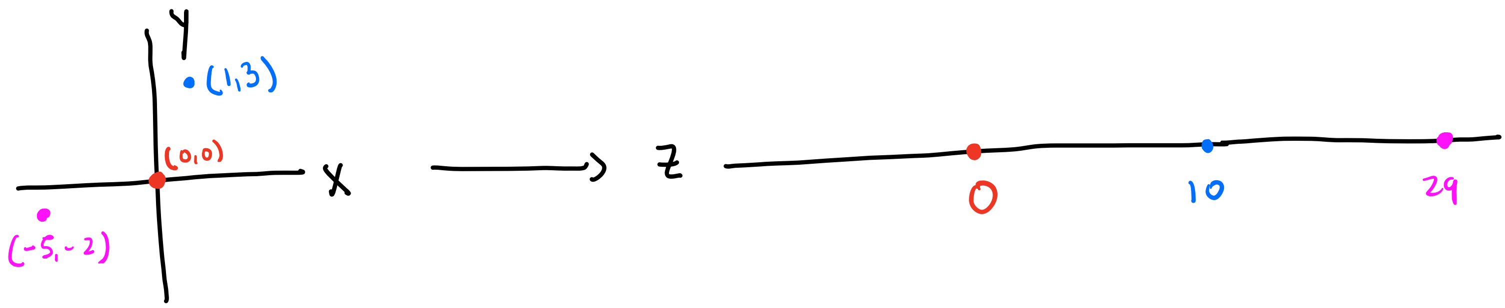
$f(x)$ and $\vec{r}(t)$ have been single variable fctns b.c. they take in only one input.

$f(x,y) = x^2 + y^2$, $\mathbb{R}^2 \xrightarrow{} \mathbb{R}$. The range of f is all possible outputs.

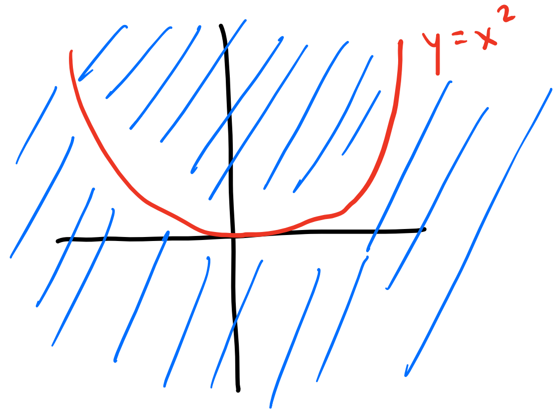
domain codomain

2 inputs 1 output



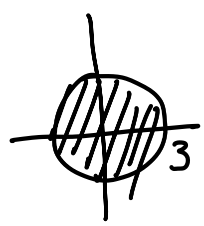
$g(x,y) = \dfrac{xy}{y - x^2}$ domain is everywhere where the denominator isn't zero.

domain(g): $\{ y - x^2 \neq 0 \}$



$h(x,y) = \sqrt{9 - x^2 - y^2}$.

domain is where $9 - x^2 - y^2 \geq 0 \Rightarrow 9 \geq x^2 + y^2$.



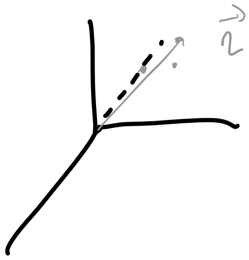
<u>Graphs</u> The graph of a function in 2 variables is

$$\{ (x, y, f(x,y)) \mid (x,y) \text{ is in domain of } f \} \subseteq \mathbb{R}^3$$

and we usually denote $z = f(x,y)$. So the graph of f exists in $\mathbb{R}^3$.

Ex 1. Graph $f(x,y) = 3x - 2y + 4$.

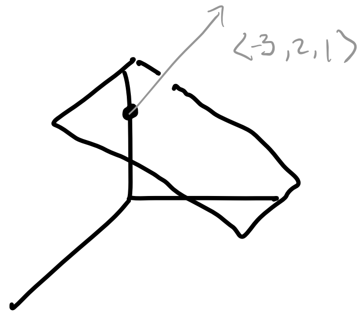
$z = 3x - 2y + 4 \implies -3x + 2y + z = 4$



$\vec{n} = \langle -3, 2, 1 \rangle$

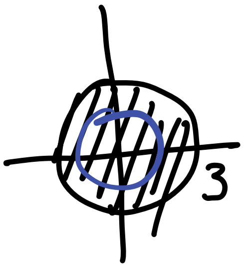
a pt on plane is $(0,0,4)$

intercepts.



Ex 2. $h(x,y) = \sqrt{9 - x^2 - y^2}$

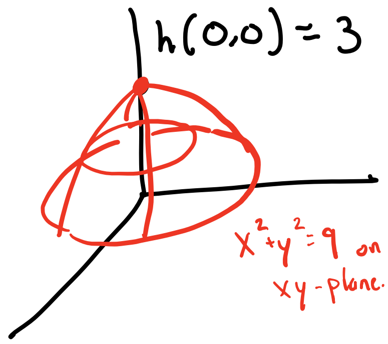
Strategy to draw graphs: Take 2 extremes and fill what's inbetween by continuity.



At $(0,0)$, $h(0,0) = 3$

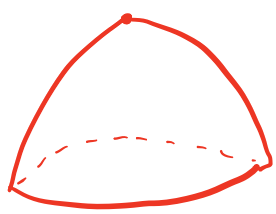
At $x^2 + y^2 = 9$, $h(x,y) = 0$

At $x^2 + y^2 = 5$, $h(x,y) = \sqrt{9-5} = 2$



Confirm by setting $z = \sqrt{9 - x^2 - y^2}$

$\implies z^2 + x^2 + y^2 = 9.$



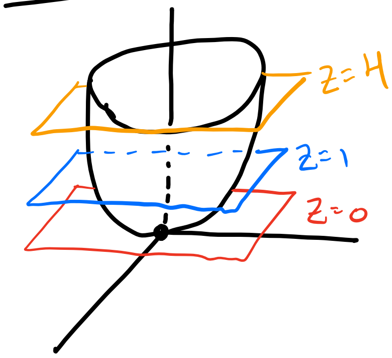
upper hemisphere of the sphere $x^2 + y^2 + z^2 = 3$

Neat fact: Stereographic projection.

Level Curves / Contour Maps

Look at traces parallel to the xy-plane

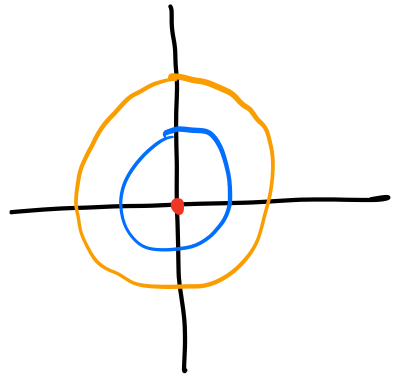
Ex 3. $f(x,y) = x^2 + y^2$



$4 = x^2 + y^2$

$1 = x^2 + y^2$

$0 = x^2 + y^2$

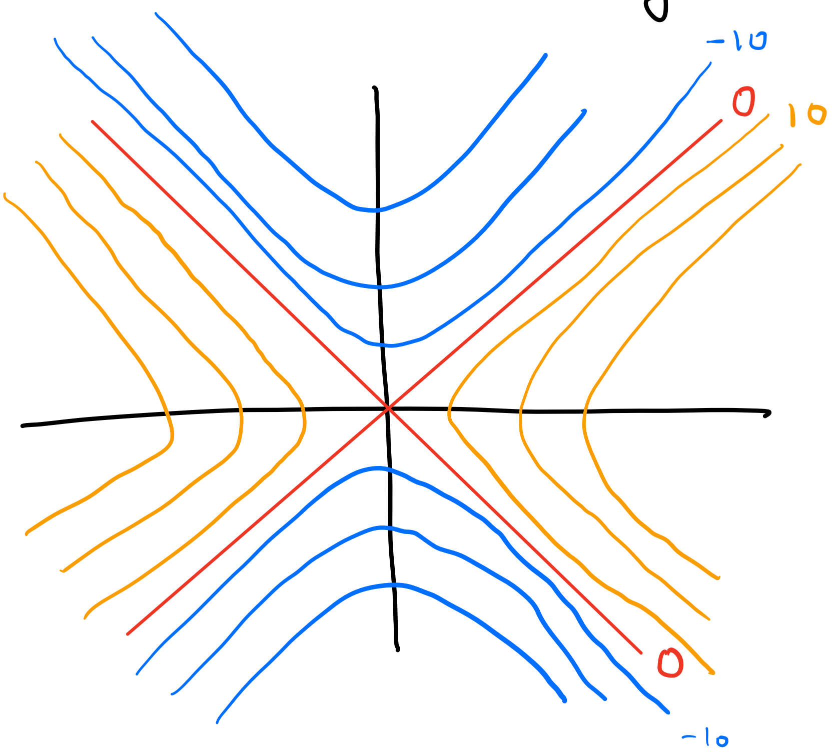


Ex 4. $g(x,y) = x^2 - 3y^2$

1) Suppose $0 = x^2 - 3y^2$. Then we may factor $x^2 - 3y^2$ as $(x-\sqrt{3}y)(x+\sqrt{3}y)$

2) Suppose $0 > c = x^2 - 3y^2$. Then we have a hyperbole opening up and down.

3) Suppose $0 < c = x^2 - 3y^2$. Then we have a hyperbole opening left and right.



1) velocity vs. speed

$r'(t_0)$ $\qquad$ $\|r'(t_0)\|$

length $\qquad$ number

2) $\sqrt{4+\frac{1}{t^2}+4t^2} = \sqrt{\left(2t+\frac{1}{t}\right)^2} = 2t+\frac{1}{t}$

directly factor

arclength $S = \int_a^b \|r'(t)\|\,dt.$