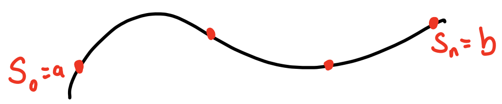
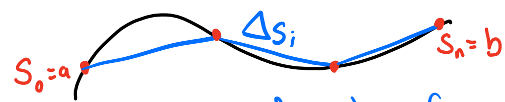
Chapter 15 focused on integrating over regions. Line integrals will integrate over curves. We've seen something like this before.

<u>Recall</u> given $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ on an interval $[a,b]$, Denote $C = \vec{r}(t)$ for $a \le t \le b$. We have

$$\text{length of } \vec{r}(t) \text{ over } [a,b] = \int_a^b \|\vec{r}\,'(t)\|\,dt$$



after specifying subintervals $\longrightarrow$

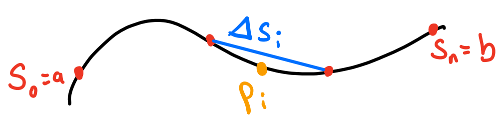


we took length of each subinterval

and then let $n \to \infty$, accumulating lengths to give length of curve.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \Delta s_i = \int_a^b \|\vec{r}\,'(t)\|\,dt$$

Suppose we are given a function $f(x,y,z)$. In addition to computing the length of a sub-interval Δs_i, pick a sample point p_i and sum up, we obtain the integral



$$\lim_{n\to\infty} \sum_{i=1}^{n} f(p_i)\,\Delta s_i = \int_C f(x,y,z)\,ds$$

For $f(x,y,z)=1$, we have $\int_C ds = \text{length}(C) = \int_a^b \|\vec{r}'(t)\| dt$

"$ds = \|r'(t)\| dt$"

So to compute line integrals of f over a curve C, parameterized by $\vec{r}(t)$ on $[a,b]$, we have

$$\boxed{\int_C f(x,y,z)\, ds = \int_a^b f(\vec{r}(t)) \cdot \|r'(t)\| dt}$$

<u>Ex 1</u>. Compute $\int_C f ds$ where $f = x^2 z$ and $\vec{r}(t) = \langle e^t, \sqrt{2}t, e^{-t} \rangle$ for $0 \le t \le 1$.

$$\int_C f ds = \frac{1}{2}(e^2+1)$$

<u>Ex 2</u>. An object is cut out by region contained in $y = 2x^2$ and $y = 8$. It has a density function $\rho(x,y) = \frac{y}{x}$ g/cm. Set up mass density of object on the boundary.