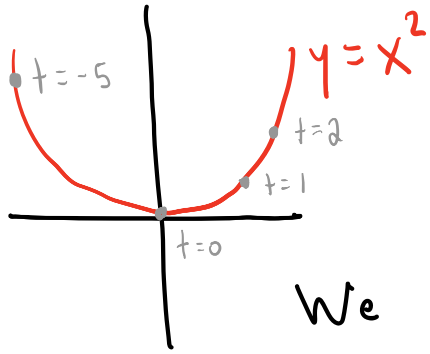
We can easily parameterize curves in $\mathbb{R}^2$

Last time we saw $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$ for $0 \leq t \leq 2\pi$ gives the unit circle.

To parameterize the curve cut out by $y = x^2$, set $x = t$.



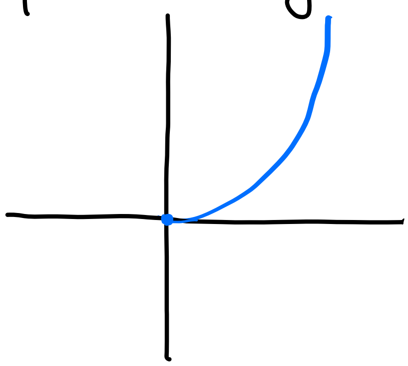
particle's path given by $\vec{r}(t) = \langle t, t^2 \rangle$

$-\infty < t < \infty$

We could set $y = t$. Then $x = \sqrt{t}$

Smaller domain.

$\vec{r}(t) = \langle \sqrt{t}, t \rangle$ but $0 \leq t < \infty$ and we obtain only half the graph



Ex 1. Give two parameterizations of line given by $3x + 2y = 6$ between $(2,0)$ and $(-2,6)$, one w/ $x = t$ and other w/ $y = t$

① $x = t$

$3t + 2y = 6$

$\Rightarrow y = \frac{-3}{2}t + 3$

$\vec{r}_1(t) = \langle t, 3 - \frac{3}{2}t \rangle$

$-2 \leq t \leq 2$

② $y = t$

$0 \leq t \leq 6$

$\vec{r}_2(t) = \langle 2 - \frac{2}{3}t, t \rangle$

$0 \leq t \leq 6$

A line in $\mathbb{R}^3$ is described:

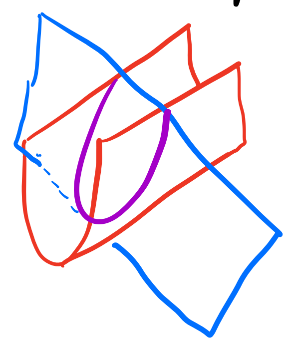
1) algebraically by a single variable vector parameterization
2) geometrically by the intersection of two planes

A curve in $\mathbb{R}^3$ is described:

1) algebraically by a single variable vector parameterization $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$
2) geometrically by the intersection of two surfaces

ex 2 Parameterize curve given by the system

$$\begin{cases} z = y^2 & \text{(hard taco shell)} \\ 2x + y - 3z = 6 & \text{(plane)} \end{cases}$$



Choose $y = t \Rightarrow z = t^2$

$\Rightarrow 2x + t - 3t^2 = 6 \Rightarrow x = 3 - \frac{1}{2}t + \frac{3}{2}t^2$

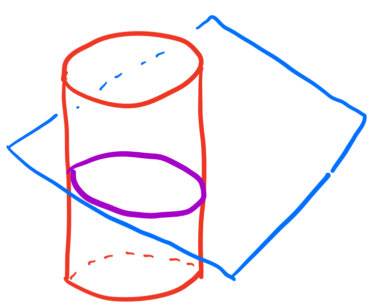
So the curve of intersection has the parameterization

$$\vec{r}(t) = \langle 3 - \tfrac{1}{2}t + \tfrac{3}{2}t^2, t, t^2 \rangle$$

Does this pt hit the pt $(6, -1, 1)$?

Ex 3. Parameterize curve given by the system

$$\begin{cases} x^2 + y^2 = 4 \\ 3x - z = 2 \end{cases}$$



$x = 2\cos(t),\ y = 2\sin(t) \Rightarrow z = -2 + 6\cos(t)$

$\vec{r}(t) = \langle 2\cos(t), 2\sin(t), 6\cos(t) - 2 \rangle$

$0 \leq t \leq 2\pi$.

Ex 4. Does the curve $\langle t^2+2t, 1-5t \rangle$ in $\mathbb{R}^2$ intersect the pt $(8,-9)$?