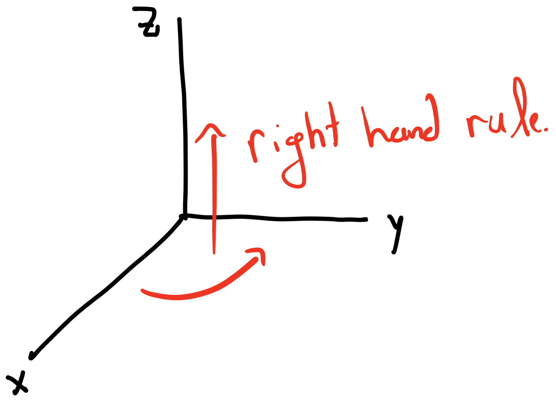
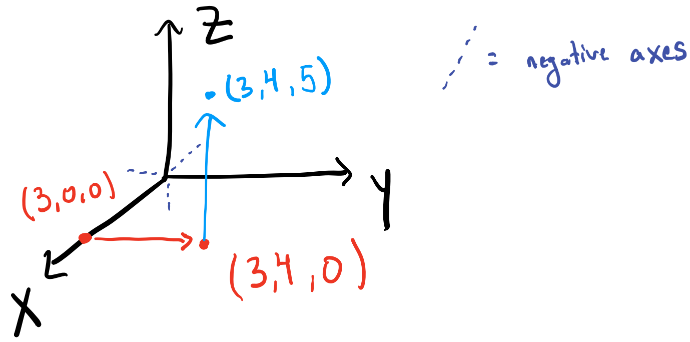
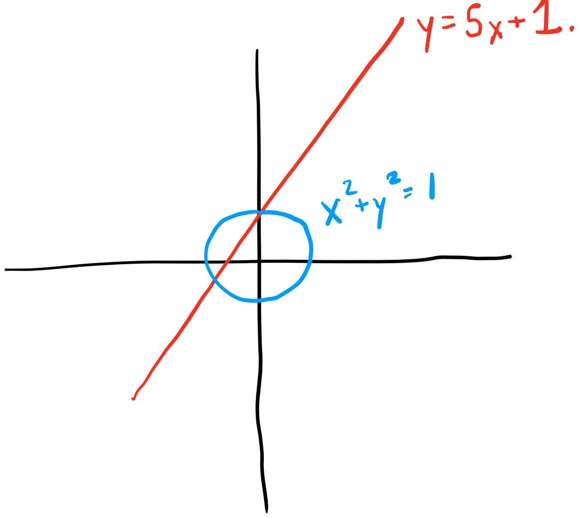
3d-Space: Surfaces, vectors and curves

$\mathbb{R}^3$ = Set of triplets of real numbers (a,b,c)

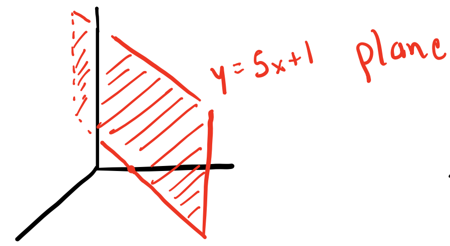


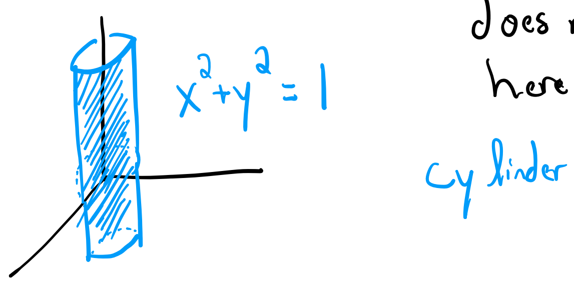


$\mathbb{R}^2$



$\mathbb{R}^3$





the z-coordinate does not matter here!

$y=mx+b$
$ax+by=c$ is a line in $\mathbb{R}^2$

dim 1

any polynomial = 0 is a curve

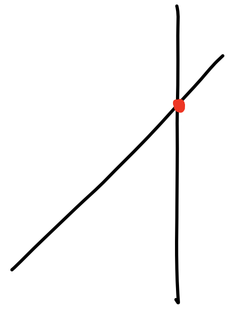
$ax+by+cz=d$ is a plane in $\mathbb{R}^3$

dim 2

any polynomial = 0 is a surface in $\mathbb{R}^3$

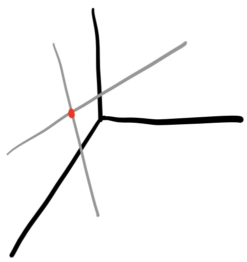
Intersections of stuffs

$\mathbb{R}^2$



line $\cap$ line $=$ pt   if lines aren't parallel or same.

$\mathbb{R}^3$



line $\cap$ line $=$ pt   if lines aren't parallel or skew or same.

line $\cap$ plane $=$ line, pt, or nothing. When?

plane $\cap$ plane $=$ line or nothing. Why not a pt?

What about the equation of a line? Geometrically, it is the intersection of 2 planes.
Algebraically, it is a $\boxed{\text{function}}$, need vectors to describe

<u>Vector</u>: Same as before but in one dimension higher. $\langle a, b, c \rangle$

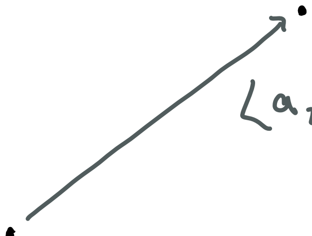
For a vector $\vec{v} = \langle a, b, c \rangle$, the magnitude is given by

$$\|\vec{v}\| = \sqrt{a^2 + b^2 + c^2}$$

direction?

Distance between 2 pts (a_1, b_1, c_1) and (a_2, b_2, c_2) is the length of the vector between them

(a_2, b_2, c_2)



$\langle a_2 - a_1, b_2 - b_1, c_2 - c_1 \rangle$

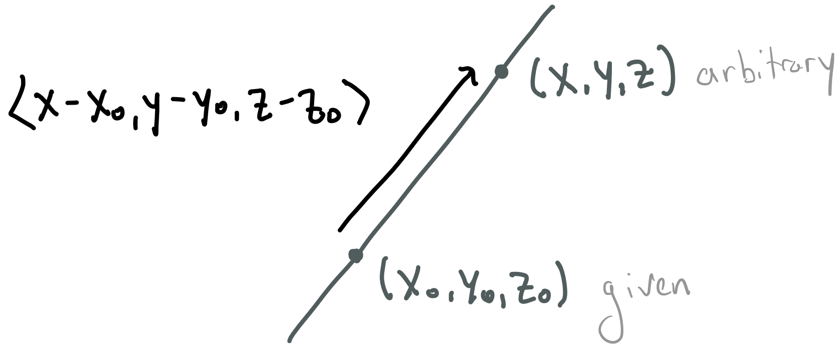
length is $\sqrt{(a_2-a_1)^2 + (b_2-b_1)^2 + (c_2-c_1)^2}$

(a_1, b_1, c_1)

For a line in $\mathbb{R}^2$, it suffices to have
- a pt ✓
- slope (direction) ?

For a line in $\mathbb{R}^3$, it suffices to have
- a pt $\langle x_0, y_0, z_0 \rangle$
- $\boxed{\text{direction vector}}$ $\langle a, b, c \rangle$

" pts in the direction of $\langle a,b,c \rangle$ "



$\Rightarrow \langle x-x_0, y-y_0, z-z_0 \rangle = t\langle a,b,c \rangle$ for some $t \in \mathbb{R}$.

$\Rightarrow \langle x,y,z \rangle = \underbrace{\langle x_0,y_0,z_0 \rangle}_{\text{given}} + \underset{\text{variable!}}{t}\,\underbrace{\langle a,b,c \rangle}_{\text{given}}$

$\Rightarrow r(t) = \langle x_0, y_0, z_0 \rangle + t\langle a,b,c \rangle$

(position) vector parametrization of the line L.

ex 1 $P = (1,0,2)$ Compute v.p. of $L = \overline{PQ}$

$Q = (2,5,1)$

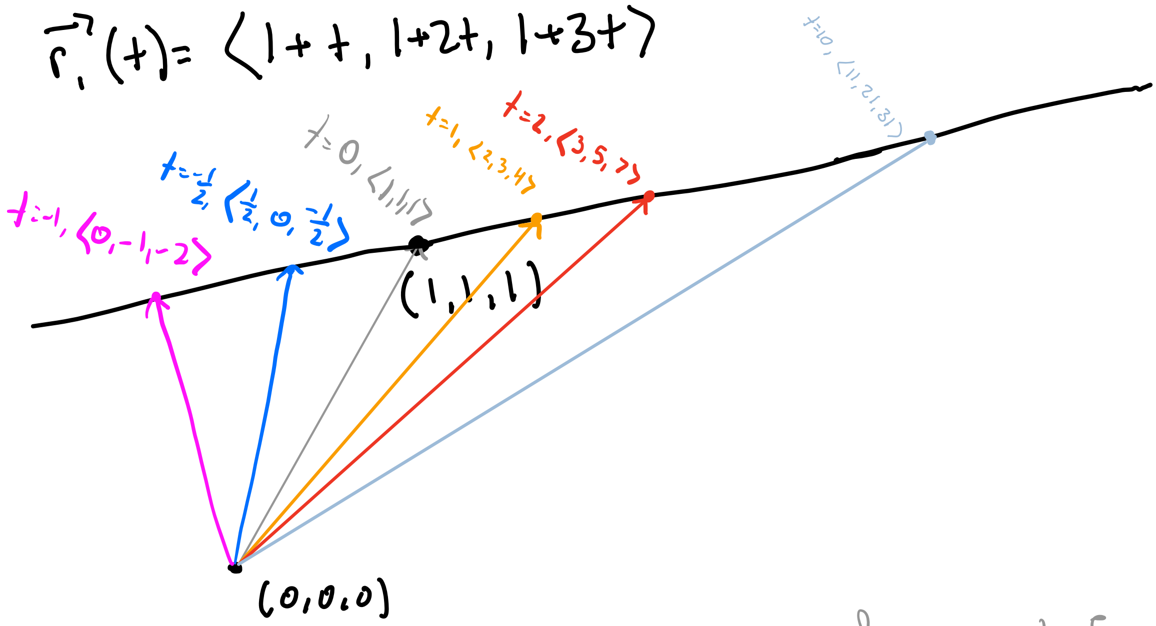
Parametric eqns of the line L are if we separated each component.

$x(t) = 1 + t$

$y(t) = 5t$

$z(t) = 2 - 3t$

$\vec{r}_1(t) = \langle 1+t,\ 1+2t,\ 1+3t \rangle$



Let's take a new point on the line $t=5$ gives pt $(6, 11, 16)$ and direction vector. $\langle \frac{1}{2}, 1, \frac{3}{2} \rangle$

$\vec{r}_2(t) = \langle 6, 11, 16 \rangle + t \langle \frac{1}{2}, 1, \frac{3}{2} \rangle$

t	$\vec{r}_1(t)$	$\vec{r}_2(t)$
0	$\langle 1,1,1 \rangle$	$\langle 6,11,16 \rangle$
1	$\langle 2,3,4 \rangle$	$\langle 6.5,12,17.5 \rangle$
2	$\langle 3,5,7 \rangle$	$\langle 7,13,19 \rangle$
6	$\langle 7,13,19 \rangle$	$\langle 9,17,25 \rangle$
5	$\langle 6,11,16 \rangle$	

So $\vec{r}_1(t)$ and $\vec{r}_2(t)$ are different parameterization's for the same line.

ex 2. When do lines $\vec{r}_1(t) = \langle 1-3t, t, 0 \rangle$ intersect?
$\vec{r}_2(t) = \langle 2t, 1, 1+t \rangle$

$t=1 \quad s=-1$

ex 3. When does the line $\vec{r}(t) = \langle 3+t, 2-3t, 1+2t \rangle$ hit the xy-plane? What pt on the xy-plane?

$t = \frac{-1}{2} \quad \langle 2.5, 3.5, 0 \rangle$

What if
$$x(t) = \cos(t)$$
$$y(t) = \sin(t)$$
$$z(t) = t$$
?

Is this a line?
No, but its a curve.