Transformation of the equatorial proper motion to the galactic system

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ABSTRACT

I present simple analytical equations to transform proper motion vectors from equatorial to Galactic coordinates.

Subject headings: stars: kinematics – astrometry – reference systems

The proper motions of celestial bodies are typically measured in the equatorial system of coordinates ($\mu_{\alpha\star} = \mu_{\alpha} \cos \delta$ and μ_{δ}). The great majority of objects for which proper motions give astrophysical information lie in the Galaxy. Thus, to analyze these data one needs to transform the measured values to the Galactic coordinate system ($\mu_{l\star} = \mu_l \cos b$ and μ_b). One way to do that is by representing the proper motion as a vector in a three-dimensional Cartesian system and applying a coordinate system rotation i.e., by multiplying the vector by a matrix, and projecting the result onto the sphere. The conversion of the coordinates between the equatorial system (α , δ) and the Galactic one (l, b) is described by the following equations (e.g., Binney & Merrifield 1998):

$$\sin b = \cos \delta \cos \delta_G \cos (\alpha - \alpha_G) + \sin \delta \sin \delta_G \tag{1}$$

$$\sin(l_{\Omega} - l)\cos b = \cos \delta \sin(\alpha - \alpha_G) \tag{2}$$

$$\cos(l_{\Omega} - l)\cos b = \sin \delta \cos \delta_G - \cos \delta \sin \delta_G \cos (\alpha - \alpha_G)$$
 (3)

where α_G and δ_G are the equatorial coordinates of the North Galactic Pole and l_{Ω} denotes the Galactic longitude of the ascending node of the Galactic plane. The values of these quantities are (Perryman & ESA 1997):

$$\alpha_G = 192^{\circ}85948, \qquad \delta_G = 27^{\circ}12825, \qquad l_{\Omega} = 32^{\circ}93192$$
 (4)

The transformation of the proper motions can be represented as a rotation:

$$\begin{pmatrix} \mu_{l\star} \\ \mu_b \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \mu_{\alpha\star} \\ \mu_{\delta} \end{pmatrix}$$
 (5)

where $A_{11} = \partial \mu_{l\star}/\partial \mu_{\alpha\star}$ etc. The values of A_{21} and A_{22} can be trivially found by differentiating the equation (1). At the same time, calculating A_{11} and A_{12} by the same method results in some tedious algebra. Instead of doing this, one can use the fact that the rotation matrix has to be orthogonal matrix with the determinant of 1 i.e., $A_{11} = A_{22}$ and $A_{12} = -A_{21}$. The resulting transformation is:

$$\begin{pmatrix} \mu_{l\star} \\ \mu_b \end{pmatrix} = \frac{1}{\cos b} \begin{pmatrix} C_1 & C_2 \\ -C_2 & C_1 \end{pmatrix} \begin{pmatrix} \mu_{\alpha\star} \\ \mu_{\delta} \end{pmatrix}$$
 (6)

where the coefficients C_1 , C_2 are given by:

$$C_1 = \sin \delta_G \cos \delta - \cos \delta_G \sin \delta \cos (\alpha - \alpha_G) \tag{7}$$

$$C_2 = \cos \delta_G \sin \left(\alpha - \alpha_G\right) \tag{8}$$

The quantity $\cos b$ can be calculated as

$$\cos b = \sqrt{C_1^2 + C_2^2} \tag{9}$$

The above equations define the way to transform proper motions from equatorial to Galactic coordinates even without direct calculation of the Galactic coordinates of the object. The equations for transformation of proper motions do not appear in many textbooks. They are included in Smart (1938) and were later reproduced by Bovy (2011).

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