A multivariable fctn has multiple inputs and one output.

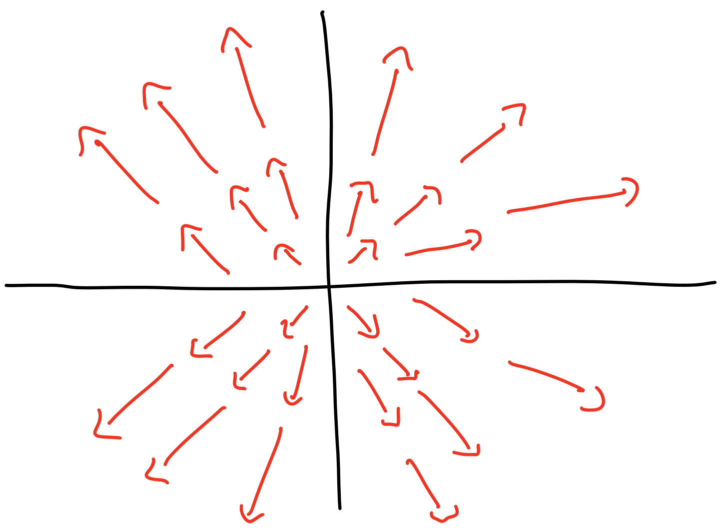
A vector-valued fctn has one input and multiple outputs.

A vector field is a fctn with multiple inputs and multiple outputs.

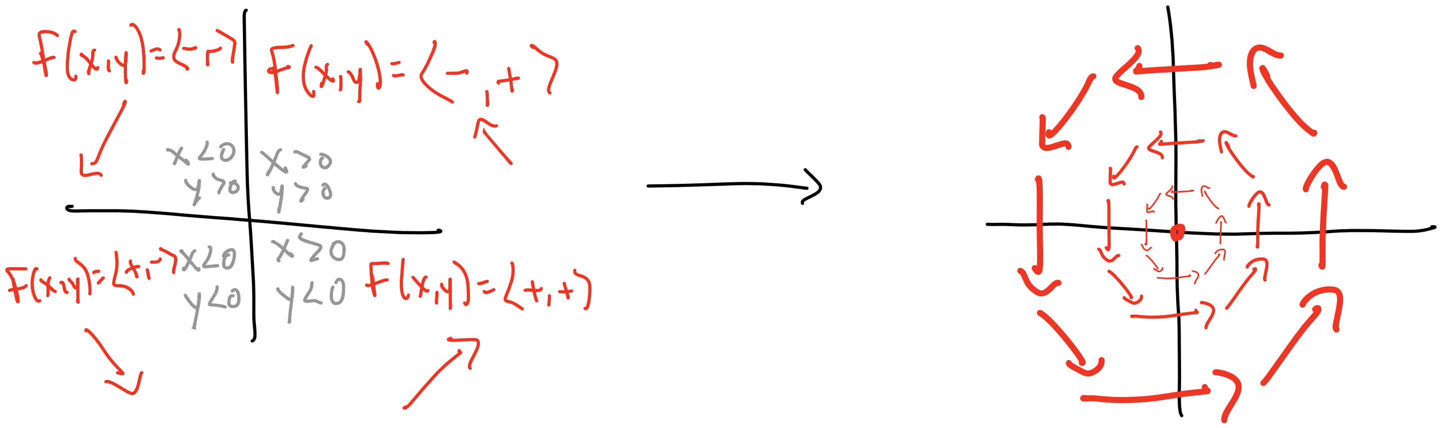
ex1. gradient of a fctn $f(x,y)=x^2+y^2$

$\nabla f(x,y) = \langle 2x, 2y \rangle$

For every point $(x,y) \in \mathbb{R}^2$, we obtain a vector



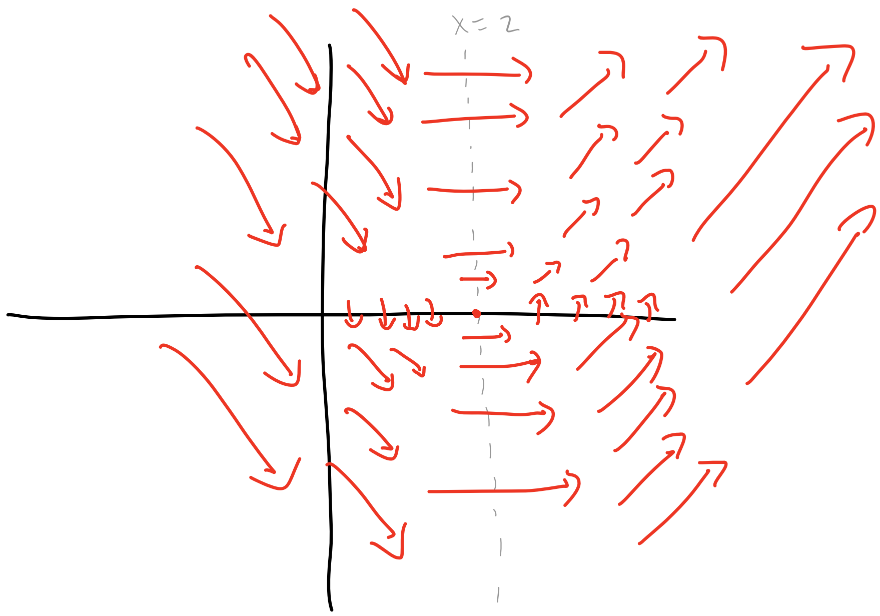
ex2 $F(x,y) = \langle -y, x \rangle$. To draw vector fields, one may observe the signs of coordinates in each quadrant.



Vector fields describe the flow of an object(s) or quantity.

<u>ex 3</u>. $F(x,y)=\langle y^2, x-2 \rangle$

- $y^2 > 0$ for any y, so vectors will face towards right.
- if $x=2$, have horizontal vectors
- if $x>2$, vectors are pointing up
- if $x<2$, vectors are pointing down



So a fctn can admit a vector field, via the gradient. We can call this vector field a gradient field.

$$f(x,y,z) \longrightarrow \nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

Can we obtain a function given a vector field?

$$f(x,y,z) \text{ such that } \xleftarrow{??} \vec{F} = \langle F_1(x,y,z), F_2(x,y,z), F_3(x,y,z) \rangle$$

$F_1 = f_x$

$F_2 = f_y$

$F_3 = f_z$?

Not always!!

nonexamples

ex 2 $F(x,y) = \langle -y, x \rangle$

$\int -y\,dx = -xy$, $\int x\,dy = xy$, these differ by xy

ex 3 $F(x,y) = \langle y^2, x-2 \rangle$

Suppose $f_x = y^2$ and $f_y = x-2$. By Clairaut's, we get

$f_{xy} = f_{yx} \Rightarrow 2y \neq 1$, a contradiction. So an f cannot exist.

examples

- $F = \langle y, x+z^2, 2yz \rangle$

$\int y\,dx = \boxed{xy}$, $\int x+z^2\,dy = \boxed{xy+yz^2}$, $\int 2yz\,dz = \boxed{yz^2}$

So $f(x,y,z) = xy + yz^2$ is a <u>potiential</u> function of F.

- $F = \langle 2xze^{x^2}, 0, e^{x^2} \rangle$

$\int 2xze^{x^2}\,dx = \boxed{ze^{x^2}}$ $\qquad \int e^{x^2}\,dz = \boxed{ze^{x^2}}$

$f(x,y,z) = ze^{x^2}$

$g(x,y,z) = ze^{x^2} + 5$

<u>Fact</u> Potiential fctns for the same vector field differ by a constant.

(2-dimn'l)

Test for non-conservativeness

Given $F = \langle F_1, F_2 \rangle$,

if $\dfrac{\partial F_1}{\partial y} \neq \dfrac{\partial F_2}{\partial x}$, then F is not conservative

This test is because of Clairauts!

(3-dimn'l) Given $F = \langle F_1, F_2, F_3 \rangle$,

if $\dfrac{\partial F_1}{\partial y} \neq \dfrac{\partial F_2}{\partial x}$, $\dfrac{\partial F_1}{\partial z} \neq \dfrac{\partial F_3}{\partial x}$ or $\dfrac{\partial F_2}{\partial z} \neq \dfrac{\partial F_3}{\partial y}$,

then F is not conservative.