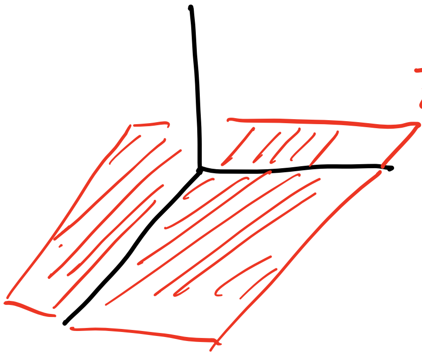
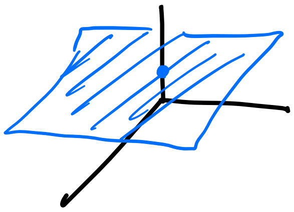
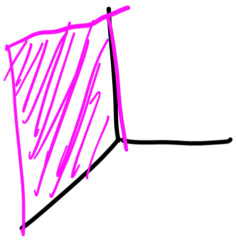
Planes are given by a linear equation $ax+by+cz=d$.



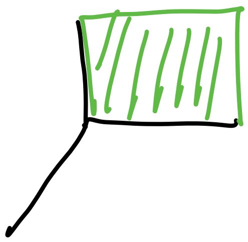
$z=0$ is the usual xy-plane



$z=3$ is xy-plane but translated 3 units upward

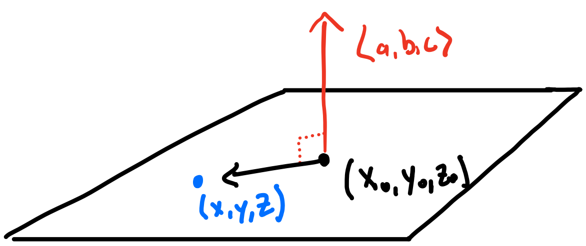


$y=0$ is xz-plane



$x=0$ is yz-plane

To write the eqn of a plane in $\mathbb{R}^3$, one needs a pt (x_0, y_0, z_0) and a normal vector $\langle a,b,c\rangle$ at the pt.

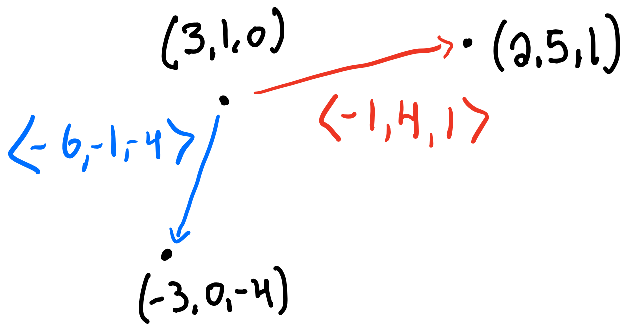


normal vector means a vector that is perpendicular to any other vector on the plane.

$$\langle a,b,c\rangle \cdot \langle x-x_0, y-y_0, z-z_0\rangle = 0$$

$$\Rightarrow \quad \left.\begin{array}{c} a(x-x_0)+b(y-y_0)+c(z-z_0)=0 \\ \text{or} \\ ax+by+cz=d \quad \text{where } d=ax_0+by_0+cz_0 \end{array}\right\} \text{standard scalar form of a plane}$$

Ex 1. Consider the plane P containing the pts $(3,1,0)$, $(2,5,1)$ and $(-3,0,-4)$. Find an equation of the plane P.



$$\begin{vmatrix} i & j & k \\ -6 & -1 & -4 \\ -1 & 4 & 1 \end{vmatrix}$$

$$= (-1+16)i - (-6-4)j + (-24-1)j$$

$$15i + 10j - 25j$$

$\Rightarrow \quad 3(x-3) + 2(y-1) - 5z = 0$

$\Rightarrow \quad 3x + 2y - 5z = 11$

Q: Does $6x + 4y - 10z = 22$ describe the same plane?

$\Rightarrow$ yes. why?

Planes in $\mathbb{R}^3$ must be parallel or intersect.

↳ if their normal vectors are parallel.

Ex 2 Find where the planes intersect $\begin{cases} 2x + y + z = 7 \\ x - y + 3z = 5 \end{cases}$ $\quad \langle 2,1,1\rangle$, $\langle 1,-1,3\rangle$

Method 1: pick whatever we want for any variable.

$z = 0$.

$\begin{matrix} 2x + y = 7 \\ x - y = 5 \end{matrix} \Rightarrow (4,-1,0)$ is a pt.

Method 2: find 2 pts that lie on the line.

Ex 3. Find the pt where the plane $3x-9y+2z=7$ and line $r(t)=\langle 1,2,1\rangle + t\langle 2,0,-1\rangle$.

$\Rightarrow (11,2,-4)$