Calvin Koder Point estimation given other known points 2016
and distances from knowns to unknown

Part 0: Duolateration

Given: point B, point C, distance |AB|, distance |AC|
Find: point A

0 Translate $\triangle ABC$ to $\triangle A'B'C'$ with shift $(-B.x, -B.y)$

0 $A' = (A.x - B.x,\ A.y - B.y)$
1 $B' = (0, 0)$
2 $C' = (C.x - B.x,\ C.y - B.y)$
3 $\measuredangle ABC = \measuredangle A'B'C'$

1 Rotate $\triangle A'B'C'$ to $\triangle A''B''C''$ about axis B' s.t. $C''.y = 0$

0 $B'' = B'$
1 $C'' = (C''.x, 0)$
2 $\|C''B''\| = \|CB\|$
3 $\|C''B''\| = \sqrt{(C''.y - B''.y)^2 + (C''.x - B''.x)^2}$
4 $\|C''B''\| = \sqrt{C''.x^2}$ (Substitute from 1.0, 0.1, 1.1)
5 $\|C''B''\| = |C''.x|$
6 $|C''.x| = \|CB\|$
7 $C''.x = \|CB\|$ OR $\underbrace{C.x = -\|CB\|}_{\text{disregard}}$, disregard because positive orientation is simpler
8 $C''.x = \|CB\|$
9 $C'' = (\|CB\|, 0)$

2 Calculate $\measuredangle A'B'C''$

0 $\measuredangle A'B'C'' = \measuredangle A'B'C' + \measuredangle C'B'C''$
1 $\measuredangle A'B'C'' = \measuredangle ABC + \measuredangle C'B'C''$ (sub 0.3)
2 $\|AC\|^2 = \|AB\|^2 + \|BC\|^2 - 2\|AB\|\,\|BC\| \cos(\measuredangle ABC)$
3 $\measuredangle ABC = \pm \operatorname{acos}\left[\dfrac{\|AB\|^2 + \|BC\|^2 - \|AC\|^2}{2\|AB\|\,\|BC\|}\right]$
4 $\|B'C''\| = \|B'C'\| \cos(\measuredangle C'B'C'')$
5 $\measuredangle C'B'C'' = \pm \operatorname{acos}\left[\dfrac{\|B'C''\|}{\|B'C'\|}\right]$

2 Calculate $\measuredangle A'B'C''$ (cont.)

6 $\measuredangle A'B'C'' = \pm|\measuredangle ABC| \pm |\measuredangle C'B'C''|$ (w/ given values)

7 $\measuredangle A'B'C'' = |\measuredangle ABC| + |\measuredangle C'B'C''|$ or $\measuredangle ABC = -|\measuredangle ABC| - |\measuredangle C'B'C''|$

3 Calculate A'

0 $A' = (|A'B'|\cos(\measuredangle A'B'C''), |A'B'|\sin(\measuredangle A'B'C''))$

1 $|A'B'| = |AB|$

2 $A' = (|AB|\cos(\measuredangle A'B'C''), |AB|\sin(\measuredangle A'B'C''))$

or

$A' = (|AB|\cos(-\measuredangle A'B'C''), |AB|\sin(-\measuredangle A'B'C''))$

3 $A' = (|AB|\cos(\measuredangle A'B'C''), |AB|\sin(\measuredangle A'B'C''))$

or

$A' = (|AB|\cos(\measuredangle A'B'C''), -|AB|\sin(\measuredangle A'B'C''))$

4 Calculate A

0 $A = A' + B$

1 $A = (|AB|\cos(\measuredangle A'B'C'') + B.x, |AB|\sin(\measuredangle A'B'C'') + B.y)$

or

$A = (|AB|\cos(\measuredangle A'B'C'') + B.x, -|AB|\sin(\measuredangle A'B'C'') + B.y)$

Shorthand notation: $A_0 = A_+$, $A_1 = A_-$