ME 492 Modeling a Burn Wire Circuit
February 16, 2018

Gerald Recktenwald
gerry@pdx.edu

1 Design Concept

Figure 1 shows a simplified representation of the burn wire. A piece of fine gage, is stretched across a length of nylon monofilament that acts as a restraint to the coiled helical antenna. The metal wire acts as a heater with the goal of melting through the monofilament an releasing the antenna.

2 Thermal Models

2.1 Energy Equation for the Wire

The energy equation for the wire is

$$mc\frac{dT}{dt} = P - Q \tag{1}$$

where m is the mass of the wire, c is the specific heat of the wire material, T is the temperature of the wire, which is assumed to be spatially uniform, t is time, P is the power input from electric resistance heating, and Q is the heat loss to the ambient. We neglect Q, which will give an optimistic design. The power input is

$$P = I^2 R = \frac{V^2}{R} \tag{2}$$

where I is the current flowing through the wire, R is the electrical resistance of the wire, and V is the voltage across the ends of the wire.

The wire resistance is determined by the geometry and electrical properties of the wire material according to

$$R = \frac{\rho_e L}{A_c} \tag{3}$$

where ρ_e is the electrical resistivity of the wire material, L is the length of the wire over which V is applied, and $A_c = (\pi/4)d_w^2$ is the cross-sectional area of the wire with diameter d_w.

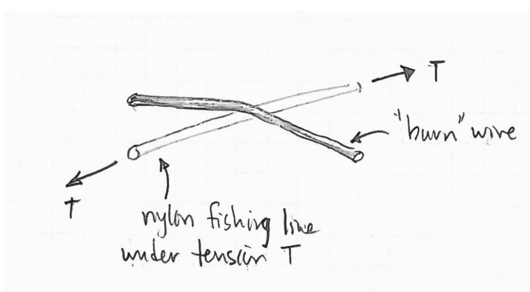

Figure 1: Burn wire lying on top of a nylon monofilament. The T in the sketch is the tension force in the monofilament.

Notice that if P is constant and $Q = 0$, the wire temperature increases linearly with time. For convenience, define the heating rate as

$$H = \frac{T_f - T_i}{\Delta t}. \tag{4}$$

In our design calculations, we can just specify H from the initial and final temperatures and the time it takes the wire to heat from T_i to T_f in a time interval Δt.

Substituting Equation (4) for the time derivative in the energy equation and rearranging slightly gives

$$H = \frac{P}{mc}. \tag{5}$$

Equation (5) will be used to estimate the voltage, current and power requirements for different combinations of wire diameter and length.

2.2 Electric Resistance Heating of the Wire

Voltage:

$$H = \frac{P}{mc} = \frac{V^2}{R\rho_m c A_c L} = \frac{V^2}{\dfrac{\rho_e L}{A_c}\rho_m c A_c L} = \frac{V^2}{\rho_e \rho_m c L^2} \tag{6}$$

Solving the preceding equation for V gives

$$V = L\sqrt{Hc\rho_m\rho_e}. \tag{7}$$

Current:

$$H = \frac{P}{mc} = \frac{I^2 R}{\rho_m A_c L} = \frac{I^2 \dfrac{\rho_e L}{A_c}}{\rho_m c A_c L} = \frac{I^2 \rho_e}{\rho_m c A_c^2} = \frac{16}{\pi^2}\frac{I^2\rho_e}{\rho_m c d_w^4} \tag{8}$$

Solving the preceding equation for I gives

$$I = \frac{\pi}{4} d_w^2 \sqrt{\frac{\rho_m}{\rho_e} Hc} \tag{9}$$

Combining Equation (7) and Equation (9) gives a formula for power consumption as a function of wire diameter and length.

$$P = VI \ = \ L\sqrt{Hc\rho_m\rho_e}\ \frac{\pi}{4} d_w^2 \sqrt{\frac{\rho_m}{\rho_e} Hc} \ = \ \frac{\pi}{4} d_w^2 L H c \rho_m \tag{10}$$

2.3 Sample Calculations

Use the properties

$$1.0 \times 10^{-6} \le \rho_e \le 1.5 \times 10^{-6}\ \Omega \cdot \mathrm{m}, \qquad \rho_m = 840\,\frac{\mathrm{kg}}{\mathrm{m}^3}, \qquad c = 450\,\frac{\mathrm{J}}{\mathrm{kg\,K}}$$

The melting point of nylon is in the range $190 \le T \le 350$°C.

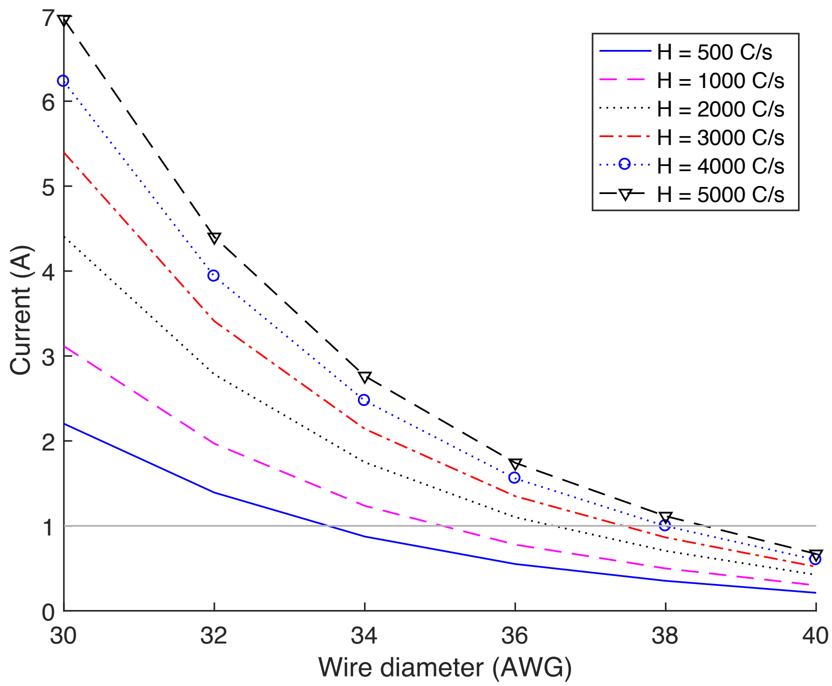

Figure 2: Current as a function of wire diameter.

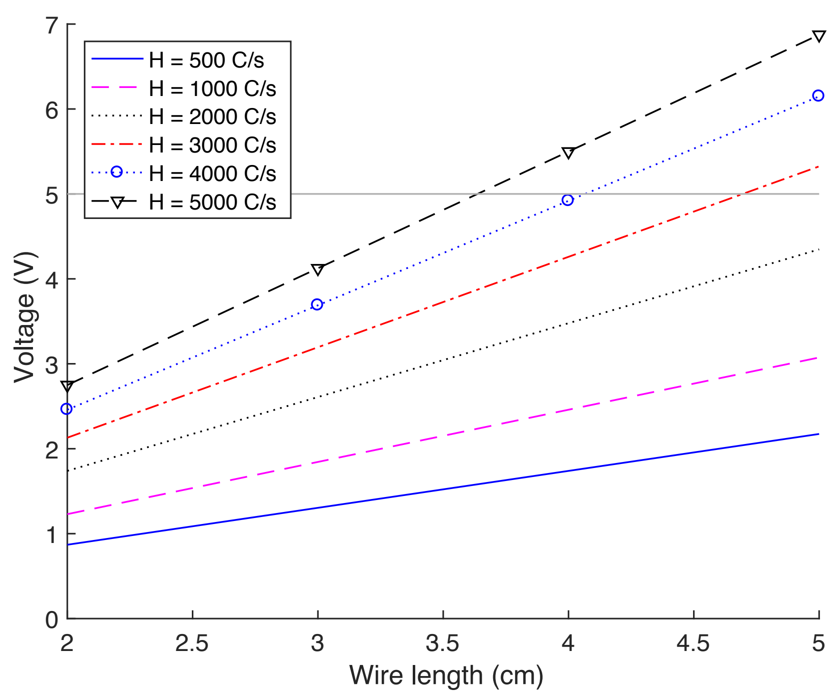

Figure 3: Voltage as a function of wire length.

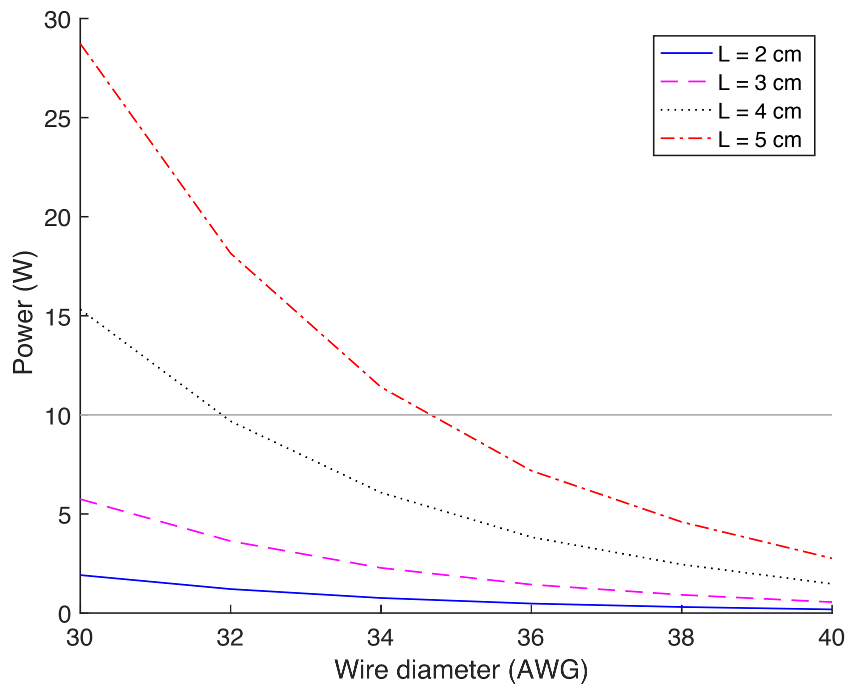

Figure 4: Power consumption as a function of wire length and diameter.