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2026/2/5 DS 3001 Assignment 2

Q6 1. $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N} a+bX_i$

$\frac{1}{N}\sum_{i=1}^{N} a = a \qquad \frac{1}{N}\sum_{i=1}^{N} bX_i = b\cdot\frac{1}{N}\sum_{i=1}^{N} X_i$

$\therefore m(a+bX) = a+b\left(\frac{1}{N}\sum_{i=1}^{N} X_i\right) = a+b\times m(X)$

2. $cov(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(x_i - m(x))$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$

$= s^2$

3. $cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(a+bY_i - m(a+bY))$

we know from 1): $m(a+bY) = a+b\times m(Y)$

$\therefore (a+bY_i - m(a+bY)) = (a+bY_i - (a+b\cdot m(Y))$

$= (a+bY_i - a - bm(Y)$

$= (bY_i - bm(Y)$

$= b(Y_i - m(Y)$

$\therefore cov(X, a+bY) = b\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(Y_i - m(Y)\right.$

$= b\times cov(X,Y)$

4. $cov(a+bX, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(a+bX_i - m(a+bX))(a+bY_i - m(a+bY))$

From 3: $(a+bY_i - m(a+bY_i)) = b(Y_i - m(Y)$

$\therefore cov(a+bX, a+bY) = \frac{1}{N}\sum_{i=1}^{N} b(x_i - m(x))\,b(Y_i - m(Y))$

$= b^2 cov(X,Y)$

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2-26/215 DS 3001 Assignment 2 Q6 (cont.)

Q6 5. Does med(a+bX) = a+b med(X)?

* When $b > 0$, ordering of X unchanged on either side.
* So when we take med(a+bX) compared to med(X), this median point is just shifted by a & scaled by b, i.e. a+b med(X)
* Conclusion: (Yes)

vIQR of

Does a+b(X) equal to a+bxIQR(X)?

* (No) IQRs do not scale linearly like this.
* When we take the IQR of a+b(X), we are taking the IQR of a set, X, that is shifted by a & scaled by X.
* This is Not the same IQR as when we take set X's IQR and scale it by b and shift it by a.

E.g. X's 1Q is 25, 3Q is 75, IQR: 50.
Assume a = 10 & b = 2.

IQR of a+b(X): 1Q = 60, 3Q = 160
IQR = 100

a+b·IQR(X): IQR(X) = 25
a+b·IQR(X) = 60
60 != 100
∴, IQR(a+b(X)) != a+bx(QR(X)

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2026/2/15 DS 3001 Assignment 2 Q6 (Cont.)

Q6 6.

Example x^2

x	1	2	3	4	5	6	7	8	9
x^2	1	4	9	16	25	36	49	64	81

Mean of terms 1-5 of x^2: $\frac{55}{5} = 11$

$(m(x))^2$: $m(x)$ for 1-5 of $x = 3$

$(m(x))^2 = 3^2 = 9$

$11 \neq 9$

Mean $\frac{1}{9}\sum_{i=1}^{9} x^2 = \frac{285}{9} = 31.667$

$(m(x))^2$: $\frac{1}{9}\sum_{i=1}^{9} x = 5 = m(x)$

$(m(x))^2 = 5^2 = 25$

$31.667 \neq 25$

$\therefore m(x^2) \neq (m(x))^2$

Example

x	1	2	3	4	5	6	7	8	9
$\sqrt{x}$	1	$\sqrt{2}$	$\sqrt{3}$	2	$\sqrt{5}$	$\sqrt{6}$	$\sqrt{7}$	$2\sqrt{2}$	$\sqrt{3}$

Mean: $\frac{1}{9}\sum_{i=1}^{9} \sqrt{x} \approx \frac{18.27}{9} \approx 2.03$

$\sqrt{m(x)}$: $m(x) = \frac{1}{9}\sum_{i=1}^{9} x = 5$

$\sqrt{m(x)} = \sqrt{5} \approx 2.23$

$2.03 \neq 2.23$

$\therefore m(\sqrt{x}) \neq \sqrt{m(x)}$