

Parallelizing an All-Pairs Shortest Path Algorithm

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Introduction

In this report we describe the implementation and tuning of a parallel all-pairs shortest path algorithm.

Profiling the given code

VTune Amplifier

Given a working implementation of parallelized Floyd-Warshall algorithm, we can further increase the performance by profiling the computation. This will identify areas in the code that run much slower than the rest, giving opportunities to implement speedups. We used the VTune Amplifier to profile our code, calling our executable through the profiler with

```
amplxe-cl -collect hotspots ./path.x
```

Function	Module	CPU Time
square	path.x	42.751s
__kmp_barrier	libiomp5.so	9.623s
__kmpc_reduce_nowait	libiomp5.so	3.498s
__kmp_fork_barrier	libiomp5.so	2.738s
genrand	path.x	0.030s
fletcher16	path.x	0.020s
__intel_ssse3_rep_memcpy	path.x	0.020s
__kmp_join_call	libiomp5.so	0.010s
__kmp_launch_thread	libiomp5.so	0.010s
__kmp_fork_call	libiomp5.so	0.010s

The hotspots analysis results tell us that the most expensive compute-time occurs in *square()*. This is expected because *square()* is the only method with nested loops carrying out the algorithm (*write_matrix()* only prints values before and after our algorithm has run).

There are two other methods in our top CPU time list from path.c however any effort put towards optimizing those would not be well spent because they are orders of magnitude faster than *square()*.

We can dig deeper into *square()*'s performance by running the command

```
amplxe-cl -report hotspots -source-object function="square"
```

Source Line	Source	CPU Time
47	for (int j = 0; j < n; ++j) {	
48	for (int i = 0; i < n; ++i) {	0.020s
49	int lij = lnew[j*n+i];	0.020s
50	for (int k = 0; k < n; ++k) {	8.699s
51	int lik = l[k*n+i];	25.457s
52	int lkj = l[j*n+k];	
53	if (lik + lkj < lij) {	3.759s
54	lij = lik+lkj;	
55	done = 0;	4.796s

This tells us the CPU time spent at many points within the method. We can see the majority of the time is spent retrieving a value from the array in the innermost loop. Observe that the matrix accesses are not optimized because we fetch non-contiguous memory each iteration and it is taking a long time. We can re-order the loops to better access our desired values and improve the runtime.

Vectorization Report

If we analyze the given code's vectorization report, we can try to adjust some code to take advantage of the compiler's vectorization capabilities. Compiling the code with flags

```
-qopt-report=5 -qopt-report-phase=vec
```

will produce a report summarizing the compiler's ability to vectorize our code. In summary, the given code `path.c` is reported to be mostly vectorized. Most estimated potential speedup values were greater than 5, yielding a large factor of speedup. Some loops were not vectorizable because they contained print statements. Another non-vectorizable loop was in `fletcher16()`, but our profiling results show this method to be insignificant in runtime comparison to `square()` so we will focus our efforts towards `square()`.

OpenMP Scaling Study

We did both strong scaling and weak scaling study using the provided OpenMP code.

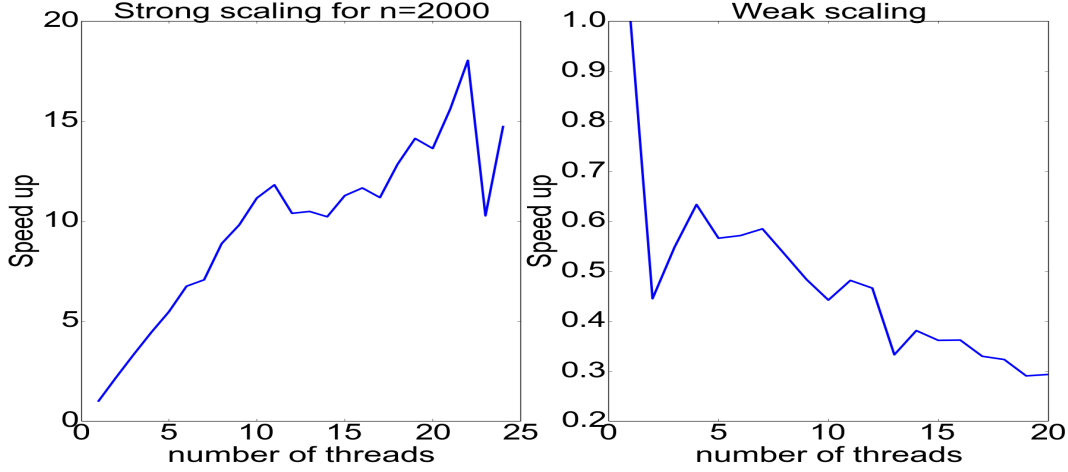
Strong Scaling

In the strong scaling, we compare parallel to serial time for a fixed size problem. Since the graph is randomly generated, for each experiment, we did 10 simulation and use the average time as their running time. Here we choose the problem size $n = 2000$ and the speedup is summarized in left plot of Figure 1. As can be seen from the left plot of Figure 1, when we increase the number of threads from 1 to 12, the speed up grows linearly with respect to the number of threads. After 12 threads, the speedup doesn't grow linearly due to the excess amount of overheading and synchronization.

Weak Scaling

In weak scaling, you need to increase both the number of processors and the amount of jobs such that each processor has the same amount of jobs. In this problem, the time complexity is $O(n^3 \log n)$

Figure 1: scaling study



and we let each processor has approximately $O(2000^3 \log(2000))$ amount of work. The results are summarized in the right figure of Figure 1. As can be seen from the right figure of Figure 1, the speedup is actually decreasing when you increase the amount of work and the number of processors. This could due to the overhauling and synchronization when the number of processors is large.

MPI Implementation

We implemented a parallel version of the shortest path algorithm using the Message Passing Interface (MPI). In our implementation, each processor is responsible for updating a portion of the shortest path matrix l . Specifically, we partition by column, so that the processor labelled k (for $k = 1, \dots, p$), updates l_{ij} for $j \in J_k$, where the $(J_k)_{1 \leq k \leq p}$ define a partition of $\{1, 2, \dots, n\}$. With this strategy the computation pattern for updating the shortest path matrix remains the same, we are simply looping over a smaller sets of columns.

After each processor has finished updating it's corresponding columns in the update $l_{ij}^{s+1} = \min_k \{l_{ik}^s + l_{jk}^s\}$, we call *MPI_Allgather* so that each processor gets the fully updated l^{s+1} for the next step. Each processor also keeps a local "done" flag that will be true if nothing was updated. In order to determine whether to terminate or not, we call *MPI_Allreduce* with a logical AND operator on the local done flags.

Predicting Speedup

We considered a simple alpha-beta model to predict the speedup of our MPI implementation. This models the communication time as

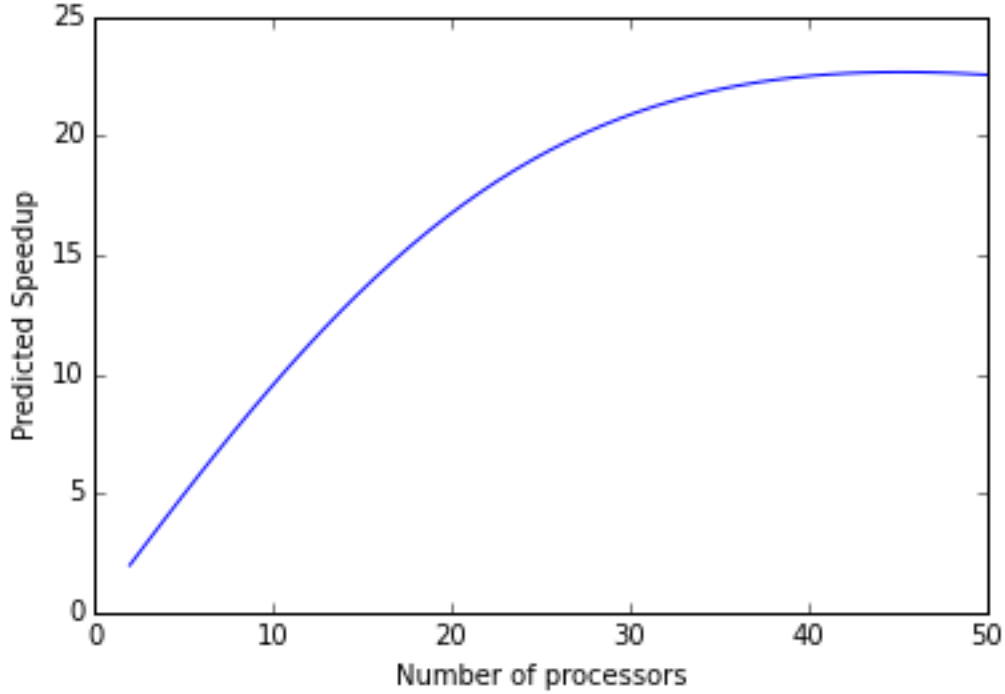
$$t_{comm} = \alpha + \beta * M$$

where α is latency, β is inverse bandwidth, and M is the message size. On the totient cluster on a single node, the time per kilobyte to communicate was previously estimated ¹ to be

$$0.28 + 0.097 \times kB \quad \mu s$$

¹<http://cornell-cs5220-f15.github.io/slides/2015-10-08-model.html>

Figure 2: Predicted MPI Speedup



Thus the total time for a single iteration in seconds can be modeled as

$$\begin{aligned}
 t_{total} &= t_{serial} + t_{comm} \\
 &= O\left(\left(\frac{n}{p}\right)^3\right) + p^2 \frac{0.28 + 0.097\left(\frac{4n^2}{1000p}\right)}{10^6}
 \end{aligned}$$

We estimated the correct value of t_{serial} by comparing against serial timing experiments, and we estimate the number of iterations to be $\log n$. A plot of estimated speedup for $n = 2016$ can be seen in Figure 2.

As seen in the plot, we should expect to get roughly linear speedup up until around 40 processors, after which communication costs would start to overtake the gains from paralellization.

Unfortunately, as can be seen in the following section, the actual results do not match the predicted. We started seeing the speedup dropping off when the number of processors was much lower

Scaling Study

We performed both strong and weak scaling studies for the MPI version of the algorithm. The results for $n = 2016$, averaged over 10 independent trials, can be seen in the Figures below. Compared with OpenMP, the MPI version performs relatively poorly in strong scaling experiments, but much better in weak scaling experiments for modest number of processors (4-8).

Figure 3: MPI Strong Scaling

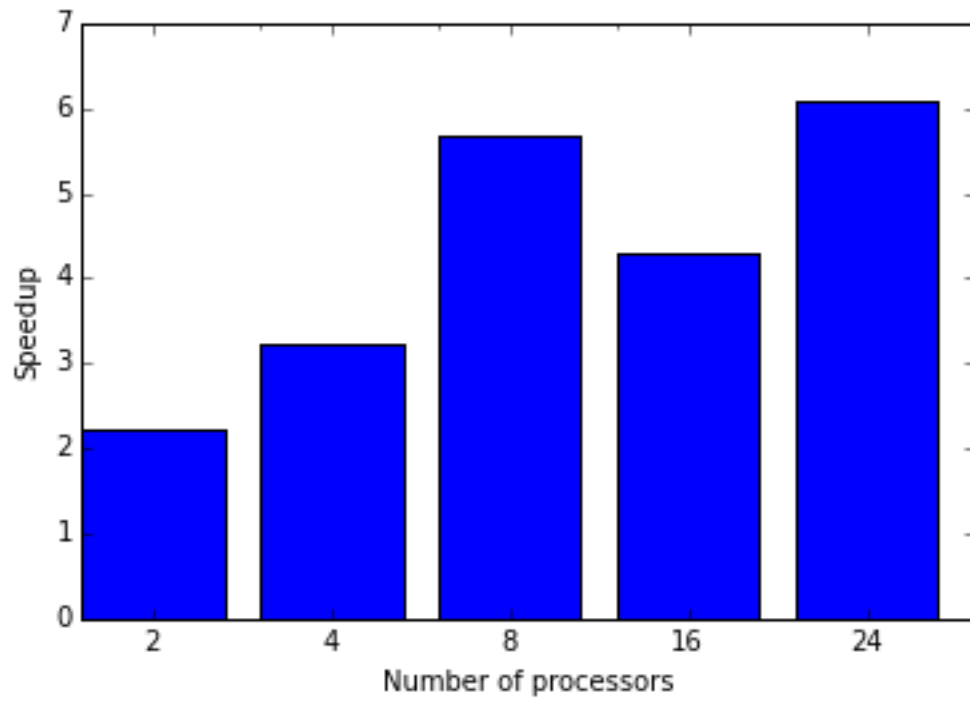


Figure 4: MPI Weak Scaling

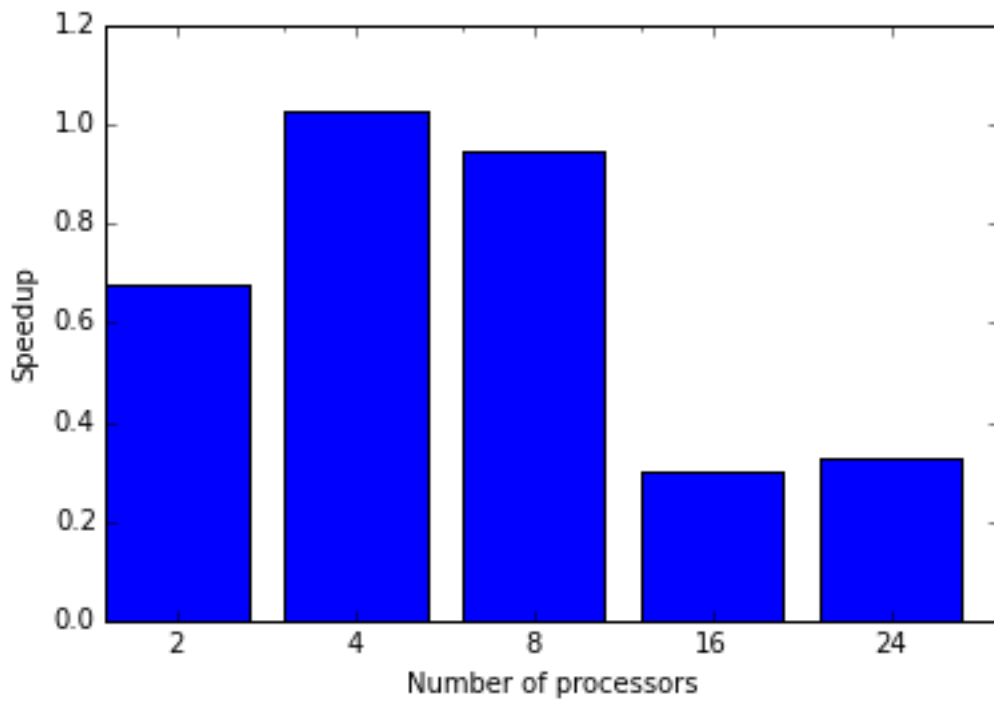
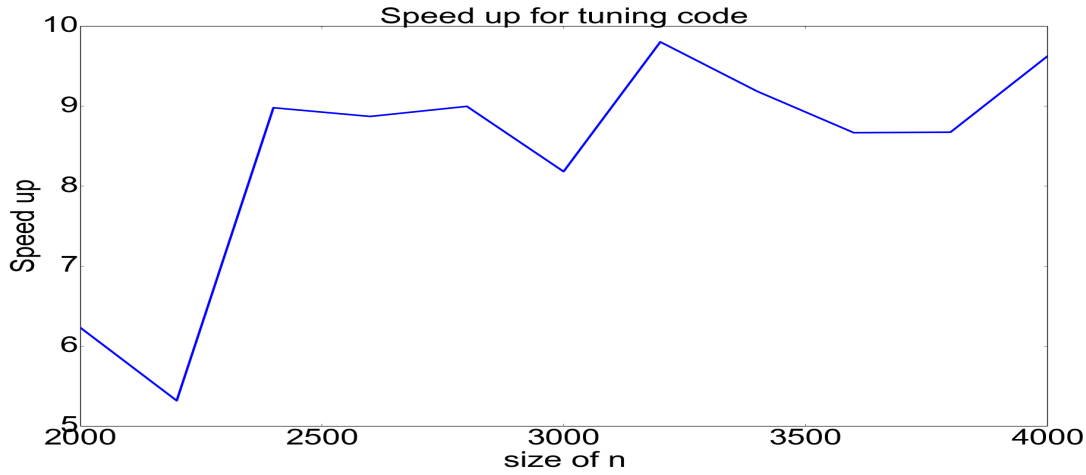


Figure 5: speed up for the tuned code



Tuning

We chose to focus on tuning the OpenMP version of the code.

Since this problem is closely related to the matrix multiplication, we used the same tricks as we did in tuning matrix multiplication, which are copy optimization and blocking. Using copy optimization, we store the transpose of the shortest path matrix to increase the possibility of cache hits. Implementing it will give us 3x to 4x speed up. Similarly, by using blocking, we can better parallelize independent jobs and increase the possibility of cache hits. Combining these two methods together, we achieve the speedup seen in Figure 5. As you can see, the speedup can be up to 10x. We found the optimal block size to be about 120.

The benchmark is the original code using OpenMP with 24 threads.

Figure 6: Running VTune Amplifier on optimized code shows improved memory access

Source Line	Source	CPU Time
50	int i, j, k;	
51	int done = 1;	1.881s
52	for (i = 0; i < M; ++i) {	
53	for (j = 0; j < N; ++j) {	0.060s
54	int cij = C[j*lda+i];	0.270s
55	for (k = 0; k < K; ++k) {	2.202s
56	int Aki = A[i * lda + k];	0.160s
57	int Bjk = B[j * lda + k];	0.030s
58	if (Aki + Bjk < cij) {	1.299s
59	cij = Aki + Bjk;	
60	done = 0;	0.090s
61	}	
62	}	
63	C[j*lda+i] = cij;	0.020s