

Formal definition convex function: $tf(a) + (t-1)f(b) \geq f(ta + (1-t)b)$, $\forall a, \forall b, t \in [0, 1]$

Proof of Linearity of expectation:

$$\begin{aligned}
 \mathbb{E}[aX + bY + c] &= \sum_{x,y} P(x,y)(ax + by + c) \\
 &= \sum_{x,y} P(x,y)ax + \sum_{x,y} P(x,y)by + c \sum_{x,y} P(x,y) \\
 &= a \sum_{x,y} P(y|x)P(x)x + b \sum_{x,y} P(x|y)P(y)y + c \quad \text{pooper} \quad (1) \\
 &= a \sum_x P(x)x \underbrace{\sum_y P(y|x)}_1 + b \sum_y P(y)y \underbrace{\sum_x P(x|y)}_1 + c \\
 &= a\mathbb{E}[x] + b\mathbb{E}[y] + c
 \end{aligned}$$