Data Analysis for Mechanical Engineering

Type I and II Errors and Power of a Test

William 'Ike' Eisenhauer

Department of Mechanical and Materials Engineering
Portland State University
Portland, Oregon 97223

wde@pdx.edu

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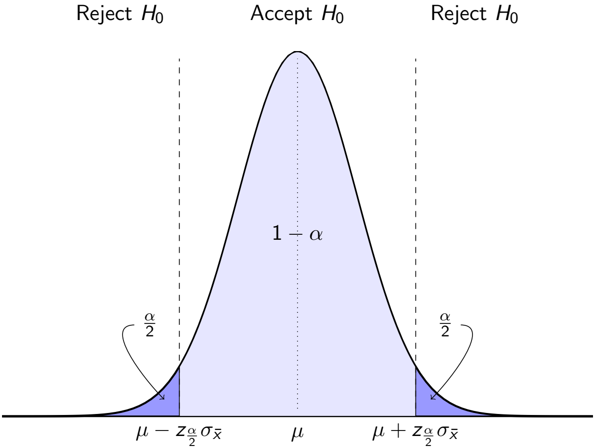
Type I Error



$$\alpha = 1 - \Phi(\mu - z_{\frac{\alpha}{2}}\sigma_{\bar{x}} < \bar{x} < \mu + z_{\frac{\alpha}{2}}\sigma_{\bar{x}}; \mu, \sigma_{\bar{x}})$$

Type I Error: False Alarm

Recall, Type I is Rejecting H_0 when the reality is H_0. The probability of doing this is α

$$\alpha = 1 - \Phi(\mu - z_{\frac{\alpha}{2}}\sigma_{\bar{x}} < \bar{X} < \mu + z_{\frac{\alpha}{2}}\sigma_{\bar{x}}; \mu, \sigma_{\bar{x}})$$

Type II Error: Missed Alarm

- Type II is Accepting (or Failing to Reject) H_0 when the reality is H_A. The probability of doing this is β
- One minor issue, you CAN ensure you NEVER make a Type II Error.
- How?
- Not very useful though is it?

Type II Error: Example

Let's look at an example to help clarify before formalizing this:

Example

- 16 Gears are samples and diameters measured and averaged
- Population variance is known to be 20
- You hypothesize that the population average is 600 at the 0.05 significance level
- What is the chance of making a Type II Error if the real average is 608?

Type II Error: Example

Example

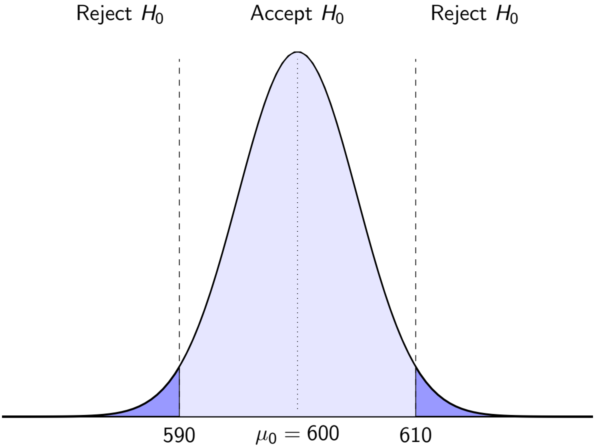
Start with Acceptance Region of H_0

$$\mu - z_{0.025}\frac{\sigma}{\sqrt{N}} < \bar{X} < \mu + z_{0.025}\frac{\sigma}{\sqrt{N}}$$

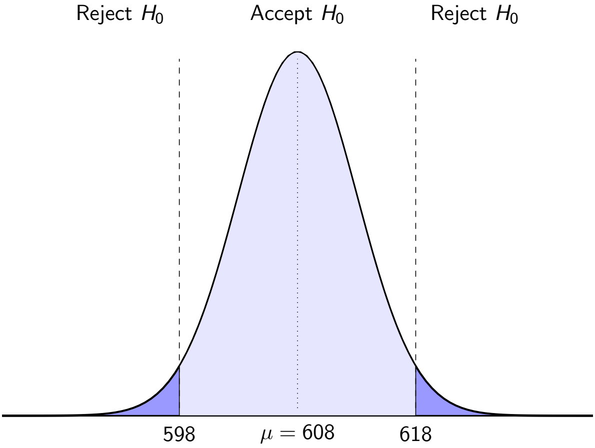
$$600 - (1.96)\frac{20}{\sqrt{16}} < \bar{X} < 600 + (1.96)\frac{20}{\sqrt{16}}$$

$$590 < \bar{X} < 610$$

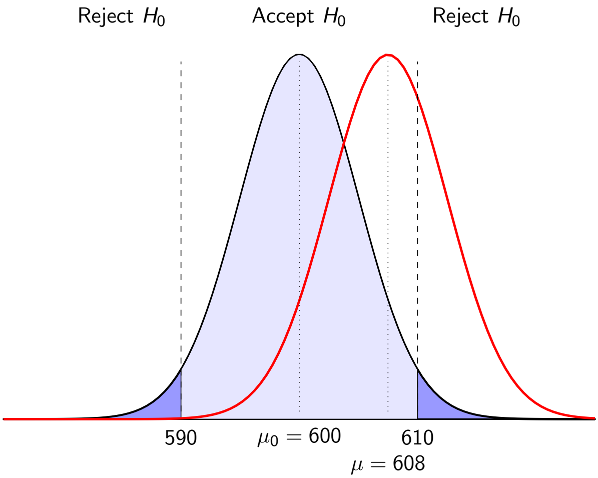
So we 'Think' the distribution of sample means should be



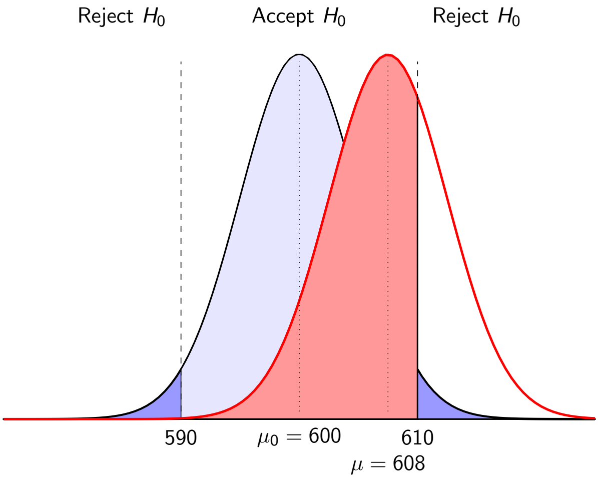
But the reality is



Overlaying Them



Accept μ_0 even though we are wrong!



Type II Error: Missed Alarm

Type II is Accepting H_0 when the reality is H_A. The probability of doing this is β

$$\beta = \Phi(\mu - z_{\frac{\alpha}{2}}\sigma_{\bar{x}} < \bar{X} < \mu + z_{\frac{\alpha}{2}}\sigma_{\bar{x}}; \mu, \sigma_{\bar{x}})$$

$$\beta = \Phi(590 < \bar{X} < 610; 608, 5)$$

$$\beta = \Phi(\frac{590 - 608}{5} < Z < \frac{610 - 608}{5})$$

$$\beta = \Phi(-3.60 < Z < 0.40) = 0.6554$$

65.5% Chance of Type II Error

Power of the Test (P)

Another use of β is to determine the power of a test.

Power of a Test (P)

The Probability of Detecting a Specified Difference or Effect

$$P = 1 - \beta$$

Power is useful for determining the minimal sample size needed beforehand to have a good shot at detecting a actual significant difference.

Sample Size

$$N \geqslant \left(\frac{(z_\beta + z_{\frac{\alpha}{2}})\sigma}{\delta} \right)^2$$

Note: Assumes you know the population standard deviation, or your sample is above 30. δ can either be difference between a known value [one means] or two sample means [two means]

Sample Size Example

Horsepower differences

How many motors are needed to detect a shift from a mean output of $40hp$ to $40 \pm 2hp$ with a probability of 0.90. Significance level needed is 0.05. The population standard deviation is known to be $2.2hp$ and the distribution of hp is normal.

$$\mu = 40$$

$$\delta = 2$$

$$P = 1 - \beta = 0.9$$

$$\alpha = 0.05$$

$$\sigma = 2.2$$

Sample Size Example

$$\beta = 0.1 \Rightarrow z_\beta = z_{0.1} = 1.28$$

$$\alpha = 0.05 \Rightarrow z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$$

$$N \geqslant \left(\frac{(z_\beta + z_{\frac{\alpha}{2}})\sigma}{\delta} \right)^2$$

$$N \geqslant \left(\frac{(1.28 + 1.96)(2.2)}{2} \right)^2$$

$$N \geqslant (3.24(1.1))^2 \Rightarrow N \geqslant 12.70 = 13$$

Power in R

R Code

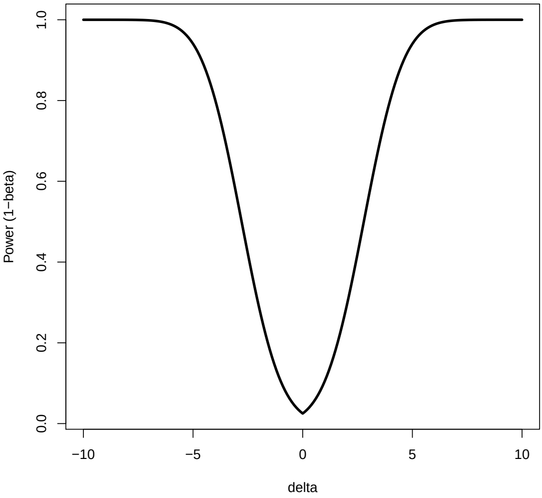
```
power.t.test(100, delta=5, sd=10, sig.level=0.05)

##
##      Two-sample t test power calculation
##
##               n = 100
##           delta = 5
##              sd = 10
##       sig.level = 0.05
##           power = 0.9404
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

Power Graphs in R

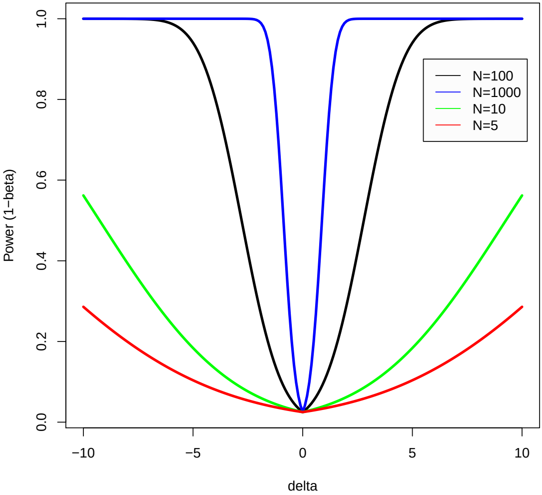
R Code

```
plot(type="l",seq(-10,10,by=0.1),
  power.t.test(
    100,
    delta=seq(-10,10,by=0.1),
    sd=10,
    sig.level=0.05)$power,
  xlab="delta",
  ylab="Power (1-beta)",
  lwd="3",
  col="black"
)
```

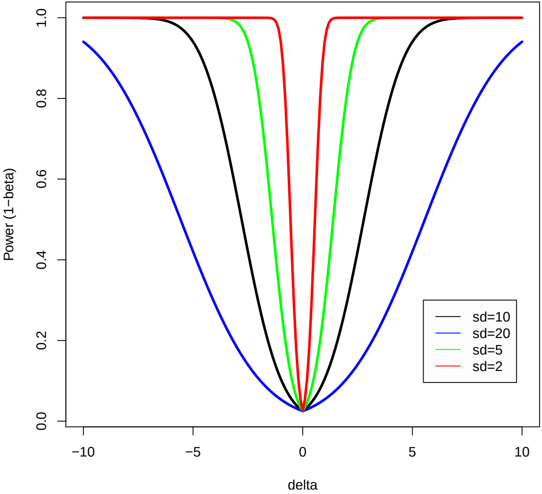
Power Graphs in R



Power Graphs in R - Changing N



Power Graphs in R - Changing σ



Power Graphs in R - Changing α