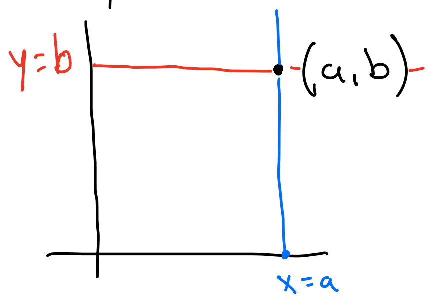
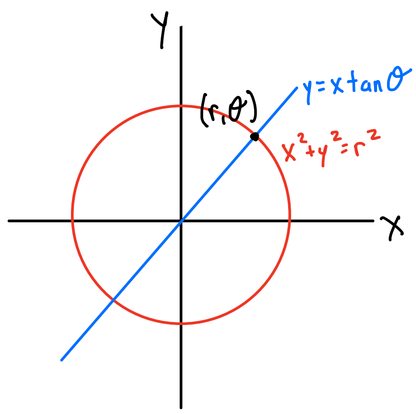
Coordinates in x,y are called rectangular coordinates.



The pt (a,b) in rectangular coordinates means the pt is the intersection of the lines $x=a$ and $y=b$.

We can swap to polar coordinates to encode different information. A pt $p=(r,\theta)$ in polar coordinates means p lies on the intersection of $x^2+y^2=r^2$ and $y=x\tan\theta$



We are able to go from polar coordinates to rectangular coordinates <u>or</u> from rectangular coordinates to polar coordinates.

Polar to rectangular given (r,θ) in polar coordinates, then in rectangular coordinates we have

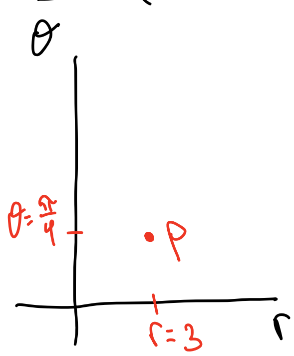
$$x = r\cos\theta$$
$$y = r\sin\theta$$

Rectangular to polar given (x,y) in rectangular coordinates, then in polar coordinates we have

$$r = \sqrt{x^2+y^2}$$
$$\tan\theta = \frac{y}{x}, \quad x \neq 0$$

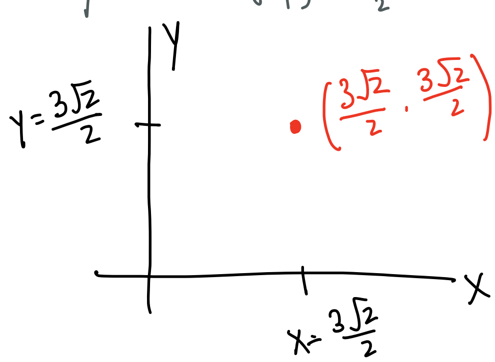
(otherwise, $\theta = \pm\frac{\pi}{2}$ depending on sign of y)

Ex. $(P \to R)$ Consider $p = (3, \frac{\pi}{4})$ in polar, express in rectangular.

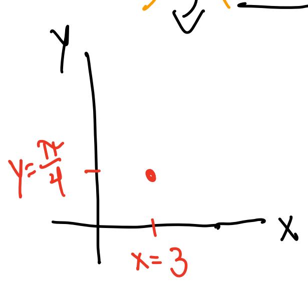


change of coordinates (correctly) $\longrightarrow$

$$x = 3\cos\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}$$
$$y = 3\sin\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}$$



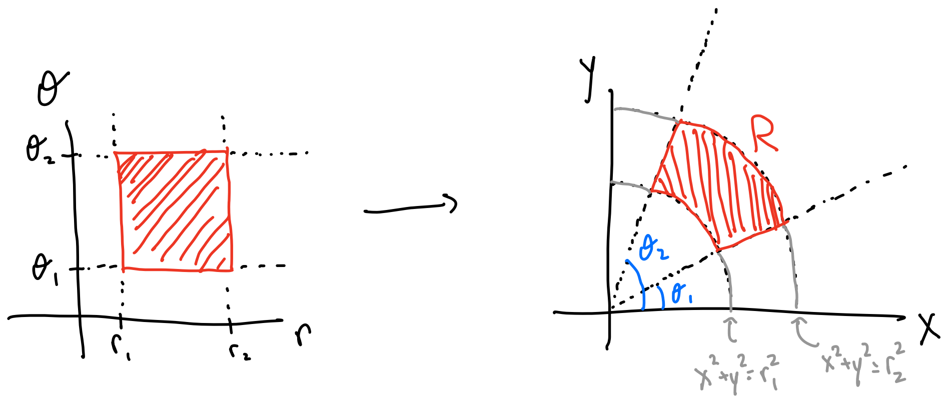
not!



Ex ($R \to P$) Consider $q = \left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ in rectangular, express in polar.

$r = 1 \qquad \theta = \frac{5\pi}{6}$

A polar rectangle is a rectangle $[r_1, r_2] \times [\theta_1, \theta_2]$ in polar coordinates. This looks different when expressed in cartesian coords.



So a fctn $f(x,y)$ expressed in polar coordinates is

$\longrightarrow f(r\cos\theta, r\sin\theta)$

and the change of variables formula says

$$\iint_R f(x,y)\,dA = \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

Ex. Compute $\iint_R \frac{1}{\sqrt{x^2+y^2}}\,dA$ where R is region bouded by $x^2+y^2=4$, $x^2+y^2=1$ and the lines $y=x$ and $x=0$.

$\frac{\pi}{4}$

Ex. Recall a cardiod may be given by $r=2(1-\cos\theta)$.

$$\int_0^{2\pi}\int_0^{2(1-\cos\theta)} r\,dr\,d\theta = \int_0^{2\pi} \frac{4(1-2\cos\theta+\cos^2\theta)}{2}\,d\theta$$

$$= \int_0^{2\pi} 2-4\cos\theta+2\underbrace{\cos^2\theta}_{=\left(\frac{1}{2}+\frac{\cos(2\theta)}{2}\right)}\,d\theta$$

$$= \int_0^{2\pi}(3-4\cos\theta+\cos 2\theta)\,d\theta$$

$$= 3\theta-4\sin\theta-\frac{\sin(2\theta)}{2}\Big|_0^{2\pi} = \boxed{6\pi}$$