ME 488: Design of Experiments

Lecture 1: Introduction to Design of Experiments

William 'Ike' Eisenhauer

Fall 2017

Department of Mechanical and Materials Engineering
Portland State University
Portland, Oregon 97223

wde@pdx.edu

Introduction to Design of Experiments

Assumptions

Assumptions

Assumptions

- What are assumptions?

Assumptions

- What are assumptions?
- Why do we, as Engineers, make assumptions?

Assumptions

- What are assumptions?
- Why do we, as Engineers, make assumptions?
- What happens we reality smacks us in the face?

Assumptions

- What are assumptions?
- Why do we, as Engineers, make assumptions?
- What happens we reality smacks us in the face?
- How should we, as Engineers, respond?

Design of Experiments

Goal

Determine an **efficient and economic plan** of systematic discovery to answer key engineering questions regarding **factor influence**, **optimal parametrization**, or **robust behavioural response**

Course Adminstrivia

Instructor: William 'Ike' Eisenhauer, MEng

Background

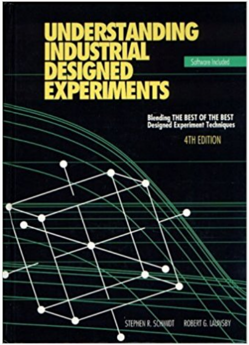
- Professor in SySE, ETM, Statistics and MME Departments
- Consulting Director for Corios

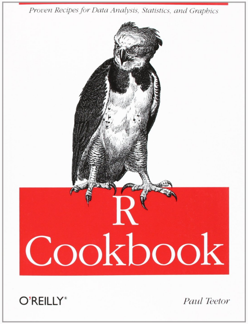
Contact Info

- Office: Engineering Building, Room 462
- Phone 503-496-2634
- E-mail: wde@pdx.edu
- Office Hours: By Appointment

Texts

One 'required' and one recommended text, additional materials will be available on-line. Six copies of Schmidt are on Reserve.





Grading

Proportion

- Homework: 25%
- Midterm/Final: 10% Low - 40% High Method
- Final Project [5 or 6 person team]: 25%

Key Dates

- MidTerm Exam Planned [Oct 25 2017]
- Final Exam [Dec 5 2017]
- Final Project [Dec 8 2017]

Warning

- Assignments will due one week after being assigned and NO late assignments will be accepted, without **PRIOR** arrangements
- Homework is required to be done on engineering paper (or a printed representation) in engineering format.
- You are not required to use LaTeX, however LaTeX files for each homework assignment will be provided.
- Submit at a MAXIMUM 1 PDF and 1 R File per week. Format of R files will be posted this week. ANY other format submitted or additional files beyond the allotment will NOT be graded.

Exams

Open Book and Note!

- Both exams are open book and open note
- Laptop computers are authorized
- However, they are be closed to the Internet
- If your wireless is viewed to be on while in possession of an exam:
    - FORFIET your EXAM
    - 0 for Grade
    - NO QUESTIONS WILL BE ASKED!
    - NO EXCEPTIONS WILL BE ACCEPTED!

Project

Cow-Ta-Pult

- Group
- Design and Construct
- Experiment and Analyze
- Details Next Week

Other issues...

The Use of R Software

- It is your responsibility to become familiar with the basics of the R software package, there are numerous tutorials available.
- For students not familiar with R, the self-paced labs from STAT353 will be made available to you in D2L
- R will be required for homework and your projects. It is available for no cost on most OS platforms.
- If the need is high, an attempt will be made to schedule an extra hour or two in the lab to assist those not familiar with the R environment.

Course Websites (URLs on Syllabus)

- D2L homework/projects submission and to view grades
- Links to supplementary material available on the D2L
- Course Announcements are posted as News Items on D2L
- Presentations are generally posted the night prior to class
- Presentations do not constitute the entirety of information required to be successful in this class.
- Each week's readings are posted on D2L

Questions...

Any Questions?

Approach to Design of Experiments

Goal

Determine an **efficient and economic plan** of systematic discovery to answer key engineering questions regarding **factor influence**, **optimal parameterization**, or **robust behavioural response**

Some really dumb ways to do this

What are some ways to get fired, or at least be viewed as incompetent, when attempting an experiment?

Homework Hint!

Homework Hint

- A quick rundown on the Game of Guess Who for those who never got a chance to play it as a kid.

Steps

1. Determine Objective
2. Identify Experimental Units
3. Define Meaningful and Measurable Response Variable(s)
4. List Independent or Lurking Variables
5. Run Pilot Tests
6. Choose Experimental Design
7. Determine number of replicates Required
8. Randomize Experimental Conditions to Experimental Units
9. Describe Method for Data Analysis
10. Review Timetable and Budgetary Considerations

Modeling

Definition (Model)

Concise mathematical way of describing information content of data.

$$Data = Model + Error$$

$$y_i = \hat{y}_i + \varepsilon$$

Definition (Model)

Concise mathematical way of describing information content of data.

$$Data = Model + Error$$

$$y_i = \hat{y}_i + \varepsilon$$

Example

5000 normally distributed observations of generator output have a mean of $42hp$ and a standard deviation of 2.3. Identify the data, model, and error in this situation.

Definition (Model)

Concise mathematical way of describing information content of data.

$$Data = Model + Error$$

$$y_i = \hat{y}_i + \varepsilon$$

Example

5000 normally distributed observations of generator output have a mean of $42hp$ and a standard deviation of 2.3. Identify the data, model, and error in this situation.

$$Data = (X_1, X_2, ..., X_{5000})$$

$$Model = \bar{X} = 42 \qquad Error = \varepsilon \sim \mathcal{N}(0, 2.3)$$

Models involving quantitative variables

- Empirical models [e.g. Reynold's Number]
- First principles based models [e.g. Newtonian physics]

Definition (Occam's Razor)

The simplest model that explains the data is **probably** the best model

Models in Design of Experiments

Models involving quantitative variables

- Empirical models [e.g. Reynold's Number]
- First principles based models [e.g. Newtonian physics]

Definition (Occam's Razor)

The simplest model that explains the data is **probably** the best model

A vast majority of Design of Experiments is picking the right model, once that is done the rest is **'just'** analysis. Typically, a statistical analysis

A VERY Quick Statistical Review

Definition(s)

- The science of learning from data, and of measuring, controlling, and communicating uncertainty [ASA]
- A branch of mathematics dealing with the collection, analysis, interpretation, and presentation of masses of numerical data [MW]
- The mathematics of ignorance [Morris Kline]

Remembering how to stay out of the ’Statistics Hell’ ruled by this guy....

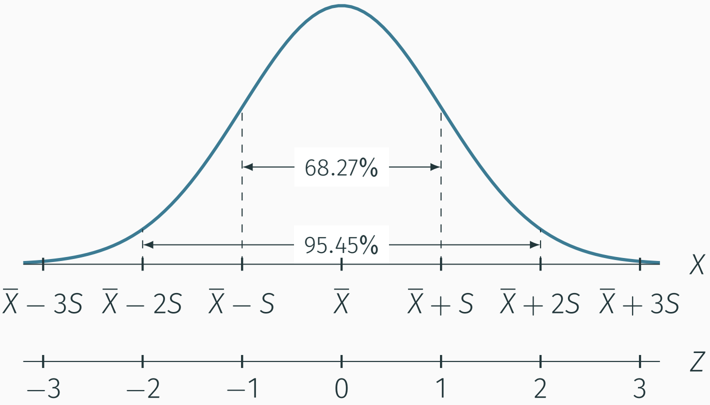
Remembering how to stay out of the 'Statistics Hell' ruled by this guy....

Second Most Frightening Mountain ever...

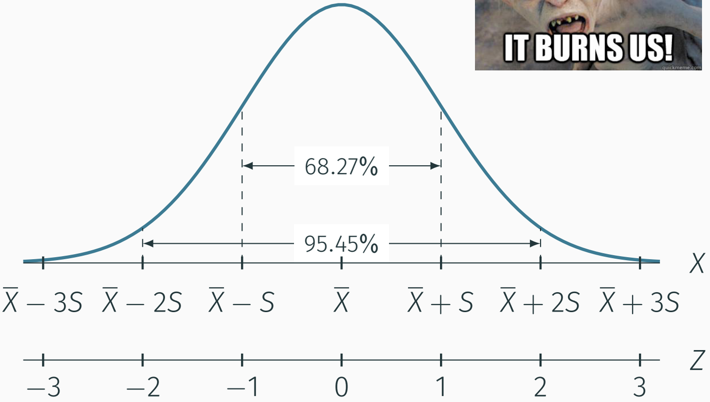
Second Most Frightening Mountain ever...

Most Frightening Mountain ever...

Most Frightening Mountain ever...



Most Frightening Mountain ever...



(Worst case) Scenario

Aluminum tubing is being extruded from a pair of supposedly identical machines.

Scenario (cont.)

The plant engineer suspects something is wrong with one of the machines. She sends a technician to take some samples from a few days' runs and he compiles the following data:

- Machine 1
  - 10 samples
  - Mean length of tubing = 1456ft variance of length = 100
- Machine 2
  - 20 samples
  - Mean length of tubing = 1450ft variance of length = 50

What inference tests will you use and at approximately what level of evidence is there to suggest that the machines are generating different amounts of tubing?

Calculate Sample Descriptive Statistics

$$\begin{aligned} n_1 &= 10 & n_2 &= 20 \\ \bar{x}_1 &= 1456ft & \bar{x}_2 &= 1450ft \\ {s_1}^2 &= 100 & {s_2}^2 &= 50 \end{aligned}$$

Construct Hypotheses

Two hypotheses to consider.

1. Population means (μ) different?
2. Population variances (σ) different?

Construct Hypotheses

The bOring result is that population means, and population variances are the same.
The Amazing result is that population means, and population variances are different.

$$H_0 : \mu_1 = \mu_2 \qquad H_0 : {\sigma_1}^2 = {\sigma_2}^2$$
$$H_A : \mu_1 \neq \mu_2 \qquad H_A : {\sigma_1}^2 \neq {\sigma_2}^2$$

Establish Significance Level

In this scenario, this is what we are basically trying to determine; or in this case, with using statistical tables, what range are we working in.

Determine Applicable Test and Test Statistic

1. One Mean or Two Means? - **TWO**
2. Population Standard Deviation(s) Known? - **NO**
3. Population Standard Deviation(s) Same? - **Not Sure** - We will assume they are and test that assumption later.
4. $n > 30$? - **NO**

Use a **t-test** with t-scores for two means for the first one, and an **F-test** for the second.

Determine Applicable Test and Test Statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S_\varepsilon \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad where \quad S_\varepsilon = \sqrt{\frac{(n_1 - 1){s_1}^2 + (n_2 - 1){s_2}^2}{n_1 + n_2 - 2}}$$

$$F = \frac{{s_1}^2}{{s_2}^2}$$

Establish Decision Criterion

This is really what we are trying to find, so we will just skip this section.

Comparing Two Means

Calculate Test Statistic based on Sample Data

$$S_\varepsilon = \sqrt{\frac{(n_1-1){s_1}^2 + (n_2-1){s_2}^2}{n_1+n_2-2}} = \sqrt{\frac{(10-1)(100)+(20-1)(50)}{10+20-2}}$$

$$S_\varepsilon = \sqrt{\frac{1850}{28}} = \boxed{8.128}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S_\varepsilon\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{1456-1450}{S_\varepsilon\sqrt{\frac{1}{10}+\frac{1}{20}}}$$

$$t = \frac{6}{S_\varepsilon(0.3873)} = \boxed{1.906}$$

$$F = \frac{{s_1}^2}{{s_2}^2} = \frac{100}{50} = \boxed{2}$$

Comparing Two Means

Make Decision and Report Conclusions

$$\nu = n_1 + n_2 - 2 = 28$$

You can pull this (most likely incorrectly) from a dead-tree table or just use R

```
> (1-pt(1.906,28))/2
[1] 0.01674083
```

Listing 1: *Using R to determine percentile of Student's T Distribution*

If we reject the claim that the population means are the same, we will be wrong about 2 times out of 100

Make Decision and Report Conclusions

$$\nu_1 = n_1 - 1 = 9$$
$$\nu_2 = n_2 - 1 = 19$$

```
> pf(2,9,19)
[1] 0.9025868
```

Listing 2: *Using R to determine percentile of F Distribution*

If we reject the claim that the population variances are the same, we have more than a 90% chance of being wrong.

Business conditions dictate if risk is qualified. Ideally, prior to sending out technician. But *Statistics Hell* looms!

Using R for a t-test

R makes this testing easy, but you have to be careful and have all the data!. [Different data/problem]

```
>  t.test(mydata$Horsepower, otherdata, var.equal = TRUE)

      Two Sample t-test

data:  mydata$Horsepower and otherdata
t = 2.397, df = 56, p-value = 0.01989
alternative hypothesis: true difference in means is not equal
    to 0
95 percent confidence interval:
  5.59519 62.53275
sample estimates:
mean of x mean of y
  173.214   139.150
```

Listing 3: *Equal Variance Two Means Test in R*

Note: $df = (50 + 8 - 2) = 56$

Types of Errors

Type I Error [False Alarm]

Occurs when we believe that there is an effect in the population when, in fact, there is not. In terms of out hypothesis, it is claim that amAzing (H_A is true) things are going on, when the reality is bOring (H_0 is true). Probability of making this error is a called α.

Type I Error [False Alarm]

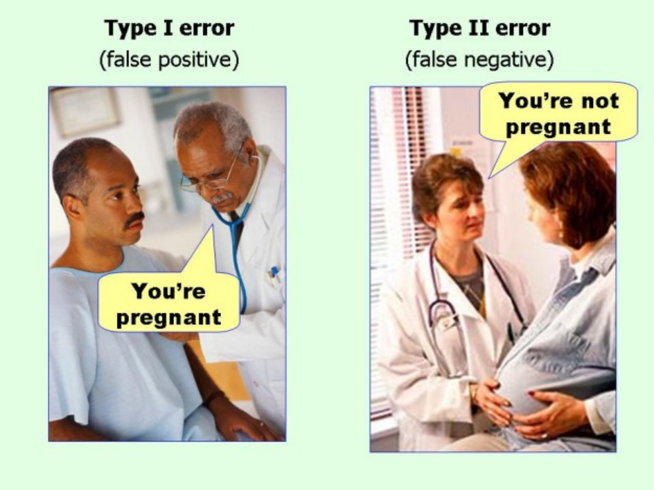
Occurs when we believe that there is an effect in the population when, in fact, there is not. In terms of out hypothesis, it is claim that amAzing (H_A is true) things are going on, when the reality is bOring (H_0 is true). Probability of making this error is a called α.

Type II Error [Missed Alarm]

Occurs when we believe that there no effect in the population when, in fact, there is. In terms of out hypothesis, it is claim that only bOring (H_O is true) things are going on, when the reality is amAzing (H_A is true), things are going on. Probability of making this error is a called β.

Type I and Type II Errors

Type I and Type II Errors



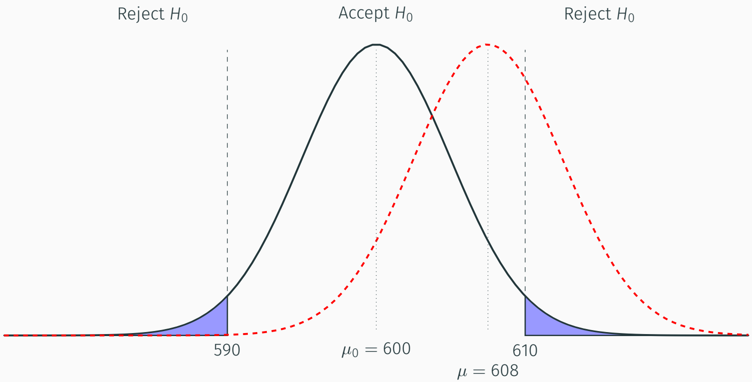
Credit: Jim Thornton, Nottingham, UK

Types of Errors

	H_0 True	H_A True
H_0 Believed	No Error	Type II
H_A Believed	Type I	No Error

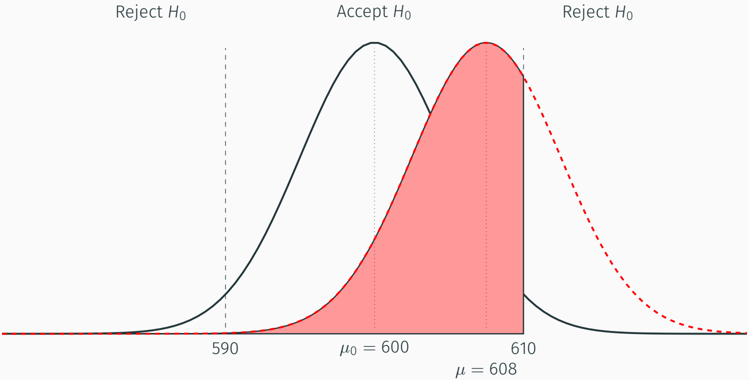
Type I: False Alarm

Area that indicates probability of a Type I error, if $H_0 : \mu = 600$ were true.



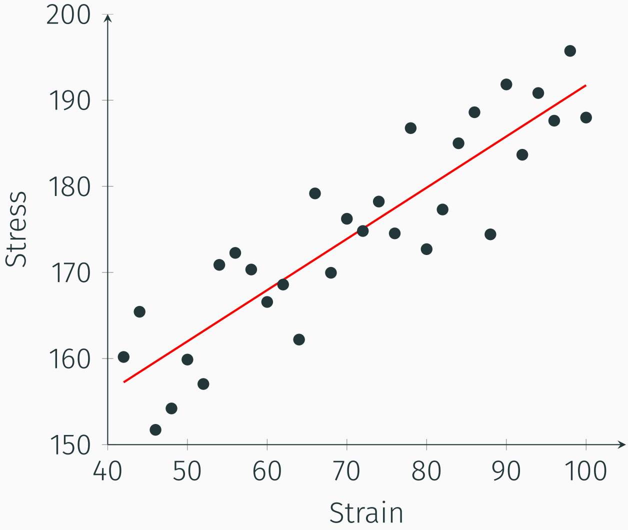
Type II: Missed Alarm

Area that indicates probability of a Type II error, if $H_0 : \mu = 600$ and $\mu_{population} = 608$.



Regression (Linear)

Regression



Remember

- Minimize the Sum of the Squares of the Errors
- Origin Assumption needs Validity
- We will come back to this concept a lot, make sure you have it down cold.

Equation Form

$$y = \beta_0 + \beta_1 x_1 + \varepsilon$$

Multiple Linear Regression

Same Idea Just Bigger

- Multiple factors
- Each coefficient can be analyzed seperately
- Maintain magnitudes with 3 orders to prevent numerical problems

Equation Forms

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n + \varepsilon$$
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \ldots + \beta_{n!+1} x_1 \ldots x_n + \varepsilon$$

Simple One

Simple? Yes it is. Now you have the equation that R has determined will minimize the MSE. But is it useful? We don't know we need to look deeper.

```
> lm(Stress ~ Strain, data=dfrm)
```

Listing 4: Linear Modeling in R

More details

The key info we need is stored in the **summary()** of our model.

```
> m <- lm(Stress ~ Strain, data=dfrm)
> summary(m)
```

Listing 5: Linear Model Summarization in R

Origin Assumption

Just force the Intercept to be 0

```
> m <- lm(Stress ~ Strain + 0, data=dfrm)
> m
> summary(m)
```

Listing 6: *Linear Model Origin Assumption*

R summary of a Regression looks something like this:

```
> summary(model)

Residuals:
    Min       1Q   Median       3Q      Max
-2192.1  -974.5  -121.1   838.6  3602.6

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 1401.0445  2197.2563   0.638  0.52401
x1             0.8910     2.7800   0.321  0.74871
x3            -0.6842     1.2399  -0.552  0.58132
x5            -1.9535     2.5699  -0.760  0.44753
x7            -4.7583     0.1607 -29.615  < 2e-16 ***
x9             8.5646     1.7890   4.787 2.24e-06 ***
x11          -11.5116     4.3129  -2.669  0.00786 **
```

Listing 7: Regression Output in R (first part)

The rest of the R summary of a Regression looks something like this:

```
Residual standard error: 1217 on 493 degrees of freedom
Multiple R-squared: 0.7544,^^IAdjusted R-squared: 0.7514
F-statistic: 252.4 on 6 and 493 DF,  p-value: < 2.2e-16

> confint(model)
                  2.5 %      97.5 %
(Intercept) -2916.097249 5718.186249
x1             -4.571095    6.353167
x3             -3.120431    1.751991
x5             -7.002812    3.095836
x7             -5.074027   -4.442649
x9              5.049680   12.079548
x11           -19.985608   -3.037678
>
```

Listing 8: Regression Output in R (second part)

Due for Next Time

Pre-Assessment Quiz on D2L

- All students are required to take Pre-Assessment quiz on the D2L site by midnight next Monday
- The actual score is not part of your grade, but it is required
- You are NOT expected to be able to answer all the questions correctly, just try your best. Some topics are covered in this class; some topics should have been taken in your Statistics class
- If, however, you score below 20% on the pre-assessment, please prepare to discuss with me after next class to ensure you have the needed foundations to be successful in this class

Due for Next Time

Homework

- Homework is mathematical refresher concepts that you should be familiar with, might be rusty for some.
- Homework #001 is posted. Due prior to next class. See online for format.