Stellar Evolution

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1 Stellar Evolution

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```
In [41]: # Import statements
         import numpy as np
         import matplotlib.pyplot as plt
         import scipy.stats
         import scipy.interpolate
```

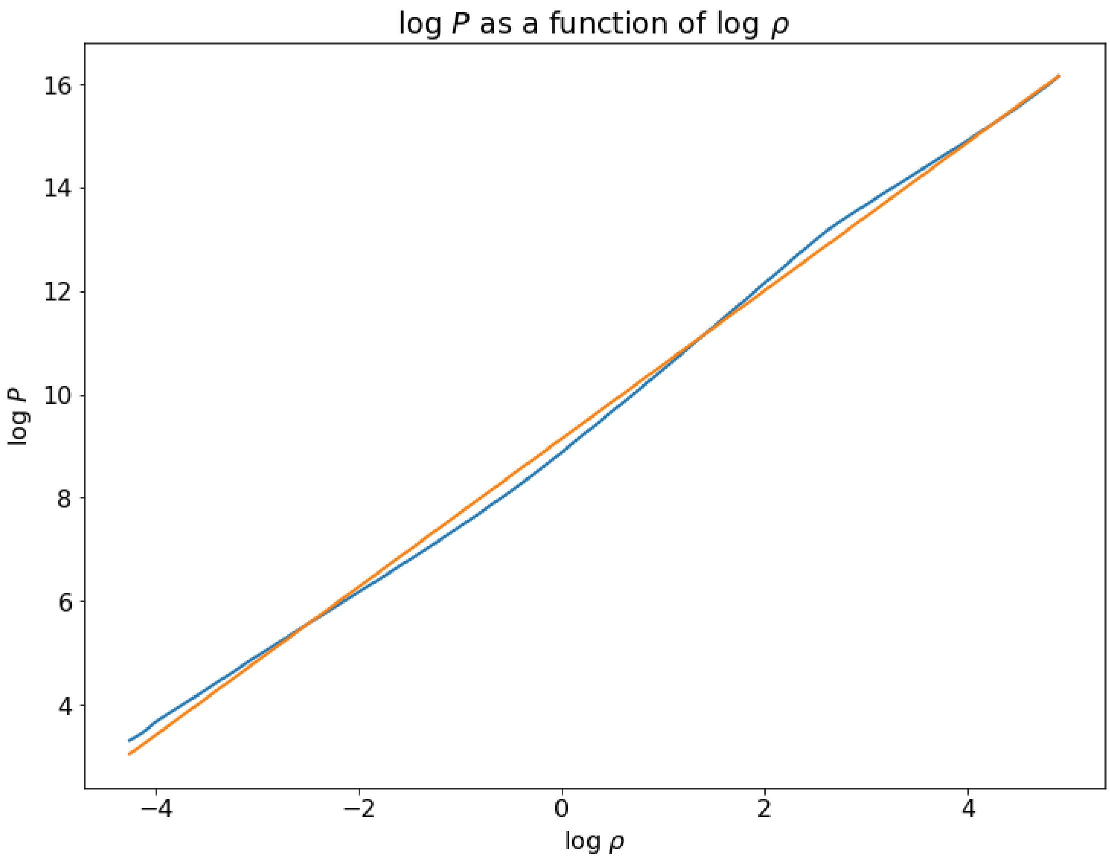
```
In [42]: # Matplotlib settings
         plt.rcParams['figure.figsize'] = [12, 9]
         plt.rcParams['font.size'] = 16
         plt.rcParams['lines.linewidth'] = 2
```

```
In [43]: # Convenience function to return a specific column from the data file
         def getcol(data, idx):
             return data[:,idx]
```

Plot a line graph of $\log P$ as a function of $\log \rho$. Estimate the gradient of the curve, and from that calculate an approximate polytropic index for the star.

```
In [44]: data = np.loadtxt('structure_00000.txt')

         # numbers of columns in data file
         col_pressure = 3
         col_density = 4

         log_density = np.log10(getcol(data, col_density))
         log_pressure = np.log10(getcol(data, col_pressure))

         # use linear fit to get gradient
         m, c, _, _, _ = scipy.stats.linregress(log_density, log_pressure)

         plt.figure()
         ax = plt.gca()
         ax.set_xlabel(r'$\log\ \rho$')
         ax.set_ylabel(r'$\log\ P$')
```

```
ax.set_title(r'$\log\ P$ as a function of $\log\ \rho$')
plt.plot(log_density, log_pressure)
plt.plot(log_density, [m * x + c for x in log_density])
plt.show()
```



The polytropic index n is given by $n = 1/(\gamma - 1)$, where $P = K\rho^{\gamma}$.
We have $\log(P) = m \log(\rho) + c$. Hence the gradient m is equal to γ.

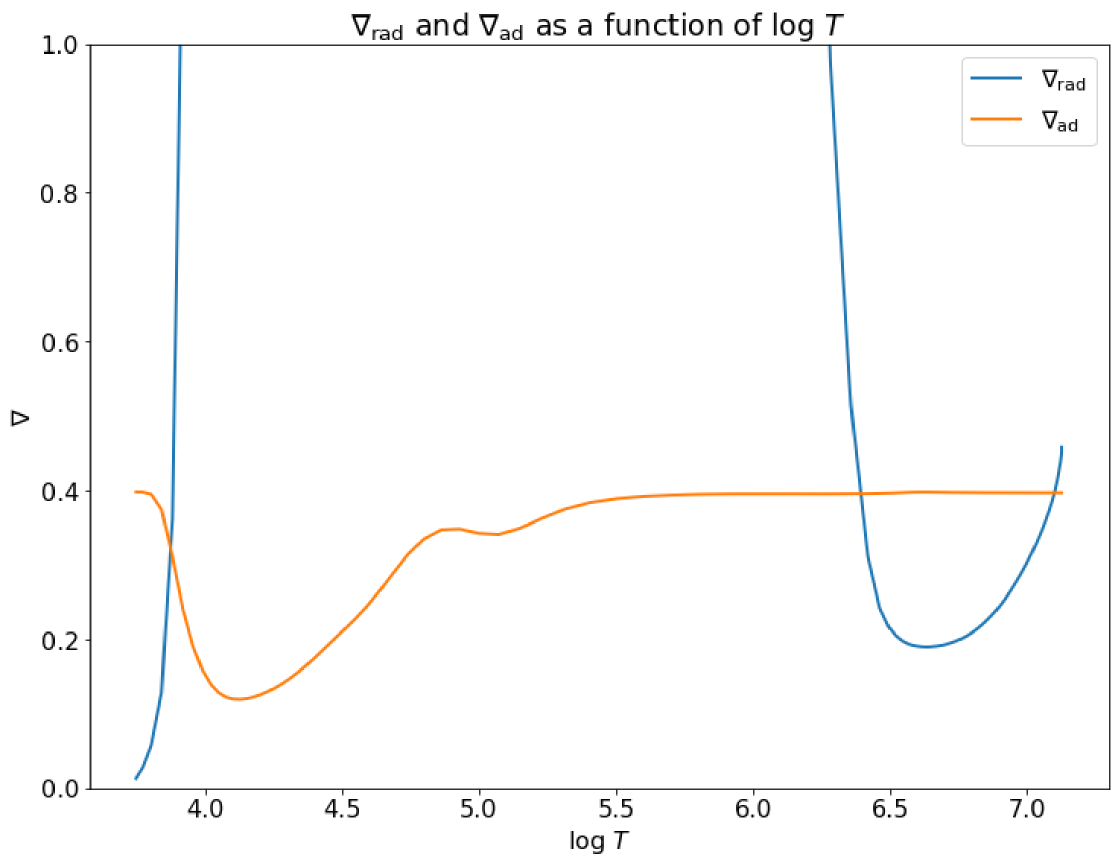
```
In [45]: polytropic_index = 1 / (m - 1)
         print('Polytropic index is {:.3f}'.format(polytropic_index))
```

```
Polytropic index is 2.307
```

Make a line plot of the radiative temperature gradient $\nabla_{\rm rad}$ and adiabatic temperature gradient $\nabla_{\rm ad}$ (where $\nabla = \mathrm{d}\log T/\mathrm{d}\log P$) as a function of $\log T$. Hence identify the size of the outer convection zone expressed as a fraction of the total radius of the star.

```
In [46]: # numbers of columns in data file
         col_rad_temp_grad = 15
         col_adb_temp_grad = 10
```

```
col_temp = 5

rad_temp_grad = getcol(data, col_rad_temp_grad)
adb_temp_grad = getcol(data, col_adb_temp_grad)
log_temp = np.log10(getcol(data, col_temp))

plt.figure()
ax = plt.gca()
ax.set_xlabel(r'$\log\ T$')
ax.set_ylabel(r'$\nabla$')
ax.set_ylim(0, 1)
ax.set_title(r'$\nabla_\mathrm{rad}$ and $\nabla_\mathrm{ad}$ as a function of $\log\ T
plt.plot(log_temp, rad_temp_grad, label='$\\nabla_\\mathrm{rad}$')
plt.plot(log_temp, adb_temp_grad, label='$\\nabla_\\mathrm{ad}$')
plt.legend()
plt.show()
```



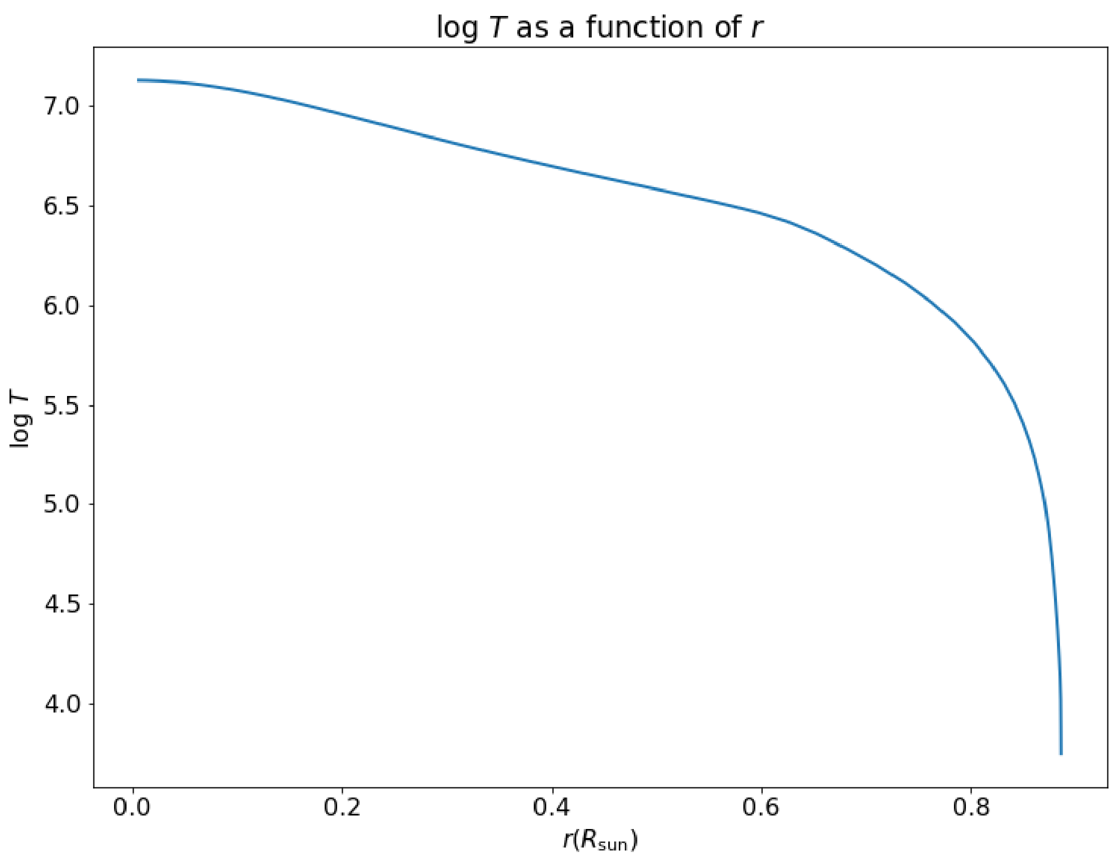

```
In [47]: # we are interested in regions where the radiative gradient exceeds the adiabatic one
         # by eye, we also need to exclude the tiny convective region in the core
         log_temp_filter = (rad_temp_grad > adb_temp_grad) & (log_temp < 6.5)
```

```
convective = log_temp[log_temp_filter]

log_temp_lower = convective[0]
log_temp_upper = convective[-1]

print('The convective region extends between log T values of {:.3g} and {:.3g}'
      .format(log_temp_lower, log_temp_upper))
```

```
The convective region extends between log T values of 3.88 and 6.36
```

To find the corresponding radius coordinates, we plot $\log\ T$ against r.

```
In [48]: col_radius = 1
         radius = getcol(data, col_radius)

         plt.figure()
         ax = plt.gca()
         ax.set_xlabel(r'$r (R_\mathrm{sun})$')
         ax.set_ylabel(r'$\log\ T$')
         ax.set_title(r'$\log\ T$ as a function of $r$')
         plt.plot(radius, log_temp)
         plt.show()
```



Linear interpolation of the curve above can be used to find the radial coordinates corresponding to the start and end of the convective region.

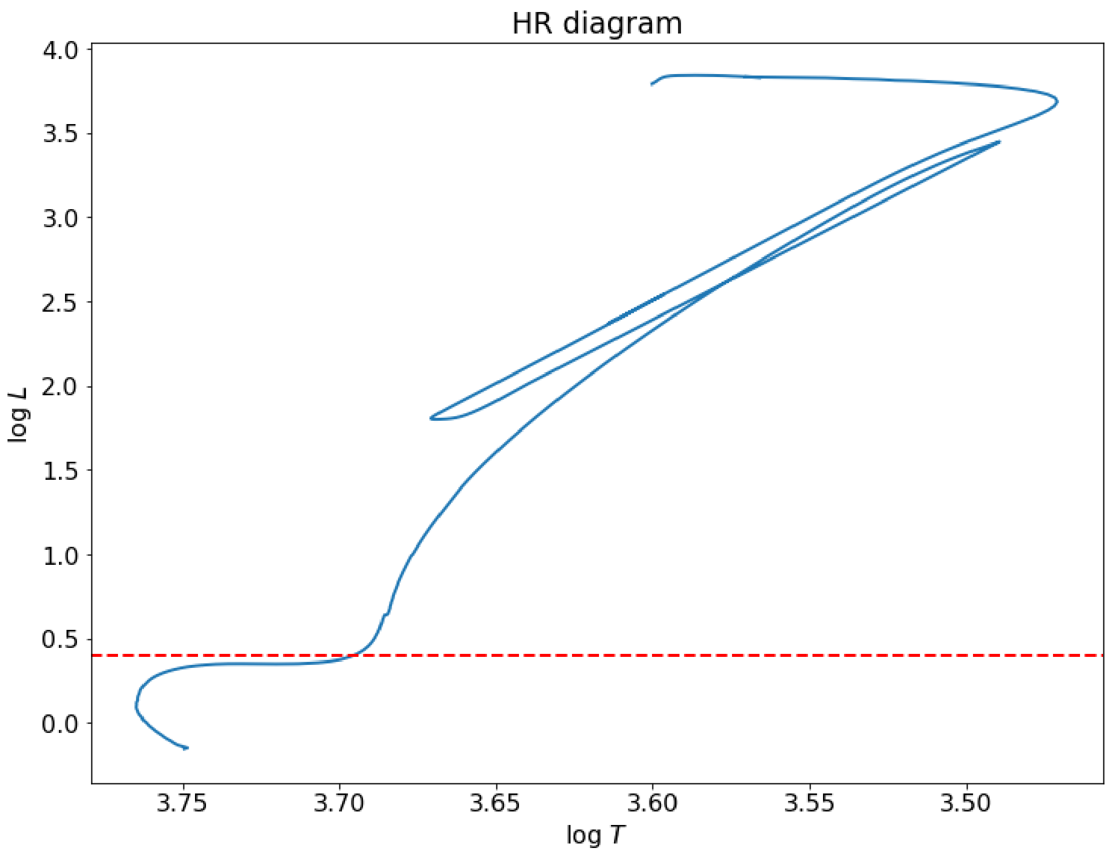
```
In [49]: r_func_log_temp = scipy.interpolate.interp1d(log_temp, radius)
         r_inner = float(r_func_log_temp(log_temp_upper))
         r_outer = float(r_func_log_temp(log_temp_lower))

         print('Outer convective zone extends between {:.3g} and {:.3g} R_sun'
               .format(r_inner, r_outer))
```

```
Outer convective zone extends between 0.652 and 0.886 R_sun
```

Estimate the age of the star and the mass of the helium core at the point where the star begins its first ascent of the red giant branch.

To find this point, need to draw HR diagram.

```
In [50]: summary = np.loadtxt('summary.txt')

         # numbers of columns in data file
         col_temperature = 5
         col_luminosity = 3

         # these quantities are already reported as logs
         log_temp = (getcol(summary, col_temperature))
         log_luminosity = (getcol(summary, col_luminosity))

         plt.figure()
         ax = plt.gca()
         ax.set_xlabel(r'$\log\ T$')
         ax.set_ylabel(r'$\log\ L$')
         ax.set_title('HR diagram')
         ax.invert_xaxis()
         plt.plot(log_temp, log_luminosity)
         plt.axhline(0.4, color='r', linestyle='--')
         plt.show()
```



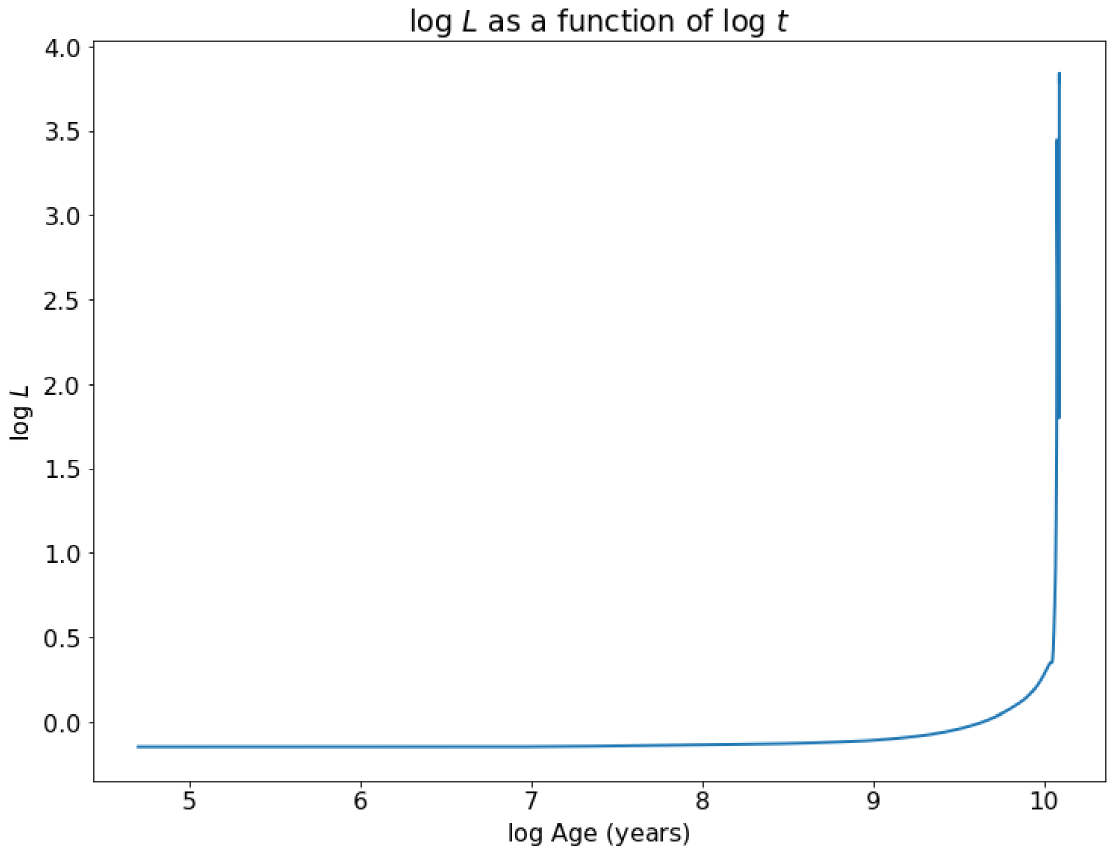
From HR diagram above, start of first RGB ascent is at $\log(L/L_{\mathrm{sun}}) \sim 0.4$.

```
In [51]: rgb_log_l = 0.4
```

Plot $\log\ L$ against $\log\ t$ to find corresponding age.

```
In [52]: col_age = 1
         log_age = np.log10(getcol(summary, col_age))

         plt.figure()
         ax = plt.gca()
         ax.set_xlabel(r'$\log\ \mathrm{Age}$ (years)')
         ax.set_ylabel(r'$\log\ L$')
         ax.set_title(r'$\log\ L$ as a function of $\log\ t$')
         plt.plot(log_age, log_luminosity)
         plt.show()
```

```
/usr/local/lib/python3.5/dist-packages/ipykernel_launcher.py:2: RuntimeWarning: divide by zero e
```

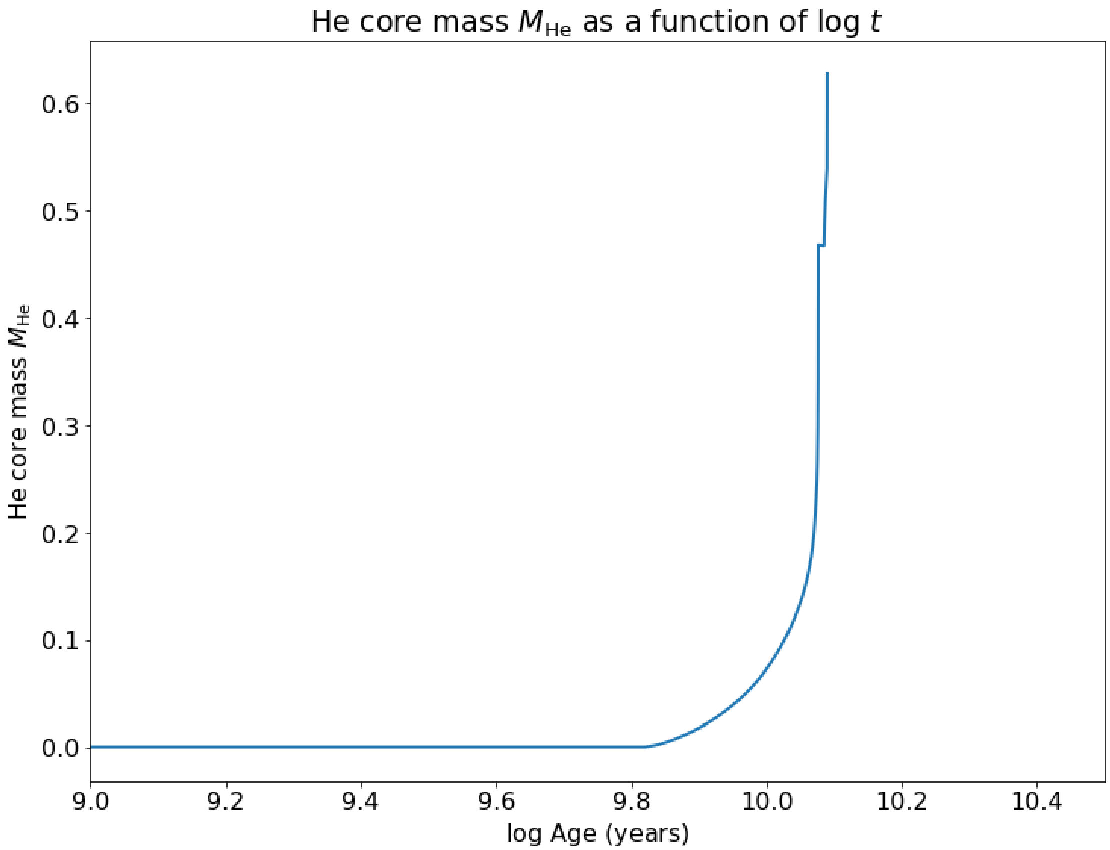



```
In [53]: log_age_func_log_l = scipy.interpolate.interp1d(log_luminosity, log_age)
         rgb_log_age = float(log_age_func_log_l(rgb_log_l))

         print('Age at start of RGB ascent is {:.3g} yrs'.format(10**rgb_log_age))
```

```
Age at start of RGB ascent is 1.13e+10 yrs
```

Finally, plot age against helium core mass.

```
In [54]: col_he_core_mass = 23
         he_core_mass = getcol(summary, col_he_core_mass)

         plt.figure()
         ax = plt.gca()
         ax.set_xlabel(r'$\log\ \mathrm{Age}$ (years)')
         ax.set_xlim(9, 10.5)
         ax.set_ylabel(r'He core mass $M_\mathrm{He}$')
         ax.set_title(r'He core mass $M_\mathrm{He}$ as a function of $\log\ t$')
         plt.plot(log_age, he_core_mass)
         plt.show()
```




```
In [55]: he_core_mass_func_log_age = scipy.interpolate.interp1d(log_age, he_core_mass)
         rgb_he_core_mass = float(he_core_mass_func_log_age(rgb_log_age))

         print('He core mass at start of RGB ascent is {:.3g} M_sun'.format(rgb_he_core_mass))
```

```
He core mass at start of RGB ascent is 0.14 M_sun
```