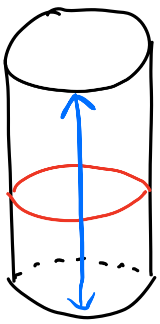
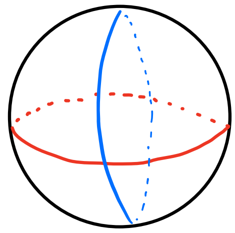
Parameterizing a curve means to express it using 1 variable

Parameterizing a surface means to express it using 2 variables

exs.

Let R be given





$G(\theta, z) = (\cos(\theta), \sin(\theta), z)$

$0 \le \theta \le 2\pi$

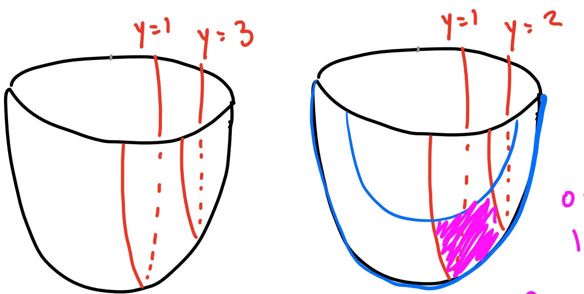
$-\infty \le z \le \infty$

$G(\theta,\phi) = (R\cos(\theta)\sin(\phi), R\sin(\theta)\sin(\phi), R\cos(\phi))$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi$

ex 2. Parameterize surface S given by $f(x,y) = 3x^2+y^2$ on the region $0 \le x \le 3$, $1 \le y \le 2$

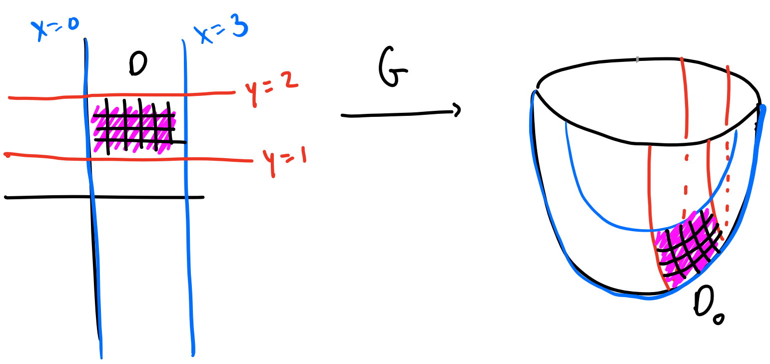


$0 \le x \le 3$
$1 \le y \le 2$

$G(u,v) = (u, v, 3u^2+v^2)$

$z = 3x+y^2$

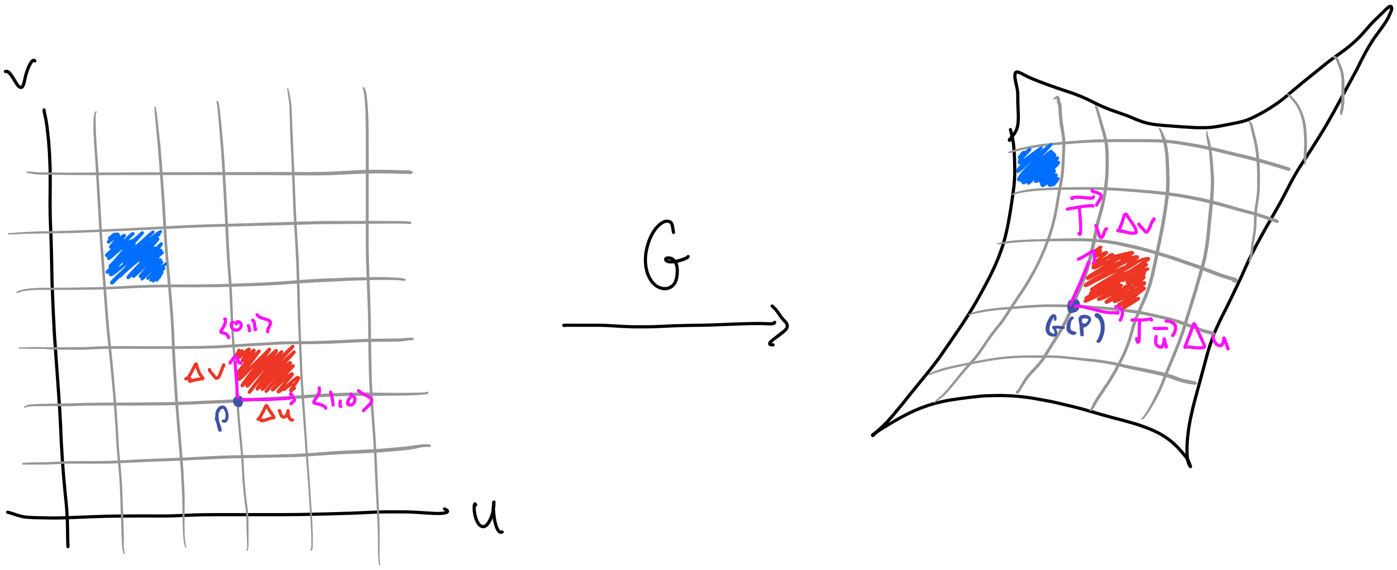
b) What is the area of that patch on S?



Recall $\vec{u}, \vec{v}$,
$\|\vec{u} \times \vec{v}\|$
= area of

Interim:

We can splice up D into rectangles, take area of each one, and then sum up all these areas. One way to subdivide D_0 is by tangent vectors to S.



has area $\Delta u \Delta v$. To approximate the area of G( ), we linearize G, using the partial derivatives G_u and G_v.

Consider the tangent vectors $\vec{T_u} = \frac{\partial G}{\partial u}$ and $\vec{T_v} = \frac{\partial G}{\partial v}$ at P. To cover the whole ,

scale $\vec{T_u}$ by Δu and $\vec{T_v}$ by $\vec{v}$. Therefore

G( ) has area $\|(\vec{T_u}\Delta u) \times (\vec{T_v}\Delta v)\|$

$$= \boxed{\|\vec{T_u} \times \vec{T_v}\| \cdot \Delta u \Delta v}$$

$$\iint 1 \, dS = \iint_D \|T_u \times T_v\| \, du\, dv$$

Back to example for $G(u,v)=(u,v,3u^2+v^2)$

$$G_u(u,v)=\langle 1,0,6u\rangle$$
$$G_v(u,v)=\langle 0,1,2v\rangle$$

$$G_u\times G_v=\langle -6u,-2v,1\rangle$$
$$\|G_u\times G_v\|=\sqrt{36u^2+4v^2+1}$$
$$\text{area}(D_0)=\int_0^3\int_1^2\|G_u\times G_v\|\,du\,dv=\boxed{\int_0^3\int_1^2\sqrt{36u^2+4v^2+1}\,du\,dv}$$

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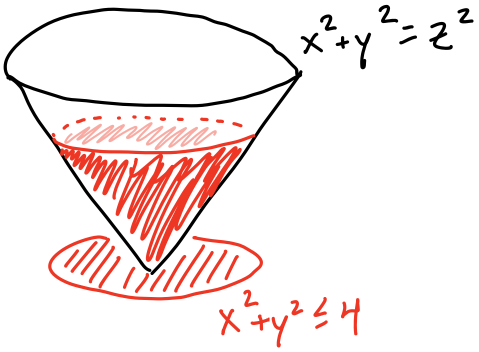
To integrate a function $f(x,y)$ over a surface area, simply compose with $G(u,v)$

General form of a surface integral of f over S

$$\boxed{\iint_S f(x,y,z)\,dS=\iint_D f(G(u,v))\,\|T_u\times T_v\|\,du\,dv}$$

Ex 3. Calculate $\iint_S x^2 z \, dS$ where S is portion of $x^2+y^2=z^2$ lying above $x^2+y^2 \leq 4$.



Cylindrical coords make sense. First, parameterize S!

$x^2+y^2=z^2 \implies r^2=z^2 \implies r=z$ ← gives a parameterization.

$x^2+y^2\leq 4 \implies r^2 \leq 4 \implies r=2.$ ← gives a bound

Let $G(r,\theta)=(r\cos\theta, r\sin\theta, r)$ for $0\leq\theta\leq 2\pi$ and $0\leq r\leq 2$.

$T_r = \frac{\partial G}{\partial r} = \langle \cos\theta, \sin\theta, 1\rangle$ and $T_\theta = \langle -r\sin\theta, r\cos\theta, 0\rangle$

Then $T_r \times T_\theta = \langle -r\cos\theta, -r\sin\theta, -r\rangle$ and so $\|T_u \times T_v\| = \sqrt{2}\, r.$