Concept Review

Orientation and Position

How do you describe Orientation and Position?

Angular orientation or attitude of a vehicle in three-dimensional (3D) space is typically described using a body-fixed reference frame, which is attached to the vehicle body - translating and rotating with it. The orientation of that body-fixed reference frame relative to a globally fixed reference frame will represent the vehicle’s attitude. There are multiple ways to represent or describe the orientation. Three common methods will be described in this document: Rotation Matrix, Euler Angle and Quaternions.

Finally, to capture both the position and rotation of a vehicle body in 3D space, the rotation matrix representation is expanded into a 4x4 Homogeneous Transformation matrix, which will also be introduced in this document

Rotation Matrix

For three-dimensional (3D) motion, a rotation matrix [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=R%250) is a 3 x 3 matrix that represents the orientation (rotation) of a Cartesian reference frame with respect to the inertial reference frame. The 3-element column vectors of a rotation matrix can be interpreted as consisting of the Cartesian unit basis vectors [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bu%7D_x%250), [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bu%7D_y%250) and [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bu%7D_z%250) expressed with respect to the inertial reference frame,

|  |  |
| --- | --- |
|  | (1) |

This means the rotation matrix can be used to change the basis of a vector from being expressed with respect to the rotated reference frame [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bv%7D_1%250) to being with respect to the inertial (original/unrotated) reference frame [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bv%7D_0%250):

|  |  |
| --- | --- |
|  | (2) |

For example, the [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=x%250) unit basis vector   of the rotated reference frame expressed with respect to the inertial frame   can be obtained by right multiplying the [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=x%250) unit vector to the rotation matrix [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=R%250),

|  |  |
| --- | --- |
|  | (3) |

To obtain the reverse, the rotation matrix can be inverted. For rotation matrices, which are orthogonal matrices with determinant equals to 1, its inverse is the same as its transpose, as such,

|  |  |
| --- | --- |
|  | (4) |

When the rotations are about the basis axes of the reference frame (i.e., , , or axes), the resulting rotation matrices have the following basic forms:

|  |  |
| --- | --- |
|  | (5) |

Rotation matrices for compound rotations, that is rotations obtained by successive rotations, can be obtained by multiplying the intermediate rotation matrices together. For example, the rotation matrix for a rotation sequence of first rotating about the [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=z%250)-axis by [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cpsi%250), followed by a rotation about the resulting [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=y%250)-axis by [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Ctheta%250) can be obtained as follows:

|  |  |
| --- | --- |
|  | (6) |

Euler Angles

Euler angles are a set of three (successive) rotation angles used to describe (parametrize) orientation of a rigid body. It was introduced by Leonhard Euler. There are many variations in the selection of the three angles based on the different combinations of 3 axes selected to describe the rotations. Also, the successive rotations can be described with respect to either fixed (extrinsic) or relative (intrinsic) axes/reference frames.

Rotations about each of the primary axis:

|  |  |
| --- | --- |
| A picture containing text, businesscard  Description automatically generated | A picture containing sky  Description automatically generated |
| Positive rotation in x-axis | Negative Rotation in x-axis |
| Figure 1: Rotations about X (roll ) | |
|  |  |
| Positive rotation in y-axis | Negative rotation in y-axis |
| Figure 2: Rotations about y (pitch ) | |
| **A picture containing text  Description automatically generated** | **A picture containing text  Description automatically generated** |
| Positive rotation in z-axis | Negative rotation in z-axis |
| Figure 3: Rotations about z (yaw ) | |

For the selection of the combination of axes, they can generally be classified into two groups: Classic Euler and Roll-Pitch-Yaw Euler (also called Tait-Bryan) angles. In Classic Euler angles, the first and third rotation axes are the same, e.g., Z-X-Z rotation. In Roll-Pitch-Yaw Euler angles, the rotations are about different axes, e.g., Z-Y-X rotation, which is the common choice for Euler angles for unmanned vehicles,

|  |  |
| --- | --- |
|  | (7) |

Here, the rotation is obtained through relative (intrinsic) rotations - by first rotating about the [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=z%250)-axis by [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cpsi%250), then about the resulting [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=y%250)-axis by [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Ctheta%250) and finally about the resulting [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=x%250)-axis by [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cphi%250).

Quaternions

Quaternions are a set of four real numbers commonly used to represent rotations in three-dimensional (3D) space. It was first described by William Rowan Hamilton. A typical definition of a quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250) for rotation has the following form:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%20%3D%20q_0%20%2B%20q_1%20i%20%2B%20q_2%20j%20%2B%20q_3%20k%250) | (8) |

The coefficients [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=q_0%250), [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=q_1%250), [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=q_2%250), and [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=q_3%250) are the quaternion parameters, and [](https://www.codecogs.com/eqnedit.php?latex=i%250), [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=j%250) and [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=k%250) are the quaternion basis/units with this property:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=i%5E2%20%3D%20j%5E2%20%3D%20k%5E2%20%3D%20i%20j%20k%20%3D%20-1%250) | (9) |

A compact vector form is usually used to express the quaternion:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%20%5Cbar%7Bq%7D%20%3D%20%5Cbegin%7Bbmatrix%7D%7B%20q_0%20%5C%5C%20q_1%20%5C%5C%20q_2%20%5C%5C%20q_3%20%5Cend%7Bbmatrix%7D%20%250) | (10) |

For rotation representation, the first quaternion parameter [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=q_0%250) can be interpreted as the scalar part and the remaining 3 element parameters be interpreted as the vector part. A quaternion has only non-zero scalar part is called a scalar quaternion. Similarly, a vector quaternion has zero scalar part but non-zero vector part. With this interpretation, the conjugate [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%5E%7B*%7D%250) of the quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250) can be expressed as having negative vector part of [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250):

|  |  |
| --- | --- |
| [Shape  Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%20%5Cbar%7Bq%7D%5E%7B*%7D%20%3D%20%5Cbegin%7Bbmatrix%7D%7B%20q_0%20%5C%5C%20-q_1%20%5C%5C%20-q_2%20%5C%5C%20-q_3%20%5Cend%7Bbmatrix%7D%20%250) | (11) |

Multiplication (product) between two quaternions [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bp%7D%5Cotimes%5Cbar%7Bq%7D%250) can be expressed as:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bp%7D%5Cotimes%5Cbar%7Bq%7D%20%3D%20(p_0%20%2B%20p_1%20i%20%2B%20p_2%20j%20%2B%20p_3%20k)%20(q_0%20%2B%20q_1%20i%20%2B%20q_2%20j%20%2B%20q_3%20k)%250) | (12) |

The right hand side can be expanded and simplified with the identity property in eq (9). This product is also referred to as the Hamilton product. Algebraically, this can be expressed in the form of a matrix multiplication between a matrix function [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=Q(%5Cbar%7Bp%7D)%250) of the first quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bp%7D%250) and the second quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250) expressed in vector form:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bp%7D%5Cotimes%5Cbar%7Bq%7D%20%3D%20Q(%5Cbar%7Bp%7D)%20%5Cbar%7Bq%7D%250) | (13) |

Where the matrix function [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=Q(%5Cbar%7Bp%7D)%250) for a quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bp%7D%250) is defined as:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=Q(%5Cbar%7Bp%7D)%20%3D%20%5Cbegin%7Bbmatrix%7D%7Bp_0%20%26%20-p_1%20%26%20-p_2%20%26%20-p_3%20%5C%5C%20p_1%20%26%20p_0%20%26%20-p_3%20%26%20p_2%20%5C%5C%20p_2%20%26%20p_3%20%26%20p_0%20%26%20-p_1%20%5C%5C%20p_3%20%26%20-p_2%20%26%20p_1%20%26%20p_0%7D%5Cend%7Bbmatrix%7D%250) | (14) |

Notice that the quaternion product is non-commutative, i.e., [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bp%7D%5Cotimes%5Cbar%7Bq%7D%5Cneq%5Cbar%7Bq%7D%5Cotimes%5Cbar%7Bp%7D%250).

The norm [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%7C%7C%5Cbar%7Bq%7D%7C%7C%250) can be defined as the square root of the product between the quaternion and its conjugate:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%7C%7C%5Cbar%7Bq%7D%7C%7C%20%3D%20%5Csqrt%7B%5Cbar%7Bq%7D%5Cotimes%5Cbar%7Bq%7D%5E%7B*%7D%7D%20%3D%20%5Csqrt%7Bq%5E%7B2%7D_0%20%2B%20q%5E%7B2%7D_1%20%2B%20q%5E%7B2%7D_2%20%2B%20q%5E%7B2%7D_3%7D%250) | (15) |

A quaternion with [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%7C%7C%5Cbar%7Bq%7D%7C%7C%20%3D%201%20%250) is called an unit quaternion. To represent a rotation, a quaternion has to be an unit quaternion.

A quaternion inverse is defined as its conjugate divided by its norm squared:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%5E%7B-1%7D%20%3D%20%5Cfrac%7B%5Cbar%7Bq%7D%5E%7B*%7D%7D%7B%7C%7C%5Cbar%7Bq%7D%7C%7C%5E2%7D%250) | (16) |

Therefore, for unit quaternions, [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%5E%7B-1%7D%20%3D%20%5Cbar%7Bq%7D%5E%7B*%7D%250). The multiplication between a quaternion and its inverse would result in an unit scalar quaternion (with norm of 1) of the form:

|  |  |
| --- | --- |
| [Diagram  Description automatically generated](http://www.sciweavers.org/tex2img.php?bc=White&fc=Black&im=jpg&fs=78&ff=txfonts&edit=0&eq=%20%5Cbar%7Bq%7D%5Cotimes%5Cbar%7Bq%7D%5E%7B-1%7D%20%3D%20%5Cbar%7Bq%7D%5E%7B-1%7D%5Cotimes%5Cbar%7Bq%7D%20%3D%20%5Cbegin%7Bbmatrix%7D%7B%201%20%5C%5C%200%20%5C%5C%200%20%5C%5C%200%20%5Cend%7Bbmatrix%7D%20%250) | (17) |

This means that when a quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250) represents a rotation, its inverse [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%5E%7B-1%7D%250) represents the reverse rotation. When using to represent rotation or orientation in 3D space, the quaternion basis [](https://www.codecogs.com/eqnedit.php?latex=i%250), [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=j%250) and [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=k%250) can also be interpreted as the Cartesian basis vectors. This means a vector [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bv%7D%20%3D%20%5Cbegin%7Bbmatrix%7D%7Bv_x%20%26%20v_y%20%26%20v_z%7D%5Cend%7Bbmatrix%7D%5E%7BT%7D%250) can be represented as a vector quaternion [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D_%7Bv%7D%20%3D%20%5Cbegin%7Bbmatrix%7D%7B0%20%26%20v_x%20%26%20v_y%20%26%20v_z%7D%5Cend%7Bbmatrix%7D%5E%7BT%7D%250). Then, a rotation represented by a quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250) applying to a vector [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bv%7D_0%250) will result in a rotated vector [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bv%7D_1%250) given as follows:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbegin%7Bbmatrix%7D%7B0%5C%5C%5Cvec%7Bv%7D_1%7D%5Cend%7Bbmatrix%7D%20%3D%20%5Cbar%7Bq%7D%5Cotimes%5Cbegin%7Bbmatrix%7D%7B0%5C%5C%5Cvec%7Bv%7D_0%7D%5Cend%7Bbmatrix%7D%5Cotimes%5Cbar%7Bq%7D%5E%7B-1%7D%250) | (18) |

The above is referred to as the conjugate operation.  Notice that both [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Carrow%7Bv%7D_1%250) and [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Carrow%7Bv%7D_0%250) are expressed with respect to the global inertial frame. To change the basis of a vector between rotated frames (represented by the quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250)), e.g., finding the global coordinate [A black rectangle with a black background

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5E%7B0%7D%5Cvec%7Br%7D%250) of a vector expressed in the rotated frame [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5E%7B1%7D%5Cvec%7Br%7D%250), one would apply the reverse rotation [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%5E%7B-1%7D%250) to [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5E%7B1%7D%5Cvec%7Br%7D%250) to obtain [A black rectangle with a black background

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5E%7B0%7D%5Cvec%7Br%7D%250):

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbegin%7Bbmatrix%7D%7B0%5C%5C%5E%7B0%7D%5Cvec%7Br%7D%7D%5Cend%7Bbmatrix%7D%20%3D%20%5Cbar%7Bq%7D%5E%7B-1%7D%5Cotimes%5Cbegin%7Bbmatrix%7D%7B0%5C%5C%5E%7B1%7D%5Cvec%7Br%7D%7D%5Cend%7Bbmatrix%7D%5Cotimes%5Cbar%7Bq%7D%250) | (19) |

For successive rotations represented by [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D_1%250), [A black rectangle with a black background

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D_2%250), … [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D_n%250), the resultant quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250) can be expressed as:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%20%3D%20%20%20%5Cbar%7Bq%7D_n%20%5Cotimes%20%5Cdots%20%5Cotimes%20%5Cbar%7Bq%7D_2%20%5Cotimes%20%5Cbar%7Bq%7D_1%250) | (20) |

A useful identity relationship for unit quaternion products, that is quaternion multiplication between unit quaternions [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bp%7D%250) and [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250) is:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%20(%5Cbar%7Bp%7D%20%5Cotimes%20%5Cbar%7Bq%7D)%5E%7B*%7D%20%3D%20%5Cbar%7Bq%7D%5E%7B*%7D%20%5Cotimes%20%5Cbar%7Bp%7D%5E%7B*%7D%250) | (21) |

The rotation represented by the quaternion [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%250) can be related to the axis of rotation (represented by the unit vector [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Chat%7Be%7D%250)) and the rotation angle [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Ctheta%250), based on Euler’s rotation theorem:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cbar%7Bq%7D%20%3D%20%5Cbegin%7Bbmatrix%7D%20%5Ccos%7B%5Cfrac%7B%5Ctheta%7D%7B2%7D%7D%20%5C%5C%20%5Chat%7Be%7D%5Csin%7B%5Cfrac%7B%5Ctheta%7D%7B2%7D%7D%20%5Cend%7Bbmatrix%7D%250) | (22) |

Homogenous Transformation

A homogeneous transformation [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=T%250) is a [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=4%20%5Ctimes%204%250) matrix encompassing both translational [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bp%7D%250) and rotational [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=R%250) components. It has the following form:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%20T%20%3D%20%5Cbegin%7Bbmatrix%7D%20R%20%26%20%5Cvec%7Bp%7D%20%5C%5C%20%5Cvec%7B0%7D_%7B1%5Ctimes%203%7D%20%26%201%20%5Cend%7Bbmatrix%7D%20%250) | (23) |

Where [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=R%250) is the rotation matrix and [Shape

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bp%7D%250) is a position vector. The homogeneous transformation [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=T%250) represents the 6 degree-of-freedom (DOF) relative motion between two reference frame in 3D space. Similar to the rotation matrix, the homogeneous transformation matrix can be used to obtain the inertial coordinate [A black rectangle with a black background

Description automatically generated with low confidence](https://www.codecogs.com/eqnedit.php?latex=%5E%7B0%7D%5Cvec%7Br%7D%250) of a vector [Shape

Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%5E%7B1%7D%5Cvec%7Br%7D%250), expressed in a translated and rotated frame:

|  |  |
| --- | --- |
| [Shape  Description automatically generated with medium confidence](https://www.codecogs.com/eqnedit.php?latex=%20%5Cbegin%7Bbmatrix%7D%20%5E%7B0%7D%5Cvec%7Br%7D%20%5C%5C%201%20%5Cend%7Bbmatrix%7D%20%3D%20T%20%5Cbegin%7Bbmatrix%7D%20%5E%7B1%7D%5Cvec%7Br%7D%20%5C%5C%201%20%5Cend%7Bbmatrix%7D%20%250) | (24) |

Logo

Description automatically generated

© 2022 Quanser Inc., All rights reserved.

Quanser Inc.

119 Spy Court

Markham, Ontario

L3R 5H6

Canada

info@quanser.com

Phone: 19059403575

Fax: 19059403576

Printed in Markham, Ontario.

For more information on the solutions Quanser Inc. offers, please visit the web site at: <http://www.quanser.com>

This document and the software described in it are provided subject to a license agreement. Neither the software nor this document may be used or copied except as specified under the terms of that license agreement. Quanser Inc. grants the following rights: a) The right to reproduce the work, to incorporate the work into one or more collections, and to reproduce the work as incorporated in the collections, b) to create and reproduce adaptations provided reasonable steps are taken to clearly identify the changes that were made to the original work, c) to distribute and publicly perform the work including as incorporated in collections, and d) to distribute and publicly perform adaptations. The above rights may be exercised in all media and formats whether now known or hereafter devised. These rights are granted subject to and limited by the following restrictions: a) You may not exercise any of the rights granted to You in above in any manner that is primarily intended for or directed toward commercial advantage or private monetary compensation, and b) You must keep intact all copyright notices for the Work and provide the name Quanser Inc. for attribution. These restrictions may not be waved without express prior written permission of Quanser Inc.