ME 488: Design of Experiments

Lecture 5: Block Designs

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Randomized Block Designs

Error Control

- To eliminate natural variation, one typically wishes for homogeneity
- Reduces σ^2 and increases $P = 1 - \beta$
- But this counters generalizability of results

Blocking

- Heterogeneous experimental units
- Homogeneous sub-groupings [Blocks]
- Randomly assign treatments within the subgroups
- Blocks can be physical, temporal, etc.

Confounding

- Blocking reduces variability
- Especially a variability that cannot be overcome
- Attempt to confounded or alias with a higher order interaction

Blocking

- t Treatments
- b Blocks
- $N = t \times b$ experimental units: RCB
- Randomized Complete Block Design
- $N > t \times b$ experimental units: GCB
- General Complete Block Design
- Normally, RCB is preferred over GCB

RCB Effects Model

$$y_{ij} = \mu + b_i + \tau_j + \epsilon_{ij}$$

$b\tau_{ij}$ is NOT included. Why not?

Model used for RCB

$b\tau_{ij}$ IS the error term!

Measuring this interaction is the point of the experiment.

What is the experimenter looking for?

Using an ANOVA, you are looking for an $F \gg 1$ on Treatments

Latin Squares

More than One Blocking Factor

- Last time we saw the power of Blocking
- What if we have TWO blocking factors that are of concern?
- Need a design that ensures the Blocking Factors are Orthogonal

Latin Squares

- Latin square: Set of all triples (r,c,s), where $1 \leqslant r, c, s \leqslant n$
- All ordered pairs (r,c) are distinct
- All ordered pairs (r,s) are distinct
- All ordered pairs (c,s) are distinct.

Just Think Baby Sudoku!

Not quite as bad as this, but same idea...

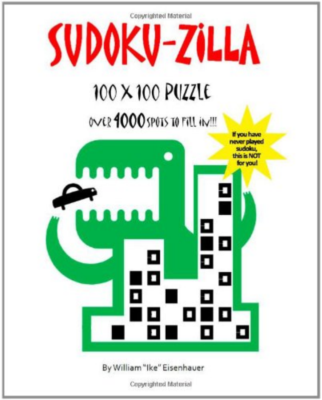

Figure 1: *Sudokuzilla! Eisenhauer 2010.*

Comparison

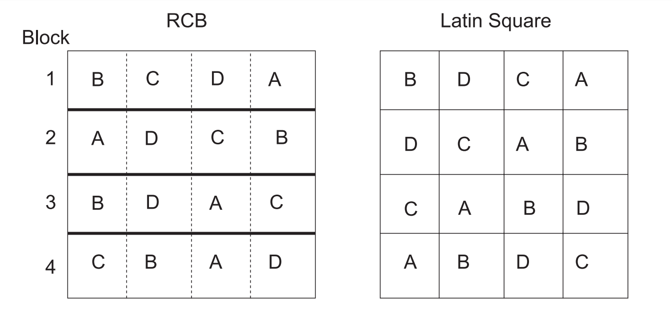

Figure 2: *Lawson (Fig. 4.3)*

Not this LSD

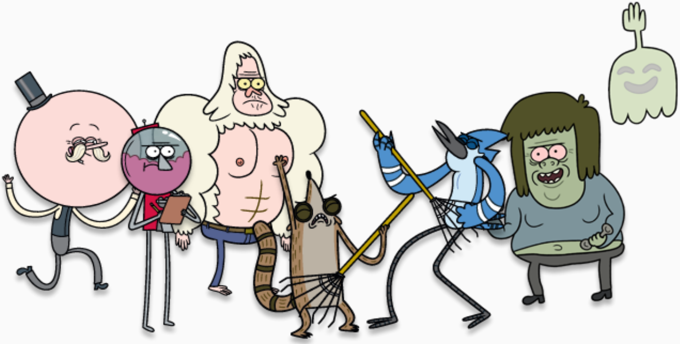

Figure 3: *Regular Show. Quintel. Fair Use Claim*

This one

$$y_{ij} = \mu + r_i + c_j + \tau_k + \epsilon_{ijk}$$

Again, no interactions terms whatsoever.

Restrictions

- Block Factors must be independent
- $\nu_r = \nu_c$: Must be a Latin SQUARE

Cycle Through the Letters Needed

$$\begin{pmatrix} A & B & C & D & E \\ B & C & D & E & A \\ C & D & E & A & B \\ D & E & A & B & C \\ E & A & B & C & D \end{pmatrix}$$

But you need to randomize and not use this grid everytime. Randomize over the Rows, then Randomize over the Columns. This ensures the Latin Square nature to be maintained.

Same way as an RBC

Nothing different, just two blocks listed instead of 1.

Three Blocking Factors?

Graeco-Latin Squares

Superimpose two Latin Squares

$$\begin{pmatrix} A & B & C \\ B & C & A \\ C & A & B \end{pmatrix} \qquad \begin{pmatrix} A & B & C \\ C & A & B \\ B & C & A \end{pmatrix}$$

Graeco-Latin Squares

Make sure to change the Latin letter to Greek

$$\begin{pmatrix} A\alpha & B\beta & C\gamma \\ B\gamma & C\alpha & A\beta \\ C\beta & A\gamma & B\alpha \end{pmatrix}$$

Hyper-Graeco-Latin Squares

- Superimpose 3 mutually orthogonal 4x4 Latin Squares = HGLS
- Superimpose Any 2 of those 3 4x4 Latin Squares = GLS

More Blocking Factors?

Well...

There is no known general formula for the number of mutually orthogonal $n \times n$ sized Latin Squares

- It is known it can not exceed $n - 1$, this upper bound is reached if n is prime.
- Minimum is 2, except for $n = 2$ or $n = 6$ where it is 1
- Known values are OEIS AOOO1438 $(\infty, \infty, 1, 2, 3, 4, 1, 6, 7, 8)$