Cosmology Homework

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Question 1

a) The **comoving distance** d_x is given by:

$$d_x(z) = \frac{c}{H_0} \int_0^z \frac{\mathrm{d}z'}{E(z')}, \tag{1}$$

where

$$E(z) = \left[\Omega_m(1+z)^3 + \Omega_\Lambda\right]^{1/2}$$

b) The **angular diameter distance** d_A is given by:

$$d_A(z) = \frac{d_x(z)}{1+z} \tag{2}$$

c) The **luminosity distance** d_L is given by:

$$d_L(z) = (1+z)^2 d_A(z) \tag{3}$$

d) The **age of the Universe** t is given by:

$$t(z) = \frac{1}{H_0} \int_z^\infty \frac{\mathrm{d}z'}{(1+z')E(z')} \tag{4}$$

Equations (1) to (4) are plotted for $\{z \mid 0 \le z \le 10\}$ in Fig. 1 below. Since these equations only depend on h via the prefactor $1/H_0$, this term may be factored out to give results independent of the value of h.

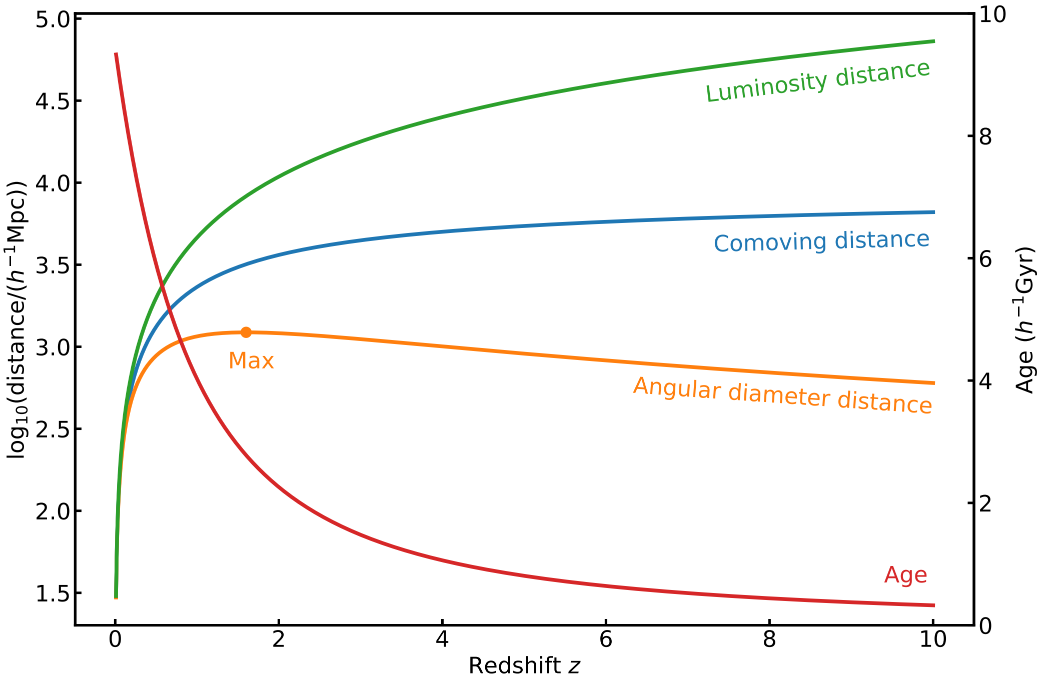

Figure 1: Distances (left axis) and age (right axis) for a flat Universe with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. The maximum value of the angular diameter distance d_A occurs at $(z, d_A) = (1.61, 1.22\,h^{-1}\mathrm{Gpc})$.

Question 2

If stars are formed at redshift $z = 4$, their age at $z = 3$ is equal to the difference in the ages of the Universe at these two redshifts. Hence, denoting the stars' age as Δt:

$$\begin{aligned}\Delta t &= t(z{=}3) - t(z{=}4) \\ &= 418\,h^{-1}\mathrm{Myr}\end{aligned} \tag{5}$$

Question 3

Lyman-α sources with rest-frame wavelength $\lambda_e = 1215.67\,$Aø will be detected if their observed (redshifted) wavelength λ_o falls within the range that the filter transmits. Hence the survey will collect light from objects distributed over a range of redshifts. Using $z = \lambda_o/\lambda_e - 1$, this range is found to be $2.99 < z < 3.01$.

The physical size of the telescope field at these redshifts ℓ can be found using the angular diameter distance:

$$\ell(z) = \theta d_A(z) \tag{6}$$

Evaluating Eq. (6) for the redshift range of the survey gives $\ell(z{=}2.99) = 1.621\,h^{-1}\mathrm{Mpc}$ and $\ell(z{=}3.01) = 1.617\,h^{-1}\mathrm{Mpc}$ respectively; as these values are similar their average may be treated as a constant value for ℓ over the redshift range considered.

The comoving and physical volumes are then found by multiplying ℓ^2 by the corresponding value for the depth of the survey:

$$V_{\mathrm{comoving}} = \ell^2 \times (d_x(z{=}3.01) - d_x(z{=}2.99)) \tag{7}$$

$$V_{\mathrm{physical}} = \ell^2 \times (d_r(z{=}3.01) - d_r(z{=}2.99)) \tag{8}$$

where the physical distance d_r is given by:

$$d_r(z) = \frac{c}{H_0}\int_0^z \frac{\mathrm{d}z'}{(1+z')E(z')} \tag{9}$$

Evaluating Eqs. (7) and (8) gives the results $V_{\mathrm{comoving}} = 43.5\,h^{-3}\mathrm{Mpc}^3$ and $V_{\mathrm{physical}} = 10.9\,h^{-3}\mathrm{Mpc}^3$.

To find the intrinsic surface brightness of a distant galaxy, the observed surface brightness in mag/arcsec^2 first needs to be converted into physical units. Apparent magnitude is defined as:

$$m_B = M_{\odot,B} - 2.5\log_{10}(L_B) + 5\log_{10}(d/10\mathrm{pc}) \tag{10}$$

where the B subscript indicates magnitudes and luminosities in the corresponding band are considered (as this band overlaps with the wavelength of the filter).

The luminosity may be expressed as $L_B = (d\ \mathrm{arcsec})^2 S_{\mathrm{phys}}$, where S_{phys} is the surface brightness in physical units of $L_{\odot,B}/\mathrm{pc}^2$. Substituting this into Eq. (10) gives:

$$\mu = M_{\odot,B} + 21.572 - 2.5\log_{10} S_{\mathrm{phys}} \tag{11}$$

where μ is the apparent surface brightness in magnitude units. Using $M_{\odot,B} = 5.48$ and substituting the given $\mu = 28$, Eq. (11) may be rearranged to obtain $S_{\mathrm{phys}} = 0.418\,L_{\odot,B}/\mathrm{pc}^2$.

From the notes:

$$S_i = 4\pi(1+z)^4 S_a \tag{12}$$

where S_i and S_a are respectively the intrinsic and apparent surface brightnesses in physical units. Taking $z = 3$, Eq. (12) gives $S_i = 1340\,L_{\odot,B}/\mathrm{pc}^2$.

Finally, Eq. (11) may be used to convert this back into magnitude units, yielding the result:

$$\mu = 19.2\,\mathrm{mag/arcsec}^2$$