Linear Regression

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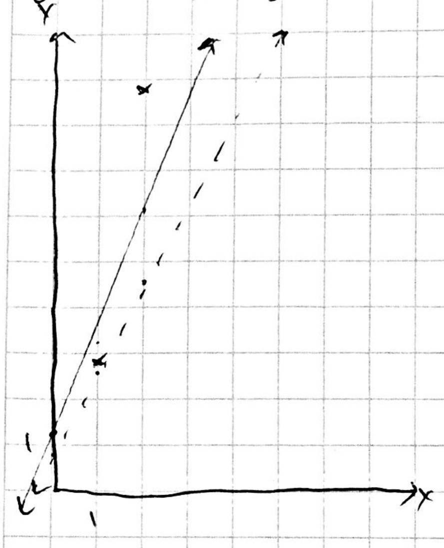
1. $\theta = [1.3, 2.9]$



2. Using the simplification $\hat{y} = \theta x$. (ignoring y intercept)

$[1.0, 2.0] \to \hat{y} = 2.9(1) = 2.9$

$[3.0, 1.0] \to \hat{y} = 2.9(3.0) = 8.7$

3. See graph above (x's)

4. $MSQ = \frac{(2.9-2.0)^2 + (8.7-1.0)^2}{2} = \boxed{30.05}$

5. $\alpha = .1$, $x_0 = 1$

$\theta_0 = 1.3 - \left[\frac{1}{2}(1(2.9-2)) + \frac{1}{2}(1(8.7-1))\right] \times 0.1 = 0.87$

$\theta_1 = 2.9 - \left[\frac{1}{2}(1(2.9-2)) + \frac{1}{2}(3(8.7-1))\right] \times 0.1 = 1.7$

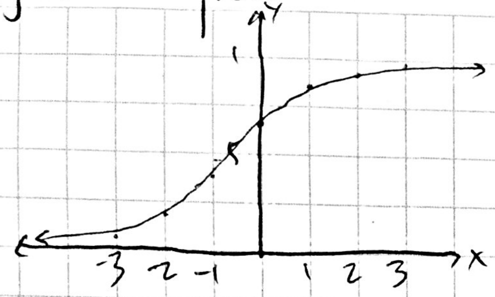
6. See above (dotted line)

7. $[1.0, 2.0] \to \hat{y} = 1.7(1) + 0.87 = \boxed{2.57}$

$(3.0, 1.0) \to \hat{y} = 1.7(3) + 0.87 = \boxed{5.97}$?

Logistic Regression

1.



$x_1 = \{-3, -2, -1, 0, 1, 2, 3\}$
$x_0 = 1$
$\theta = [.8, 1.1]$

$x_1 = -3$: $z = 0.8 + 1.1(-3) = -2.5$
$\hat{y} = \dfrac{1}{1+e^{2.5}} = 0.0759$

$x_1 = -2$: $z = 0.8 + 1.1(-2) = -1.4$
$\hat{y} = \dfrac{1}{1+e^{1.4}} = 0.198$

$x_1 = -1$: $z = 0.8 + 1.1(-1) = -0.3$
$\hat{y} = \dfrac{1}{1+e^{0.3}} = 0.426$

$x_1 = 0$: $z = 0.8$
$\hat{y} = \dfrac{1}{1+e^{-0.8}} = 0.690$

$x_1 = 1$: $z = 0.8 + 1.1(1) = 1.9$
$\hat{y} = \dfrac{1}{1+e^{-1.9}} = 0.870$

$x_1 = 2$: $z = 0.8 + 1.1(2) = 3$
$\hat{y} = \dfrac{1}{1+e^{-3}} = 0.952$

$x_1 = 3$: $z = 0.8 + 1.1(3) = 4.1$
$\hat{y} = \dfrac{1}{1+e^{-4.1}} = 0.984$

2. Given $[1.1, 0]$, $[2.7, 1]$

$[1.1, 0]$: $z = 0.8 + 1.1(1.1) = 2.01$
$\hat{y} = \dfrac{1}{1+e^{-2.01}} = 0.882$

$[2.7, 1]$: $z = 0.8 + 1.1(2.7) = 3.77$
$\hat{y} = \dfrac{1}{1+e^{-3.77}} = 0.978$

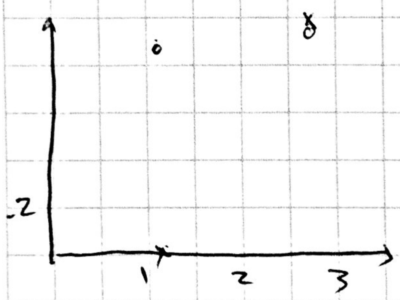
Logistic Regression

2. Cont.

y_1 log loss: $0 \cdot \log(0.882) + 1(\log(0.118)) = \cancel{2.12}\ -0.928$

y_2 log loss: $1 \cdot \log(0.978) + 0(\log(0.022)) = \cancel{-0.022}\ -.0047$

log loss $= -\frac{1}{2} \cdot (\overset{-0.928\ -0.0047}{\cancel{2.12} - \cancel{0.0227}}) = \cancel{[illegible]}\ \boxed{0.5125}$

3. 

x = actual

o = expected

4. $d\theta_0 = 1/n\ \Sigma(\hat{y}_0 - y_0)x_0 = 1/2(1(.880-0) + 1(.978-1)) = 0.43$

$d\theta_1 = 1/2(1.1(0.880-0) + 2.7(0.978-1)) = 0.455$

$\theta_0 = 0.8 - .1(0.43) = \boxed{0.757}$

$\theta_1 = 1.1 - .1(0.455) = \boxed{1.055}$

Logistic Regression

5. Using new theta $[0.757, 1.055]$

$x = -3$: $z = 0.757 + 1.055(-3) = -2.408$

$\hat{y} = \frac{1}{1+e^{2.408}} = 0.083$

$x = -2$: $z = 0.757 + 1.055(-2) = \cancel{2.}\ -1.353$

$\hat{y} = \frac{1}{1+e^{1.353}} = 0.2057$

$x = -1$: $z = 0.757 + 1.055(-1) = -0.298$

$\hat{y} = \frac{1}{1+e^{0.298}} = 0.426$

$x = 0$ $z = 0.757$

$\hat{y} = \frac{1}{1+e^{-0.757}} = 0.681$

$x = 1$ $z = 0.757 + 1.055 = 1.812$

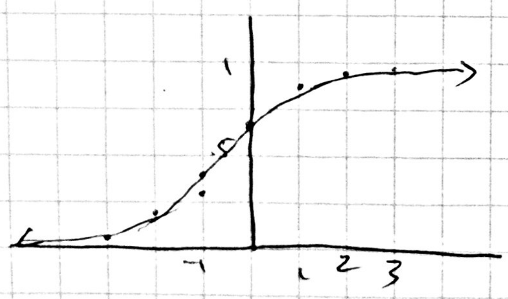
$\hat{y} = \frac{1}{1+e^{-1.812}} = 0.860$

$x = 2$ $z = 0.757 + 1.055(2) = 2.867$

$\hat{y} = \frac{1}{1+e^{-2.867}} = 0.946$

$x = 3$ $z = 0.757 + 1.055(3) = 3.922$

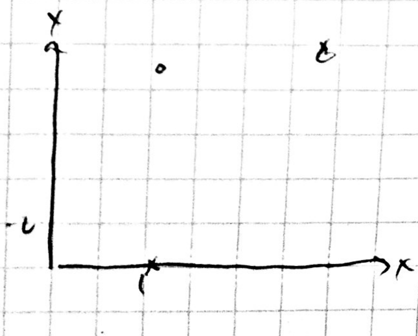
$\hat{y} = \frac{1}{1+e^{-3.922}} = 0.981$



actual
x = ~~expected~~
o = expected

6. $z_1 = 0.757 + 1.055(1.1) = 1.9175$

$\hat{y} = \frac{1}{1+e^{-1.9175}} = 0.872$



$z_2 = 0.757 + 1.055(2.7) = 3.606$

$\hat{y} = \frac{1}{1+e^{-3.606}} = 0.974$