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Concept Review

Sensor Noise

How to interpret sensor data?

Many types of sensors exist for observing various physical properties of the world around us: encoders measure position, accelerometers measure acceleration, thermocouples measure temperature, and the list goes on. What all sensors have in common is that they all contain some amount of error in their measured outputs. Random time-varying errors in a sensor's output signal are typically called 'sensor noise.' Sensor noise is caused primarily by electrical interference, although quantization errors (caused by converting analog signals to a digital-representation) also play a role. This document discusses how to interpret sensor data and model sensor noise.

Additive Gaussian White Noise

For most real-world systems, the random variations in a sensor's output signal are well modelled as additive Gaussian white noise. The term additive means that a sensor's output (y_m) is a summation of two terms: the true signal being measured (y), and a random noise term (w).

$$y_m = y + w$$

As a random variable, the noise term (in this case w) is assumed to be normally distributed, having a probability density function in the shape of a Gaussian function (commonly referred to as a bell curve and shown in fig. 1).

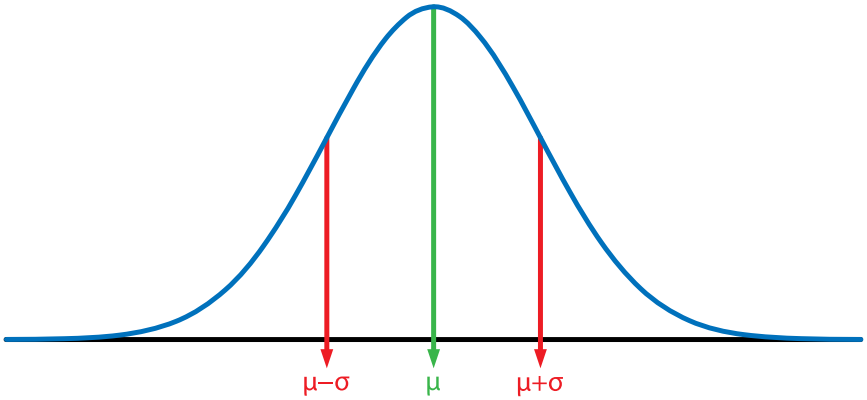

Figure 1: Gaussian function with relevant parameters highlighted.

The equation for a Guassian curve is determined by two parameters (μ and σ) and is defined as follows:

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)$$

μ represents the mean and defines the centre location of the curve (the point with the highest probability). σ is referred to as the 'standard deviation' and quantifies the level of variation, or width of the curve. Figure 2 shows how variations in μ and σ affect the resulting curve produced.

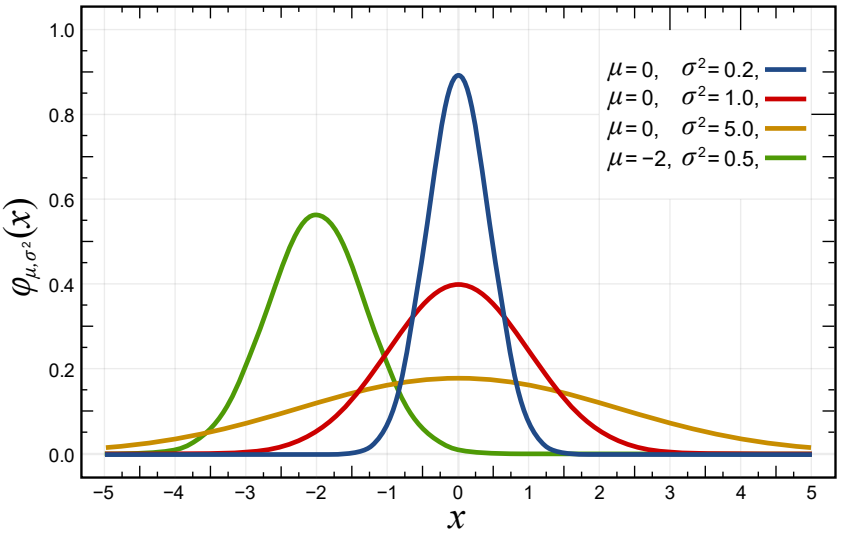

Figure 2: Gaussian functions with various parameter values.

Estimating Noise Parameters from Sensor Data

Figure 3 shows a sample recording of 30 seconds of measurements from two accelerometers, one orientated vertically, and one oriented horizontally. The accelerometers where sitting stationary on a table for the duration of the recording process. In these two plots, the presence of noise is clearly visible as the random gitters in what should otherwise be a constant value. By plotting the histogram of the recorded data (fig 4.), it can be seen that the captured data does indeed appear to be normally distributed.

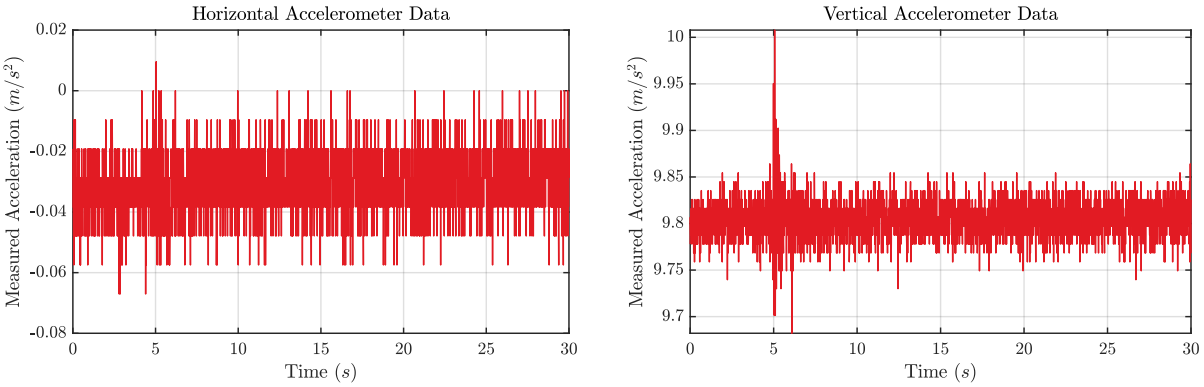

Figure 3: Accelerometer measurements over time.

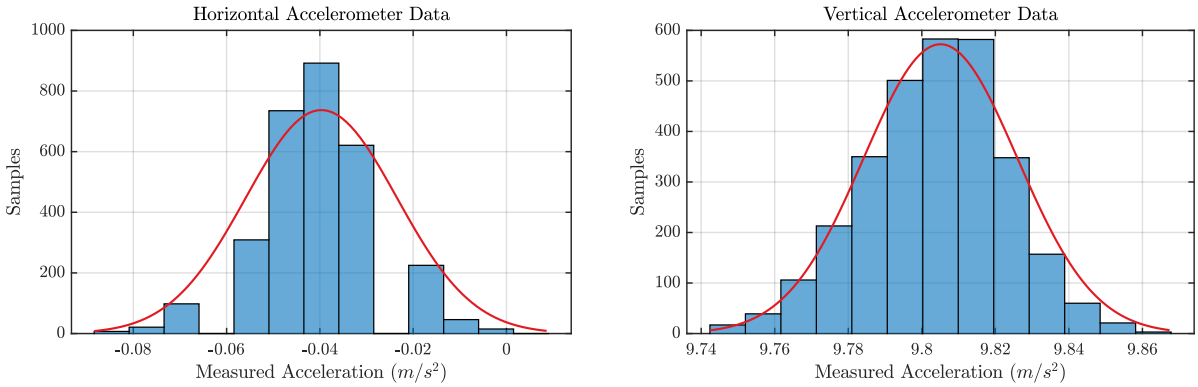

Figure 4: Histograms from measured accelerometer data, fitted to normal distributions.

Given the recorded sensor data, we can identify the shape of the underlying distribution by estimating the two parameters that define a gaussian function: μ and σ. The mean μ can be calculated as follows:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} y_{m,i}$$

where $y_{m,i}$ denotes the ith data sample and N is the number of samples recorded. Once the mean has been calculated, the standard deviation can be calculated as follows:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y_{m,i} - \mu\right)^2}$$

For the recorded accelerometer data shown in fig. 3, the identified Gaussian noise profiles are plotted in fig. 4 overtop of the histogram.

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