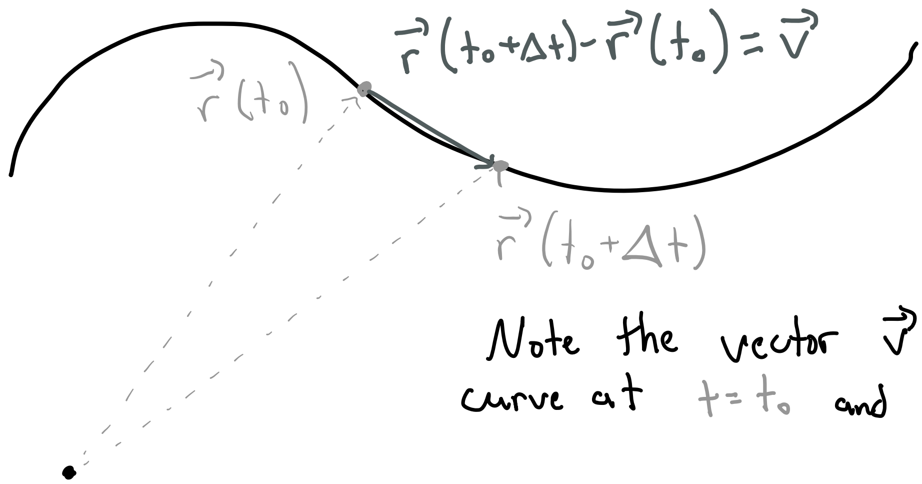
A curve in $\mathbb{R}^3$ is given by a vector parameterization

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

where $x(t), y(t), z(t)$ are single var. functions from $\mathbb{R}$ to $\mathbb{R}$.



Note the vector $\vec{v}$ is secant to the curve at $t = t_0$ and $t = t_0 + \Delta t$

As $\Delta t \to 0$, the tip of $\vec{r}(t_0+\Delta t) - r(t_0)$ inches closer to $\vec{r}(t_0)$ on the curve. Taking the limit $\Delta t \to 0$, we obtain a vector that is tangent to the curve at $t = t_0$.

<u>define</u> $$\frac{d}{dt}\vec{r}(t) = \vec{r}\,'(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

In terms of the functions $x(t), y(t), z(t)$.....

$$\boxed{\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle}$$

<u>Ex 1</u>. Compute $r''(3)$ where $\vec{r}(t) = \langle \ln(t), t, t^2 \rangle$.

$r''(3) = \langle \frac{-1}{9}, 0, 2 \rangle$

But don't forget abt continuity! $\lim_{t \to t_0} \vec{r}(t) = \vec{r}(t_0)$

$$\vec{r}(t) \text{ is cont. at } t=t_0 \iff \begin{array}{l} x(t) \text{ is cont. at } t=t_0 \\ y(t) \text{ is cont. at } t=t_0 \\ z(t) \text{ is cont. at } t=t_0 \end{array}$$

$\vec{r}(t)$ is differentiable at $t=t_0 \iff x(t), y(t), z(t)$ are differentiable at $t=t_0$.

<u>Fact</u> Differentiation rules carry over from Calc 1.

- $\frac{d}{dt}\left(\vec{r}_1(t) + \vec{r}_2(t)\right) = \vec{r}_1'(t) = \vec{r}_2'(t)$ (sum)
- $\frac{d}{dt}\left(c\,\vec{r}(t)\right) = c\, r'(t)$ (constant scalar)

$f(t): \mathbb{R} \longrightarrow \mathbb{R}$, a regular scalar-valued fctn.

- $\frac{d}{dt}\left(f(t)\vec{r}_1(t)\right) = f'(t)\,\vec{r}_1(t) + f(t)\,\vec{r}_1'(t)$ (scalar product)
- $\frac{d}{dt}\left(\vec{r}(f(t))\right) = \vec{r}'(f(t)) \cdot f'(t)$ (chain rule)

<u>New differentiation rules</u>

- $\frac{d}{dt}\left(\vec{r}_1(t)\cdot\vec{r}_2(t)\right) = \vec{r}_1\,'(t)\cdot\vec{r}_2(t) + \vec{r}_1(t)\cdot\vec{r}_2\,'(t)$ (dot product)
- $\frac{d}{dt}\left(\vec{r}_1\times\vec{r}_2\right) = \vec{r}_1\,'(t)\times\vec{r}_2(t) + \vec{r}_1(t)\times r_2'(t)$ (cross product)

Ex 2. Compute $\frac{d}{dt}\left(\vec{r}(f(t))\right)$ $\quad \vec{r}(t) = \langle t^3, t^2, t\rangle$, $f(t) = \cos(t)$

$r'(t) = \langle 3\cos^2(t), 2\cos(t), 1\rangle \cdot (-\sin(t))$

Integration

Similarly, $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$ is integrable if $x(t)$, $y(t)$ and $z(t)$ are integrable. So

$$\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b x(t)\,dt,\ \int_a^b y(t)\,dt,\ \int_a^b z(t)\,dt \right\rangle$$

Ex3. The velocity of a particle is given by

$$\frac{d\vec{r}}{dt} = \left\langle 1 - 6\sin(3t), \frac{1}{5}t \right\rangle$$

Where is the particle at $t=4$ if $\vec{r}(0) = \langle 4, 1 \rangle$?

$r(4) = \langle 6 + 2\cos(12), 2.6 \rangle$

FTOC for vector-valued functions

If $\vec{r}(t)$ is continuous on $[a,b]$ and $\frac{d}{dt}\vec{R}(t) = \vec{r}(t)$, then

$$\int_a^b \vec{r}(t)\,dt = R(b) - R(a)$$

part 2

$$\frac{d}{dt}\int_a^t \vec{r}(s)\,ds = \vec{r}(t)$$