Formal definition convex function: $tf(a) + (t-1)f(b) \geq f(ta + (1-t)b)$, $\forall a, \forall b, t \in [0,1]$
The value of Linearity of Expectation is that there is no need for the Random Variables to independent with respect to each other, this is the reason we try to create linear models so heavily. Proof:

$$\begin{aligned}
\mathbb{E}[aX + bY + c] &= \sum_{x,y} P(x,y)(ax + by + c) \\
&= \sum_{x,y} P(x,y)ax + \sum_{x,y} P(x,y)by + c\sum_{x,y} P(x,y) \\
&= a\sum_{x,y} P(y|x)P(x)x + b\sum_{x,y} P(x|y)P(y)y + c \\
&= a\sum_{x} P(x)x \underbrace{\sum_{y} P(y|x)}_{\text{sums to 1}} + b\sum_{y} P(y)y \underbrace{\sum_{x} P(x|y)}_{\text{sums to 1}} + c \\
&= a\mathbb{E}[x] + b\mathbb{E}[y] + c
\end{aligned} \tag{1}$$

if X and Y are independent, then $\mathbb{E}[XY] = \mathbb{E}[X][Y]$
$\mathrm{Var}[X] = \mathbb{E}[(X - \mathbb{E}[X])^2] = \sum_{x \in X}(x - \mathbb{E}[X]^2)P(x)$

$$\begin{aligned}
\mathrm{Var}[X] &= \mathbb{E}[(X - \mathbb{E}[X])^2] \\
&= \mathbb{E}[X^2 - 2X\mathbb{E}[X] + \mathbb{E}[X]^2] \\
&= \mathbb{E}[X^2] - 2\mathbb{E}[X\mathbb{E}[X]] + \mathbb{E}[\mathbb{E}[X]^2] \\
&= \mathbb{E}[X^2] - 2\mathbb{E}[X]\mathbb{E}[X] + \mathbb{E}[X]^2 \\
&= \mathbb{E}[X^2] - \mathbb{E}[X]^2
\end{aligned} \tag{2}$$