Question 1: Plackett-Burman

⋆ Given

A gold placer mining operation in the Klondike is operating a complex wash plant / sluice system. The claim owner wants to determine the key factors that drive the final gold output. There are 28 factors he would like tested. You have interested him in a Plackett-Burman design. Answer his following questions:

1. How many different settings selections [aka runs] are required?
2. Is there anyway without reducing the number of factors tested, to reduce the number of times the slice gate angle is changed; as it is an all day event?

He seems a bit irritated with your answer to #1. Explain to him how, if he is willing to drop one factor, you can modify the PB design significantly? If each run costs $50,000 to complete, show how much money he can save.

⋆ Solution

1. 28 factors requires at least 29 runs, but PB designs require runs in multiples of 4, so need 32 runs
2. PB not as sensitive to randomization issues, so sort the design by sluice gate setting so only one change is required during the experiment
3. If we drop to 27 factors, a PB requiring only 28 runs can be made, so $32 - 28 = 4$ runs can be saved at $50,000 each, saves $200,000

Question 2: Star Points

⋆ Given

- A: Speed of Cutting Blade [2000rpm, 5000rpm]
- B: Negative Pressure Environmental Vacuum [-1atm, -2atm]
- D: Duty Cycle of Cutting Blade [2 min on 3 min off, 4 min on 1 min off]

⋆ Find

What are the actual values needed to be tested at the α (star points) for a CCC in the above?

⋆ Solution

Center Point

1. $A_0 = 3500$
2. $B_0 = -1.5$
3. $C_0 = 3$ min on (2 min off)

Points we are looking at are:

1. $\alpha^* = \sqrt[4]{2^3} = 1.682$
2. $\{+\alpha^*, 0, 0\} = \{3500 + 1500(1.682), -1.5, 3\} = \{6023, -1.5, 3\}$
3. $\{-\alpha^*, 0, 0\} = \{3500 - 1500(1.682), -1.5, 3\} = \{977, -1.5, 3\}$
4. $\{0, +\alpha^*, 0\} = \{3500, -1.5 + 0.5(1.682), 3\} = \{3500, -0.659, 3\}$
5. $\{0, -\alpha^*, 0\} = \{3500, -1.5 - 0.5(1.682), 3\} = \{3500, -2.341, 3\}$
6. $\{0, 0, +\alpha^*\} = \{3500, -1.5, 3 + 1(1.682)\} = \{3500, -1.5, 4.682\}$
7. $\{0, 0, -\alpha^*\} = \{3500, -1.5, 3 - 1(1.682)\} = \{3500, -1.5, 1.318\}$

Alternatively, there is a different set of star points based on conceptuaizing the $\sqrt{k}$ concept:

1. $\alpha^* = \sqrt{3} = 1.732$
2. $\{+\alpha^*, 0, 0\} = \{3500 + 1500(1.732), -1.5, 3\} = \{6098, -1.5, 3\}$
3. $\{-\alpha^*, 0, 0\} = \{3500 - 1500(1.732), -1.5, 3\} = \{902, -1.5, 3\}$
4. $\{0, +\alpha^*, 0\} = \{3500, -1.5 + 0.5(1.732), 3\} = \{3500, -0.634, 3\}$
5. $\{0, -\alpha^*, 0\} = \{3500, -1.5 - 0.5(1.732), 3\} = \{3500, -2.366, 3\}$
6. $\{0, 0, +\alpha^*\} = \{3500, -1.5, 3 + 1(1.732)\} = \{3500, -1.5, 4.732\}$
7. $\{0, 0, -\alpha^*\} = \{3500, -1.5, 3 - 1(1.732)\} = \{3500, -1.5, 1.268\}$

Question 3: RSM

⋆ Given

Given a first pass experiment [2^2 BFFE with two center points] resulted in the following model equation:

$$\hat{y} = -35.9 + 2.52x_1 - 6.42x_2 + 2.35x_1x_2$$

⋆ Find

1. What should be the next 4 points tested?

2. After locating and setting up a secondary experiment the following was determined to be an viable model

$$\hat{y} = 70.549 + 2.326w_1 - 5.771w_1w_2 - 4.781{w_1}^2 + 3.062{w_2}^2$$

   The junior engineer deleted the regression model. Determine if there is a stationary point within this range and what it is, if it exists. [You do not to say what kind it is]

⋆ Solution

$$\vec{g} = \nabla\hat{y} = \left(\frac{\partial\hat{y}}{x_1}, \frac{\partial\hat{y}}{x_2}\right)\bigg|_{x_1,x_2=0} = (2.52, -6.42)$$
$$\vec{g_{scaled}} = \left(\frac{2.52}{|2.52|}, \frac{-6.42}{|2.52|}\right) = (1, -2.55)$$

1. Next four test points are: (+1,-2.55) , (+2,-5.10) , (+3,-7.65), (+4,-10.20)

$$\frac{\partial\hat{y}}{w_1} = 2.326 - 5.771w_2 - 9.562w_1 = 0$$
$$\frac{\partial\hat{y}}{w_2} = -5.771w_1 + 6.124w_2 = 0$$
$$(w_1, w_2)_{stationary} = (0.1574, 0.1483)$$

Question 4: Putting it All Together

Using an example of a complex situation where there have been 15 factors identified that could impact the output response. Explain the process you would follow in designing an experiment to evaluate and address that situation in terms of identifying and optimizing the critical factors and interaction to maximize the response value. Include in your discussion the use of the following techniques/terms we have studied this term.

- Type I Error
- Level of Significance
- ANOVA
- Regression
- Factorial Experiments
- Fractional Factorial Experiments
- Randomization
- F Value
- Adjusted r^2
- Generators/Defining Words
- Aliasing/Resolution
- Interaction Plots
- Tukey Test/Plots
- FrF2 Package
- Design Matrix

⋆ Solution

Any coherent explination that covers at least 75% of the terms correctly without misuse of any of the terms is full credit. Use of all terms correctly in a narrative [i.e. no bullet points] gains extra credit of 5 points. Failure to use at least 75% of the terms is -5, any misused term is -1 each.

Question 5: Simulation

- 10 Points if there is signficiant effort including at least two different types of designs
- 7 Points if there is only one type of design
- 5 Points if there is only trial and Error
- 3 Points if not logicla approach
- Bonus 5 points if able to achieve >45 and prove it

END OF ASSIGNMENT