A partial derivative measures the change of a function w.r.t. to one variable. In order to do this, we leave other variables fixed.

Let $f(x,y): \mathbb{R}^2 \longrightarrow \mathbb{R}$. Take derivative w.r.t. x and leave y fixed.

$f_x(x,y): \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$$f_x(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h} = \frac{\partial f}{\partial x} \stackrel{?}{=} \frac{df}{dx}$$

The function $f_x(x,y)$ measures the rate of change of $f(x,y)$ in the x-direction $\langle 1,0 \rangle$. I.e. if at the pt (a,b), x changes by an amt Δx and y is constant, the value of f will change by approx $f_x(a,b) \cdot \Delta x$,

$$\boxed{\Delta f \approx f_x(a,b)\,\Delta x}$$

ex. $f_x(0,0) = 5$, $f(0,0) = 10$, $\Delta x = \frac{1}{10} \Rightarrow \Delta f = \frac{1}{2}$. Then $f(.1,0) = 10.5$

Similarly

$$f_y(x,y) = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

measures the rate of change of $f(x,y)$ in the y-direction $\langle 0,1 \rangle$.

Ex 1. $R(x,y) = x^3 - 2xy + 3y$

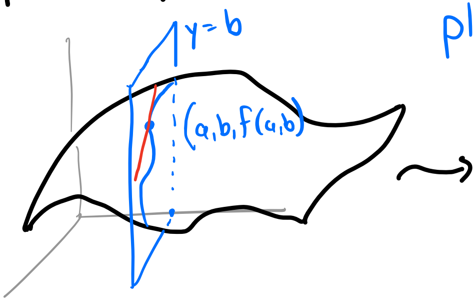
$R_x = 3x^2 - 2y$ $\qquad\qquad$ $R_y = -2x + 3$

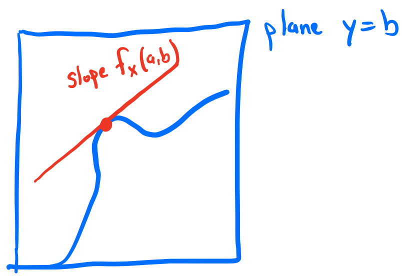
Ex 2. $f(x,y) = e^{x+5y} + x^3y^2$

$f_x = 1 \cdot e^{x+5y} + 3x^2y^2, \quad f_y = 5e^{x+5y} + 2x^3y$ //

$f_x(a,b)$ also measures slope of tangent line at $(a,b,f(a,b))$ in the plane $y=b$.

plane intersected with a surface is a curve.





Ex 2. $P(L,K) = (1.02)L^{3/4}K^{1/4}$ = production of boxes based on L laborers and K capital.

K in thousands

$P(100,60) \approx 90$ boxes. If we have 100 employess and \$60,000 in capital, we'll make about 90 boxes.

$P_L(100,60) \approx 0.67$ boxes per employee.

$\approx \frac{2}{3}$

If we currently have 100 employees and \$60,000 capital but we change number of employees, we can make .67 boxes per employee.

$P_K(100,60) \approx .37 \frac{\text{boxes}}{\$10{,}000}$

$\approx \frac{1}{3}$

If we currently have 100 employees and \$60,000 capital but we change number of employees, we can make .37 boxes per \$10,000.

Approximate $P(106,58)$: hire 6 employees and spend 20,000 capital.

$$\Delta P \approx \frac{2}{3}\cdot 6 + \frac{1}{3}(-2) = \frac{10}{3} \text{ boxes}$$

=

If at the pt (a,b) and we change x, y by Δx and Δy, then

$$\Delta f = f(a+\Delta x, b+\Delta y) - f(a,b) \approx f_x(a,b)\Delta x + f_y(a,b)\Delta y$$

Higher order derivatives

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx} \qquad \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = f_{yx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = f_{xy} \qquad \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = f_{yy}$$

Clairouts $f_{xy} = f_{yx}$ for nice enough functions $f : \mathbb{R}^2 \to \mathbb{R}$.

i.e. order of differentiation doesn't matter

ex. $R(x,y) = x^3 - 2xy + 3y$

$$R_x = 3x^2 - 2y \qquad\qquad R_y = -2x + 3$$

$R_{xx} = 6x$ $\quad$ $R_{xy} = -2$ $\quad$ $R_{xx} = -2$ $\quad$ $R_{yy} = 0$