

## Guides and Resources: Hardware - QDrone

## **Propulsion System**

This document provides information on the QDrone's propulsion system.

## Motors and Propellers

The QDrone uses the Cobra 2100Kv (size 2206) motors (Figure 1a) with dual-blade polycarbonate 6045 propellers (Figure 1b). The specifications are listed in Table 1.





a. Cobra 2100Kv motors

b. 6045 polycarbonate propellers

Figure 1: Motor and Propellers

Table 1: Motor and Propeller Specifications		
Item	Description	
Motors		
Kv	2100 RPM/V	
Stator diameter/thickness	22.00 mm / 6.00 mm	
Stator slots/magnet poles	12 / 14	
Max continuous current	25 Amps	
Time constant	40 ms	
Propellers		
Diameter	6.00 Inches	
Pitch	4.50 Inches	
Material	Polycarbonate	

## Response Curves

The motor propellor combination was characterized using a dynamometer to yield the following response curves. The command sent to the ESC is represented by  $\boldsymbol{u}$ , the Throttle Command (%). Experimentally, the relationship between the applied motor command (%) and the corresponding command voltage applied to the motors by the ESC is,

$$u = \frac{V}{V_d} \tag{6}$$

Where  $V_d$  is the battery voltage. The angular velocity of the propeller (6045 durable polycarbonate) is linearly related to the commanded voltage as,

$$V = \frac{1}{K_{v,eff}} (\omega - \omega_c) \tag{7}$$

Here,  $\omega_c$  is an angular velocity offset in RPM and  $K_{v,eff}$  is the effective motor speed constant in RPM/V. The parameters  $\omega_c$  and  $K_{v,eff}$  are obtained by fitting a linear polynomial (Figure 3 linear fit) to angular velocity (RPM) and voltage command (V) data collected from a test on a dynamometer (Figure 3 raw data).

The thrust  $F_m$  produced by the rotating propeller has a squared relationship with the angular velocity of the propeller, and can be experimentally estimated as,

$$F_m = C_t(\omega + \omega_f)^2 + F_b \tag{8}$$

Where  $C_t$  is the motor force constant in N/RPM² of the motor/propeller combination,  $\omega_f$  is another angular velocity offset and  $F_b$  is the force offset in N. The parameters  $C_t$  and  $F_b$  are obtained by fitting a quadratic polynomial (Figure 4 quadratic fit) to thrust (N) and angular velocity (RPM) data collected from hover flights of the QDrone with a varying payload (Figure 4 raw data).

Thus, the commanded force for each motor can be mapped to the commanded voltage. The required voltage corresponds to a motor command that compensates for the current battery voltage level.

**Note**: The angular velocity offset  $\omega_c$  is obtained experimentally by fitting a linear polynomial to the voltage command vs. angular velocity curve. This results in a non-zero angular velocity at a zero voltage command. This non-zero angular velocity will map to a non-zero thrust, which is not practical for use in our control model, where a zero voltage command should be mapped to a zero thrust generated. Thus, another angular velocity offset  $\omega_f$  is introduced. Here,  $\omega_f$  was calculated using the equation,

$$\omega_f = \sqrt{\frac{-F_b}{C_t}} - \omega_c$$

which is obtained by solving equations 7 and 8 for  $\omega_f$  with V=0 and  $F_m=0$ . This is also illustrated in Figures 3 and 4 below.

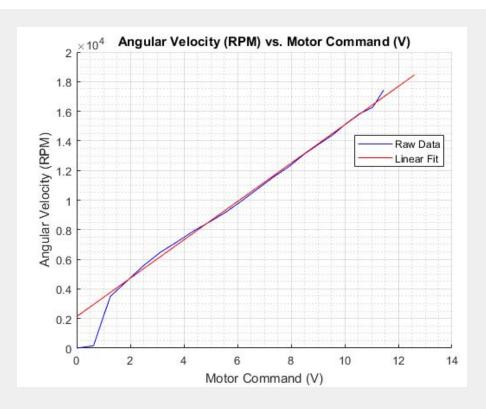


Figure 3: Motor Command (V) vs. Angular Velocity (RPM) - angular velocity of 2132.6 RPM at a 0 voltage command

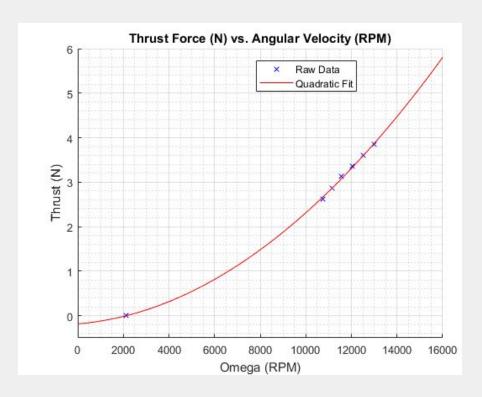


Figure 4: Angular Velocity (RPM) vs. Thrust Generated per Motor (N) - 0 thrust generated at An angular velocity of 2132.6 RPM

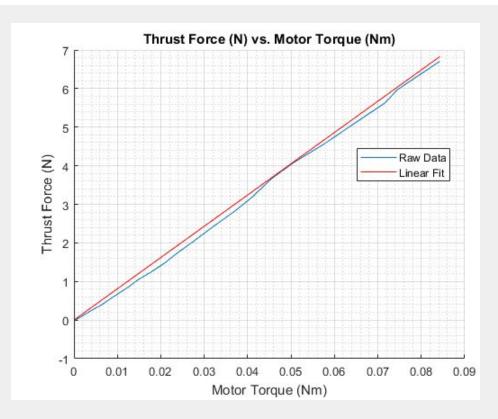


Figure 5: Motor Torque (Nm) vs. Thrust Generated per Motor (N)

Finally, the motor torque  $au_m$  is linearly related to the motor force  $F_m$  by

$$F_m = k_\tau \tau_m \tag{9}$$

Where  $k_{\tau}$  is the motor thrust-torque constant. This is obtained by fitting a linear polynomial (Figure 5 linear fit) to the motor torque vs. thrust generated data (Figure 5 raw data).

The parameters  $K_{v,eff}$ ,  $\omega_c$ ,  $\omega_f$ ,  $C_t$ ,  $F_b$  and  $k_\tau$  obtained have been summarized in Table 2 below.

Table 2: QDrone non-linear model parameter estimates			
Dimensions			
$K_{v,eff}$	Effective motor speed constant	1295.4 RPM/V	
$\omega_c$	Voltage to Angular velocity offset	2132.6 RPM	
$\omega_f$	Angular velocity to force offset	1004.5 RPM	
$C_t$	Motor force constant	2.0784 x 10 <sup>-8</sup> N/RPM <sup>2</sup>	
$F_b$	Motor force offset	-0.2046 N	
$k_{ au}$	Motor thrust-torque constant	81.0363 N/Nm	

Thus, given the generalized force vector, one can find the corresponding motor forces as,

$$\vec{F}_{m} = \begin{bmatrix} F_{m,1} \\ F_{m,2} \\ F_{m,3} \\ F_{m,4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2L_{roll}} & \frac{1}{2L_{pitch}} & \frac{k_{\tau}}{4} \\ \frac{1}{4} & -\frac{1}{2L_{roll}} & -\frac{1}{2L_{pitch}} & -\frac{k_{\tau}}{4} \\ \frac{1}{4} & \frac{1}{2L_{roll}} & \frac{1}{2L_{pitch}} & -\frac{k_{\tau}}{4} \\ \frac{1}{4} & \frac{1}{2L_{roll}} & -\frac{1}{2L_{pitch}} & \frac{k_{\tau}}{4} \end{bmatrix} \begin{bmatrix} F \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix}$$

$$(10)$$

From here, the angular velocity is obtained by,

$$\vec{\omega} = \sqrt{\frac{1}{C_t} (\vec{F}_m - \begin{bmatrix} F_b & F_b & F_b & F_b \end{bmatrix}^T)} - \begin{bmatrix} \omega_f & \omega_f & \omega_f & \omega_f \end{bmatrix}^T$$
(11)

where  $\vec{\omega} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & \omega_4 \end{bmatrix}^T$  . From here, the required motor voltage vector is given by,

$$\vec{V} = \frac{1}{K_{v,eff}} (\vec{\omega} - \begin{bmatrix} \omega_c & \omega_c & \omega_c \end{bmatrix}^T)$$
(12)

Where  $\vec{V} = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \end{bmatrix}^T$  is a vector of motor voltages. Lastly, the required motor command is then.

$$\vec{u} = \frac{1}{V_d} \vec{V} \tag{13}$$

Thus, a final controller command as a thrust force (N) and 3 rotation torques (Nm) can be converted to a set of motor commands (% PWM pulse from 0 to 1) using the equations in (10) and (11) above.

The maximum system force/torque will be [30.67 N, 1.6373 Nm, 1.3476 Nm, 0.1892 Nm] for Throttle thrust, Roll torque, Pitch torque and Yaw torque (TRPY) with a 3S LiPo battery, 6045 props and 2206 Cobra motors (2100 Kv). Note that this mapping results in a trim of 53.8% as well.