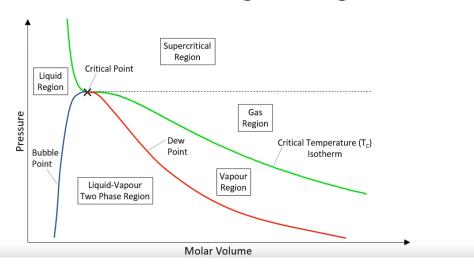
Equations_Of_State_pt1

March 19, 2023

Pressure-Volume Diagram: Regions



Liquid-Vapour Two Phase Region: Coexistence of liquid and vapour, encapsulated between dewpoint and bubble point line.

Vapour Region: Between Dew Point and Critical Temperature (Tc) Isotherm, where our substance is in the gas phase but below the critical temperature

Dashed line represents the critical pressure

1 Ideal Gas Law

1.1 Assumptions:

- Volume occupied by gas molecules is negligible
- There is no intermolecular attraction or repulsion between the gas molecules

1.2 Limitations:

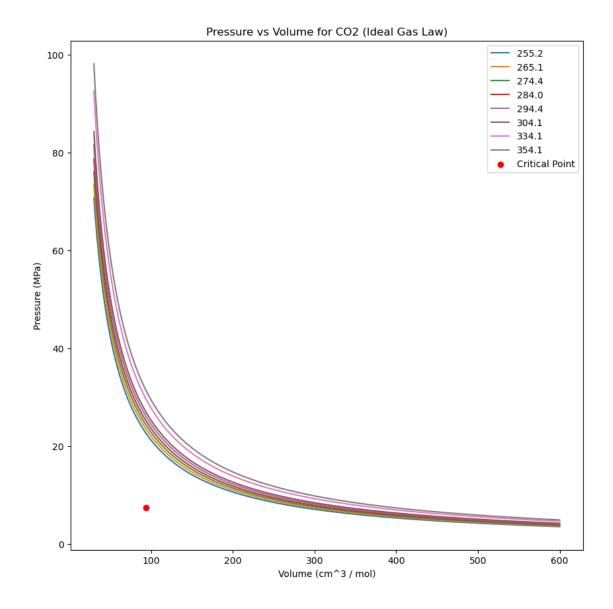
- Unreliable at low temperatures and/or high pressures
- Cannot predict vapour-liquid coexistence

```
[4]: import pandas as pd import matplotlib.pyplot as plt
```

```
[5]: # Constant Values
     R = 8.314 \# cm^3 MPa / K mol
     list_of_volumes = [i for i in range(30, 601, 1)]
     list_of_temperatures = [255.2, 265.1, 274.4, 284.0, 294.4, 304.1, 334.1,354.1]
[6]: critical_pressure = 7.377 # MPa
     critical volume = 94.120 # cm^3 / mol
     critical_temperature = 304.128 # K
[7]: def ideal_gas_law(gas_constant, list_of_volumes, list_of_temperatures):
         df = pd.DataFrame(columns = [str(i) for i in list of temperatures], index = 1
      →list_of_volumes)
         for index, rows in df.iterrows():
             for column in df:
                 df[column][index] = (float(column) * gas_constant) / index
         return df
[8]: | ideal_gas_df = ideal_gas_law(R, list_of_volumes, list_of_temperatures)
[9]: ideal_gas_df
[9]:
              255.2
                         265.1
                                    274.4
                                               284.0
                                                          294.4
                                                                     304.1 \
          70.724427
     30
                    73.468047
                                76.045387
                                           78.705867
                                                      81.588053
                                                                 84.276247
     31
          68.442994
                     71.09811
                                 73.59231
                                           76.166968 78.956181 81.557658
     32
          66.30415
                    68.876294
                                 71.29255
                                            73.78675
                                                        76.4888 79.008981
     33
                                 69.13217
                                           71.550788 74.170958
          64.294933 66.789133
                                                                  76.61477
     34
          62.403906 64.824747
                                67.098871
                                           69.446353 71.989459 74.361394
     . .
     596
          3.559954
                     3.698056
                                 3.827788
                                            3.961705
                                                       4.106781
                                                                  4.242093
     597
          3.553991
                     3.691862
                                 3.821376
                                            3.955069
                                                       4.099902
                                                                  4.234987
     598
          3.548048
                     3.685688
                                 3.814986
                                            3.948455
                                                       4.093046
                                                                  4.227905
     599
          3.542125
                     3.679535
                                 3.808617
                                                       4.086213
                                                                  4.220847
                                            3.941863
     600
          3.536221
                     3.673402
                                 3.802269
                                            3.935293
                                                       4.079403
                                                                  4.213812
              334.1
                         354.1
     30
          92.590247
                    98.132913
     31
          89.603465
                    94.967335
     32
          86.803356 91.999606
     33
          84.172952
                    89.211739
     34
          81.697276 86.587865
     . .
     596
          4.660583
                     4.939576
     597
          4.652776
                      4.931302
     598
           4.644996
                      4.923056
     599
          4.637241
                      4.914837
```

```
600 4.629512 4.906646
```

[571 rows x 8 columns]



Critical Point is for Co2 - Isotherm goes nowhere near it

2 Peng-Robinson Equation of State

Peng-Robinson equation of state

$$P = \frac{RT}{V-b} - \frac{a}{V(V+b) + b(V-b)} \qquad a = 0.45724 \frac{\alpha R^2 T_c^2}{P_c} \quad b = 0.07780 \frac{RT_c}{P_c}$$

$$\alpha = \left(1 + \kappa (1 - \sqrt{T/T_c})^2 \right) \quad \kappa = 0.37464 + 1.54226\omega - 0.26992\omega^2$$
 where P is the pressure (Pa); V is the molar volume (m³ mol⁻¹); V is the gas constant (8.314 J mol⁻¹ K⁻¹); V is the absolute temperature (K); V is the critical pressure for the component of interest (Pa); V is the critical temperature for the component of interest (K); V is the acentric factor for the component of interest.

Omega {w} = accentric factor, shape of the molecule. Completely spherical is 0, most noble gases

```
[11]: # Accentric factor is usually from literature
w = 0.224
kappa = 0.37464 + 1.54226 * w - 0.26992 * w**2
alpha = lambda T: (1 + kappa * (1 - (T / critical_temperature)**0.5))**2
```

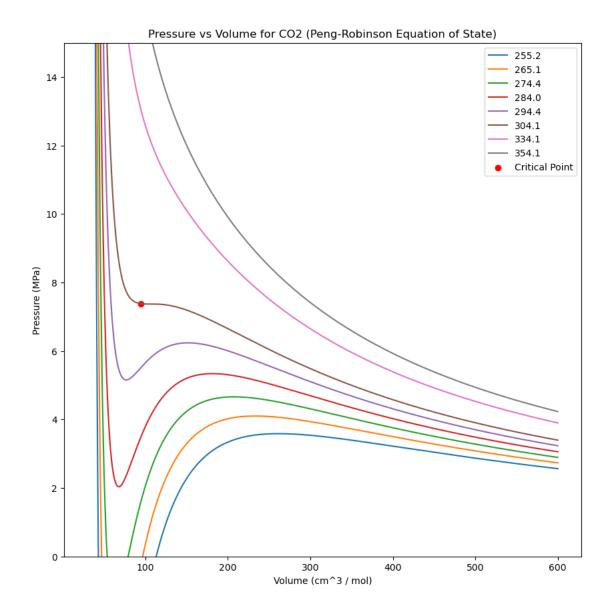
```
[12]: bc = 0.07780 * R * critical_temperature / critical_pressure
print(bc)
```

26.666513614965435

```
[13]: ac = 0.45724 * R**2 * critical_temperature**2 / critical_pressure print(ac)
```

396275.55680960533

return df



```
[17]: # Find critical point for Peng-Robinson Equation of State using fsplve from scipy.optimize import fsolve

Tc = 304.128
Pc = 7.377
R = 8.314
w = 0.224
kappa = 0.37464 + 1.54226 * w - 0.26992 * w**2
alpha = lambda T: (1 + kappa * (1 - (T / Tc)**0.5))**2

bc = 0.07780 * R * Tc / Pc
print(bc)
```

```
ac = 0.45724 * R**2 * Tc**2 / Pc
print(ac)
```

26.666513614965435 396275.55680960533

```
[18]: from scipy.optimize import fsolve
      def f(Vc):
         Pc = 7.377 \# example value
          R = 8.314 \# J/(mol*K)
          Tc = 304.128 \# K
          A = ac * Pc / (R * Tc)**2 # calculate A using ac, Pc, R, and Tc
          #print(A)
          B = bc * Pc / (R * Tc) # calculate B using bc, Pc, R, and Tc
          #print(B)
          Z = Pc*Vc/(R*Tc)
          print(Z)
          return 1*Z**3 + (-(1-B))*(Z**2) + (A-3*B**2-2*B) * Z - (A*B-B**2-B**3)
      # initial guess for Vc
      Vc_guess = 0.1
      # solve for Vc when f(Vc) equals 0
      Vc_solution = fsolve(f, Vc_guess)
      print("Vc =", Vc_solution)
```

```
[0.00029175]
```

[0.00029175]

[0.00029175]

[0.00029175]

[0.02946692]

[0.08781725]

[0.14460107]

[0.18368107]

[0.21428775]

[0.2371007]

[0.25446203]

[0.26762591]

[0.27772322]

[0.28561995]

[0.29210604]

[0.29810179]

```
[0.30564994]
     [0.33179765]
     [0.31052141]
     [0.31052142]
     [0.33969658]
     [0.31284225]
     [0.32013604]
     [0.32269987]
     [0.32126178]
     [0.32139022]
     [0.32137893]
     [0.32137903]
     [0.32137903]
     Vc = [110.15498908]
[19]: f(110.16)
     0.32139364461915276
[19]: 8.53852726023785e-09
[23]: A = ac * Pc / (R * Tc)**2
[25]: B = bc * Pc / (R * Tc)
[21]: import CubicEquationSolver as ces
      #1*Z**3 + (-(1-B))*(Z**2) + (A-3*B**2-2*B) * Z - (A*B-B**2-B**3)
[38]: Z_roots = ces.solve(1, -0.94805090, 0.23069070, -0.01496327)
[34]: T = 284.00 \# K
      P = 4.6 \# MPa
[39]: Z_roots
[39]: array([0.61016106, 0.10555004, 0.2323398])
[50]: Vc_root1 = Z_roots[1] * R * T / P
      print(f' At Z = {Z_roots[1]}, the Volume is {Vc} cm^3/mol. This is the liquid_
       →phase (Plot on graph)')
      At Z = 0.10555004295080034, the Volume is 119.25981783492531 cm<sup>3</sup>/mol. This is
     the liquid phase (Plot on graph)
[51]: Vc_root0 = Z_roots[0]* R * T / P
      print(f' At Z = {Z_roots[0]}, the Volume is {Vc} cm^3/mol. This is the Vapour_\( \)
       →phase (Plot on graph)')
```

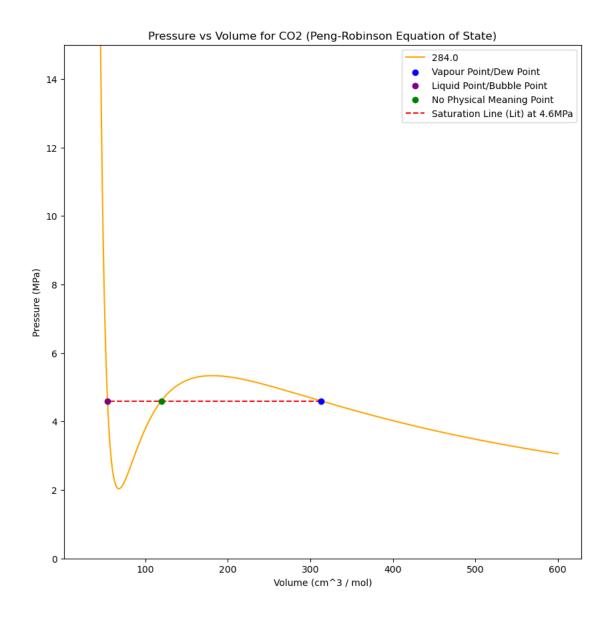
At Z = 0.6101610560175712, the Volume is 119.25981783492531 cm³/mol. This is the Vapour phase (Plot on graph)

```
[52]: Vc_root2 = Z_roots[2]* R * T / P
print(f' At Z = {Z_roots[2]}, the Volume is {Vc} cm^3/mol. There is no physical

→meaning (Plot on graph)')
```

At Z = 0.2323398010316285, the Volume is 119.25981783492531 cm³/mol. There is no physical meaning (Plot on graph)

```
[67]: # Plot preos_df, each column of the dataframe as a line on the same plot with
      index as x-axis and pressure as y-axis, reduce y axis from 0 to 15 €
     # Plot the critical point as a red dot
     plt.figure(figsize = (10, 10))
     for column in preos_df:
         if column == '284.0':
             plt.plot(preos_df.index, preos_df[column], label = column, color =_u
      plt.xlabel('Volume (cm^3 / mol)')
     plt.ylabel('Pressure (MPa)')
     plt.ylim(0, 15)
     plt.title('Pressure vs Volume for CO2 (Peng-Robinson Equation of State)')
     # plot the Z roots and Vc on the same plot and the dot on top of the 284 line
     plt.scatter(Vc_root0, P, color = 'blue', label = 'Vapour Point/Dew_
      →Point',zorder=3)
     plt.scatter(Vc_root1, P, color = 'purple', label = 'Liquid Point/Bubble Point', u
       ⇒zorder=3)
     plt.scatter(Vc_root2, P, color = 'green', label = 'No Physical Meaning Point', u
       ⇒zorder=3)
     #Draw a dotted line connecting the roots at P
     plt.plot([Vc_root0, Vc_root1], [P, P], color = 'red', linestyle = '--', label =
      plt.legend()
     plt.show()
```





3 Mixing Rules

Mixing Rules

$$a = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j a_{ij} \qquad b = \sum_{i=1}^{n} x_i b_i \qquad a_{ij} = (1 - k_{ij})(a_i a_j)^{1/2}$$

where a is the Peng-Robinson attraction parameter for the mixture; a_i is the Peng-Robinson attraction parameter for component i; b is the Peng-Robinson covolume for the mixture; b_i is the Peng-Robinson covolume for component i; and k_{ij} is the Peng-Robinson binary interaction parameter.

```
[68]: R = 8.314 \# J/(mol*K)
      T = 233.2 \# K
      P = 0.1 \# MPa
[69]: import pandas as pd
      import numpy as np
      import matplotlib.pyplot as plt
[79]: df_data = pd.read_csv("./Critical_Constants_and_Acentric_Factors.csv")
[80]: df data
[80]:
                                       Element
                                                 Tc(K) Pc(bar)
                                                                 Vc(cm3/mol)
      0
                                         Argon 150.80
                                                          48.70
                                                                         74.9 0.001
      1
                                       Bromine 588.00
                                                         103.00
                                                                        127.2 0.108
      2
                                      Chlorine 416.90
                                                          79.80
                                                                        123.8 0.090
      3
                                      Fluorine 144.30
                                                          52.20
                                                                         66.3 0.054
                                                           2.27
      4
                                      Helium-4
                                                  5.19
                                                                        57.4 -0.365
      71
                                    1-Propanol 536.80
                                                          51.70
                                                                        219.0 0.623
      72
                                                          56.30
                                                                        254.0 0.243
                                      Pyridine 620.00
      73
         Trichlorotrifluoroethane (Freon 113)
                                                487.30
                                                          34.10
                                                                        325.5 0.256
      74
             Trichlorofluoromethane (Freon 11) 471.20
                                                          44.10
                                                                        247.8 0.189
      75
                                Trimethylamine 433.30
                                                          40.90
                                                                        254.0 0.205
      [76 rows x 5 columns]
[72]: molecules = ['Methane', 'Ethane', 'Propane']
```

```
[73]: mol_fraction = [0.1, 0.3, 0.6]
[104]: # Let's create a table where the molecules are the columns and the rows are the
       →mol_fraction, critical pressure, critical temperature, and acentric factor
       df = pd.DataFrame(columns = molecules, index = ['Mol Fraction', 'Critical_
        →Pressure (MPa)', 'Critical Temperature (K)', 'Acentric Factor', 'bi', 'ki', □
        df.loc['Mol Fraction'] = mol fraction
       #Critical Pressure and temperature from df_data using the molecules from the
        ⇔column Element and pressure converting from bar to MPa
       df.loc['Critical Pressure (MPa)'] = df_data.loc[df_data['Element'].
        ⇔isin(molecules), 'Pc(bar)'].values * 0.1
       df.loc['Critical Temperature (K)'] = df_data.loc[df_data['Element'].
        ⇔isin(molecules), 'Tc(K)'].values
       df.loc['Acentric Factor'] = df_data.loc[df_data['Element'].isin(molecules),__

¬'w'].values
       # Find bi for each molecule - 0.0778*R*Tc/Pc
       df.loc['bi'] = 0.0778 * R * df.loc['Critical Temperature (K)'] / df.
        →loc['Critical Pressure (MPa)']
       #Find ki for each molecule - 0.37464 + 1.54226*w - 0.26992*w**2
       df.loc['ki'] = 0.37464 + 1.54226 * df.loc['Acentric Factor'] - 0.26992 * df.
        ⇔loc['Acentric Factor']**2
       #Find ai which involves alpha ai for each molecule - 0.45724*R**2*Tc**2/Pc* (1
       \hookrightarrow + ki(1 - (T/Tc)**0.5))**2
       df.loc['ai'] = 0.45724 * R**2 * df.loc['Critical Temperature (K)']**2 / df.
        →loc['Critical Pressure (MPa)'] * (1 + df.loc['ki'] * (1 - (T/df.
        →loc['Critical Temperature (K)'])**0.5))**2
```

[105]: df

[105]: Methane Ethane Propane Mol Fraction 0.1 0.3 0.6 Critical Pressure (MPa) 4.88 4.6 4.25 Critical Temperature (K) 305.4 190.4 369.8 Acentric Factor 0.099 0.011 0.153 bi 40.479844 26.773104 56.28175 ki 0.524678 0.391572 0.604287 686683.081716 228701.643792 1285770.861809 ai

3.1 Interactions Parameter

i, j	Methane	Ethane	Propane
Methane	0	0.00340	0.01070
Ethane	0.0340	0	0.00900

```
i, j Methane Ethane Propane
Propane 0.0170 0.00900 0
```

```
[106]: df interaction = pd.DataFrame(columns = molecules, index = molecules)
       df_interaction = df_interaction.fillna(0.0)
       df interaction = df interaction.astype(float)
       df_interaction.loc['Methane', 'Methane'] = 0.0
       df_interaction.loc['Methane', 'Ethane'] = 0.00340
       df_interaction.loc['Methane', 'Propane'] = 0.01070
       df_interaction.loc['Ethane', 'Methane'] = 0.00340
       df_interaction.loc['Ethane', 'Ethane'] = 0.0
       df_interaction.loc['Ethane', 'Propane'] = 0.00900
       df_interaction.loc['Propane', 'Methane'] = 0.01070
       df_interaction.loc['Propane', 'Ethane'] = 0.00900
       df_interaction.loc['Propane', 'Propane'] = 0.0
[107]: df_interaction
[107]:
                Methane Ethane Propane
       Methane
                 0.0000 0.0034
                                  0.0107
       Ethane
                 0.0034 0.0000
                                  0.0090
                 0.0107 0.0090
                                  0.0000
       Propane
[108]: | # Create table of aij using the interaction table and the ai table
       df_aij = pd.DataFrame(columns = molecules, index = molecules)
       # Formulae for aij - sqrt(ai*aj)*(1 - kij)
       for i in molecules:
           for j in molecules:
               df_{aij.loc[i, j]} = (df.loc['ai', i] * df.loc['ai', j])**0.5 * (1 - ___

¬df_interaction.loc[i, j])
       df_aij
[108]:
                      Methane
                                      Ethane
                                                     Propane
       Methane
                686683.081716 394942.344225
                                               929582.571402
       Ethane
                394942.344225
                               228701.643792
                                               537390.626968
       Propane 929582.571402 537390.626968 1285770.861809
[114]: # Multiple the aij table by the mol fraction table
       df_aij_mol_fraction = pd.DataFrame(columns = molecules, index = molecules)
       for i in molecules:
           for j in molecules:
               print (i, j)
               print (df_aij.loc[i, j])
               print (df.loc['Mol Fraction', i])
               print (df.loc['Mol Fraction', j])
```

```
df_aij_mol_fraction.loc[i, j] = df_aij.loc[i, j] * df.loc['Mol⊔

→Fraction', i] * df.loc['Mol Fraction', j]

df_aij_mol_fraction
```

```
Methane Methane
686683.0817162682
0.1
0.1
Methane Ethane
394942.34422541014
0.1
0.3
Methane Propane
929582.5714019976
0.1
0.6
Ethane Methane
394942.34422541014
0.3
0.1
Ethane Ethane
228701.64379200127
0.3
0.3
Ethane Propane
537390.6269677967
0.3
0.6
Propane Methane
929582.5714019976
0.6
0.1
Propane Ethane
537390.6269677967
0.6
0.3
Propane Propane
1285770.861809112
0.6
0.6
```

[114]: Methane Ethane Propane
Methane 6866.830817 11848.270327 55774.954284
Ethane 11848.270327 20583.147941 96730.312854
Propane 55774.954284 96730.312854 462877.510251

```
[117]: # Peng-Robinson Equation of State for a mixture of 3 components
       # Sum all values in the aij_mol_fraction table and it will equal a(T)
       a = df_aij_mol_fraction.sum().sum()
       print(a)
      819034.5639398942
[119]: # Sum all values in the bi_mol_fraction table and it will equal b(T)
       b = (df.loc['bi'] * df.loc['Mol Fraction']).sum()
       print(b)
      45.84896575534325
      3.1.1 Cubic Equation
      m(3) * Z^3 + m(2) * Z^2 + m(1) * Z + m(0) = 0
[121]: # Find A and B
       A = a * P / (R**2 * T**2)
       B = b * P / (R * T)
       print(A)
       print(B)
      0.021788387838798252
      0.0023647812713837404
[127]: # Find the Z_roots for the mixture
       m3 = 1
[122]: m2 = -1 - B
[123]: m1 = A - 3 * B**2 - 2 * B
[125]: mO = -A * B + B**2 + B**3
[128]: print( m3, m2, m1, m0)
      1 -1.0023647812713838 0.017042048724646312 -4.591935672607873e-05
[129]: Z_roots_mix = ces.solve(m3, m2, m1, m0)
```

[130]: Z_roots_mix

[130]: array([0.9851125 , 0.00335386, 0.01389842])

3.1.2 Finding the fugacity coefficient for all components

```
[139]: # Create table of xi (mole fraction) and df_aij where mol fraction * aij
       df_xi = pd.DataFrame(columns = molecules, index = molecules)
       for i in molecules:
           for j in molecules:
               print (i, j)
               print (df.loc['Mol Fraction', i])
               print (df_aij.loc[i, j])
               df_xi.loc[i, j] = df.loc['Mol Fraction', i] * df_aij.loc[i, j]
       df_xi
      Methane Methane
      0.1
      686683.0817162682
      Methane Ethane
      394942.34422541014
      Methane Propane
      0.1
      929582.5714019976
      Ethane Methane
      0.3
      394942.34422541014
      Ethane Ethane
      228701.64379200127
      Ethane Propane
      0.3
      537390.6269677967
      Propane Methane
      929582.5714019976
      Propane Ethane
      0.6
      537390.6269677967
      Propane Propane
      0.6
      1285770.861809112
[139]:
                      Methane
                                      Ethane
                                                    Propane
                                39494.234423
      Methane
                 68668.308172
                                                92958.25714
       Ethane
                                68610.493138
                118482.703268
                                              161217.18809
      Propane 557749.542841 322434.376181 771462.517085
[171]: # Sum each column in the df_xi table
       df_xi.sum(axis = 0)
       df_sum_xi = pd.DataFrame(columns = molecules, index = ['Sum_xi'])
```

```
df_sum_xi.loc['Sum_xi'] = df_xi.sum(axis = 0)
df_sum_xi
```

[171]: Methane Ethane Propane Sum_xi 744900.55428 430539.103741 1025637.962316

```
[178]: | # Find the fugacity coefficients for each component at Z root mix 0 and 2
       # Column is Z, methan_fug, ethane_fug, propane_fug
       # Index is liq_phase, no_physical_meaning, vap_phase
       df_fug = pd.DataFrame(columns = [str('Z')] + molecules, index = ['liq_phase', __

¬'no_physical_meaning', 'vap_phase'])
       # Column Z is the Z roots mix
       df_fug.loc['liq_phase', 'Z'] = Z_roots_mix[1]
       df fug.loc['no physical meaning', 'Z'] = Z roots mix[2]
       df_fug.loc['vap_phase', 'Z'] = Z_roots_mix[0]
       # Fugacity coefficients for each component at Z root mix 0, using df_sum_xi and
        \hookrightarrow df_aij
       # Formulae for fugacity coefficients - exp(((bi/b)*(Z-1)) - (log(Z-B)) - (A/B))
        \Rightarrow (2*sqrt(2)*B))*(2*(df_sum_xi.loc['Sum_xi', i])*df_aij.loc[i, i])/a)*LN((Z+u)
        \hookrightarrow 2.414*B)/(Z-0.414*B))
       for i in molecules:
           # Let's break it up for liq_phase
           # First part of the equation
           first_part = (df.loc['bi', i] / b) * (1-df_fug.loc['liq_phase', 'Z'])
           print(first_part)
           # Second part of the equation
           second_part = np.log(df_fug.loc['liq_phase', 'Z'] - B)
           # Third part of the equation
           third part 1 = A / (2 * 2**0.5 * B)
           third_part_2 = ((2 * df_sum_xi.loc['Sum_xi', i]/a)- (df.loc['bi', i]/b))
           third_part_3 = np.log((df_fug.loc['liq_phase', 'Z'] + 2.414 * B) / (df_fug.
        →loc['liq_phase', 'Z'] - 0.414 * B))
           third_part = third_part_1 * third_part_2 * third_part_3
           # Fugacity coefficient for liq_phase
           df_fug.loc['liq_phase', i] = np.exp(first_part - second_part - third_part)
           # For vap phase
           # First part of the equation
           first_part = (df.loc['bi', i] / b) * (1-df_fug.loc['vap_phase', 'Z'])
           print(first_part)
           # Second part of the equation
           second_part = np.log(df_fug.loc['vap_phase', 'Z'] - B)
```

```
# Third part of the equation
           third_part_1 = A / (2 * 2**0.5 * B)
           third_part_2 = ((2 * df_sum_xi.loc['Sum_xi', i]/a)- (df.loc['bi', i]/b))
           third_part_3 = np.log((df_fug.loc['vap_phase', 'Z'] + 2.414 * B) / (df_fug.
        →loc['vap_phase', 'Z'] - 0.414 * B))
           third_part = third_part_1 * third_part_2 * third_part_3
           # Fugacity coefficient for vap_phase
           df_fug.loc['vap_phase', i] = np.exp(first_part - second_part - third_part)
      0.8799343570732929
      0.013144103885122388
      0.5819828362969195
      0.008693424456215113
      1.2234297617004004
      0.01827509944881167
[179]: df_fug
[179]:
                                   Ζ
                                        Methane
                                                     Ethane
                                                               Propane
       liq_phase
                            0.003354 41.059377 235.50114 13.084299
       no_physical_meaning 0.013898
                                                                   NaN
                                            NaN
                                                       {\tt NaN}
       vap_phase
                            0.985113
                                       1.009945
                                                    1.01591
                                                              1.007535
[177]: import preos
  []:
```