Formal definition convex function: $tf(a) + (t-1)f(b) \geq f(ta + (1-t)b)$, $\forall a, \forall b, t \in [0,1]$
Proof of Linearity of expectation:

$$\begin{aligned}
\mathbb{E}[aX + bY + c] &= \sum_{x,y} P(x,y)(ax + by + c) \\
&= \sum_{x,y} P(x,y)ax + \sum_{x,y} P(x,y)by + c\sum_{x,y} P(x,y) \\
&= a\sum_{x,y} P(y|x)P(x)x + b\sum_{x,y} P(x|y)P(y)y + c \\
&= a\sum_{x} P(x)x \underbrace{\sum_{y} P(y|x)}_{1} + b\sum_{y} P(y)y \underbrace{\sum_{x} P(x|y)}_{1} + c \\
&= a\mathbb{E}[x] + b\mathbb{E}[y] + c
\end{aligned} \tag{1}$$

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