An Empirical Estimation of the IS-LM Model in the Post-War U.S.

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Abstract

This paper investigates how well the empirical estimation captures some of the comparative statics in the basic IS-LM model. While the IS-LM model interprets the relationship between interest rate and output reasonably well, it is crucial to see how far the theoretical model is from the real-world revelations. A VAR model and several impulse response functions display the comparative statics based on the US time series data. Our result shows the empirical estimations primarily justify the theory-based comparative statics in the presence of variations.

Keywords: IS-LM Model; VAR; Impulse Response Function; GDP; Interest Rate

1. Introduction

The IS-LM model has a crucial foundation in modern macroeconomic theory and model. The primary, theoretical version of the IS-LM model is part of many introductory textbooks (e.g., Dornbusch and Fischer, 1987; Hall and Taylor, 1988; Farmer, 2002; Mankiw, 2019). Many governments and commercial firms use the IS-LM curve for policymaking and economic forecasting.

The IS-LM model is highly relevant to the fundamentals of various areas of economics. Cottrell and Darity (1991) study the IS-LM model under increasing returns within industrial economics and international trade. Bordo and Schwartz (2003) discuss monetarist objections to the IS-LM model. Findlay (2010) explores the connection between slopes and the effectiveness of the fiscal and monetary policy. There are also big chunks of literature studying the fundamental modeling, different texts' articulations, and possible revision routes for the IS-LM model, for practical and pedagogical purposes (Colander, 2003; King, 2000; King, 1993).

This paper investigates some of the basic comparative statics of the IS-LM model. It uses the US post-war time-series data to verify how well those theoretical comparative statistics capture real-world scenarios.

2. Literature Review

2.1 IS-LM Model and empirical studies

Based on the explanations and illustrations of several textbooks (Farmer, 2002; Wade, 2003; Mankiw, 2019), the IS-LM model is a two-dimensional macroeconomic tool that shows the relationship between interest rate and output. The IS curve is defined to be the set of all Y and r combinations that clear the goods market. The LM curve is defined as the set of all Y and r combinations that clear the aggregate money market.

For the IS curve, the Keynesian cross is the main path to the IS-LM model. The principle between interest rate and planned investment is that interest rate is the cost of borrowing to finance an acquisition. Therefore, an increase in the interest rate reduces planned investment. Thus, the investment-interest rate curve slopes downward. On the other hand, the reduced investment leads to reduced planned expenditure, leading to a fall in the output. Therefore, the IS curve is downward-sloping and convex to the origin.

IS: $$Y=C(Y-T)+I(r)+G \quad (1)$$

For the LM curve, the money supply is theorized as an exogenous variable. What determines the money demand is the interest rate and output. Increased interest rate leads to less money demand because people want to hold less of their wealth. In comparison, increased output leads to more money demand because higher expenditure encourages people to spend more. LM curve, therefore, equalizes money demand and money supply.

LM: $$M/P = L(r,Y) \quad (2)$$

The intersection of the IS and LM curves models general equilibrium where supposed simultaneous equilibria occur in goods and the assets markets. In the short run, the IS-LM model explains changes in national income when the price level is fixed short run. The model also shows why an aggregate demand curve can shift.

2.2 Derived Hypotheses

Based on the basic IS-LM curve, I propose the following five hypotheses. H1 focuses on the investment-interest-rate interaction. H2, H3, H4 focus on the components of the IS equation. H5 focuses on the combined IS-LM model.

H1: Higher interest rate leads to reduced investment.
H2: Higher consumption leads to higher output.
H3: Higher investment leads to higher output.
H4: Higher government spending leads to higher output.
H5: Higher money supply leads to higher output.

3. Data

3.1 Data Collection

The data is gathered from multiple datasets available in the Gretl database server. *Gdpc1*, *pcec96*, *gpdic1*, *gcec1*, *aaa*, *mzmsl*, and *gdppidxq* are imported from fedstl, the FRED economic database for economic research of the Federal Reserve Bank of St. Louis. *Fedfund* is imported from fedbog, the Federal Reserve Board database. Both fedstl and fedbog are the official database for U.S. macroeconomic data and indices frequently used by econometricians.

Gdpc1 refers to the Real Gross Domestic Product, in the unit of billions of chained 2012 dollars, collected quarterly, a proxy for the income level/output in the seasonally adjusted annual rate. *Pcec96* refers to the Real Personal Consumption Expenditures, in the unit of billions of chained 2012 dollars, collected quarterly, in the seasonally adjusted annual rate, a proxy for the consumption. *Gpdic1* refers to the Real Gross Private Domestic Investment, in the unit of billions of chained 2012 dollars, collected quarterly, a proxy for the investment in the seasonally adjusted annual rate. *Gcec1* refers to the Real Government Consumption Expenditures and Gross Investment, in the unit of billions of chained 2012 dollars, collected quarterly, in the seasonally adjusted annual rate, a proxy for the government spending. *AAA* refers to Moody's seasoned Aaa Corporate Bond Yield, in the unit of percentage, collected monthly, not seasonally adjusted, a proxy for the interest rate. *Mzmsl* refers to the Money-Zero-Maturity Money Stock, seasonally adjusted, collected monthly, a proxy for the nominal money supply in the unit of billions of dollars. *Gdppidxq* refers to the Gross Domestic Product: Chain-type Price Index (discontinued), in the unit of 2012=100 billions of dollars, seasonally adjusted, collected quarterly, a proxy for the price level. *Fedfund* refers to the Federal Funds Rate (effective), in the unit of percentage, collected monthly, a proxy for the interest rate. Table 1. below shows the descriptive information of the eight variables in their original format.

Table 1. *Raw Variable Description*

	Real Gross Domestic Product	Real Personal Consumption Expenditures	Real Gross Private Domestic Investment	Real Government Consumption Expenditures and Gross Investment	Moody's Seasoned Aaa Corporate Bond Yield	Money Zero Maturity Money Stock	Gross Domestic Product: Chain-type Price Index (discontinued)	Federal Funds Rate (effective)
Raw variable name	*gdpc1*	*pcecc96*	*gpdic1*	*gcec1*	*Aaa*	*mzmsl*	*gdppidxq*	*fedfund*
Unit	Billions of chained 2012 dollars	Billions of chained 2012 dollars	Billions of chained 2012 dollars	Billions of chained 2012 dollars	Percentage	Billions of dollars	2012-100 billion of dollars	Percentage
Time-series range	1947:1 - 2020:2	1947:1 - 2020:2	1947:1 - 2020:2	1947:1 - 2020:2	1919:1 - 2020:6	1959:1-2020:6	1947:1-2017:2	1954:7-2020:10
Frequency	quarterly	quarterly	quarterly	quarterly	monthly	monthly	quarterly	monthly
Proxy	Income level or output	Consumption	Investment	Government spending	Interest rate	Nominal money supply	Price level	Interest rate
Database	St. Louis Fed	St. Louis Fed	St. Louis Fed	St. Louis Fed	St. Louis Fed	St. Louis Fed	St. Louis Fed	Federal Reserve Board

The four expenditure-type variables—real GDP, real Personal Consumption Expenditures, real Gross Private Domestic Investment, real Government Consumption Expenditures, and Gross Investment—are reasonably good representations of GDP, consumption, investment, and government spending, respectively, based on the theoretical IS-LM model in textbooks, although Some empirical studies of IS-LM model use GNP as the proxy for income or output (Gali, 1992). They are all in real terms and take into account the same year's chained dollars. Using both *aaa* and *fedfund* is because the federal funds rate is the interest rate controlled by the Fed, while the longer-term Aaa Corporate Bond Yield is of more relevance to firms' investment decisions. Using Money Zero Maturity Money Stock as the nominal money supply is a measure of liquid money, readily available cash from bills and banknotes, checking, savings, and money market accounts while excluding CDs or time deposits. It captures the idea of money supply better than the somewhat narrow M1 and arguably better than M2, in which time deposits are included yet money market funds are excluded. Using *gdppidxq* is because it considers product substitutions made by consumers and other changes in their spending habits, which is therefore considered a more accurate inflation gauge than the traditional fixed-weighted Consumer Price Index.

For consistent frequency of our variables, *mzmsl* is compacted from monthly to quarterly by summing the values. In contrast, *aaa* and *fedfund* are packed from monthly to quarterly by using end-of-period values. Also, I created a new variable series *M*, referring to Real Money Supply, by computing *mzmsl/gdppidxq*, essentially dividing nominal money supply by the price level. After that, limit the time-series sample period to be 1960 quarter 1 to 2015 quarter 1 to restrict the overall variances and avoid years like 2020 where the coronavirus drove macroeconomic indices wild.

3.2 Data Description

In this subsection, I will provide brief descriptions of seven variables that will all be included as endogenous variables in the VAR model later. They are real GDP, real consumption, real investment, real government spending, Moody's Aaa corporate bond yield, effective federal funds rate, and real money supply. Meanwhile, I will also observe the trend in each time-series plot and roughly determine whether the process is non-stationary or not, and if so, what kind of non-stationary process each plot may fall under. I will verify them in more detail through the ADF-test later.

Gdpc1, the real Gross Domestic Product, has 221 valid observations from 1960 quarter 1 to 2015 quarter 1 with no missing values. The mean is 9292.5 billion dollars, the minimum is 3232.0 billion dollars, the maximum is 17306.0 billion dollars, and the standard deviation is 4256.2 billion dollars. The GDP across the 55 years is following a roughly increasing trend, with a few slight downturns in 1975, 1980, 1982, 1990, and 2008, which corresponds to the 1973-75 recession, 1980 recession, 1981-82 recession, early 1990s recession, and the Great Recession, respectively.

The GDP plot is presumably non-stationary because it seems not to oscillate around a constant mean, while the roughly steady increasing trend may tell us it is a random walk, with or without drift, and with a deterministic trend ($Y_t = \alpha + Y_{t-1} + \beta t + \varepsilon_t$ or $Y_t = Y_{t-1} + \beta t + \varepsilon_t$). The summary statistics table and time-series plot for GDP are as follows.

Table 2. *Summary Statistics, using the observations 1960:1 - 2015:1 for the variable gdpc1 (221 valid observations)*

Mean	Median	Minimum	Maximum
9292.5	8533.6	3232.0	17306.0
Std. Dev.	C.V.	Skewness	Ex. kurtosis
4256.2	0.45802	0.33564	-1.2425
5% Perc.	95% Perc.	IQ range	Missing obs.
3576.2	16219.	7643.2	0

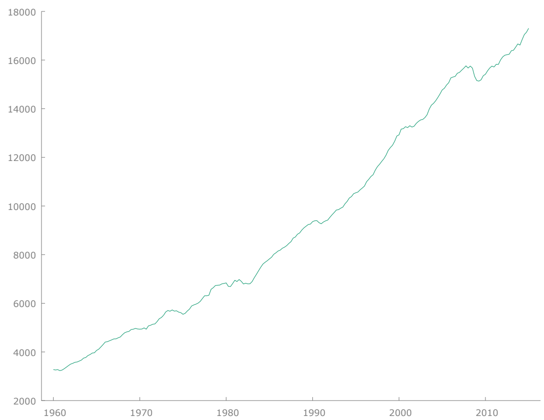


Figure 1. *Time-series plot for gdpc1*

Pcecc96, the real Personal Consumption Expenditures, has 221 valid observations from 1960 quarter 1 to 2015 quarter 1 with no missing values. The mean is 6084.4 billion dollars, the minimum is 1994.9 billion dollars, the maximum is 11798.0 billion dollars, and the standard deviation is 2990.2 billion dollars. The time-series curve of the consumption across the 55 years has a similar increasing trend like that of GDP, with a few downturns in 1975, 1980, 1982, 1990, and 2008, which corresponds to the 1973-75 recession, 1980 recession, 1981-82 recession, early 1990s recession, and the Great Recession, respectively. Like GDP, the consumption plot is presumably non-stationary because it seems not to oscillate around a constant mean, while the roughly steady increasing trend may tell us it is a random walk, with or without drift, and with the deterministic trend ($Y_t = \alpha + Y_{t-1} + \beta t + \varepsilon_t$ or $Y_t = Y_{t-1} + \beta t + \varepsilon_t$). The summary statistics table and time-series plot for consumption are as follows.

Table 3. *Summary Statistics, using the observations 1960:1 - 2015:1 for the variable pcecc96 (221 valid observations)*

Mean	Median	Minimum	Maximum
6084.4	5543.7	1994.9	11798.0
Std. Dev.	C.V.	Skewness	Ex. kurtosis
2990.2	0.49146	0.39520	-1.2013
5% Perc.	95% Perc.	IQ range	Missing obs.
2163.6	11006.	5288.8	0

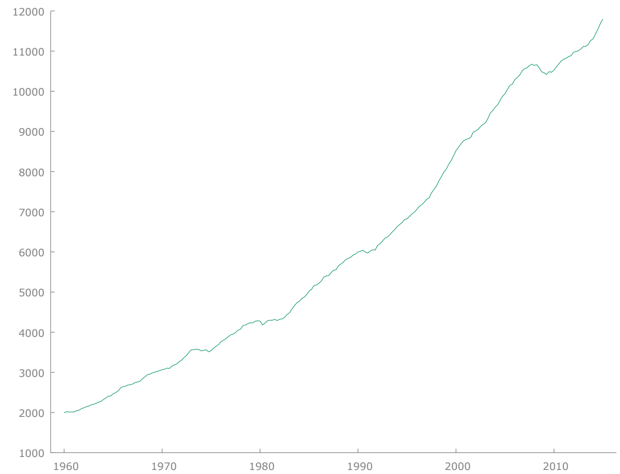


Figure 2. *Time-series plot for pcecc96*

Gpdic1, the real Gross Private Domestic Investment, has 221 valid observations from 1960 quarter 1 to 2015 quarter 1 with no missing values. The mean is 1381.8 billion dollars, minimum is 313.84 billion dollars, maximum is 3127.3 billion dollars, and standard deviation is 795.69 billion dollars. The time-series curve of the investment across the 55 years is fluctuating around a roughly increasing trend, with a few major downturns in 1975, 1980, 1982, 1990, 2001, and 2008, which may corresponds to the market downturns and low investment incentives in 1973-75 recession, 1980 recession, 1981-82 recession, early 1990s recession, early 2000s recession, and the Great Recession, respectively. Unlike GDP or consumption, the investment plot looks more unpredictable because of the major rises and falls, and therefore it may be the type of non-stationary process of random walk with drift and deterministic trend ($Yt = \alpha + Yt\text{-}1 + \beta t + \varepsilon t$). The summary statistics table and time-series plot for investment are as follows.

Table 4. *Summary Statistics, using the observations 1960:1 - 2015:1 for the variable gpdic1 (221 valid observations)*

Mean	Median	Minimum	Maximum
1381.8	1158.1	313.84	3127.3
Std. Dev.	C.V.	Skewness	Ex. kurtosis
795.69	0.57585	0.51590	-1.1095
5% Perc.	95% Perc.	IQ range	Missing obs.
398.19	2752.3	1491.3	0

Figure 3. *Time-series plot for gpdic1*

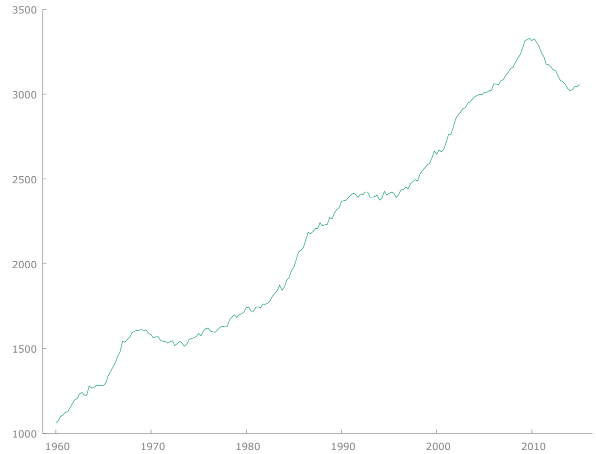
Gcec1, the real Government Consumption Expenditures and Gross Investment, has 221 valid observations from 1960 quarter 1 to 2015 quarter 1 with no missing values. The mean is 2177.6 billion dollars, minimum is 1062.4 billion dollars, maximum is 3328.1 billion dollars, and standard deviation is 662.58 billion dollars. The time-series curve of the government spending across the 55 years is follows roughly an increasing trend, with a few uprushes in, for example, 1968 and 2009, which may correspond to the arm race expenditure and relief programs during the Great Recession. Like investment, the government spending plot looks more spurious following perhaps a very loosely increasing trend, and therefore it may as well be the type of non-stationary process of random walk with drift and deterministic trend ($Yt = \alpha + Yt\text{-}1 + \beta t + \varepsilon t$). The summary statistics table and time-series plot for government spending are as follows.

Table 5. *Summary Statistics, using the observations 1960:1 - 2015:1*

for the variable gcec1 (221 valid observations)

Mean	Median	Minimum	Maximum
2177.6	2209.0	1062.4	3328.1
Std. Dev.	C.V.	Skewness	Ex. kurtosis
662.58	0.30428	0.16451	-1.2789
5% Perc.	95% Perc.	IQ range	Missing obs.
1225.2	3220.2	1171.1	0

Figure 4. *Time-series plot for gcec1*

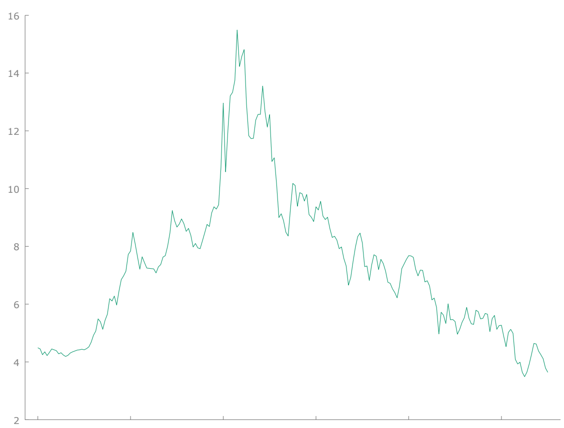


Aaa, the Moody's Seasoned Aaa Corporate Bond Yield, has 221 valid observations from 1960 quarter 1 to 2015 quarter 1 with no missing values. The mean is 7.3307 percent, minimum is 3.4900 percent, maximum is 15.490 percent, and standard deviation is 2.5500 percent. The time-series curve of the Aaa corporate bond yield across the 55 years follows a loosely quadratic pattern, where it peaked in early 1980s and started to fall gradually in the next 40 years. The plot may fall under a random walk with quadratic deterministic trend, with or without drift, or just pure random walk, with or without drift (*Yt = α + Yt-1 + β1t + β2t^2+ εt or Yt = α + Yt-1 + εt*), but as an macroeconomic index, it should be naturally fairly unpredictable at the end of the day. The summary statistics table and time-series plot for government spending are as follows.

Table 6. *Summary Statistics, using the observations 1960:1 - 2015:1*
for the variable aaa (221 valid observations)

Mean	Median	Minimum	Maximum
7.3307	7.2200	3.4900	15.490
Std. Dev.	C.V.	Skewness	Ex. kurtosis
2.5500	0.34785	0.84481	0.43997
5% Perc.	95% Perc.	IQ range	Missing obs.
4.1930	12.651	3.4150	0

Figure 5. *Time-series plot for aaa*



Fedfund, the effective Federal Funds rate, has 221 valid observations from 1960 quarter 1 to 2015 quarter 1 with no missing values. The mean is 5.3247 percent, minimum is 0.070000 percent, maximum is 19.100 percent, and standard deviation is 3.6414 percent. The time-series curve of the federal funds rate across the 55 years does not have a clear pattern. Very loosely speaking, there may be a quadratic pattern where the federal funds rate peaked at 1981, but there are a couple of rises and falls due to different periods' monetary policies. Therefore, at best it may be random walk with drift and quadratic deterministic trend, but I suggest it maybe either pure random walk or random walk with drift ($Yt = Yt\text{-}1 + \varepsilon t$ *or* $Yt = \alpha + Yt\text{-}1 + \varepsilon t$). We may suspect the federal funds rate shows most exogeneity as compared to other variables. The summary statistics table and time-series plot for government spending are as follows.

Table 7. *Summary Statistics, using the observations 1960:1 - 2015:1 for the variable fedfund (221 valid observations)*

Mean	Median	Minimum	Maximum
5.3247	5.2400	0.070000	19.100
Std. Dev.	C.V.	Skewness	Ex. kurtosis
3.6414	0.68388	0.90580	1.4501
5% Perc.	95% Perc.	IQ range	Missing obs.
0.12000	11.421	4.2950	0

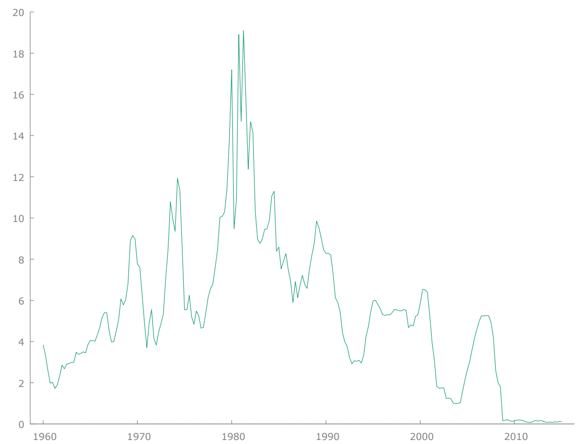


Figure 6. *Time-series plot for fedfund*

M, the real money supply computed through MZM money stock/GDP-based price index, has 221 valid observations from 1960 quarter 1 to 2015 quarter 1. The mean is 129.53 billion dollars, minimum is 49.453 billion dollars, maximum is 359.61 billion dollars, and standard deviation is 85.977 billion dollars. The time-series plot follows a roughly exponentially increasing trend over decades, where before 1990s the increase is slow and steady, while after then becoming steeper. Therefore, it may follow a non-stationary process of random walk with drift and deterministic trend ($Yt = \alpha + Yt\text{-}1 + \beta t + \varepsilon t$). The summary statistics and time-series plot of real money supply are as follows.

Table 8. *Summary Statistics, using the observations 1960:1 - 2015:1 for the variable M (221 valid observations)*

Mean	Median	Minimum	Maximum
129.53	96.802	49.483	359.61
Std. Dev.	C.V.	Skewness	Ex. kurtosis
85.977	0.66379	1.1456	0.045780
5% Perc.	95% Perc.	IQ range	Missing obs.
55.760	316.86	122.59	0

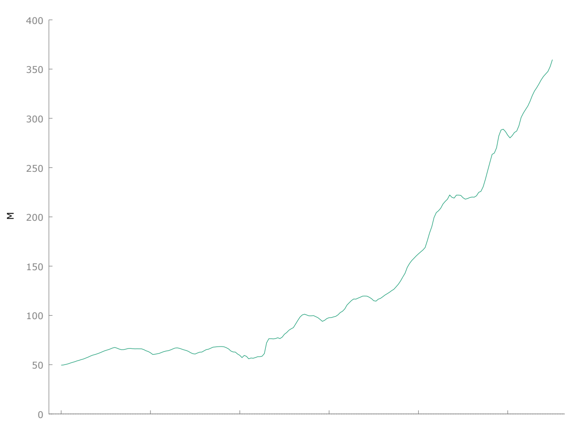


Figure 7. *Time-series plot for M*

Lastly, I pull out the correlation matrix between *gdpc1*, *pcec96*, *gpdic1*, *gcec1*, *aaa*, *fedfund*, and *M*. The correlation coefficients between expenditure-type variables, including *gdpc1*, *pcec96*, *gpdic1*, *gcec1*, and *M*, are significantly high, either 0.9 or 1.0. Those variables' times series plots are sort of self-explanatory of this pattern, where all of them more or less follow a roughly increasing trend over the years. While encountering crises or recessions, they all, to some degree, move in the same direction, which is fairly intuitive. Index-type variables, including *aaa* and *fedfund*, correlated with expenditure-type variables less remarkably, but -0.2, -0.3, or -0.5 are still relatively large values, exhibiting the simple macroeconomic rationale that when the interest rate is low, monetary policies are expansionary, and national-level expenditures are boosted, and vice versa. This correlation matrix is not as informative because the correlation between any two variables does not consider any endogeneity or lag possibilities, which is especially important when dealing with time-series data in the scope of macroeconomics. What's more informative are the VAR model and impulse response functions that take both factors into account, where the comparative statics are more unbiased and predictive.

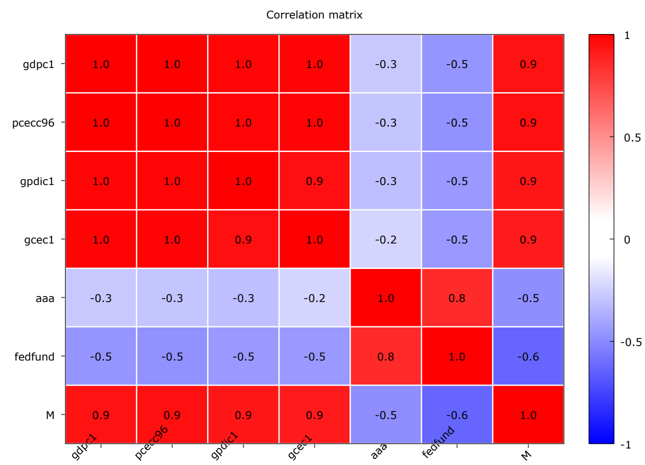


Figure 8. *Correlation matrix of seven variables of interest*

4. Methodology

To compare the empirical estimation of comparative statics to the theoretical IS-LM model, I will use the Vector Autoregression (VAR) statistical model. The VAR model is a generalization of the univariate autoregressive model (AR) for forecasting a time series vector.

The advantage of the VAR model is that it considers the situation that all variables may affect each other so that the framework breaks the limitation of univariate models like OLS (Ordinary Least Squares), where only unidirectional relationships between dependent and independent variables are allowed. In the case of our research purpose, the variables such as GDP, interest rate, consumption, and so on presumably all affect each other to certain degrees. Therefore we can treat all variables as endogenous. The VAR model also fits our purpose better than TSLS (Two-stage Least Squares) because TSLS still has the main focus onto some dependent variables despite it uses instrumental variables to address the endogeneity of independent variables. In contrast, our macro-level variables are affecting each other collectively.

4.1 Unit-root and Cointegration Tests

Before getting into the modeling, it's essential to check whether our variables are stationary or not. A stationary process has the property that the mean, variance, and autocorrelation structure do not change over time. Non-stationary data, nevertheless, are unpredictable and can hardly be modeled or forecasted in usual cases. Using non-stationary time series may be spurious since the variance evolves and goes to infinity as time goes to infinity. Therefore, I use the Augmented Dickey-Fuller (ADF) test, one of the commonly used unit root tests, to test whether or not each variable is stationary. The null hypothesis of the ADF test is that a unit root is present in the time series sample. If the null hypothesis is rejected, then the time series sample is stationary or trend-stationary, depending on whether a trend is included in the test. It is a bit lard to choose the lag length p for the ADF test. If p is too small, then the remaining serial correlation in the errors will bias the test. If p is too large, then the power of the test will suffer. The Monte Carlo experiments suggest it is better to error on the side, including too many lags. Therefore, I choose eight as the lag length, the length that may somewhat balance the unbiasedness and power of the test, though presumably not optimal.

For *gdpic1*, the real GDP, the p-value of ADF test with an only constant is 0.9993, with constant and the trend is 0.7952, both larger than the 0.05 p-value-to-beat. Including the trend is because I suspect the real GDP time series sample follows an increasing trend. Therefore, we fail to reject the null hypothesis.

A unit root is present in the real GDP time series sample, and the sample is non-stationary. The ADF test table for *gdpicl* is as follows.

For *pcecc96*, the real consumption, the p-value of ADF test with constant is 0.9995, with constant and trend is 0.8586, both larger than the 0.05 p-value-to-beat. Including the trend is because I suspect the real consumption time series sample may follow an increasing trend. Therefore, we fail to reject the null hypothesis. A unit root is present in the real consumption time series sample, and the sample is non-stationary.

For *gpdic1*, the real investment, the p-value of ADF test with constant is 0.978, with constant and trend is 0.3765, both larger than the 0.05 p-value-to-beat. Including the trend is because I suspect the real investment time series sample may follow an increasing trend. Therefore, we fail to reject the null hypothesis. A unit root is present in the real investment time series sample, and the sample is non-stationary.

For *gcec1*, the real government spending, the p-value of ADF test with constant is 0.7894, with constant and trend is 0.1879, both larger than the 0.05 p-value-to-beat. Including the trend is because I suspect the real investment time series sample may follow an increasing trend. Therefore, we fail to reject the null hypothesis. A unit root is present in the real government spending time series sample, and the sample is non-stationary.

For *aaa*, Moody's Aaa corporate bond yield, the p-value of ADF test with constant is 0.5137, with constant and trend is 0.7056, and with constant, trend, and trend squared is 0.3646, all larger than the 0.05 p-value-to-beat. Including trend squared is because I suspect the time-series sample may follow a quadratic pattern. Therefore, we fail to reject the null hypothesis. A unit root is present in the Aaa corporate bond yield time series sample, and the sample is non-stationary.

For *fedfund*, effective federal funds rate, the p-value of ADF test with constant is 0.5137, with constant and trend is 0.7056, and with constant, trend, and trend squared is 0.3646, all larger than the 0.05 p-value-to-beat. Including trend squared is because I suspect the time-series sample may follow a quadratic pattern. Therefore, we fail to reject the null hypothesis. A unit root is present in the effective federal funds rate time series sample, and the sample is non-stationary.

For *M*, the real money supply, the p-value of ADF test with constant is 1, with constant and trend is 1, both far from the 0.05 p-value-to-beat. Including the trend is because I suspect the real money supply time series sample may follow an exponentially increasing trend. Therefore, we fail to reject the null hypothesis. A unit root is present in the real money supply time series sample, which is non-stationary.

For unbiasedness and predictability of the model, we should not use the non-stationary variables but instead, transform non-stationary variables by differencing or de-trending to make them stationary or trend-stationary in that the transformed processes have their error terms' means and variances constant over time. I try to first-difference all seven variables and do the ADF tests again, and the results show all-time series samples are thereof stationary or trend-stationary. However, the problem with a first or higher-order difference and log-difference is that one might lose relevant information by doing so. For example, if I have two-time series processes that are integrated of order one and they are co-integrated, first-differencing both samples will remove the co-integration property when I run the VAR. I will test for co-integration between variables right after. In our case, first-differenced variables will make the impulse response functions and graphs less informative or predictable. Therefore, instead of taking the difference, I log-transform four expenditure-type variables—GDP, consumption, investment, government spending—and maintain the original format of two alternative interest rates and the real money supply. This is the more common transformative approach used by other empirical studies on the IS-LM model (Gali, 1992). Moreover, since all of the seven variables roughly exhibit deterministic trends over time, though less evident for *aaa* and *fedfund*, the spurious results can be avoided by de-trending, which for me is to not include the trend in the later VAR modeling.

The table (see Appendix) is a Engle-Granger cointegration test for my transformed variables. The idea is that if my variables are non-stationary, there may be the possibility that a linear combination of some or all of my variables is stationary or trend-stationary. Given that my seven variables are non-stationary, the p-value of the ADF test for u^hat from the cointegration regression is larger than 0.05. Therefore we reject the null hypothesis that residuals from the cointegrating regression have unit-root, which means there is no evidence for a cointegrating relationship.

4.2 VAR Model

I use the *–lagselect* command in Gretl to help determine the best lag order the VAR model should use based on our seven variables. There are three different information criteria, and each suggests the best lag order corresponding to the smallest criterion-generated value. AIC (Akaike criterion) offers seven lags that would be optimal, whereas BIC (Schwarz Bayesian criterion) and HQC (Hannan-Quinn criterion) both suggest two lags would be the best. Because AIC may tend to over-parameterize, I choose two lags, BIC and HQC suggest. Therefore, our VAR model will use the lag order of 2. The table of three information criteria regarding the best lag order is as follows.

Table 16. *Best lag order selection by AIC, BIC, HQC*

lags	loglik	p(LR)	AIC	BIC	HQC
1	2011.22186		-18.358891	-17.475171	-18.001750
2	2170.74396	0.00000	-19.396657	-17.739682*	-18.727018*
3	2216.27793	0.00025	-19.364112	-16.933882	-18.381975
4	2271.03065	0.00000	-19.418128	-16.214643	-18.123493
5	2324.41838	0.00000	-19.459328	-15.482587	-17.852194
6	2393.76870	0.00000	-19.650410	-14.900415	-17.730779
7	2462.19162	0.00000	-19.832785*	-14.309535	-17.600656
8	2507.05031	0.00035	-19.793900	-13.497395	-17.249272

We now have essential conditions to run our VAR model. The variables are *log_gdp1c*, *log_pcecc96*, *log_gpdic1*, *log_gcec1*, *aaa*, *fedfund*, and *M*. All seven variables are set to be endogenous. The lag order is 2. The model does not include a trend for de-trending. The HC1 heteroskedasticity-robust standard errors have been applied.

The model comprises one equation per variable in the system, and there are seven equations where each variable is regressed with 1st and 2nd lags of all variables, including itself. The system of equations form is as follows.

VAR Equations 1-7

$$gdp1c_t = \mu_1 + \beta_{11,1}\text{log\_}gdp1c_{t-1} + \beta_{11,2}\text{log\_}gdp1c_{t-2} + \beta_{12,1}\text{log\_}pcecc96_{t-1} + \beta_{12,2}\text{log\_}pcecc96_{t-2} + \beta_{13,1}\text{log\_}gpdic1_{t-1} + \beta_{13.2}\text{log\_}gpdic1_{t-2} + \beta_{14,1}\text{log\_}gcec1_{t-1} + \beta_{14,2}\text{log\_}gcec1_{t-2} + \beta_{15,1}aaa_{t-1} + \beta_{15,2}aaa_{t-2} + \beta_{16,1}fedfund_{t-1} + \beta_{16,2}fedfund_{t-2} + \beta_{17,1}M_{t-1} + \beta_{17,2}M_{t-2} + \varepsilon_{1,t}$$

$$pcecc96_t = \mu_2 + \beta_{21,1}\text{log\_}gdp1c_{t-1} + \beta_{21,2}\text{log\_}gdp1c_{t-2} + \beta_{22,1}\text{log\_}pcecc96_{t-1} + \beta_{22,2}\text{log\_}pcecc96_{t-2} + \beta_{23,1}\text{log\_}gpdic1_{t-1} + \beta_{23,2}\text{log\_}gpdic1_{t-2} + \beta_{24.1}\text{log\_}gcec1_{t-1} + \beta_{24,2}\text{log\_}gcec1_{t-2} + \beta_{25.1}aaa_{t-1} + \beta_{25.2}aaa_{t-2} + \beta_{26,2}fedfund_{t-1} + \beta_{26,2}fedfund_{t-2} + \beta_{27,1}M_{t-1} + \beta_{27,2}M_{t-2} + \varepsilon_{2,t}$$

$$gpdic1_t = \mu_3 + \beta_{31,1}\text{log\_}gdp1c_{t-1} + \beta_{31,2}\text{log\_}gdp1c_{t-2} + \beta_{32,1}\text{log\_}pcecc96_{t-1} + \beta_{32,2}\text{log\_}pcecc96_{t-2} + \beta_{33,1}\text{log\_}gpdic1_{t-1} + \beta_{33,2}\text{log\_}gpdic1_{t-2} + \beta_{34,1}\text{log\_}gcec1_{t-1} + \beta_{34,2}\text{log\_}gcec1_{t-2} + \beta_{35,1}aaa_{t-1} + \beta_{35,2}aaa_{t-2} + \beta_{36,1}fedfund_{t-1} + \beta_{36,2}fedfund_{t-2} + \beta_{37,1}M_{t-1} + \beta_{37,2}M_{t-2} + \varepsilon_{3,t}$$

$$gcec1_t = \mu_4 + \beta_{41,1}\text{log\_}gdp1c_{t-1} + \beta_{41,2}\text{log\_}gdp1c_{t-2} + \beta_{42,1}\text{log\_}pcecc96_{t-1} + \beta_{42,2}\text{log\_}pcecc96_{t-2} + \beta_{43,1}\text{log\_}gpdic1_{t-1} + \beta_{43,2}\text{log\_}gpdic1_{t-2} + \beta_{44,1}\text{log\_}gcec1_{t-1} + \beta_{44,2}\text{log\_}gcec1_{t-2} + \beta_{45,1}aaa_{t-1} + \beta_{45,2}aaa_{t-2} + \beta_{46,1}fedfund_{t-1} + \beta_{46,2}fedfund_{t-2} + \beta_{47,1}M_{t-1} + \beta_{47,2}M_{t-2} + \varepsilon_{4,t}$$

$$aaa_t = \mu_5 + \beta_{51,1}\text{log\_}gdp1c_{t-1} + \beta_{51,2}\text{log\_}gdp1c_{t-2} + \beta_{52,1}\text{log\_}pcecc96_{t-1} + \beta_{52,2}\text{log\_}pcecc96_{t-2} + \beta_{53,1}\text{log\_}gpdic1_{t-1} + \beta_{53,2}\text{log\_}gpdic1_{t-2} + \beta_{54,1}\text{log\_}gcec1_{t-1} + \beta_{54,2}\text{log\_}gcec1_{t-2} + \beta_{55,1}aaa_{t-1} + \beta_{55,2}aaa_{t-2} + \beta_{56,1}fedfund_{t-1} + \beta_{56,2}fedfund_{t-2} + \beta_{57,1}M_{t-1} + \beta_{57,2}M_{t-2} + \varepsilon_{5,t}$$

$$fedfund_t = \mu_6 + \beta_{61,1}\text{log\_}gdp1c_{t-1} + \beta_{61,2}\text{log\_}gdp1c_{t-2} + \beta_{62,1}\text{log\_}pcecc96_{t-1} + \beta_{62,2}\text{log\_}pcecc96_{t-2} + \beta_{63,1}\text{log\_}gpdic1_{t-1} + \beta_{63,2}\text{log\_}gpdic1_{t-2} + \beta_{64,1}\text{log\_}gcec1_{t-1} + \beta_{64,2}\text{log\_}gcec1_{t-2} + \beta_{65,1}aaa_{t-1} + \beta_{65,2}aaa_{t-2} + \beta_{66,1}fedfund_{t-1} + \beta_{66,2}fedfund_{t-2} + \beta_{67,1}M_{t-1} + \beta_{67,2}M_{t-2} + \varepsilon_{6,t}$$

$$M_t = \mu_7 + \beta_{71,1}\text{log\_}gdp1c_{t-1} + \beta_{71,2}\text{log\_}gdp1c_{t-2} + \beta_{72,1}\text{log\_}pcecc96_{t-1} + \beta_{72,2}\text{log\_}pcecc96_{t-2} + \beta_{73,1}\text{log\_}gpdic1_{t-1} + \beta_{73,2}\text{log\_}gpdic1_{t-2} + \beta_{74,1}\text{log\_}gcec1_{t-1} + \beta_{74,2}\text{log\_}gcec1_{t-2} + \beta_{75,1}aaa_{t-1} + \beta_{75,2}aaa_{t-2} + \beta_{76,1}fedfund_{t-1} + \beta_{76,2}fedfund_{t-2} + \beta_{77,1}M_{t-1} + \beta_{77,2}M_{t-2} + \varepsilon_{7,t}$$

The coefficients $\beta_{ii,t}$ captures the influence of the *t*th lag of variable y_i itself, while the coefficient $\beta_{ij,t}$ captures the influence of the *t*th lag of variable y_j on y_i. $\varepsilon_{i,t}$s are white noise processes that may be contemporaneously correlated. The more compact matrix/vector form of our VAR model is as follows.

$$\begin{bmatrix} \text{log\_}gdp1c_t \\ \text{log\_}pcecc96_t \\ \text{log\_}gpdic1_t \\ \text{log\_}gcec1_t \\ aaa_t \\ fedfund_t \\ M_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \\ \mu_7 \end{bmatrix} + \begin{bmatrix} \beta_{11,1} & \beta_{12,1} & \beta_{13,1} & \beta_{14,1} & \beta_{15,1} & \beta_{16,1} & \beta_{17,1} \\ \beta_{21,1} & \beta_{22,1} & \beta_{23,1} & \beta_{24,1} & \beta_{25,1} & \beta_{26,1} & \beta_{27,1} \\ \beta_{31,1} & \beta_{32,1} & \beta_{33,1} & \beta_{34,1} & \beta_{35,1} & \beta_{36,1} & \beta_{37,1} \\ \beta_{41,1} & \beta_{42,1} & \beta_{43,1} & \beta_{44,1} & \beta_{45,1} & \beta_{46,1} & \beta_{47,1} \\ \beta_{51,1} & \beta_{52,1} & \beta_{53,1} & \beta_{54,1} & \beta_{55,1} & \beta_{56,1} & \beta_{57,1} \\ \beta_{61,1} & \beta_{62,1} & \beta_{63,1} & \beta_{64,1} & \beta_{65,1} & \beta_{66,1} & \beta_{67,1} \\ \beta_{71,1} & \beta_{72,1} & \beta_{73,1} & \beta_{74,1} & \beta_{75,1} & \beta_{76,1} & \beta_{77,1} \end{bmatrix} \begin{bmatrix} \text{log\_}gdp1c_{t-1} \\ \text{log\_}pcecc96_{t-1} \\ \text{log\_}gpdic1_{t-1} \\ \text{log\_}gcec1_{t-1} \\ aaa_{t-1} \\ fedfund_{t-1} \\ M_{t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} \beta_{11,2} & \beta_{12,2} & \beta_{13,2} & \beta_{14,2} & \beta_{15,2} & \beta_{16,2} & \beta_{17,2} \\ \beta_{21,2} & \beta_{22,2} & \beta_{23,2} & \beta_{24,2} & \beta_{25,2} & \beta_{26,2} & \beta_{27,2} \\ \beta_{31,2} & \beta_{32,2} & \beta_{33,2} & \beta_{34,2} & \beta_{35,2} & \beta_{36,2} & \beta_{37,2} \\ \beta_{41,2} & \beta_{42,2} & \beta_{43,2} & \beta_{44,2} & \beta_{45,2} & \beta_{46,2} & \beta_{47,2} \\ \beta_{51,2} & \beta_{52,2} & \beta_{53,2} & \beta_{54,2} & \beta_{55,2} & \beta_{56,2} & \beta_{57,2} \\ \beta_{61,2} & \beta_{62,2} & \beta_{63,2} & \beta_{64,2} & \beta_{65,2} & \beta_{66,2} & \beta_{67,2} \\ \beta_{71,2} & \beta_{72,2} & \beta_{73,2} & \beta_{74,2} & \beta_{75,2} & \beta_{76,2} & \beta_{77,2} \end{bmatrix} \begin{bmatrix} \text{log\_}gdp1c_{t-2} \\ \text{log\_}pcecc96_{t-2} \\ \text{log\_}gpdic1_{t-2} \\ \text{log\_}gcec1_{t-2} \\ aaa_{t-2} \\ fedfund_{t-2} \\ M_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \\ \varepsilon_{6,t} \\ \varepsilon_{7,t} \end{bmatrix}$$

$$Y_t = \mu_t + B_1LY_{t-1} + B_2L^2Y_{t-2} + u_t$$

5. Results

5.1 VAR Results

There are a series of tables that constitute the full VAR model output in this section. Each table corresponds to one of the seven equations where one of the seven variables is the response variable. All seven variables, each with two lags, are the explanatory variables. The interpretation approaches here shall be clarified. The individual coefficients are of less interest since the same variable's different lags are highly correlated, leading to the issue of multicollinearity.

What is more informative is the F-test results down below the output. Taking the first regression output as an example, the first F-test has the null hypothesis that all lags of log_*gdpc1* are not associated with log_*gdpc1* itself. The second F-test has the null hypothesis that all lags of log_*pcecc96* are not associated with log_*gdpc1*. The rest are in the same manner, except for the last F-test, whose null hypothesis is that using lag two is not meaningful. If the p-values of the first eight F-tests are less than 0.05, then we can say that on a 5% significance level, there are correlations between log_*gdpc1* and all lags of itself and other variables. If the p-value of the last F-test is less than 0.05, then we can say that on a 5% significance level, using 2-order past values for this particular regression is meaningful.

Another thing to point out is the concept of Granger causality. Clive Granger defined the causality relationship based on two principles: 1. The cause happens before its effect. 2. The reason has unique information about the future values of its impact. The Granger causality is not equivalent to true causality, as the latter is a very bold statement. Still, it is relatively valuable in the context of multivariate time series models like VAR. For example, suppose we reject the null hypothesis in the second F-test of the first regression output of log_*gdpc1*. In that case, we can say all lags of log_ *pcecc96* Granger-causes log_ *gdpc1* in that the history of log_*pcecc96* adds predictive power over current log_*gdpc1*, given the history of log_*gdpc1* itself.

VAR system, lag order 2
OLS estimates, observations 1960:3-2015:1 (T = 219)
Log-likelihood = 2229.8905
Determinant of covariance matrix = 3.3773633e-018
AIC = -19.4054
BIC = -17.7805
HQC = -18.7491
Portmanteau test: LB(48) = 2717.68, df = 2254 [0.0000]

Table 17. *Regression Output for log_gdpc1*

Equation 1: l_gdpc1
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	0.0743093	0.0609560	1.219	0.2242	
l_gdpc1_1	0.591011	0.212865	2.776	0.0060	***
l_gdpc1_2	0.378098	0.199476	1.895	0.0594	*
l_pcecc96_1	0.626125	0.152475	4.106	<0.0001	***
l_pcecc96_2	−0.583497	0.145610	−4.007	<0.0001	***
l_gpdic1_1	0.0643972	0.0309521	2.081	0.0387	**
l_gpdic1_2	−0.0663088	0.0299657	−2.213	0.0280	**
l_gcec1_1	0.00816710	0.0635440	0.1285	0.8979	
l_gcec1_2	−0.0257362	0.0651170	−0.3952	0.6931	
aaa_1	−0.00111725	0.00125161	−0.8927	0.3731	
aaa_2	0.00157596	0.00121227	1.300	0.1951	
fedfund_1	0.000492769	0.000541189	0.9105	0.3636	
fedfund_2	−0.00166272	0.000515519	−3.225	0.0015	***
M_1	0.000145460	0.000199422	0.7294	0.4666	
M_2	−0.000192596	0.000199020	−0.9677	0.3343	

Mean dependent var	9.032028	S.D. dependent var	0.484075
Sum squared resid	0.009656	S.E. of regression	0.006880
R-squared	0.999811	Adjusted R-squared	0.999798
F(14, 204)	81422.66	P-value(F)	0.000000
rho	−0.042147	Durbin-Watson	2.076712

F-tests of zero restrictions:

All lags of l_gdpc1	F(2, 204) =	155.92 [0.0000]
All lags of l_pcecc96	F(2, 204) =	8.4905 [0.0003]
All lags of l_gpdic1	F(2, 204) =	2.5069 [0.0840]
All lags of l_gcec1	F(2, 204) =	0.79551 [0.4527]
All lags of aaa	F(2, 204) =	1.1863 [0.3074]
All lags of fedfund	F(2, 204) =	7.7101 [0.0006]
All lags of M	F(2, 204) =	4.4635 [0.0127]
All vars, lag 2	F(7, 204) =	5.5419 [0.0000]

The regression output of log_*gdpc1* as the dependent variable and all variables, each with two lags, as independent variables, is shown in Table 17. The p-values of the 1st, 2nd, and 6th F-tests are 0.0000, 0.0003, and 0.0006, all statistically significant at 1% significance level, and therefore we can say all lags of real GDP Granger-causes itself, all lags of real consumption Granger-causes real GDP, and all lags of federal funds rate Granger-causes real GDP, at 1% significance level. The p-value of the 7th F-test is 0.0127, only significant at 5% significance level, so that we can say all lags of real money supply Granger-causes real GDP, at 5% significance level. The p-value of the 3rd F-test is 0.0840, only significant at 10% significance level so that we can say all lags of real investment Granger-causes real

GDP at 10% significance level. The p-value of the last F-test is 0.0000, statistically significant at the 1% significance level. Thus we say including lag 1 and 2 for all variables in real GDP's regression is meaningful. The p-values of the remaining 4th and 5th F-tests are 0.4527 and 0.3074, not statistically significant at 10% significance level, so we fail to reject the null hypotheses, nor are we able to draw any correlations between lags of real government spending and real GDP, or between Aaa corporate bond yield and real GDP.

Table 18. *Regression Output for log_pcecc96*

Equation 2: l_pcecc96
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	0.0203534	0.0499705	0.4073	0.6842	
l_gdpc1_1	−0.0569882	0.188566	−0.3022	0.7628	
l_gdpc1_2	0.130306	0.180211	0.7231	0.4705	
l_pcecc96_1	1.10749	0.124070	8.926	<0.0001	***
l_pcecc96_2	−0.191901	0.116898	−1.642	0.1022	
l_gpdic1_1	0.0483970	0.0316935	1.527	0.1283	
l_gpdic1_2	−0.0334540	0.0306149	−1.093	0.2758	
l_gcec1_1	0.0100873	0.0589484	0.1711	0.8643	
l_gcec1_2	−0.0173059	0.0590229	−0.2932	0.7697	
aaa_1	−0.00239113	0.00105672	−2.263	0.0247	**
aaa_2	0.00399600	0.00101210	3.948	0.0001	***
fedfund_1	−0.000994112	0.000381680	−2.605	0.0099	***
fedfund_2	−0.000771612	0.000404440	−1.908	0.0578	*
M_1	8.04879e-05	0.000213124	0.3777	0.7061	
M_2	−0.000104484	0.000213661	−0.4890	0.6254	

Mean dependent var	8.591730	S.D. dependent var	0.520488
Sum squared resid	0.006452	S.E. of regression	0.005624
R-squared	0.999891	Adjusted R-squared	0.999883
F(14, 204)	145069.9	P-value(F)	0.000000
rho	−0.020409	Durbin-Watson	2.017319

F-tests of zero restrictions:

All lags of l_gdpc1	F(2, 204) =	1.5389 [0.2171]
All lags of l_pcecc96	F(2, 204) =	261.1 [0.0000]
All lags of l_gpdic1	F(2, 204) =	3.1675 [0.0442]
All lags of l_gcec1	F(2, 204) =	0.22066 [0.8022]
All lags of aaa	F(2, 204) =	14.207 [0.0000]
All lags of fedfund	F(2, 204) =	14.599 [0.0000]
All lags of M	F(2, 204) =	2.0709 [0.1287]
All vars, lag 2	F(7, 204) =	3.3096 [0.0023]

The regression output of log_*pcecc96* as the dependent variable and all variables, each with two lags, as independent variables, is shown in Table 18. The p-values of the 2nd , 5th, and 6th F-tests are 0.0000, 0.0000, and 0.0000, all statistically significant at 1% significance level, and therefore we can say all lags of real consumption Granger-causes itself, all lags of Aaa corporate bond yield Granger-causes real consumption, and all lags of federal funds rate Granger-causes real consumption, at 1% significance level. The p-value of the 3rd F-test is 0.0442, only significant at 5% significance level, so that we can say all lags of real investment Granger-causes real consumption, at 5% significance level. The p-value of the last F-test is significant at 1% significance level, so that we can say including lag 1 and 2 for all variables in real consumption's regression is meaningful. The p-values of the remaining F-tests are not statistically significant at 10% significance level. Therefore, we fail to reject the null hypotheses, we cannot draw any correlations between all lags of real GDP and real consumption, or between all lags of real government spending and real consumption, or between all lags of real money supply and real consumption.

Table 19. *Regression Output for log_gpdic1*

Equation 3: l_gpdic1
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−0.172070	0.248796	−0.6916	0.4900	
l_gdpc1_1	−1.54934	0.974816	−1.589	0.1135	
l_gdpc1_2	1.53895	0.938429	1.640	0.1026	
l_pcecc96_1	3.69672	0.759689	4.866	<0.0001	***
l_pcecc96_2	−3.43571	0.760740	−4.516	<0.0001	***
l_gpdic1_1	1.15675	0.145758	7.936	<0.0001	***
l_gpdic1_2	−0.295598	0.145812	−2.027	0.0439	**
l_gcec1_1	−0.314824	0.296551	−1.062	0.2897	
l_gcec1_2	0.187414	0.302900	0.6187	0.5368	
aaa_1	0.00274948	0.00593996	0.4629	0.6439	
aaa_2	−0.00317127	0.00554777	−0.5716	0.5682	
fedfund_1	0.00494056	0.00252520	1.957	0.0518	*
fedfund_2	−0.00715604	0.00253077	−2.828	0.0052	***
M_1	−2.58425e-05	0.00105212	−0.02456	0.9804	
M_2	−9.38137e-05	0.00105041	−0.08931	0.9289	

Mean dependent var	7.060796	S.D. dependent var	0.614947
Sum squared resid	0.195032	S.E. of regression	0.030920
R-squared	0.997634	Adjusted R-squared	0.997472
F(14, 204)	7195.208	P-value(F)	2.8e-266
rho	−0.057470	Durbin-Watson	2.109844

F-tests of zero restrictions:

All lags of l_gdpc1	F(2, 204) =	1.3452 [0.2628]
All lags of l_pcecc96	F(2, 204) =	12.203 [0.0000]
All lags of l_gpdic1	F(2, 204) =	347.01 [0.0000]
All lags of l_gcec1	F(2, 204) =	2.605 [0.0764]
All lags of aaa	F(2, 204) =	0.1917 [0.8257]
All lags of fedfund	F(2, 204) =	4.0884 [0.0182]
All lags of M	F(2, 204) =	1.5622 [0.2122]
All vars, lag 2	F(7, 204) =	6.1579 [0.0000]

The regression output of log*_gpdic1* as the dependent variable and all variables, each with two lags, as independent variables, is shown in Table 19. The p-values of the 2nd and 3rd and F-tests are 0.0000 and 0.0000, both statistically significant at 1% significance level, and therefore we can say all lags of real consumption Granger-causes real investment, and all lags of real investment Granger-causes itself, at 1% significance level. The p-value of the 6th F-test is 0.0182, only significant at 5% significance level, so that we can say all lags of federal funds rate Granger-causes real investment, at 5% significance level. The p-value of the 4th F-test is 0.0764, significant at 10% significance level, so that we can say all lags of real government spending Granger-causes real investment, at 10% significance level. The p-value of the last F-test is 0.0000, statistically significant at the 1% significance level, so that we conclude including lag 1 and 2 for all variables in real investment's regression is meaningful. The p-values of the remaining F-tests are not statistically significant at 10% significance level. Therefore, we fail to reject the null hypotheses, and we cannot draw any correlations between all lags of real GDP and real investment, between all lags of real government spending and real investment, between all lags of Aaa corporate bond yield and real investment, or between all lags of real money supply and real investment.

Table 20. Regression Output for log_*gcec1*

Equation 4: l_gcec1
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	0.187787	0.0749525	2.505	0.0130	**
l_gdpc1_1	−0.470153	0.282775	−1.663	0.0979	*
l_gdpc1_2	0.407537	0.275232	1.481	0.1402	
l_pcecc96_1	0.210855	0.208533	1.011	0.3131	
l_pcecc96_2	−0.184162	0.192164	−0.9584	0.3390	
l_gpdic1_1	0.0707615	0.0441711	1.602	0.1107	
l_gpdic1_2	−0.0431487	0.0445847	−0.9678	0.3343	
l_gcec1_1	1.18782	0.0991827	11.98	<0.0001	***
l_gcec1_2	−0.193019	0.0982216	−1.965	0.0508	*
aaa_1	0.000426731	0.00154939	0.2754	0.7833	
aaa_2	−0.000175188	0.00150040	−0.1168	0.9072	
fedfund_1	0.000120424	0.000657172	0.1832	0.8548	
fedfund_2	−0.000389157	0.000682168	−0.5705	0.5690	
M_1	0.000609166	0.000246264	2.474	0.0142	**
M_2	−0.000632756	0.000246806	−2.564	0.0111	**

Mean dependent var	7.643621	S.D. dependent var	0.311373
Sum squared resid	0.018445	S.E. of regression	0.009509
R-squared	0.999127	Adjusted R-squared	0.999067
F(14, 204)	18729.75	P-value(F)	0.000000
rho	−0.026850	Durbin-Watson	2.044752

F-tests of zero restrictions:

All lags of l_gdpc1	F(2, 204) =	1.5181 [0.2216]
All lags of l_pcecc96	F(2, 204) =	0.51194 [0.6001]
All lags of l_gpdic1	F(2, 204) =	4.1876 [0.0165]
All lags of l_gcec1	F(2, 204) =	1535.7 [0.0000]
All lags of aaa	F(2, 204) =	0.084757 [0.9188]
All lags of fedfund	F(2, 204) =	0.22149 [0.8015]
All lags of M	F(2, 204) =	3.7305 [0.0256]
All vars, lag 2	F(7, 204) =	1.9946 [0.0574]

The regression output of log_*gcec1* as the dependent variable and all variables, each with two lags, as independent variables, is shown in Table 20. The p-value of the 4th F-test is 0.0000, statistically significant at 1% significance level, and therefore we can say all lags of real government spending Granger-causes itself, at 1% significance level. The p-values of the 3rd and 7th F-test are 0.0165 and 0.0256, significant at 5% significance level, so that we can say all lags of real investment Granger-causes real government spending, and all lags of real money supply Granger-causes real government spending, at 5% significance level. The p-value of the last F-test is 0.0574, only significant at 10% significance level, so that we conclude including lag 1 and 2 for all variables in real government spending's regression is meaningful. The p-values of the remaining F-tests are not statistically significant at 10% significance level. Therefore, we fail to reject the null hypotheses, and we cannot draw any correlations between all lags of real GDP and real government spending, between all lags of real consumption and government, between all lags of Aaa corporate bond yield and real government spending, and between all lags of effective federal funds rate and real government spending.

Table 21. *Regression Output for aaa*

Equation 5: aaa
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−3.22840	3.57090	−0.9041	0.3670	
l_gdpc1_1	4.46164	13.8318	0.3226	0.7474	
l_gdpc1_2	−1.58351	13.2416	−0.1196	0.9049	
l_pcecc96_1	0.461014	10.3927	0.04436	0.9647	
l_pcecc96_2	−1.34656	9.75214	−0.1381	0.8903	
l_gpdic1_1	−1.46096	2.33918	−0.6246	0.5330	
l_gpdic1_2	0.679627	2.32005	0.2929	0.7699	
l_gcec1_1	−3.52806	4.58788	−0.7690	0.4428	
l_gcec1_2	2.36161	4.53581	0.5207	0.6032	
aaa_1	0.768700	0.145526	5.282	<0.0001	***
aaa_2	0.100124	0.141327	0.7085	0.4795	
fedfund_1	0.0676786	0.0556873	1.215	0.2256	
fedfund_2	0.00869297	0.0528006	0.1646	0.8694	
M_1	0.00338866	0.0135205	0.2506	0.8024	
M_2	−0.00481570	0.0133216	−0.3615	0.7181	

Mean dependent var	7.356849	S.D. dependent var	2.546805
Sum squared resid	48.28274	S.E. of regression	0.486498
R-squared	0.965854	Adjusted R-squared	0.963510
F(14, 204)	359.9134	P-value(F)	2.3e-135
rho	−0.013117	Durbin-Watson	2.021633

F-tests of zero restrictions:

All lags of l_gdpc1	F(2, 204) = 0.29396 [0.7456]
All lags of l_pcecc96	F(2, 204) = 0.062544 [0.9394]
All lags of l_gpdic1	F(2, 204) = 0.93426 [0.3946]
All lags of l_gcec1	F(2, 204) = 0.87967 [0.4165]
All lags of aaa	F(2, 204) = 176.29 [0.0000]
All lags of fedfund	F(2, 204) = 2.8788 [0.0585]
All lags of M	F(2, 204) = 0.69545 [0.5000]
All vars, lag 2	F(7, 204) = 0.18925 [0.9874]

The regression output of *aaa* as the dependent variable and all variables, each with two lags, as independent variables, is shown in Table 21. Only the p-value of the 5th F-test, 0.0000, is statistically significant at 1% significance level, which means all lags of Aaa corporate bond yield Granger-causes itself. The remaining p-values are all not significant at 10% significance level, so we reject the null hypotheses, saying that we are not able to conclude any correlations between all lags of real GDP, real consumption, real investment, real government spending, effective federal funds rate, real money supply and Aaa corporate bond yield. This shows that Aaaa corporate bond yield might exhibit the most exogeneity among all seven variables in the VAR model. This matters in terms of the Cholesky order I will determine later in the forecast section.

Table 22. *Regression Output for fedfund*

Equation 6: fedfund
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	−0.254984	8.55411	−0.02981	0.9762	
l_gdpc1_1	97.3048	42.7590	2.276	0.0239	**
l_gdpc1_2	−87.2376	36.8322	−2.369	0.0188	**
l_pcecc96_1	−28.7334	26.6834	−1.077	0.2828	
l_pcecc96_2	20.9016	23.7020	0.8819	0.3789	

l_gpdic1_1	−13.1471	8.25708	−1.592	0.1129	
l_gpdic1_2	13.8794	7.78742	1.782	0.0762	*
l_gcec1_1	−28.5271	16.8019	−1.698	0.0911	*
l_gcec1_2	24.8695	15.6255	1.592	0.1130	
aaa_1	0.101423	0.426278	0.2379	0.8122	
aaa_2	−0.0114514	0.402487	−0.02845	0.9773	
fedfund_1	0.719385	0.145749	4.936	<0.0001	***
fedfund_2	0.0999690	0.178168	0.5611	0.5753	
M_1	−0.0715461	0.0309366	−2.313	0.0217	**
M_2	0.0680467	0.0313297	2.172	0.0310	**

Mean dependent var	5.340594	S.D. dependent var	3.654130
Sum squared resid	284.2405	S.E. of regression	1.180396
R-squared	0.902352	Adjusted R-squared	0.895651
F(14, 204)	285.9976	P-value(F)	1.2e-125
rho	−0.046834	Durbin-Watson	2.089608

F-tests of zero restrictions:

All lags of l_gdpc1	F(2, 204) = 2.8145 [0.0623]
All lags of l_pcecc96	F(2, 204) = 0.69083 [0.5023]
All lags of l_gpdic1	F(2, 204) = 2.0362 [0.1332]
All lags of l_gcec1	F(2, 204) = 1.6048 [0.2035]
All lags of aaa	F(2, 204) = 0.27392 [0.7607]
All lags of fedfund	F(2, 204) = 49.502 [0.0000]
All lags of M	F(2, 204) = 3.5206 [0.0314]
All vars, lag 2	F(7, 204) = 2.3448 [0.0253]

The regression output of *fedfund* as the dependent variable and all variables, each with two lags, as independent variables, is shown in Table 22. The p-value of the 6th F-test, 0.0000, is statistically significant at 1% significance level, which means all lags of effective federal funds rate Granger-causes itself. The p-value of the 7th F-test, 0.0314, is statistically significant at the 5% significance level, so we say all lags of real money supply Granger-causes the federal funds rate. The remaining p-values are all not significant at 10% significance level, so we reject the null hypotheses, saying that we are not able to conclude any correlations between all lags of real GDP, real consumption, real investment, real government spending, Aaa corporate bond yield and federal funds rate. This shows that federal funds rate also exhibits evident exogeneity among all seven variables, which matters in terms of the Cholesky order as well.

Table 23. *Regression Output for M*

Equation 7: M
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	2.97182	10.3727	0.2865	0.7748	
l_gdpc1_1	−120.825	43.8016	−2.758	0.0063	***
l_gdpc1_2	126.916	44.4759	2.854	0.0048	***
l_pcecc96_1	67.2834	33.4598	2.011	0.0457	**
l_pcecc96_2	−71.3731	30.6885	−2.326	0.0210	**
l_gpdic1_1	15.2608	6.69101	2.281	0.0236	**
l_gpdic1_2	−14.3808	6.42777	−2.237	0.0263	**
l_gcec1_1	19.5351	12.4502	1.569	0.1182	
l_gcec1_2	−23.2202	12.8206	−1.811	0.0716	*
aaa_1	−1.23281	0.315566	−3.907	0.0001	***

aaa_2	1.04138	0.308569	3.375	0.0009	***
fedfund_1	−0.191390	0.0951637	−2.011	0.0456	**
fedfund_2	0.329554	0.0962966	3.422	0.0008	***
M_1	1.62205	0.0683500	23.73	<0.0001	***
M_2	−0.618434	0.0688855	−8.978	<0.0001	***

Mean dependent var	130.2558	S.D. dependent var	86.02739
Sum squared resid	446.1035	S.E. of regression	1.478777
R-squared	0.999723	Adjusted R-squared	0.999705
F(14, 204)	51124.31	P-value(F)	0.000000
rho	0.032055	Durbin-Watson	1.930813

F-tests of zero restrictions:

All lags of l_gdpc1	F(2, 204) =	4.0737 [0.0184]
All lags of l_pcecc96	F(2, 204) =	2.91 [0.0567]
All lags of l_gpdic1	F(2, 204) =	2.6327 [0.0743]
All lags of l_gcec1	F(2, 204) =	2.2147 [0.1118]
All lags of aaa	F(2, 204) =	7.9193 [0.0005]
All lags of fedfund	F(2, 204) =	5.9272 [0.0031]
All lags of M	F(2, 204) =	42181 [0.0000]
All vars, lag 2	F(7, 204) =	18.444 [0.0000]

The regression output of *M* as the dependent variable and all variables, each with two lags, as independent variables, is shown in Table 23. The p-values of the 5th, 6th, 7th F-test are 0.0005, 0.0031, 0.0000, all statistically significant at 1% significance level, and therefore we can say all lags of real money supply Granger-causes Aaa corporate bond yield, all lags of real money supply Granger-causes federal funds rate, and all lags of real money supply Granger causes itself, at 1% significance level. The p-value of the 1st F-test is 0.0184, significant at 5% significance level, so that we can say all lags of real money supply Granger-causes real GDP, at 5% significance level. The p-values of the 2nd and 3rd F-test are 0.0567 and 0.0743, only significant at 10% significance level, so that we conclude all lags of real consumption Granger-causes real money supply, and all lags of real investment Granger causes real money supply. The p-value of the last F-test is 0.000, statistically significant at the 1% significance level, which means including lag 1 and 2 for all variables in real money supply's regression is meaningful. The p-values of the 4th F-test is 0.1118, not statistically significant at the 10% significance level, so we reject the null hypothesis, and we cannot say anything between all lags of real government spending and real money supply.

Table 24. *Test for 2-lags for the whole system*

For the system as a whole:
Null hypothesis: the longest lag is 1
Alternative hypothesis: the longest lag is 2
Likelihood ratio test: Chi-square(49) = 333.292 [0.0000]

Table 24 shows that for the system, since the likelihood ratio test has a p-value of 0.0000, statistically significant at 1% significance level, so we reject the null hypothesis that the longest lag for whole system is 1, which to some extent reinforces our choice of including both lag 1 and 2 for all variables in the VAR model.

5.2 Impulse Responses and Discussion

In economics, the impulse response function is used to describe how the economy reacts over time to exogenous impulses, usually referred to as shocks. Our VAR model can use the impulse response function to see how one variable responds given one standard deviation shock in another variable over time. At the beginning of the paper, I theoretically sort out the common comparative statics in the IS-LM model. The impulse responses in our VAR model based on empirical estimation enable us to see how well the theoretical comparatives statics are verified or challenged.

The impulse response function is realized through the Cholesky decomposition. In general, the cross-equation variance-covariance matrix for residuals in VAR has the problem of contemporaneous correlation. The off-diagonal covariance entries are non-zero and sometimes have relatively high absolute values if expressed in correlation coefficients. In table 25, the upper right triangle of the matrix shows precisely such an issue, where the correlation coefficients of residuals between real GDP and real consumption, for example, is 0.641. If we don't address the issue of contemporaneous correlation, the impulse response of one variable in response to another will have less predictive power.

Table 25. *Cross-equation variance-covariance matrix for residuals*

```
Cross-equation VCV for residuals
(correlations above the diagonal)

  4.4092e-005      (0.641)      (0.779)      (0.260)      (0.251)      (0.243)     (-0.073)
  2.3086e-005  2.9460e-005      (0.262)      (0.003)      (0.200)      (0.185)      (0.028)
   0.00015428  4.2495e-005   0.00089056     (-0.015)      (0.230)      (0.246)     (-0.160)
  1.5832e-005  1.5898e-007 -4.2432e-006  8.4222e-005     (-0.071)     (-0.088)      (0.045)
   0.00078124   0.00050890    0.0032160  -0.00030503      0.22047      (0.595)     (-0.129)
    0.0018390    0.0011423    0.0083657  -0.00091760      0.31851       1.2979     (-0.169)
  -0.00069306   0.00022007   -0.0068242   0.00059224    -0.086447     -0.27532       2.0370

log determinant = -40.2294
```

The Cholesky decomposition only works if we correctly order the variables from the most exogenous to the less exogenous, known as the Cholesky ordering. I put the federal funds rate in the 1st place since it's chosen by the Fed to influence the money supply in the US economy and often changes the public's consumption and investment behaviors, thus exhibiting most exogeneity. The Aaa corporate bond yield is in the 2nd place since it's often used as an alternative to the federal funds rate as an indicator of interest rate, which affects the behaviors of consumers and investors significantly. The time-series plots of both indices somewhat confirm their homogeneity since their processes are spurious over time, and there is hardly a trend to categorize. The rest of the five expenditure-type variables are harder to determine in the Cholesky order because they are all considered substantially endogenous. Therefore, my ordering of those variables is more random. Considering that GDP and consumption consist of many determinants and can be altered by a lot of factors, I put GDP in the last place and consumption in the second last. Likewise, I put the government spending in the third last place and the investment before it. From most exogenous to least, the final Cholesky ordering is as follows: *fedfund*, *aaa*, *M*, log_*gpdic1*, log_*gcec1*, log_*pcecc96*, and log_*gdpc1*.

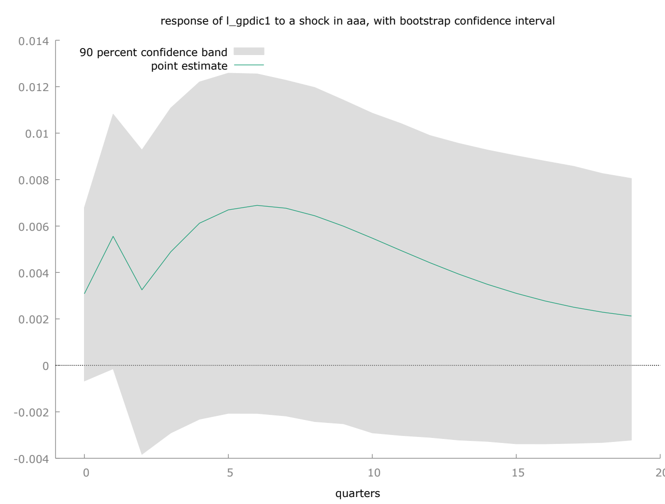
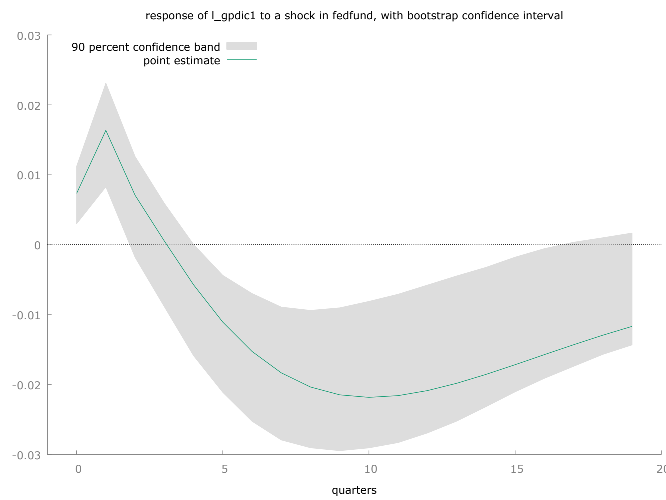


Figure 9. *Response of log_gpdic1 to the shock in aaa*

First, we check how the real investment responds to a one standard deviation shock in both interest rate indices, shown in Figures 9 and 10. In the IS-LM model, the increase in interest rate will lead to decreased investment. The impulse response graph in the case of Aaa corporate bond yield shows that the real investment will increase by nearly 0.006% in the first two quarters, fluctuate a little, peak at nearly 0.007% in the 6th quarter, and gradually fall in the next couple quarters, but never fall back to the initial level, not to mention the decrease at least in our future projection period.

This contradicts the comparative statics between interest rate and investment, perhaps because people have high expectations in the corporate bond market, especially during crises or recessions when government bonds are unfavorable. The Fed also tends to mass-purchase corporate bonds to support market functioning and ease credit conditions. Notice that the 90 percent confidence band for the response of real investment to Aaa corporate bond yield is wide, and a good portion of the bootstrap band reaches the negative territory, which means at some possibility the real investment might decrease in response to an increase in Aaa corporate bond yield, although less likely. Specifically for corporate bonds, perhaps the increased interest rate will only generate paper losses, but the value of the bonds when they are held to maturity doesn't change. People may also stay in the course and diversify the investment portfolio to offset the changing interest rates in the long term.

Figure 10. *Response of log_gpdic1 to the shock in fedfund*



The impulse response graph of real investment in response to the federal funds rate shock seems more intuitive. After an unusual increase of 0.016% in the 1st quarter, the real investment is expected to fall below zero in the 3rd quarter and continues the descending trend in the next couple quarters. The initial increase may be an estimation error. Still, it may also be due to a period of investors' reaction time where they need some room to forgo the previous zest in speculations and adapt to the new interest rate. The general idea that an increased federal funds rate leads to decreased investment is pretty applicable in reality, as stocks, bonds, and cashes all follow rather closely to the rise of the federal funds rate. However, notice that the reverse may not be accurate. When interest rates were ultra-low, like during the Great Recession and 2020 coronavirus, there were no real investment incentives to bonds and cash due to the dismal returns (Friedberg, 2018).

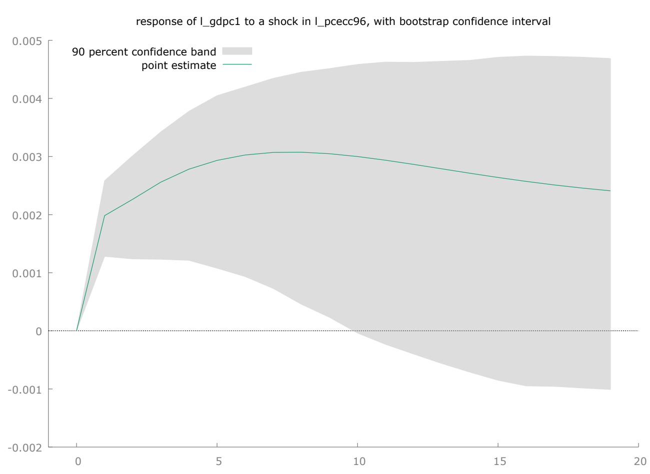
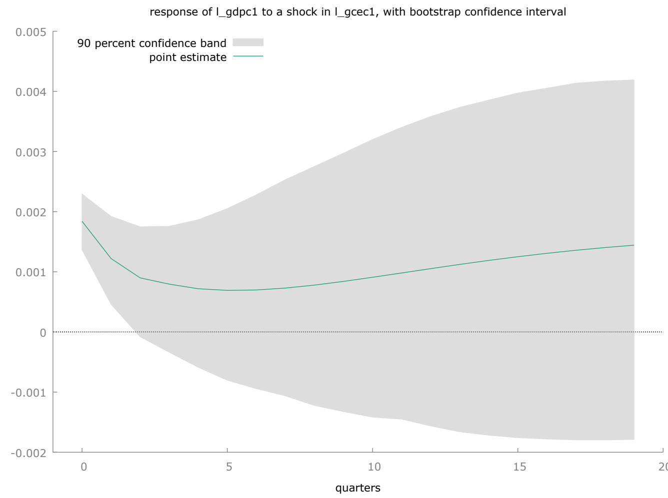


Figure 11. *Response of log_gdpc1 to a shock in log_pcecc96*

The impulse response graph of real GDP in response to the shock of real consumption, real investment, and government spending is shown in Figures 11, 12, and 13. The 1% increase in real consumption is expected to boost real GDP by 0.002% in the 1st quarter, continue the increasing trend, peak at 0.003% increase in the 7th quarter, and slow down a little after that. The shock in real investment is expected to boost the real GDP by 0.006 % in the 3rd quarter and then slow down to 0.003% in the 15th quarter, while the increasing trend maintains. The 1% increase in real government spending is expected to boost real GDP by 0.0014% immediately, falling to 0.0006% in the 5th quarter and hastening the growth again gradually afterward.

Figure 12. *Response of log_gdpc1 to a shock in log_gpdic1*



Among the three graphs, only the investment vs. GDP response graph, Figure 12, exhibits a limited leeway of how likely the investment might not boost GDP. For the consumption vs. GDP graph, the level of increase in GDP can potentially vary. Some theorists (Sumner, 2020) argue that individual consumption can only boost GDP if consumption boosts nominal GDP first and if more nominal GDP boosts real GDP. This may be realized when the central bank is Figure 13. Response of log*_gdpc1* to a shock in log*_gcec1* incompetent and when the wages are sticky. The latter should be the case, but the Fed is undoubtedly not incapable. Therefore, while it's unlikely that the increased consumption will lower the output, it often varies to what degree it boosts the output.

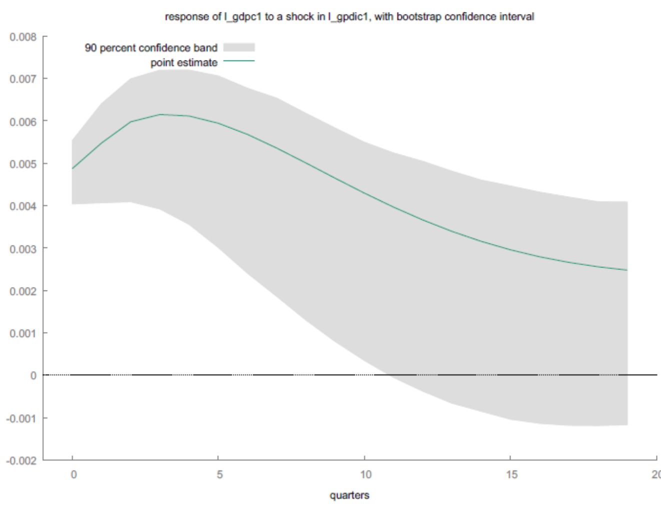
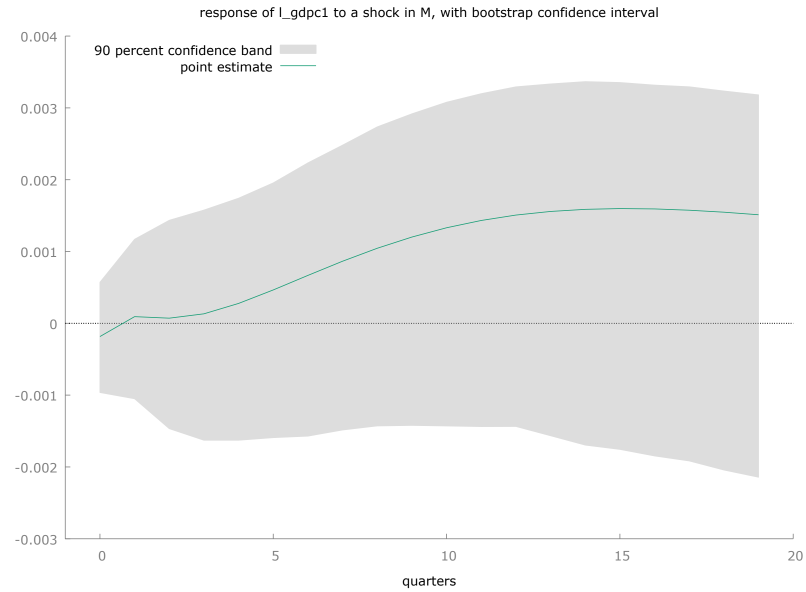


Figure 13. *Response of log_gdpc1 to a shock in log_gpdic1*

The sudden increase in government spending can often be part of the expansionary fiscal policy. In the IS-LM model, the fiscal expansion shifts the IS curve to the right and, for any given price level, raises income or output for any given price level. The impulse response graph of real GDP in response to real government spending, shown in Figure 13, generally accepts this result. At the same time, the negative segment of the 90% confidence band leaves out the possibility for increasing government spending to leave the output somehow unchanged or even lower output. The idea is that if government spending causes the unemployed to gain jobs, they will have more income to spend, leading to a final increase in GDP. However, if the economy is at full capacity, the increase in government spending would tend to crowd out the private sector leading to no net increase in GDP from private sector spending to government sector spending. Moreover, increasing government spending may sometimes decrease economic growth due to inefficient use of money.

Figure 14. *Response of log_gdpc1 to a shock in M*



The impulse response graph of real GDP in response to a shock in real money supply is shown in Figure 14. The initial reaction is surprisingly negative, showing the immediate response is that real GDP decreases by 0.00016%, but the subsequent response goes over to the positive territory, offering a roughly decennial increase of 0.001% of output or income. This loosely accepts the idea in the IS-LM model that a monetary expansion practiced as an increase in money supply raises real money balance, shifts the LM curve downward, and raises income. However, again, the bootstrap confidence band also brackets a decent negative portion, indicating the possibility that the income level might not necessarily increase. This may be because the output is expected to be fixed in the long run, so money supply affects the actual production of goods and services in the short run due to sticky prices and wages, but may only cause the prices to change in the longer run (Mathai, 2020).

6. Conclusions

The paper aims to use empirical estimation to verify some of the basic comparative statics in the IS-LM model. Theoretically, an increase in interest rate leads to decreased investment; an increase in consumption, investment, and government spending leads to increased output; an increase in money supply leads to increased output. The impulse response functions of our VAR model show that H1 is rejected when using Aaa corporate bond yield as interest rate, not rejected when using effective fed funds rate as the interest rate. This may be due to the high rate-of-returns of and investment strategies used on corporate bonds. H2, H3, and H4 are roughly accepted. At the same time, the output in response to increased consumption has some space for discussion, and the output in response to government spending may vary as well, depending on whether the economy is at full capacity or not. H5 is accepted, but some doubt remains in the long run. The idea is that the simple comparative statics in the IS-LM model all are reasonable hypothetically, while if the real-world data show contradictory results, it is crucial to think about what factors deviate the results from the theoretical basis, on top of the verifications or contradictions themselves.

7. Suggestions for further work

The use of the VAR model can be substituted for better models since VARs are atheoretical. They are not built on a particular economic theory that imposes a theoretical structure on the equations. Every variable is assumed to influence other variables symmetrically in the system. The sort of “mutual-regressive” process among all endogenous variables to some degree looks like data mining, where one reveals more than they think by over extracting relationships between variables without a theoretical basis. Our VAR is built upon the IS-LM model, but the complexity of macroeconomic variables could be addressed better in the modeling process. A structural VAR may be a possible methodology for future work.

Besides, relevant structural break tests or other out-of-sample tests could be conducted to evaluate the VAR model's forecast capacity and perhaps over-fitting issues. Breaking samples into estimation and holdout groups or using a Chow test is neither applicable in a VAR model.

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Appendix (ADF-tests and Co-integration test Outputs)

Table 9. ADF test for *gdpc1*

```
Augmented Dickey-Fuller test for gdpc1
testing down from 8 lags, criterion AIC
sample size 218
unit-root null hypothesis: a = 1

  test with constant
  including 2 lags of (1-L)gdpc1
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): 0.00166535
  test statistic: tau_c(1) = 1.49747
  asymptotic p-value 0.9993
  1st-order autocorrelation coeff. for e: -0.002
  lagged differences: F(2, 214) = 18.031 [0.0000]

  with constant and trend
  including 2 lags of (1-L)gdpc1
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0110373
  test statistic: tau_ct(1) = -1.59541
  asymptotic p-value 0.7952
  1st-order autocorrelation coeff. for e: -0.002
  lagged differences: F(2, 213) = 18.120 [0.0000]
```

Table 10. ADF test for *pcecc96*

```
Augmented Dickey-Fuller test for pcecc96
testing down from 8 lags, criterion AIC
sample size 216
unit-root null hypothesis: a = 1

  test with constant
  including 4 lags of (1-L)pcecc96
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): 0.00128929
  test statistic: tau_c(1) = 1.58846
  asymptotic p-value 0.9995
  1st-order autocorrelation coeff. for e: -0.005
  lagged differences: F(4, 210) = 20.961 [0.0000]

  with constant and trend
  including 3 lags of (1-L)pcecc96
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00606214
  test statistic: tau_ct(1) = -1.40955
  asymptotic p-value 0.8586
  1st-order autocorrelation coeff. for e: 0.023
  lagged differences: F(3, 211) = 26.634 [0.0000]
```

Table 11. ADF test for *gpdic1*

```
Augmented Dickey-Fuller test for gpdic1
testing down from 8 lags, criterion AIC
sample size 218
unit-root null hypothesis: a = 1

  test with constant
  including 2 lags of (1-L)gpdic1
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): 0.00127234
  test statistic: tau_c(1) = 0.293077
  asymptotic p-value 0.978
```

```
  1st-order autocorrelation coeff. for e: -0.004
  lagged differences: F(2, 214) = 14.974 [0.0000]

  with constant and trend
  including 2 lags of (1-L)gpdic1
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0352257
  test statistic: tau_ct(1) = -2.40567
  asymptotic p-value 0.3765
  1st-order autocorrelation coeff. for e: -0.009
  lagged differences: F(2, 213) = 17.036 [0.0000]
```

Table 12. ADF test for *gcec1*

```
Augmented Dickey-Fuller test for gcec1
testing down from 8 lags, criterion AIC
sample size 216
unit-root null hypothesis: a = 1

  test with constant
  including 4 lags of (1-L)gcec1
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00168203
  test statistic: tau_c(1) = -0.898578
  asymptotic p-value 0.7894
  1st-order autocorrelation coeff. for e: -0.008
  lagged differences: F(4, 210) = 11.682 [0.0000]

  with constant and trend
  including 4 lags of (1-L)gcec1
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0301968
  test statistic: tau_ct(1) = -2.82542
  asymptotic p-value 0.1879
  1st-order autocorrelation coeff. for e: -0.022
  lagged differences: F(4, 209) = 13.865 [0.0000]
```

Table 13. ADF test for *aaa*

```
Augmented Dickey-Fuller test for aaa
testing down from 8 lags, criterion AIC
sample size 217
unit-root null hypothesis: a = 1

  test with constant
  including 3 lags of (1-L)aaa
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0202981
  test statistic: tau_c(1) = -1.53947
  asymptotic p-value 0.5137
  1st-order autocorrelation coeff. for e: 0.003
  lagged differences: F(3, 212) = 4.716 [0.0033]

  with constant and trend
  including 3 lags of (1-L)aaa
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0239078
  test statistic: tau_ct(1) = -1.79891
  asymptotic p-value 0.7056
  1st-order autocorrelation coeff. for e: 0.004
  lagged differences: F(3, 211) = 4.501 [0.0044]

  with constant, linear and quadratic trend
  including 3 lags of (1-L)aaa
  model: (1-L)y = b0 + b1*t + b2*t^2 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0684128
  test statistic: tau_ctt(1) = -2.8641
```

```
  asymptotic p-value 0.3646
  1st-order autocorrelation coeff. for e: -0.002
  lagged differences: F(3, 210) = 5.060 [0.0021]
```

Table 14. ADF test for *fedfund*

```
Augmented Dickey-Fuller test for fedfund
testing down from 8 lags, criterion AIC
sample size 212
unit-root null hypothesis: a = 1

  test with constant
  including 8 lags of (1-L)fedfund
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0419762
  test statistic: tau_c(1) = -1.71099
  asymptotic p-value 0.4257
  1st-order autocorrelation coeff. for e: 0.013
  lagged differences: F(8, 202) = 5.597 [0.0000]

  with constant and trend
  including 8 lags of (1-L)fedfund
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0652613
  test statistic: tau_ct(1) = -2.41066
  asymptotic p-value 0.3739
  1st-order autocorrelation coeff. for e: 0.013
  lagged differences: F(8, 201) = 5.525 [0.0000]

  with constant, linear and quadratic trend
  including 5 lags of (1-L)fedfund
  model: (1-L)y = b0 + b1*t + b2*t^2 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.170268
  test statistic: tau_ctt(1) = -4.59711
  asymptotic p-value 0.004604
  1st-order autocorrelation coeff. for e: -0.010
  lagged differences: F(5, 206) = 8.207 [0.0000]
```

Table 15. ADF test for *M*

```
Augmented Dickey-Fuller test for M
testing down from 8 lags, criterion AIC
sample size 212
unit-root null hypothesis: a = 1

  test with constant
  including 8 lags of (1-L)M
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): 0.00784889
  test statistic: tau_c(1) = 4.05986
  asymptotic p-value 1
  1st-order autocorrelation coeff. for e: 0.033
  lagged differences: F(8, 202) = 22.548 [0.0000]

  with constant and trend
  including 8 lags of (1-L)M
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): 0.00431377
  test statistic: tau_ct(1) = 1.30445
  asymptotic p-value 1
  1st-order autocorrelation coeff. for e: 0.033
  lagged differences: F(8, 201) = 22.154 [0.0000]
```

```
Cointegrating regression -
OLS, using observations 1960:1-2015:1 (T = 221)
Dependent variable: l_gdpc1

             coefficient    std. error    t-ratio    p-value
  ----------------------------------------------------------
  const        2.70053      0.127648      21.16     3.72e-054 ***
  l_pcecc96    0.458609     0.0217462     21.09     5.87e-054 ***
  l_gpdic1     0.122057     0.00586394    20.81     3.84e-053 ***
  l_gcec1      0.170313     0.00942670    18.07     8.68e-045 ***
  aaa         −0.00386063   0.000500567   −7.713    4.74e-013 ***
  fedfund      0.00250826   0.000272081    9.219    3.07e-017 ***
  M            9.18070e-06  3.75519e-05    0.2445   0.8071
  time         0.00268107   0.000208461   12.86     2.31e-028 ***
  timesq      −3.55870e-06  5.70949e-07   −6.233    2.44e-09  ***

Mean dependent var   9.023517   S.D. dependent var   0.490067
Sum squared resid    0.006670   S.E. of regression   0.005609
R-squared            0.999874   Adjusted R-squared   0.999869
Log-likelihood       836.5242   Akaike criterion    −1655.048
Schwarz criterion   −1624.465   Hannan-Quinn        −1642.699
rho                  0.777937   Durbin-Watson        0.443926

Augmented Dickey-Fuller test for uhat
including 2 lags of (1-L)uhat
sample size 218
unit-root null hypothesis: a = 1

  test without constant
  model: (1-L)y = (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.175128
  test statistic: tau_ctt(7) = -3.81662
  asymptotic p-value 0.7259
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(2, 215) = 5.371 [0.0053]
```