Question 1: 2^4 Design

⋆ Given

A ship-breaking company has to be extremely careful due to high possibilities of running into asbestos during the breaking operations.

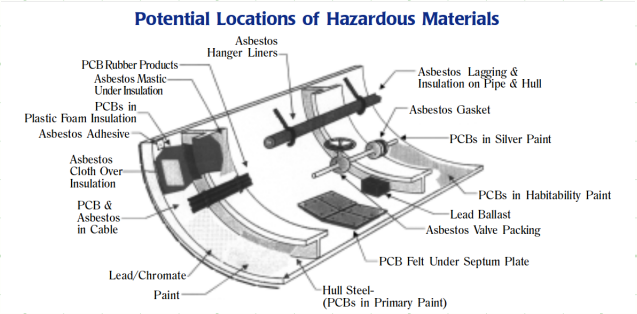

Figure 1: U.S. Department of Labor OSHA 2001

Four factors have been identified through a Cause and Effect exercise that may result in increased airborne asbestos release.

- A: Speed of Cutting Blade
- B: Negative Pressure Environmental Vacuum
- C: Method of Initial Shell Penetration
- D: Duty Cycle of Cutting Blade [i.e. thermal heating of blade]

It has been decided that each of these factors will have two levels to be tested in a 2^4 Balanced Full Factorial Design

- A: Speed of Cutting Blade [2000rpm, 5000rpm]
- B: Negative Pressure Environmental Vacuum [-1atm, -2atm]
- C: Method of Initial Shell Penetration [Plasma Weld-cut, Oxy Fuel Torch Weld-cut]
- D: Duty Cycle of Cutting Blade [2 min on 3 min off, 4 min on 1 min off]

⋆ Find

1. Create the Design, Planning, and Model Matrices for this Experiment
2. How many replicates are needed to allow at least 20 df_{error} in a model that includes all interaction effects?
3. From a purely Engineering sense, which 'two-factor' interaction would you mostly give up last?

⋆ **Solution**

Design Matrix

Std Ord	*Run Ord*	A	B	C	D
1	6	−	−	−	−
2	12	−	−	−	+
3	9	−	−	+	−
4	15	−	−	+	+
5	10	−	+	−	−
6	14	−	+	−	+
7	7	−	+	+	−
8	2	−	+	+	+
9	16	+	−	−	−
10	8	+	−	−	+
11	5	+	−	+	−
12	11	+	−	+	+
13	4	+	+	−	−
14	3	+	+	−	+
15	13	+	+	+	−
16	1	+	+	+	+

Planning Matrix

Std Ord	*Run Ord*	Speed of Blade	Vacuum	Shell Pen.	Duty Cycle
1	6	$2000rpm$	$-1atm$	PW	$2/3$
2	12	$2000rpm$	$-1atm$	PW	$4/1$
3	9	$2000rpm$	$-1atm$	OF	$2/3$
4	15	$2000rpm$	$-1atm$	OF	$4/1$
5	10	$2000rpm$	$-2atm$	PW	$2/3$
6	14	$2000rpm$	$-2atm$	PW	$4/1$
7	7	$2000rpm$	$-2atm$	OF	$2/3$
8	2	$2000rpm$	$-2atm$	OF	$4/1$
9	16	$5000rpm$	$-1atm$	PW	$2/3$
10	8	$5000rpm$	$-1atm$	PW	$4/1$
11	5	$5000rpm$	$-1atm$	OF	$2/3$
12	11	$5000rpm$	$-1atm$	OF	$4/1$
13	4	$5000rpm$	$-2atm$	PW	$2/3$
14	3	$5000rpm$	$-2atm$	PW	$4/1$
15	13	$5000rpm$	$-2atm$	OF	$2/3$
16	1	$5000rpm$	$-2atm$	OF	$4/1$

Model Matrix

Std Ord	*Run Ord*	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD
1	6	−	−	−	−	+	+	+	+	+	+	−	−	−	−	+
2	12	−	−	−	+	+	+	−	+	−	−	−	+	+	+	−
3	9	−	−	+	−	+	−	+	−	+	−	+	−	−	+	−
4	15	−	−	+	+	+	−	−	−	−	+	+	+	+	−	+
5	10	−	+	−	−	−	+	+	−	−	+	+	+	−	+	−
6	14	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+
7	7	−	+	+	−	−	−	+	+	−	−	−	+	+	−	+
8	2	−	+	+	+	−	−	−	+	+	+	−	−	−	+	−
9	16	+	−	−	−	−	−	−	+	+	+	+	+	+	−	−
10	8	+	−	−	+	−	−	+	+	−	−	+	−	−	+	+
11	5	+	−	+	−	−	+	−	−	+	−	−	+	−	+	+
12	11	+	−	+	+	−	+	+	−	−	+	−	−	+	−	−
13	4	+	+	−	−	+	−	−	−	−	+	−	−	+	+	+
14	3	+	+	−	+	+	−	+	−	+	−	−	+	−	−	−
15	13	+	+	+	−	+	+	−	+	−	−	+	−	−	−	−
16	1	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

For a 2^4 BFFE

- $df_{error} = df_{total} - \sum df_{main} - \sum df_{2way} - \sum df_{3way} - \sum df_{4way}$
- $df_{total} = (2^4)n - 1 = 16n - 1$
- $\sum df_{main} = \binom{k}{1} = \binom{4}{1} = 4$
- $\sum df_{2way} = \binom{k}{2} = \binom{4}{2} = 6$
- $\sum df_{3way} = \binom{k}{3} = \binom{4}{2} = 4$
- $\sum df_{4way} = \binom{k}{4} = \binom{4}{2} = 1$
- $df_{error} = 16n - 1 - 4 - 6 - 4 - 1 = 16n - 16$
- $n = (df_{error} + 16)/16 = (20 + 16)/16 = 2.25 \rightarrow 3$ replicates required to get at least 20 df_{error}

The AD one, the duty cycle and blade speed are highly related and likely to be a interaction factor in the release of asbestos. However, other justified answers are acceptable.

Question 2: Saving More Money through Confounding

⋆ Given

Using the last problem information

⋆ Find

1. Take your Design Matrix and Change your D factor level pattern to match the current 'three-way' ABC relationship
2. Draw up the new Model Matrix where D=ABC
3. What is the relationship between the columns that are identical to each other now as that relationship gets propagated?
4. Optional: Any ideas on how to use that relationship to cut down the number of runs needed?

⋆ Solution

Design Matrix (D=ABC)

Std Ord	*Run Ord*	A	B	C	D
1	6	−	−	−	−
2	12	−	−	−	−
3	9	−	−	+	+
4	15	−	−	+	+
5	10	−	+	−	+
6	14	−	+	−	+
7	7	−	+	+	−
8	2	−	+	+	−
9	16	+	−	−	+
10	8	+	−	−	+
11	5	+	−	+	−
12	11	+	−	+	−
13	4	+	+	−	−
14	3	+	+	−	−
15	13	+	+	+	+
16	1	+	+	+	+

Model Matrix

Std Ord	*Run Ord*	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD
1	6	−	−	−	−	+	+	+	+	+	+	−	−	−	−	+
2	12	−	−	−	−	+	+	+	+	+	+	−	−	−	−	+
3	9	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+
4	15	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+
5	10	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+
6	14	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+
7	7	−	+	+	−	−	−	+	+	−	−	−	+	+	−	+
8	2	−	+	+	−	−	−	+	+	−	−	−	+	+	−	+
9	16	+	−	−	+	−	−	+	+	−	−	+	−	−	+	+
10	8	+	−	−	+	−	−	+	+	−	−	+	−	−	+	+
11	5	+	−	+	−	−	+	−	−	+	−	−	+	−	+	+
12	11	+	−	+	−	−	+	−	−	+	−	−	+	−	+	+
13	4	+	+	−	−	+	−	−	−	−	+	−	−	+	+	+
14	3	+	+	−	−	+	−	−	−	−	+	−	−	+	+	+
15	13	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
16	1	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

The following are all the same

- A=BCD
- B=ACD
- C=ABD
- AB=CD
- AC=BD
- AD=BC

And ABCD all equal +1

Question 3: Finish the Example

⋆ Given

Look at the example from the lecture [Week 3], and continue trimming down the model. Show each step and your rationale for why you are continuing. Include the Engineering or Fundamental Principle you are following.

⋆ Find

Also for your final model:

1. What is your confidence that this is a useful model?
2. What is r^2_{adj}?
3. What Factor Levels result in the **Lowest** Acoustic Response?
4. What are the 90% confidence intervals on the coefficients?

⋆ Solution

- The model confidence is based on the p-value for F
- r^2_{adj} is returned from the regression run
- Look out for disallowed choices of factor levels, you can only pick the main ones, the interaction effects are driven by your choices of main factor levels.
- Did the student remember to change the conf.level setting?
- Were steps justified?

END OF ASSIGNMENT