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Physical System Parameters

Drone Parametrization

v 0.5

What this document covers

This document covers both the linear and non-linear mapping from the Generalized Forces to the Motor Commands, that are used in the Autonomous Vehicles Research Studio documentation and software.

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System Parameters

Some QDrone parameters that are used in the derivations in this document have been listed in Table 1 below.

Table 1: QDrone mechanical parameters		
Dimensions		
L_{roll}	Roll motor-to-motor distance	0.2136 m
L_{pitch}	Pitch motor-to-motor distance	0.1758 m
$L \times W \times H$	Net frame dimensions along x, y & z directions	0.363 m x 0.403 m x 0.139 m
h_{cg}	Vertical C.O.G location (w.r.t the ground when drone sits flat)	0.051 m
Mass and Moment of Inertia		
M_b	Battery mass	0.267 kg
M_d	Drone mass	0.854 kg
M	Total mass	1.121 kg
J_{xx}	Roll Moment of Inertia	1.0×10^{-2} kgm^2
J_{yy}	Pitch Moment of Inertia	8.2×10^{-3} kgm^2
J_{zz}	Yaw Moment of Inertia	1.48×10^{-2} kgm^2
Electrical Parameters used in linear mapping		
A_{hover}	Hover motor current	5.82 A
k_t	Motor torque constant	4.50×10^{-3} Nm/A
K_v	Motor speed constant (specification)	2100 RPM/V

Linear Force to Motor Command Mapping

Given the weight of the QDrone and the nominal full voltage of a 3S LiPo battery (12.6V), the selected propulsion system on the QDrone is able to maintain hover at around 53.8% command (u_{hover}) on a fully charged fresh battery. If linear extrapolation is used, the maximum commanded thrust force for each motor (thrust at 100% command) can be estimated by taking the QDrone takeoff weight, divided by 4 times the hover percentage command:

$$K_f = F_{max} = \frac{M \times g}{4 \times u_{\text{hover}}}$$

$$K_f = \frac{(0.854 + 0.267) \times 9.81}{4 \times 0.538} = 5.11N \tag{1}$$

Where u_{hover} was obtained from experimental data collected in a manual flight. Roll and Pitch moment arms are estimated as half the motor-to-motor distances. They are obtained by measuring the distance between the center of the motor shafts along the forward/backward and sideway direction of the QDrone frame:

$$\begin{aligned} L_{roll} &= 0.2136m \\ L_{pitch} &= 0.1758m \end{aligned} \tag{2}$$

The motor torque constant k_t is obtained from the motor manufacturer data sheet. The hover motor current A_{hover} is obtained from experimental data collected with dynamometer. The normalized yaw torque constant K_t can be estimated as follows

$$\begin{aligned} K_t &= k_t \frac{A_{hover}}{u_{hover}} \\ K_t &= 4.5 \times 10^{-3} \frac{Nm}{A} \frac{5.8239A}{0.538} \\ K_t &= 0.0487Nm \end{aligned} \tag{3}$$

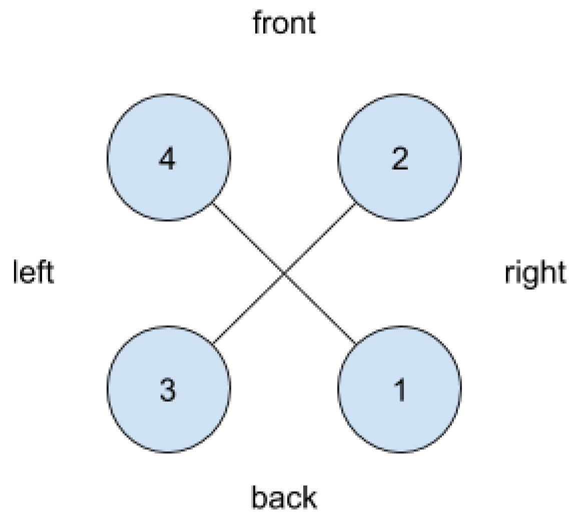

Figure 1: Motor ordering in the QDrone

The mapping matrix, from motor percentage command u_i to system force/torques,

$$\begin{bmatrix} F \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix} = \begin{bmatrix} K_f & K_f & K_f & K_f \\ -K_f\frac{L_{roll}}{2} & -K_f\frac{L_{roll}}{2} & K_f\frac{L_{roll}}{2} & K_f\frac{L_{roll}}{2} \\ K_f\frac{L_{pitch}}{2} & -K_f\frac{L_{pitch}}{2} & K_f\frac{L_{pitch}}{2} & -K_f\frac{L_{pitch}}{2} \\ K_t & -K_t & -K_t & K_t \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{4}$$

Inverting the mapping matrix to obtain the inverse relationship:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{4K_f} & -\frac{1}{2K_f L_{roll}} & \frac{1}{2K_f L_{pitch}} & \frac{1}{4K_t} \\ \frac{1}{4K_f} & -\frac{1}{2K_f L_{roll}} & -\frac{1}{2K_f L_{pitch}} & -\frac{1}{4K_t} \\ \frac{1}{4K_f} & \frac{1}{2K_f L_{roll}} & \frac{1}{2K_f L_{pitch}} & -\frac{1}{4K_t} \\ \frac{1}{4K_f} & \frac{1}{2K_f L_{roll}} & -\frac{1}{2K_f L_{pitch}} & \frac{1}{4K_t} \end{bmatrix} \begin{bmatrix} F \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix} \tag{5}$$

Thus, a final controller command as a thrust force (N) and 3 rotation torques (Nm) can be mapped to a set of motor percentage commands (% PWM pulse from 0 to 1) using the matrix in (5) above.

The maximum system force/torque (mapping to 100% command) will be [20.44 N, 1.09 Nm, 0.90 Nm, 0.10 Nm] for Throttle thrust, Roll torque, Pitch torque and Yaw torque (TRPY) with a 3S LiPo battery, 6045 props and 2206 Cobra motors (2100 Kv)..

This is linearly extrapolated about the hover force of 11.0N (53.8% of maximum thrust force of 20.44N).

Figure 2 below shows the linear and non-linear models compared. The non-linear mapping is presented in the next section.

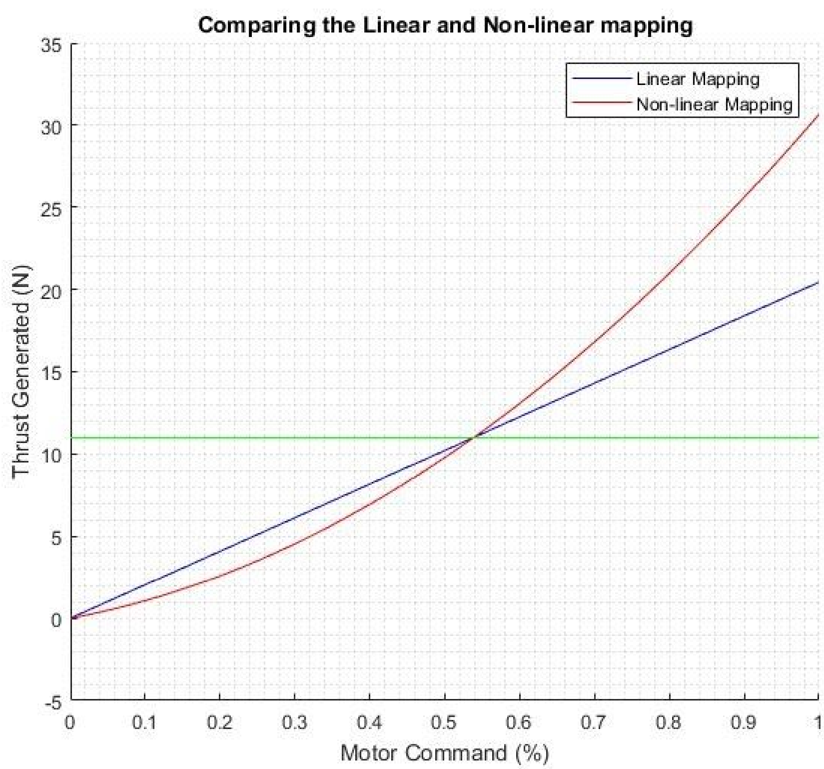

Figure 2: Linear vs. Non-linear mapping models

Non-Linear Force to Motor Command Mapping

Experimentally, the relationship between the applied motor command (%) and the corresponding command voltage applied to the motors by the ESC is,

$$u = \frac{V}{V_d} \tag{6}$$

Where V_d is the battery voltage. The angular velocity of the propeller (6045 durable polycarbonate) is linearly related to the commanded voltage as,

$$V = \frac{1}{K_{v,eff}}(\omega - \omega_c) \tag{7}$$

Here, ω_c is an angular velocity offset in RPM and $K_{v,eff}$ is the effective motor speed constant in RPM/V. The parameters ω_c and $K_{v,eff}$ are obtained by fitting a linear polynomial (Figure 3 linear fit) to angular velocity (RPM) and voltage command (V) data collected from a test on a dynamometer (Figure 3 raw data).

The thrust F_m produced by the rotating propeller has a squared relationship with the angular velocity of the propeller, and can be experimentally estimated as,

$$F_m = C_t(\omega + \omega_f)^2 + F_b \tag{8}$$

Where C_t is the motor force constant in N/RPM² of the motor/propeller combination, ω_f is another angular velocity offset and F_b is the force offset in N. The parameters C_t and F_b are obtained by fitting a quadratic polynomial (Figure 4 quadratic fit) to thrust (N) and angular velocity (RPM) data collected from hover flights of the QDrone with a varying payload (Figure 4 raw data).

Thus, the commanded force for each motor can be mapped to the commanded voltage. The required voltage corresponds to a motor command that compensates for the current battery voltage level.

Note: The angular velocity offset ω_c is obtained experimentally by fitting a linear polynomial to the voltage command vs. angular velocity curve. This results in a non-zero angular velocity at a zero voltage command. This non-zero angular velocity will map to a non-zero thrust, which is not practical for use in our control model, where a zero voltage command should be mapped to a zero thrust generated. Thus, another angular velocity offset ω_f is introduced. Here, ω_f was calculated using the equation,

$$\omega_f = \sqrt{\frac{-F_b}{C_t}} - \omega_c$$

which is obtained by solving equations 7 and 8 for ω_f with $V = 0$ and $F_m = 0$. This is also illustrated in Figures 3 and 4 below.

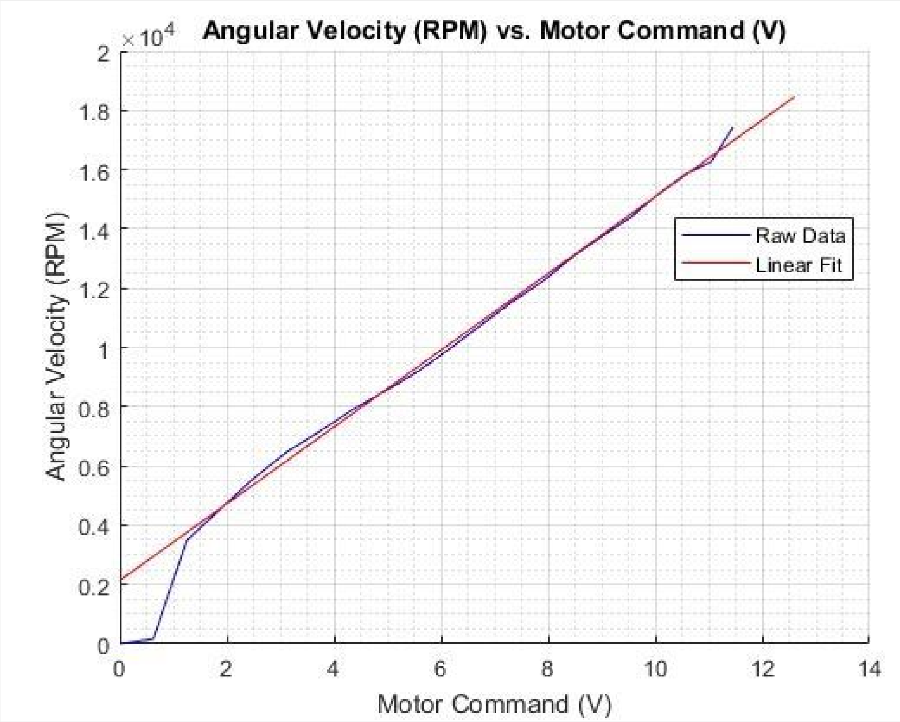

Figure 3: Motor Command (V) vs. Angular Velocity (RPM) - angular velocity of 2132.6 RPM at a 0 voltage command

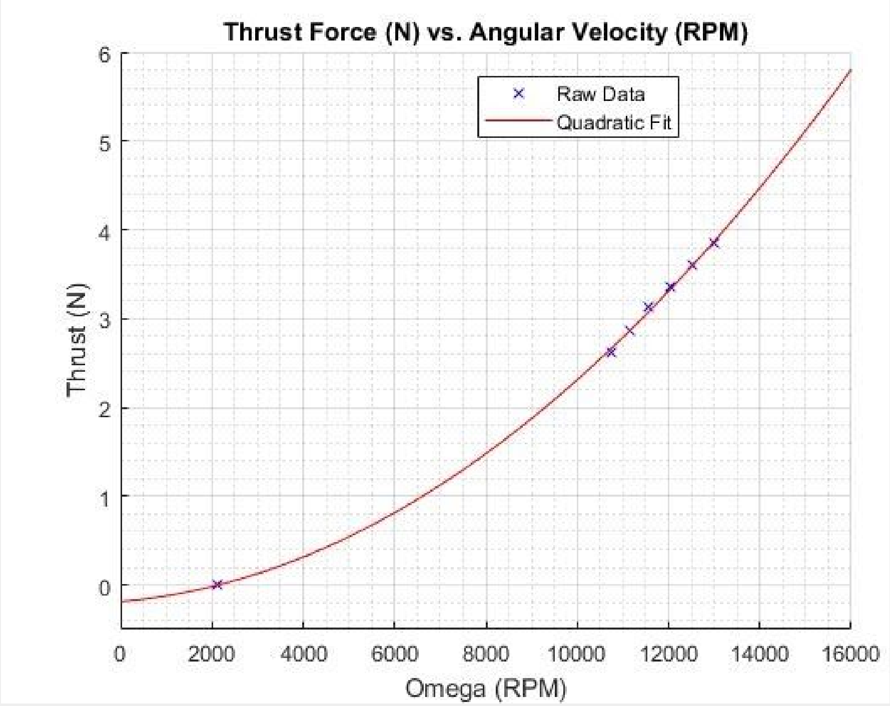

Figure 4: Angular Velocity (RPM) vs. Thrust Generated per Motor (N) - 0 thrust generated at An angular velocity of 2132.6 RPM

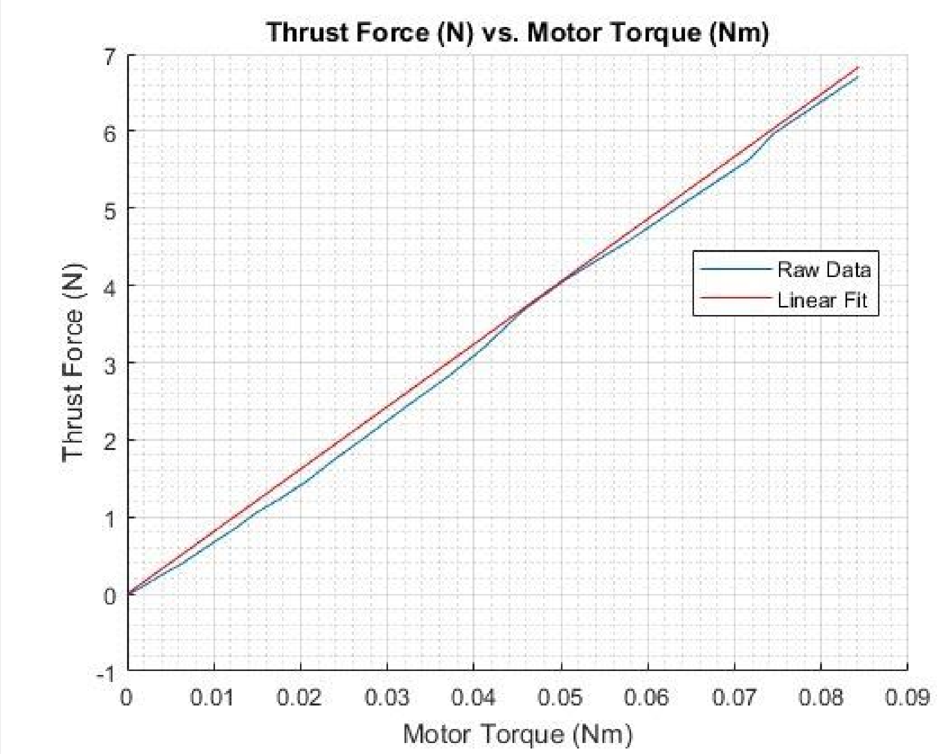

Figure 5: Motor Torque (Nm) vs. Thrust Generated per Motor (N)

Finally, the motor torque τ_m is linearly related to the motor force F_m by

$$F_m = k_\tau \tau_m \tag{9}$$

Where k_τ is the motor thrust-torque constant. This is obtained by fitting a linear polynomial (Figure 5 linear fit) to the motor torque vs. thrust generated data (Figure 5 raw data).

The parameters $K_{v,eff}$, ω_c, ω_f, C_t, F_b and k_τ obtained have been summarized in Table 2 below.

Table 2: QDrone non-linear model parameter estimates		
Dimensions		
$K_{v,eff}$	Effective motor speed constant	1295.4 RPM/V
ω_c	Voltage to Angular velocity offset	2132.6 RPM
ω_f	Angular velocity to force offset	1004.5 RPM
C_t	Motor force constant	2.0784 x 10^{-8} N/RPM^2
F_b	Motor force offset	-0.2046 N
k_τ	Motor thrust-torque constant	81.0363 N/Nm

Thus, given the generalized force vector, one can find the corresponding motor forces as,

$$\vec{F}_m = \begin{bmatrix} F_{m,1} \\ F_{m,2} \\ F_{m,3} \\ F_{m,4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2L_{roll}} & \frac{1}{2L_{pitch}} & \frac{k_\tau}{4} \\ \frac{1}{4} & -\frac{1}{2L_{roll}} & -\frac{1}{2L_{pitch}} & -\frac{k_\tau}{4} \\ \frac{1}{4} & \frac{1}{2L_{roll}} & \frac{1}{2L_{pitch}} & -\frac{k_\tau}{4} \\ \frac{1}{4} & \frac{1}{2L_{roll}} & -\frac{1}{2L_{pitch}} & \frac{k_\tau}{4} \end{bmatrix} \begin{bmatrix} F \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix} \tag{10}$$

From here, the angular velocity is obtained by,

$$\vec{\omega} = \sqrt{\frac{1}{C_t}\left(\vec{F}_m - \begin{bmatrix} F_b & F_b & F_b & F_b \end{bmatrix}^T\right)} - \begin{bmatrix} \omega_f & \omega_f & \omega_f & \omega_f \end{bmatrix}^T \tag{11}$$

where $\vec{\omega} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & \omega_4 \end{bmatrix}^T$. From here, the required motor voltage vector is given by,

$$\vec{V} = \frac{1}{K_{v,eff}}\left(\vec{\omega} - \begin{bmatrix} \omega_c & \omega_c & \omega_c & \omega_c \end{bmatrix}^T\right) \tag{12}$$

Where $\vec{V} = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \end{bmatrix}^T$ is a vector of motor voltages. Lastly, the required motor command is then,

$$\vec{u} = \frac{1}{V_d}\vec{V} \tag{13}$$

Thus, a final controller command as a thrust force (N) and 3 rotation torques (Nm) can be converted to a set of motor commands (% PWM pulse from 0 to 1) using the equations in (10) and (11) above.

The maximum system force/torque will be [30.67 N, 1.6373 Nm, 1.3476 Nm, 0.1892 Nm] for Throttle thrust, Roll torque, Pitch torque and Yaw torque (TRPY) with a 3S LiPo battery, 6045 props and 2206 Cobra motors (2100 Kv). Note that this mapping results in a trim of 53.8% as well.