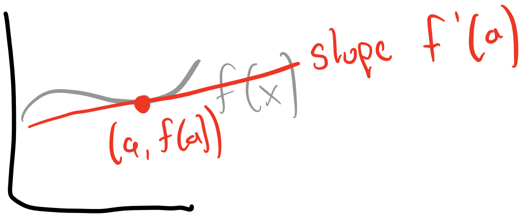
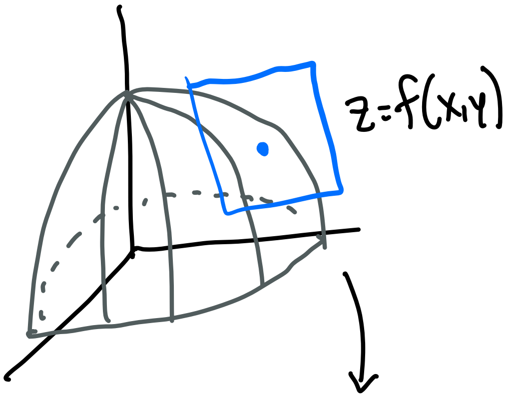
Calc 1 use tangent lines to approximate a function $f:\mathbb{R}\to\mathbb{R}$



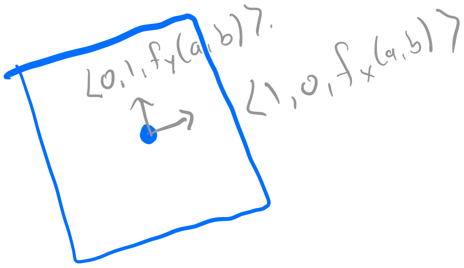
$$\boxed{L(x)-f(a)=f'(a)\,(x-a)}$$

Calc 3 use tangent planes to approximate a fctn $f:\mathbb{R}^2\to\mathbb{R}$.



- pt $(a,b,(f(a,b))$
- normal vector = cross product of

$$\langle 0,1,f_y(a,b)\rangle$$
$$\langle 1,0,f_x(a,b)\rangle$$



$$=\langle f_x(a,b),f_y(a,b),-1\rangle$$

tangent plane has eqn: $f_x(a,b)(x-a)+f_y(a,b)(y-b)-(z-f(a,b))=0$

$$\Rightarrow\quad z-f(a,b)=\underline{f_x(a,b)}(x-a)+\underline{f_y(a,b)}(y-b)$$ *if they exist!

$$L(x,y)=z=f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$$

linearization of $f(x,y)$ centered at (a,b).

Ex. Let $f(x,y)=xy^3+x^2$. Find eqn of tangent plane at $(2,-2,f(2,-2))$

$z=44-4x+24y$

We can sometimes approximate $f(x_0, y_0)$ using linearization of f at (a,b) if (x_0, y_0) is close to (a,b).

Ex. Approximate $(2.92)^2 \cdot \sqrt{4.08}$.

Find linearization of $f(x,y) = x^2\sqrt{y}$ centered at $(3,4)$

$f_x = 2x\sqrt{y}$ $\quad f_y = \frac{1}{2\sqrt{y}}x^2$ $\quad$ and $\quad f(3,4) = 18$

$f_x(3,4) = 12$ $\quad f_y(3,4) = \frac{9}{4}$

$L(x,y) = 18 + 12(x-3) + \frac{9}{4}(y-4)$

$$
\begin{aligned}
L(2.92, 4.08) &= 18 + 12(-.08) + \tfrac{9}{4}(.08) \\
&= 18 - 12\left(\tfrac{2}{25}\right) + \tfrac{9}{4}\left(\tfrac{2}{25}\right) \\
&= 18 - \tfrac{24}{25} + \tfrac{9}{2}\left(\tfrac{1}{25}\right) \\
&= 17 + \tfrac{1}{25} + 2.25\left(\tfrac{1}{25}\right) \\
&\approx 17 + .04 + 2.25\left(\tfrac{1}{25}\right) \\
&\approx 17 + .04 + .08 + .01 \\
&\approx 17.13
\end{aligned}
$$

$(2)\frac{1}{25} + (.25)\left(\frac{1}{25}\right)$

$.08 \quad + .01$

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The differential of f df is defined by

$$df = f_x(x,y)\,dx + f_y(x,y)\,dy$$

where $dx = \Delta x$ and $dy = \Delta y$. The differential measures change in height of the tangent plane for given changes dx and dy in x and y.