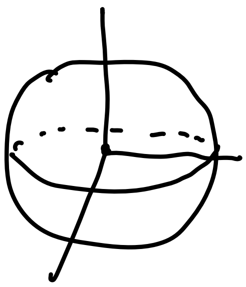
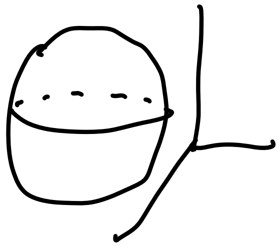
A (standard position) quadric surface is a surface given by

$$Ax^2+By^2+Cz^2+ax+by+cz+d=0$$



$x^2+y^2+z^2=1$



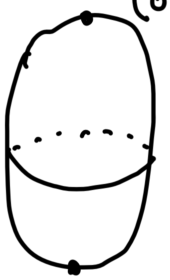
$(x-1)^2+(y+5)^2+(z-3)^2=2$

center $(1,-5,3)$

radius 2

Ellipsoid $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2+\left(\frac{z}{c}\right)^2=1$

(0,0,3)



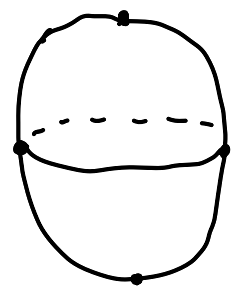
(0,0,-3)

$x^2+y^2+\left(\frac{z}{3}\right)^2=1$ stretched in the z-direction by 3.

$x^2+\left(\frac{y}{4}\right)^2+\left(\frac{z}{3}\right)^2$

(0,0,3)

(0,-4,0)



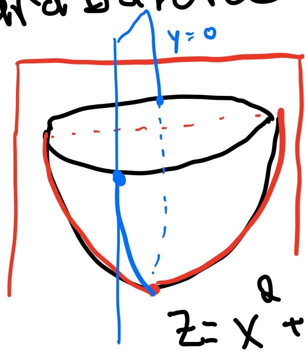
(0,4,0)

(0,0,-3)

1

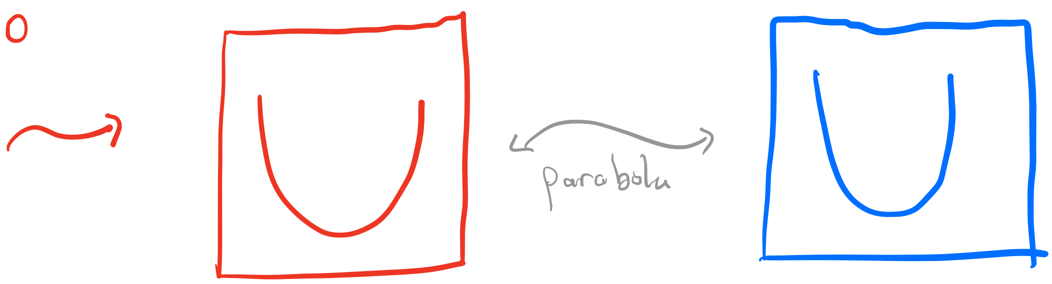
$$\boxed{\text{Ellipsoids } \left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2+\left(\frac{z}{c}\right)^2=1}$$

Paraboloids are surfaces who's vertical traces are $\boxed{\text{parabolas}}$

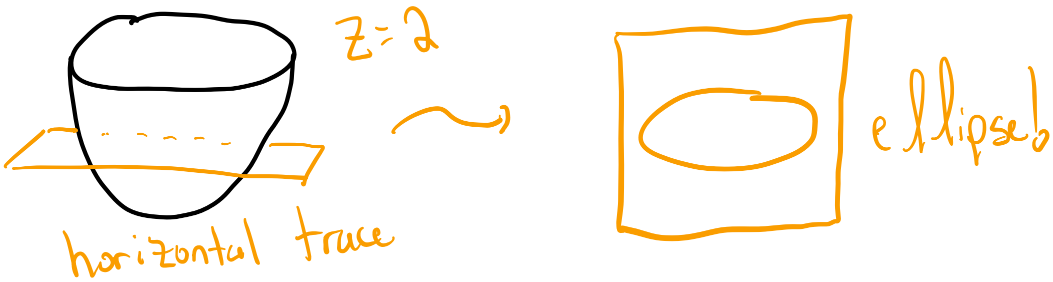


x=0

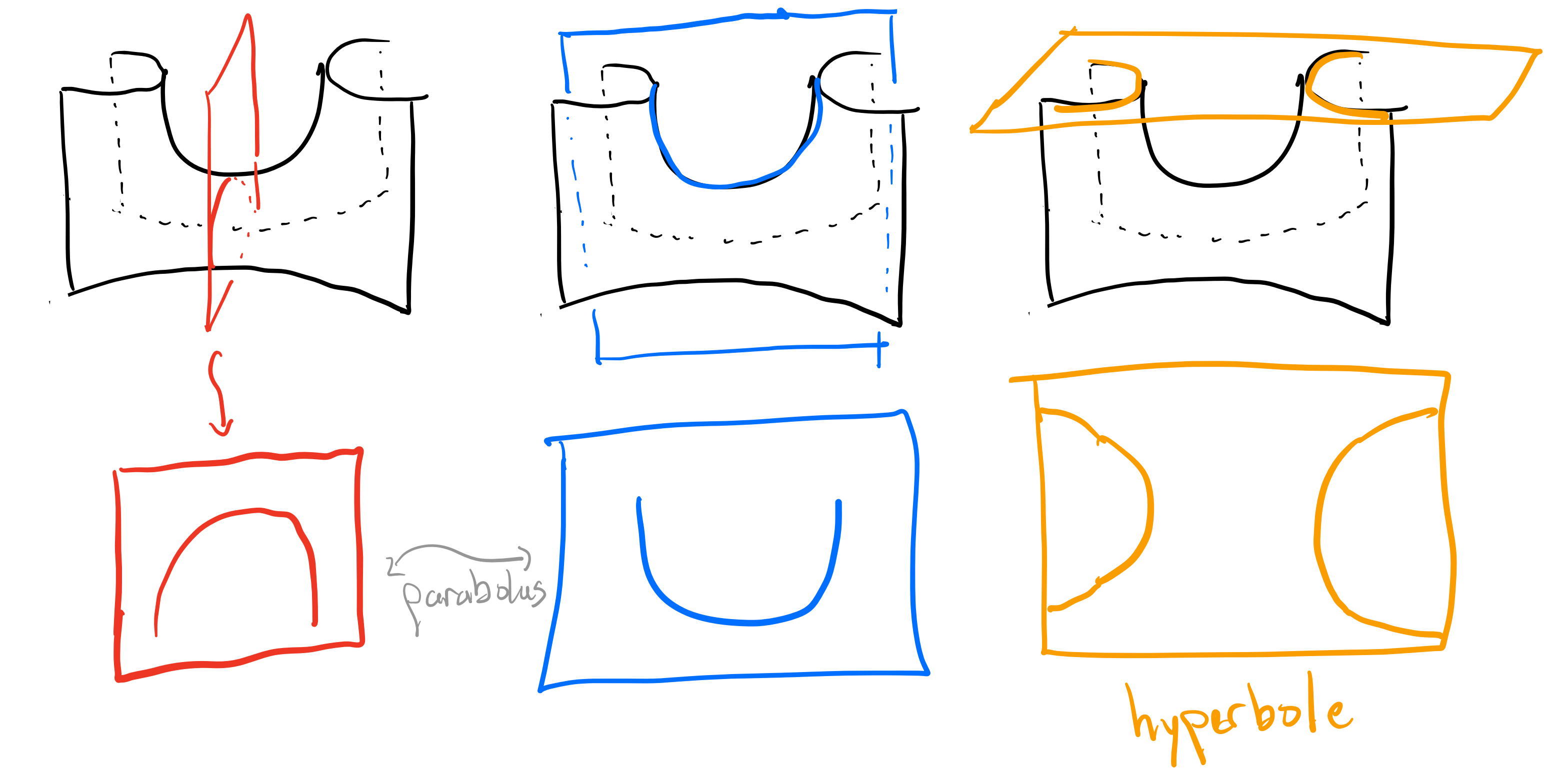
$z=x^2+y^2$



concavity is constant in every vertical trace.



For $z = x^2 - y^2$:



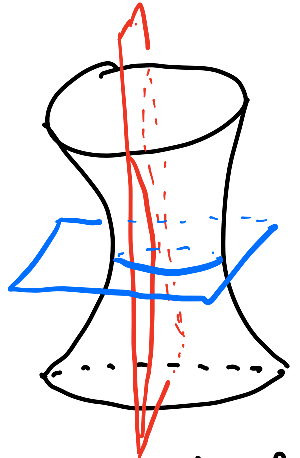
② **Paraboloids**

elliptic : $z = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2$

hyperbolic : $z = \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2$

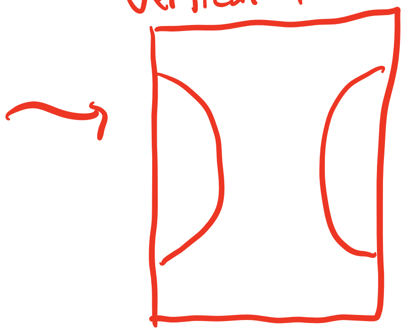
Hyperbolas → Hyperboloids, vertical traces are hyperboloids

$\mathbb{R}^2$ → $\mathbb{R}^3$



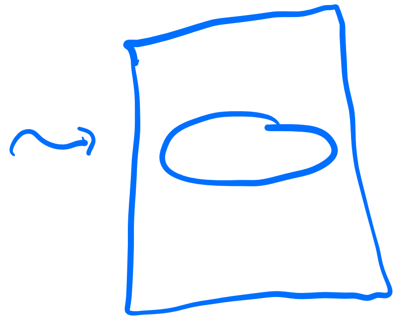
$x^2+y^2=z^2+1$

vertical trace



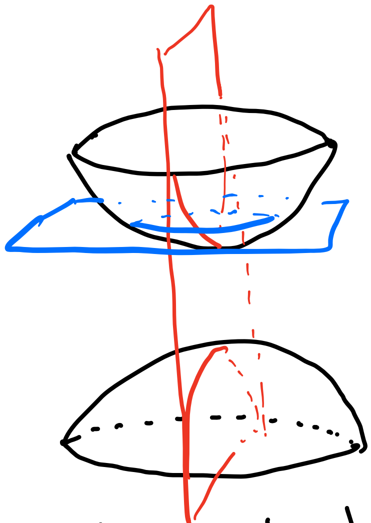
hyperbola

horizontal trace

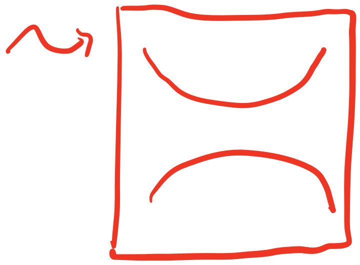


ellipse

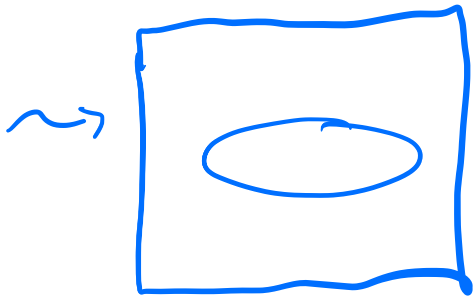
one sheet



$x^2+y^2=z^2-1$



hyperbola



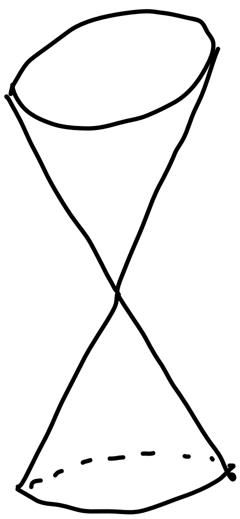
ellipse

two sheet

(3) Hyperboloids

one-sheet: $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\left(\frac{z}{c}\right)^2+1$

two-sheet: $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\left(\frac{z}{c}\right)^2-1$



(4) Double cone

$z^2=x^2+y^2$