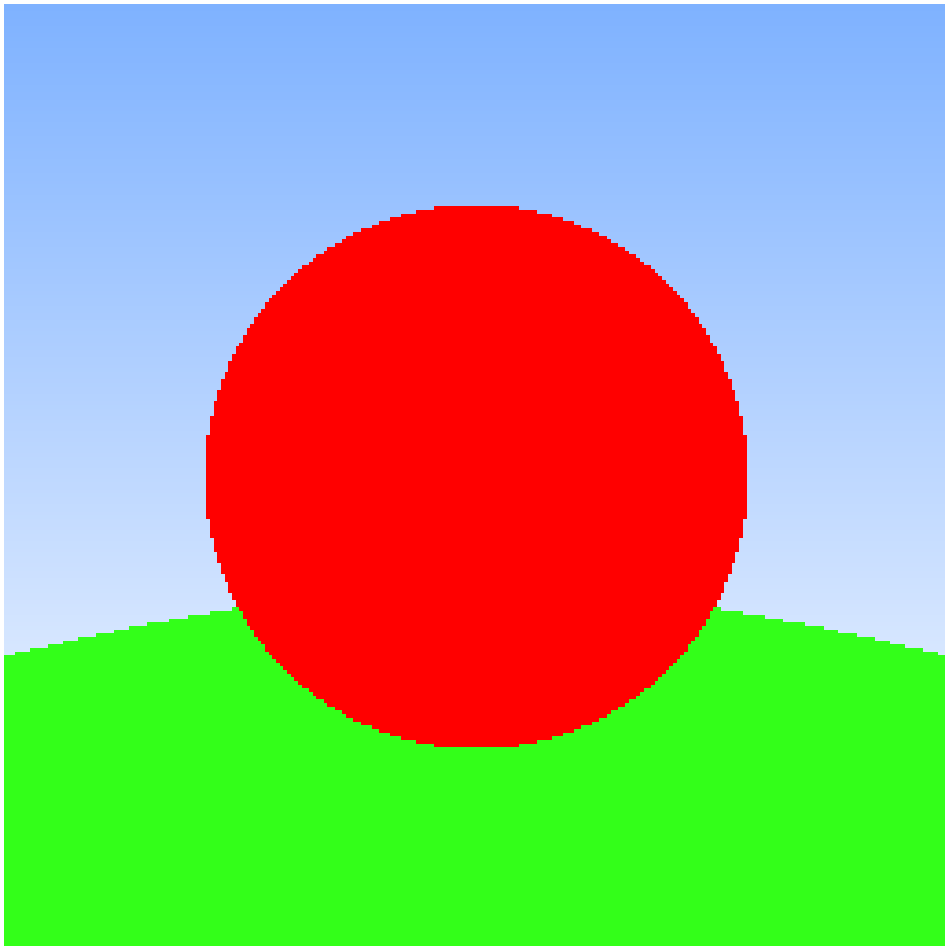
Updating Scene

- Make list of all the spheres in the scene

```
# ...
# same constants as before
spheres = [Sphere(Vector3(0, 10, 0), 5, Vector3(1, 0, 0)),
    Sphere(Vector3(0, 10, -100), 95, Vector3(0.2, 1, 0.1))]
for y in range(height):
    for x in range(width):
        ray = camera.generate_ray(x / width, y / height)
        color = get_intersection(ray, spheres)
        wrapper.write_pixel(x, y, color)
wrapper.save()
```

Updating Scene

- Make list of all the spheres in the scene



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Shading

- So far, everything looks flat. How can we give the illusion of depth?
- Right now, light stops once it hits something, which isn't very realistic
- In real life, light bounces around until it is absorbed by something
- Even though our rays start at the camera, the same effect occurs
- Changing the way rays of light bounce changes how the material looks

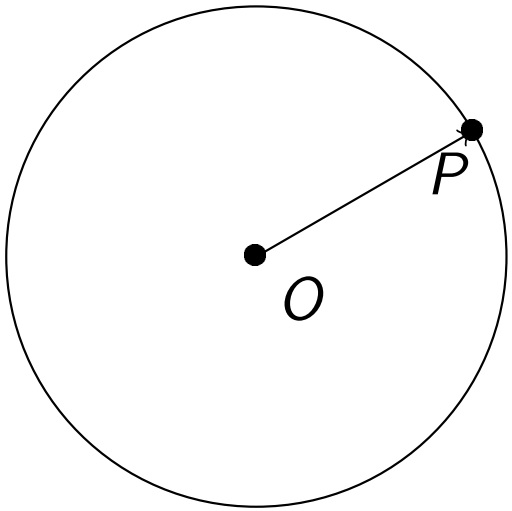
Lambertian Material

- Easiest material to simulate is called "lambertian"; models rough surfaces, looks the same from all angles
- Need to generate new direction for a ray, for lambertian material this is done with generating an arbitrary unit vector

```
def rand_on_sphere():
    # random.random() returns [0, 1),
    # map to the range [-1, 1]
    v = Vector3(2 * random.random() - 1,
                2 * random.random() - 1,
                2 * random.random() - 1)
    return v.normalize() # make unit vector
```

Calculate Normal of Sphere

- Normal is unit vector perpendicular to the surface
- Used in calculate how a ray bounces
- For sphere, normal is vector from center to point



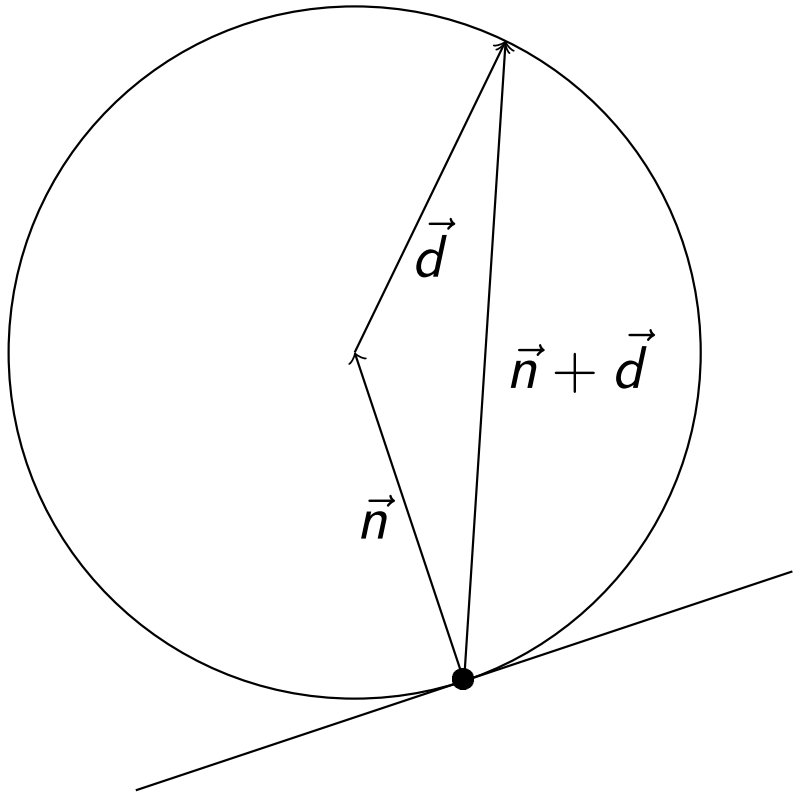
Calculate Normal of Sphere

- Normal is unit vector perpendicular to the surface
- Used in calculate how a ray bounces
- For sphere, normal is vector from center to point

```
def sphere_normal(self, p):
    return (p - self.center).normalize()
```

Generating new Ray Direction

- Given normal $\vec{n}$ and randomly generated unit vector $\vec{d}$, the new direction is $\vec{n} + \vec{d}$



Updating HitRecord and Sphere Intersection

- We must first update HitRecord to store the normal vector
- Then update sphere intersection to calculate this information

```
# ...
# rest is the same
for solution in solutions:
    if solution > 0: # new code to make HitRecord
        p = ray.at(solution)
        normal = (p - self.center).normalize()
        # add normal as parameter to HitRecord
        return HitRecord(p, self.color, solution, normal)
```

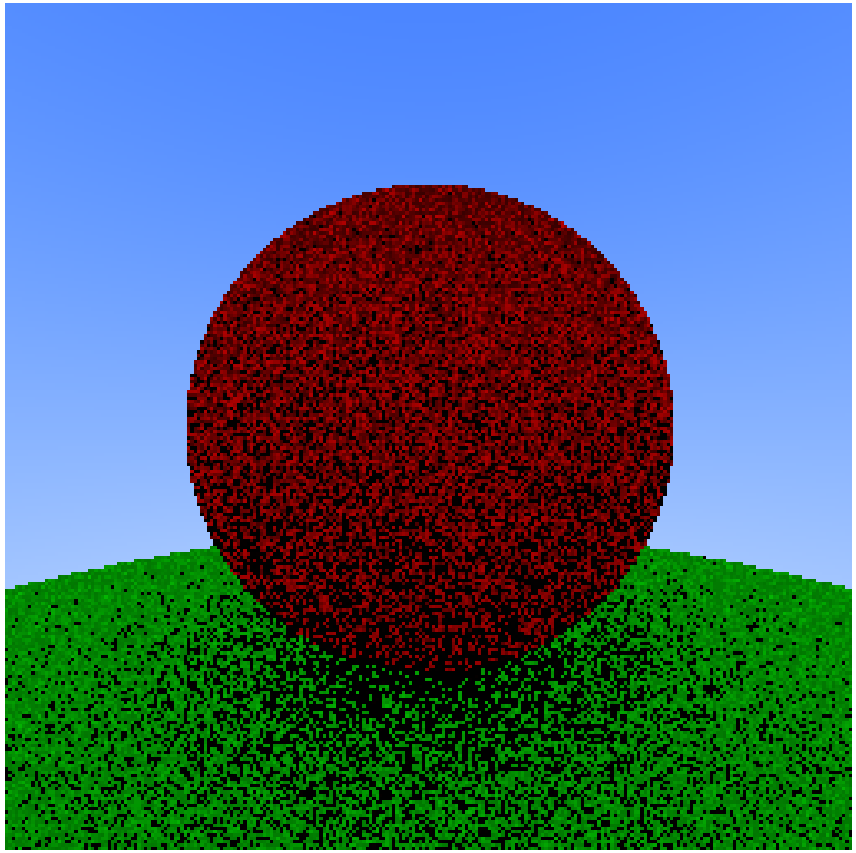
Updating get_intersection

- We must now store the depth of the traversal, how many bounces we've done, and eventually stop; call with arbitrary depth (like 20)

```
def get_intersection(ray, spheres, depth):
    if depth == 0: # too many bounces
        return Vector3(0, 0, 0) # black
    found_record = None
    # same logic to find closest intersection as before
    # ...
    # generate new ray direction
    direction = found_record.normal + rand_on_sphere()
    new_ray = Ray(found_record.point, direction)
    # li term of render equation
    next_color = get_intersection(new_ray, spheres, depth - 1)
    current_color = found_record.color
    # This is compontent-wise multiplication
    return current_color.multiply(next_color)
```

Render with Shadows

- Now the image has a lot of noise
- A single ray can't capture enough information to accurately estimate what the image should look like
- Increase accuracy by sending multiple rays per pixel



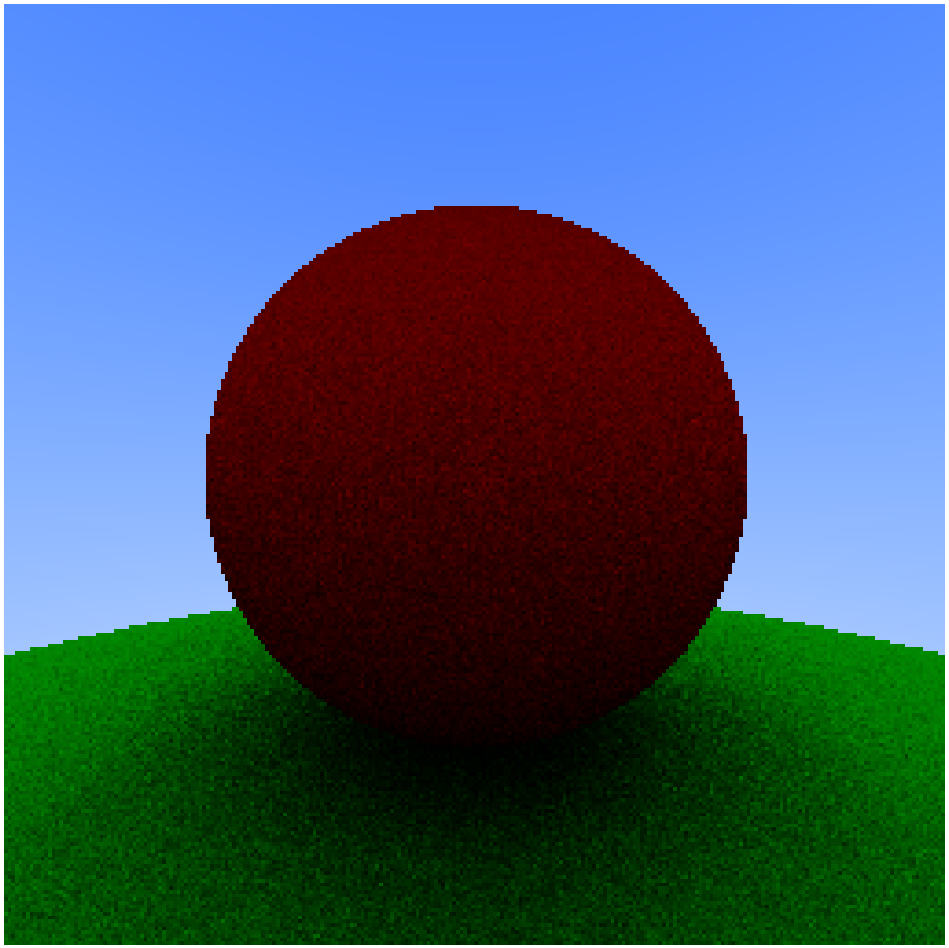
Multiple Samples

- Set color to average of all samples
- Taking n samples per pixel makes render take n times longer to finish

```
# ...
samples = 50
for y in range(height):
    for x in range(width):
        final_color = Vector3(0, 0, 0)
        for _ in range(samples):
            ray = camera.generate_ray(x / width, y / height)
            color = get_intersection(ray, spheres, 20)
            final_color += color
        final_color /= samples
        wrapper.write_pixel(x, y, final_color)
    wrapper.save() # save more often
```

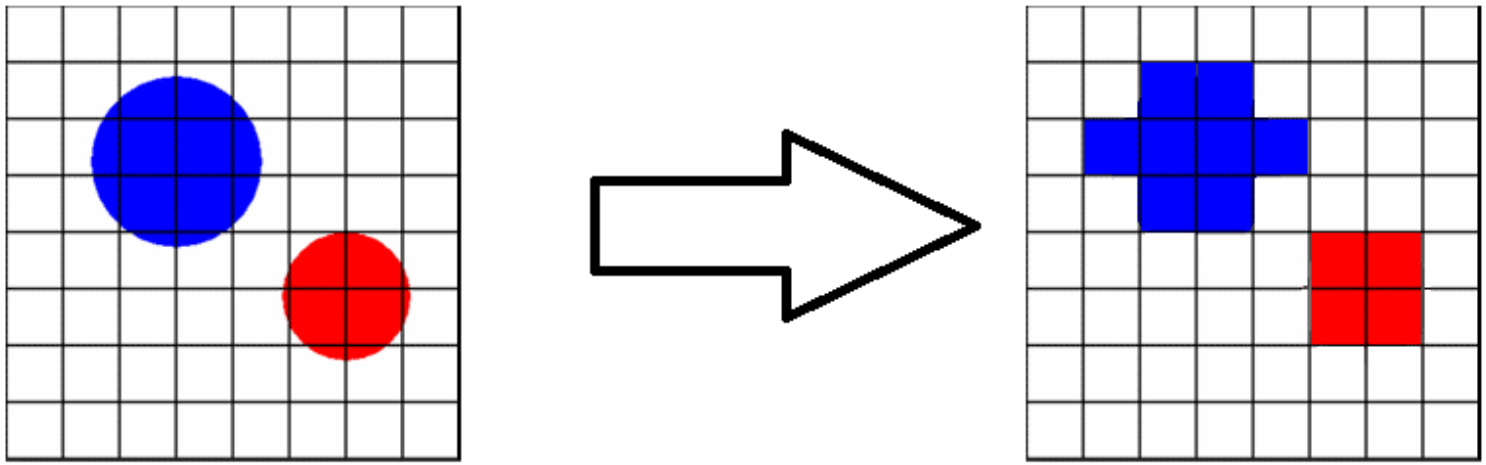
Multiple Samples

- Set color to average of all samples
- Taking n samples per pixel makes render take n times longer to finish



Anti-Aliasing

- Image still contains harsh boundaries between sphere and background
- Each pixel only contains what's at the center of the pixel



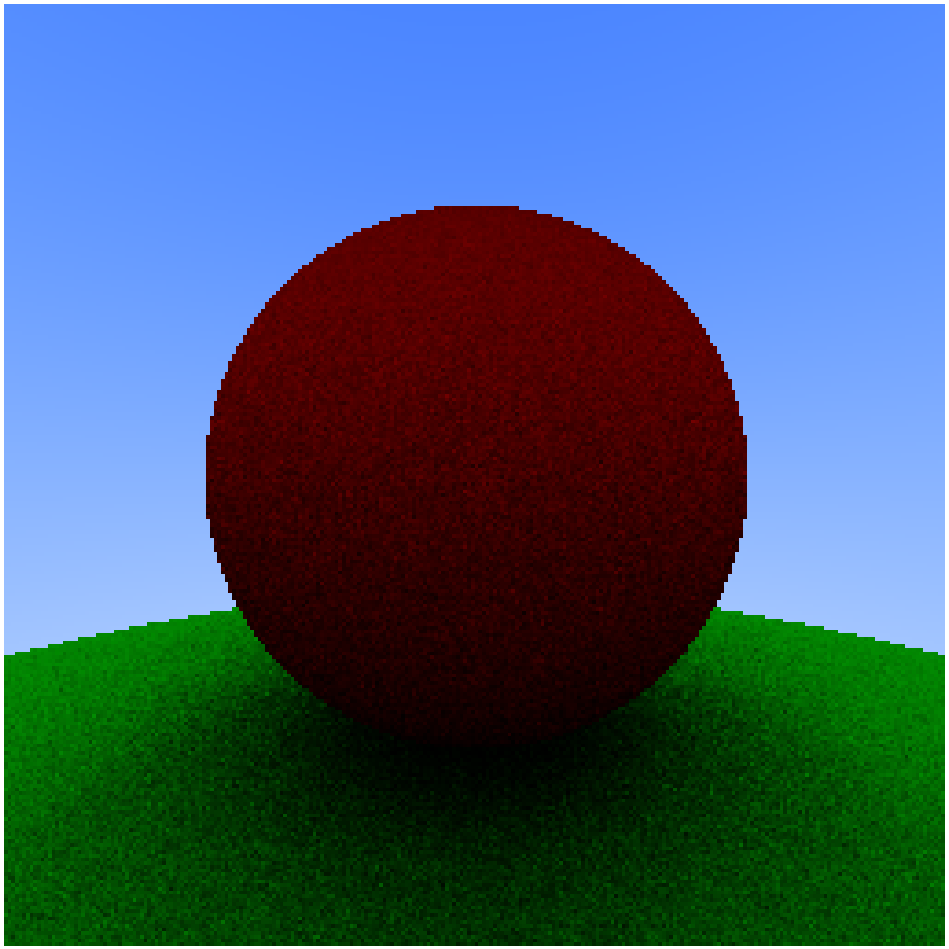
Anti-Aliasing

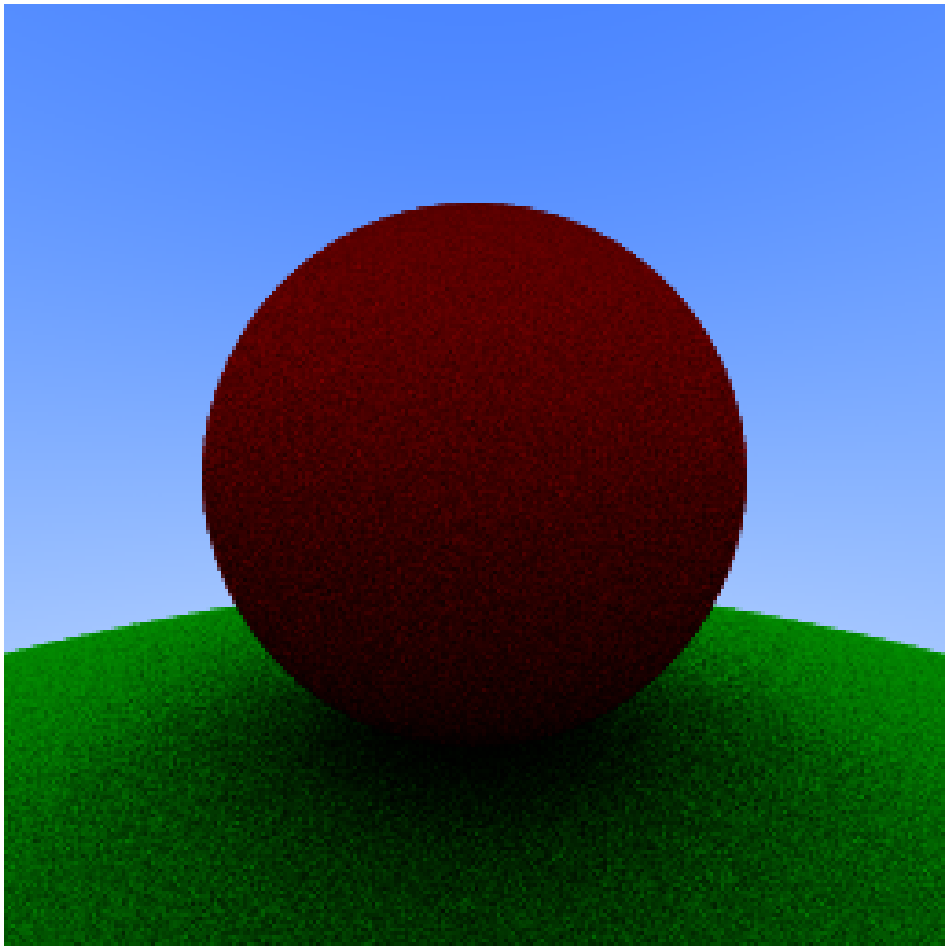
- Nudging each original ray better represents what the pixel shows

```
samples = 50
for y in range(height):
    for x in range(width):
        final_color = Vector3(0, 0, 0)
        for sample in range(samples):
            ox = x + random.random() # offset x
            oy = y + random.random() # offset y
            ray = camera.generate_ray(ox / width, oy / height)
            color = get_intersection(ray, spheres, MAX_DEPTH)
            final_color += color
        final_color /= samples
        wrapper.write_pixel(x, y, final_color)
    wrapper.save()
```

Anti-Aliasing

- Nudging each original ray better represents what the pixel shows





Lambertian Class

- We want to eventually have multiple materials, so lets abstract the material into a class
- Stores the color and has single method to return new direction for ray

```
class Lambertian:
    def __init__(self, color):
        # color is Vector3
        self.color = color

    def scatter(self, record, ray):
        # returns direction for next ray
        return record.normal + rand_on_sphere()
```

Updating Sphere Class

- Sphere class now has a `material` member instead of `color`

```
class Sphere:
    def __init__(self, center, radius, material):
        self.center = center
        self.radius = radius
        self.material = material # updated
    def intersection(self, ray):
        diff = ray.origin - self.center
        a = ray.direction.dot(ray.direction)
        b = diff.dot(2 * ray.direction)
        c = diff.dot(diff) - self.radius * self.radius
        solutions = solve_quadratic(a, b, c)
        for solution in solutions:
            if solution > 0:
                p = ray.at(solution)
                normal = (p - self.center).normalize()
                # Now we also have to update HitRecord
                return HitRecord(p, self.material, solution, normal)
        return None
```

Updating HitRecord Class

- Similarly, `HitRecord` now has a `material` member instead of `color`

```
class HitRecord:
    def __init__(self, point, material, time, normal):
        self.point = point
        self.normal = normal
        self.time = time
        self.material = material
```

Updating get_intersection Method

- Use `scatter` on the `material` in the `HitRecord` to get the new direction

```
# generate new ray direction
direction = found_record.material.scatter(found_record, ray)
new_ray = Ray(found_record.point, direction)
curr_color = found_record.material.color
next_color = get_intersection(new_ray, sphere, depth - 1)
return curr_color.multiply(next_color)
```

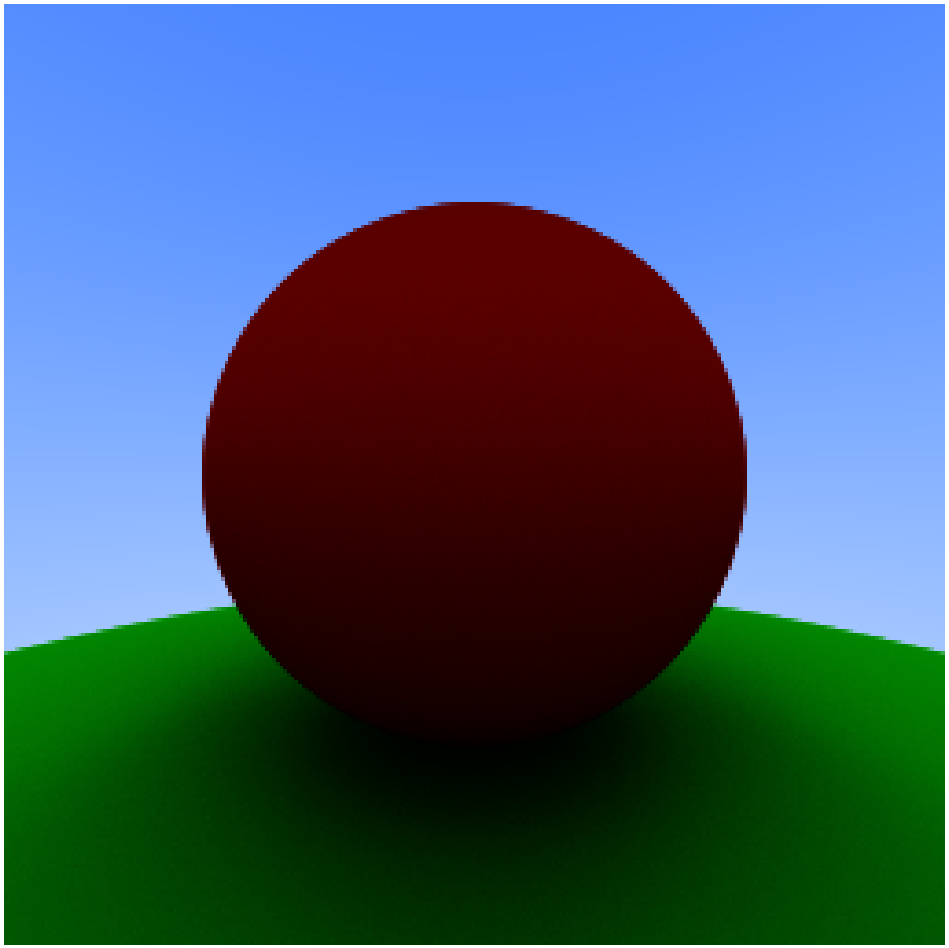
Updating Render Loop

- Construct spheres using new `Lambertian` class

```
red = Lambertian(Vector3(1, 0, 0))
green = Lambertian(Vector3(0.2, 1, 0.1))
spheres = [Sphere(Vector3(0, 10, 0), 5, red),
           Sphere(Vector3(0, 10, -100), 95, green)]
# same loop body as before
```

Updating Render Loop

- Resulting image is the same, but the code is more flexible
- This is the final scene rendered at 5,000 samples per pixel



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What next?

- Implement reflective, metallic material (reflect ray about normal vector)
- Implement transparent glass-like material (refract ray using Snell's law)
- Modify the camera and use the `focal_length` variable to create a depth-of-field effect
- Add texture coordinates so that you can wrap an image on a sphere
- Add support for more shapes
  - If you add support for triangles, you can import meshes (`.obj` files) and render arbitrary objects
- Make it multi-threaded (rendering is "embarrassingly parallelizable")

Further Reading and Resources

- Ray Tracing in a Weekend (`https://raytracing.github.io/`)
  - These slides are *heavily* inspired by this series
- Physically Based Rendering (`http://www.pbr-book.org/3ed-2018/contents.html`)
  - Very in-depth (sometimes too in-depth), but also very advanced and dense
- Advanced Global Illumination, 2nd Edition
  - Math-heavy, but also thorough (I'm currently reading this)
- An in-depth look at a variety of materials (`https://blog.selfshadow.com/publications/s2013-shading-course/hoffman/s2013_pbs_physics_math_slides.pdf`)
- List of websites with meshes that you can render:
  - `https://casual-effects.com/data/`
  - `https://threedscans.com/`
  - `https://pbrt.org/scenes-v3.html`