Question 1: Conversion to Logarithmic Additive Form

⋆ Given

Taylor's modified tool life equation is:

$$TL = \frac{C_T}{{V_c}^x \, {s_o}^y \, t^z}$$

where x, y, z, C_T are all experimentally determined parameters based on tool and work materials and machine environments.

⋆ Find

Convert to logarithmic additive form. i.e. Write out equation for $w = \ln TL$.

⋆ Solution

$$\begin{aligned}
w &= \ln TL = \ln \left[\frac{C_T}{{V_c}^x \, {s_o}^y \, t^z} \right] \\
w &= \ln TL = \ln C_T - \ln({V_c}^x \, {s_o}^y \, t^z) \\
w &= \ln TL = \ln C_T - \ln {V_c}^x - \ln {s_o}^y - \ln t^z \\
w &= \ln TL = \ln C_T - x \ln V_c - y \ln s_o - z \ln t
\end{aligned}$$

$$\boldsymbol{w = \ln TL = \ln C_T - x \ln V_c - y \ln s_o - z \ln t} \Leftarrow$$

Question 2: Partial Derivatives

★ **Given**

$$z = f(x, y) = x^4y^3 + 8x^2y + y^4 + 5x$$

★ **Find**

$$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$$

★ **Solution**

$$\frac{\partial}{\partial x} x^4y^3 + 8x^2y + y^4 + 5x$$

$$\boldsymbol{\frac{\partial z}{\partial x} = 4x^3y^3 + 16xy + 5} \Leftarrow$$

$$\frac{\partial}{\partial y} x^4y^3 + 8x^2y + y^4 + 5x$$

$$\boldsymbol{\frac{\partial z}{\partial y} = 3x^4y^2 + 8x^2 + 4y^3} \Leftarrow$$

Question 3: Partial Derivatives

⋆ Given

If, for Question 1, you set the following:

$$w = \ln TL \qquad a = \ln V_c$$
$$b = \ln s_o \qquad c = \ln t$$

So that, $w(a, b, c)$ now exists.

⋆ Find

$$\nabla w$$

⋆ Solution

$$\begin{aligned}
&\nabla w \\
&\nabla \left(\ln C_T - x \ln V_c - y \ln s_o - z \ln t\right) \\
&\nabla \left(\ln C_T - xa - yb - zc\right) \\
&\left[\frac{\partial w}{\partial a}, \frac{\partial w}{\partial b}, \frac{\partial w}{\partial c}\right] \\
&[-x, -y, -z]
\end{aligned}$$

$$\underline{\underline{\boldsymbol{[-x, -y, -z]}}} \Leftarrow$$

Question 4: Compare Means Test Interpretation

⋆ Given

Here is the R output of a two means test. Answer the following:

```
> t.test(data1, data2, var.equal=1)

  Two Sample t-test

data:  data1 and data2
t = 1.6085, df = 23, p-value = 0.1214
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.6986671  5.5833778
sample estimates:
mean of x mean of y
 22.06783  19.62548
```

⋆ Find

1. H_0
2. H_A
3. If the number of samples of data1 is 10, how many samples are in data2?
4. What is the conclusion from this test, if $\alpha = 0.05$?
5. What is the chance of a Type I error if you said the means were different?

⋆ Solution

- $H_0 : \mu_{data1} = \mu_{data2}$
- $H_A : \mu_{data1} \neq \mu_{data2}$
- $df = n_1 + n_2 - 2 \rightarrow n_2 = df + 2 - n_1 = 23 + 2 - 10 = 15$
- $p_{value} > \alpha$ ∴ cannot reject H_0
- Type I Error Rate $= p_{value} = 12.14\%$

Question 5: Regression Interpretation

⋆ Given

Here is the R output of an linear regression. Determine the following:

```
> summary(model)

Call:
lm(formula = y ~ x1 + x3 + x5 + x7 + x9 + x11, data = alldata)

Residuals:
    Min      1Q  Median      3Q     Max
-2192.1  -974.5  -121.1   838.6  3602.6

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 1401.0445  2197.2563   0.638  0.52401
x1             0.8910     2.7800   0.321  0.74871
x3            -0.6842     1.2399  -0.552  0.58132
x5            -1.9535     2.5699  -0.760  0.44753
x7            -4.7583     0.1607 -29.615  < 2e-16 ***
x9             8.5646     1.7890   4.787 2.24e-06 ***
x11          -11.5116     4.3129  -2.669  0.00786 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1217 on 493 degrees of freedom
Multiple R-squared: 0.7544, Adjusted R-squared: 0.7514
F-statistic: 252.4 on 6 and 493 DF,  p-value: < 2.2e-16

> confint(model)
                  2.5 %      97.5 %
(Intercept) -2916.097249 5718.186249
x1             -4.571095    6.353167
x3             -3.120431    1.751991
x5             -7.002812    3.095836
x7             -5.074027   -4.442649
x9              5.049680   12.079548
x11           -19.985608   -3.037678
>
```

⋆ Find

1. How many data points were used in this model development?
2. Which variable (other than intercept) would you throw out next? Why?
3. Is this a better model than the "mean" model, explain your rationale

⋆ Solution

- $df = n - N_{variables} - 1 \rightarrow n = df + N_{variables} + 1 = 493 + 6 + 1 = 500$
- x_1: Confidence Interval Contains 0 and p_{value} is highest
- $df = n_1 + n_2 - 2 \rightarrow n_2 = df + 2 - n_1 = 23 + 2 - 10 = 15$
- Yes, the F-value is extremely far away from 1 and has an extremely low p_{value}

Question 6: Design a Simple Experiment

⋆ Given

You have a balance scale and a bag of 10 bolts, one of which is heavier than the others,

⋆ Find

What is the least amount of weighings necessary to guarantee the identification of the heavier bolt. Show your experimental strategy (i.e. Action - Result - Next Action(s) Tree). Generalize, by giving the lower bound of the minimal number of weighings required for N bolts, given one is heavier than the others.

⋆ Solution

- Split into 3 groups evenly as possible, in this case A:3,B:3,C:4
- Weigh two even groups, in this case A,B
- If they are even, split the remaining goup in three as even as possible groups and repeat
- If they are not, then split the lower scape group in three as even as possible groups and repeat
- When left with 1 bolt, that is the one
- Lower bound of weighings is $\lfloor log_3 N \rfloor$
- Upper bound of weighings is $\lceil log_3 N \rceil$

Grader: Give credit -3 if they attempted a binary (2 group split), good but using the information that the direction of defect is known allows one to use 3 groups instead, more efficient.

Question 7: Design a Bit More complicated Experiment

⋆ Given

Given the following possible sets of My Little Ponies that you are playing a game of Guess Who with.

⋆ Find

What is the smallest set of YES/NO questions required to guarantee the identification of ANY choosen Pony. List your set of questions, and for each possible choosen Pony in the chart, show how many questions would be required to identify it. Remember you are not allowed to use positional based questions, as the positions could be randomized without your knowledge.

⋆ Solution

Question set has to have at least 6 questions $log_2(36) > 6 > log_2(35)$. Confirm that student has uniquely qualified every pony with question set. Spot test two of your choice.

END OF ASSIGNMENT