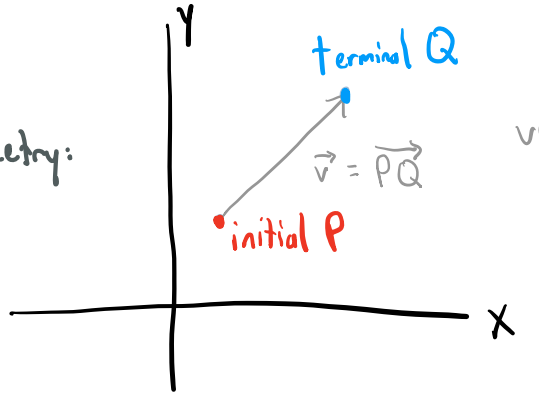
12.1 Vectors in the Plane

$\mathbb{R}$ = set of real numbers

$\mathbb{R}^2$ = set of pairs of real numbers. We usually graph this set on xy-plane.

geometry:



vector $\vec{v}$ is an arrow determined by 2 pts, one initial and one terminal. It pts in some direction and has some length.

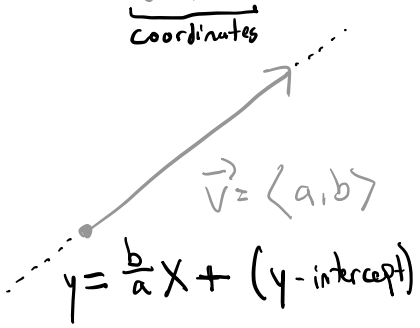
algebra: if $P = (a_1, b_1)$ (initial) $Q = (a_2, b_2)$ (terminal), then the vector $\vec{v}$ can be given by

$\vec{v} = \langle a_2 - a_1, b_2 - b_1 \rangle$.

$= \underbrace{\langle a, b \rangle}_{\text{coordinates}}$ where $a = a_2 - a_1$, $b = b_2 - b_1$

magnitude $= \|\vec{v}\| = \sqrt{a^2 + b^2}$

direction $= \frac{b}{a}$



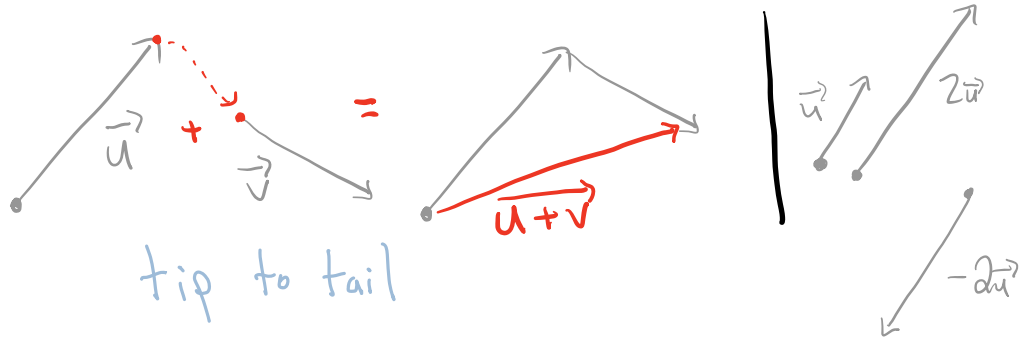
If initial P of $\vec{v}$ is the origin, the coordinates of $\vec{v}$ coincide with the terminal Q.

Vector Operations

vector addition

scalar multiplication

geometry:



algebra: $\vec{v} = \langle a, b \rangle$

$\vec{u} = \langle c, d \rangle$

$\vec{v} + \vec{u} = \langle a+c, b+d \rangle$

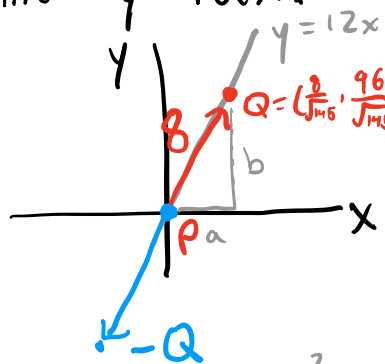
k a scalar

$k \cdot \vec{v} = \langle ka, kb \rangle$

Ex. Find a vector of length 8 w/ initial pt the origin and on the line $y = 12x$.

$\vec{v} = \overrightarrow{PQ}$, $P = (0,0)$

aside: $\overrightarrow{PQ}$, $P = (0,0)$, $Q = (a,b)$, $\overrightarrow{PQ} = \langle a, b \rangle$



$8 = \|\overrightarrow{PQ}\| = \sqrt{(b-0)^2 + (a-0)^2}$

$= \sqrt{b^2 + a^2}$

$64 = a^2 + b^2$ magnitude

$\frac{b}{a} = 12$

$b = 12a$ direction

$64 = a^2 + (12a)^2$

$64 = 145a^2$

$\frac{64}{145} = a^2$

$\pm \frac{8}{\sqrt{145}} = a$

$+ \rightarrow \left(\frac{8}{\sqrt{145}}, \frac{96}{\sqrt{145}} \right)$

$- \rightarrow \left(\frac{-8}{\sqrt{145}}, \frac{-96}{\sqrt{145}} \right)$

More on Scaling vectors

algebra

Vectors $\vec{v}$ and $\vec{u}$ are parallel if $\vec{v} = c \cdot \vec{u}$ for some $c \in \mathbb{R}^*$.

geometrically

$\vec{v}$ and $\vec{u}$ pt in the same or opposite direction.

} Parallel.

The vector $\vec{0} = \langle 0, 0 \rangle$ is the zero vector.

A vector with length 1 is a unit vector. To normalize a vector, divide by magnitude

$$\vec{v} \rightsquigarrow \frac{\vec{v}}{\|\vec{v}\|} \quad \text{length } 1$$

$$\rightsquigarrow \frac{c \cdot \vec{v}}{\|\vec{v}\|} \quad \text{length } |c| \text{ for } c \in \mathbb{R}^*.$$

Ex 2 Find a vector of length 12 that makes an angle of $5\pi/6$ with the positive x-axis.