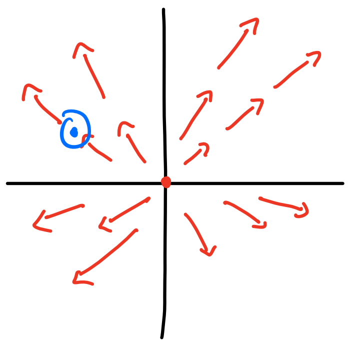
Given a vector field $F = \langle F_1, F_2, F_3 \rangle$, the divergence of F is

$$\operatorname{div}(F) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

The divergence of F is a fctn, so we can plug in pts.

Ex 1. Draw $\operatorname{div}(F)(P)$ where $F = \langle 2x, 3y \rangle$ and P are your fav. pts.



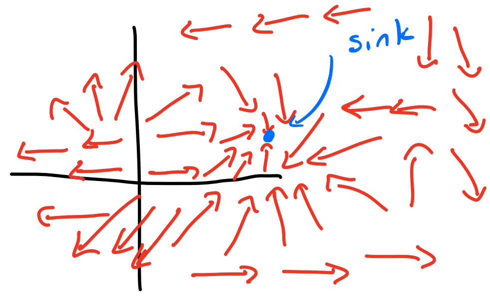
$\operatorname{div}(F)(P) = 5$ for any point P.

Why do we care about divergence? Divergence at a pt P measures the rate of change of vector field expansion (or contraction).

$\operatorname{div}(F)(P) > 0 \leadsto$
- vectors are expanding away from P
- we say P is a source
- flow is increasing away from P

ex 2.

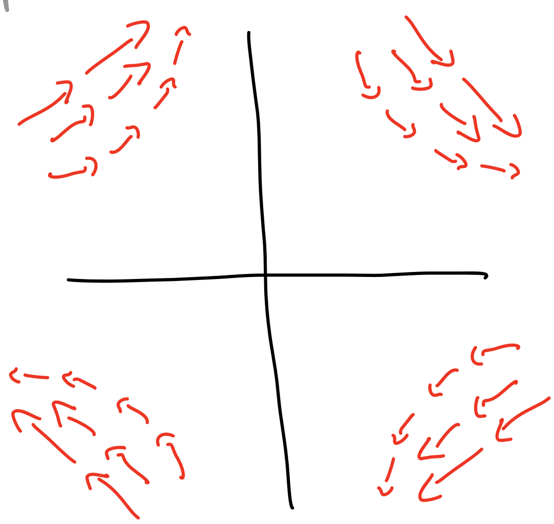
$\operatorname{div}(F)(P) < 0 \leadsto$
- vectors are compressing towards P
- we say P is a sink.
- flow is decreasing towards P



div(F)(P) = 0 $\rightsquigarrow$ flow going into P is equal to flow going out of P.

$F = \langle x^2y, -xy^2 \rangle$

div(F) = $2xy - 2xy = 0$ for every point.



Curl Given a vector field $F = \langle F_1, F_2, F_3 \rangle$, the curl of F is a vector field given by the (informal) cross product of

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \langle F_1, F_2, F_3 \rangle$$

$$= \left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right\rangle$$

Ex 2. Compute curl of $F = \langle yz, xz, xy \rangle$.

Recall that these are the Clairout conditions for conservativeness. If F is conservative, then $\text{curl}(F) = 0$.