# Heat Sink Modelling and Characterisation Group Report

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# 1 Introduction

Heatsinks are used in many ways, so the optimal design of the heat sink system has become very important. In this report, we study the heatsink and analyse how the number of fins in the heatsink influences the thermal resistance of the heatsink. We do this by making and utilizing the corresponding program to calculate the heatsink thermal resistance to optimize design.

We also designed a fan duct that was used to connect the cooling fan and heatsink together, however unfortunately this fan duct could not be 3D printed and used in the experiment. We analysed the manufacturing method and cost under different number of requirements mainly which would be best for short notice manufacturing and reduced cost manufacturing. This paper also presents a method based on polynomials to model the relationship between air flow and pressure for fans and obtains good fitting degree.

# 2 Heat Sink Modelling

## 2.1 Parametric Heat Sink Thermal Model

Overview: program 1 was designed in several stages using what had been understood from the resources. I created a class containing all data and equations we have been given. Importing and curve fitting the fan curve data came next, which enabled me to find a volume flow rate using pressure drop. Finally, taking number of fins as an argument, a loop identifies the converged volume flow rate (operating point), which is written on a text file and used by program 2 to graph the thermal resistance against number of fins.

### 2.1.1 Program 1 (heatsink_thermal_resistance.py), lines:

(7-12): Importing pandas to read excel spreadsheet, math for maths functions, numpy for arrays used in graph, matplotlib for graphing and scipy for curve fitting.

(12-28): Key for variables and constants used in program.

(29-31): Reading excel spreadsheet and adding relevant data to lists.

(33-44): We want to find air flow rate as a function of pressure, as our equations give pressure drop but require air flow rate, so I plotted Q=f(P). Then creating a polynomial to curve fit with, finding coefficients and graph to ensure it is accurate. I left it at 5$^{th}$ order as it fits well. Figure 1. shows that polynomial graph.

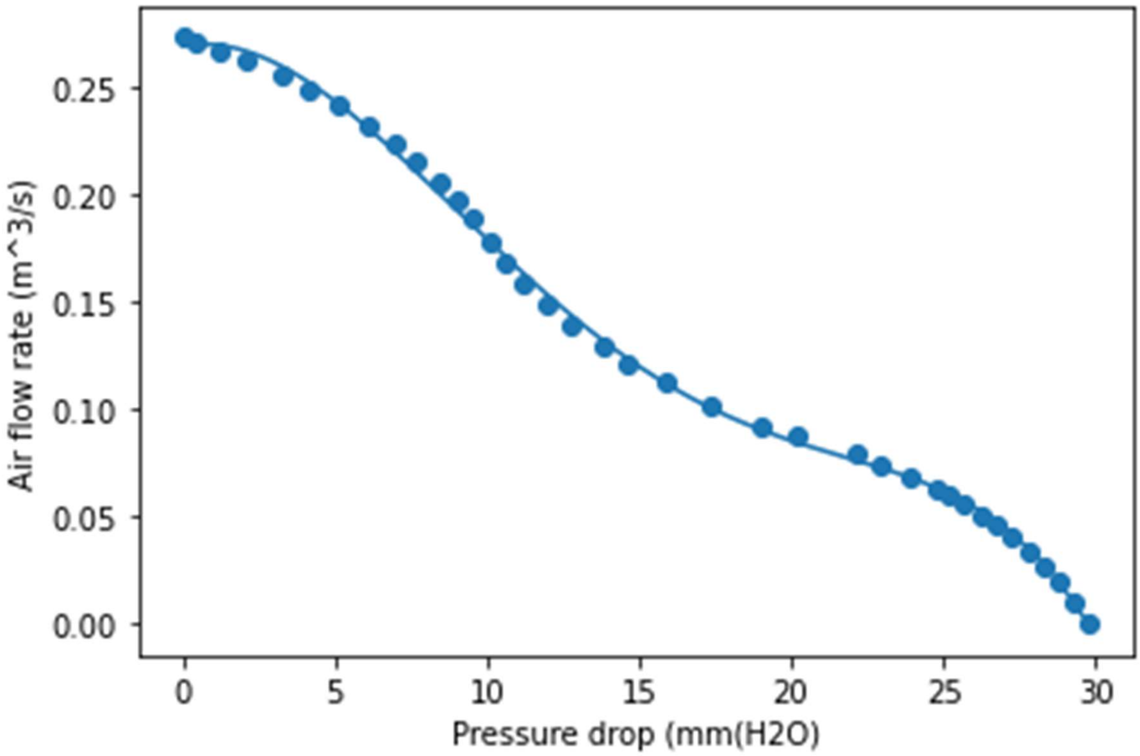


Figure 1. A Graph to show Air flow rate against Pressure drop.

(48-86): R_sink class contains all variables, constants and equations specified by academic paper and theory videos. I researched constants and temperature dependent properties were taken at 300K (room temp). I had to ensure parameters were ordered correctly so no undefined parameters are called. Pressure drop is calculated using given equations. Q2 uses the polynomial found previously to give a value for volume flow rate using pressure drop.

(88-96): User is asked to input N (number of fins). R_sink(N,Q).Q2 is called in a loop, initial value for Q argument does not affect result. This redefines Q, and the process is repeated. For N=10 and N=18, Q oscillated between two values so a variable "Previous" is defined by Q, then Q is redefined twice before they are tested for equivalence to break the loop. The oscillation could be due to elements of the equation becoming invalid. Above N=20, pressure drop grows exponentially so model could break down. The difference between values was tiny so the first one to repeat is used for convenience.

(101-106): The converged Q value is used to call R_sink().R_sink, which is displayed to the user. It is also added to a text file.

### 2.1.2 Program 2 (Thermal_resistance_graph program.py), lines:

(7-22): Reads the text file and notes thermal resistance for each N value. Thermal resistances should not change for a given N so only one resistance is taken for each. The results are displayed in Table 1., as a table, and Figure 2., as a graph.

| **Number of Fins** | **$R_{sink}$ (3s.f)** |
|---|---|
| 10 | 0.197 |
| 12 | 0.161 |
| 14 | 0.138 |
| 16 | 0.121 |
| 18 | 0.108 |

Table 1. $R_{sink}$ Values for each Number of Fins.

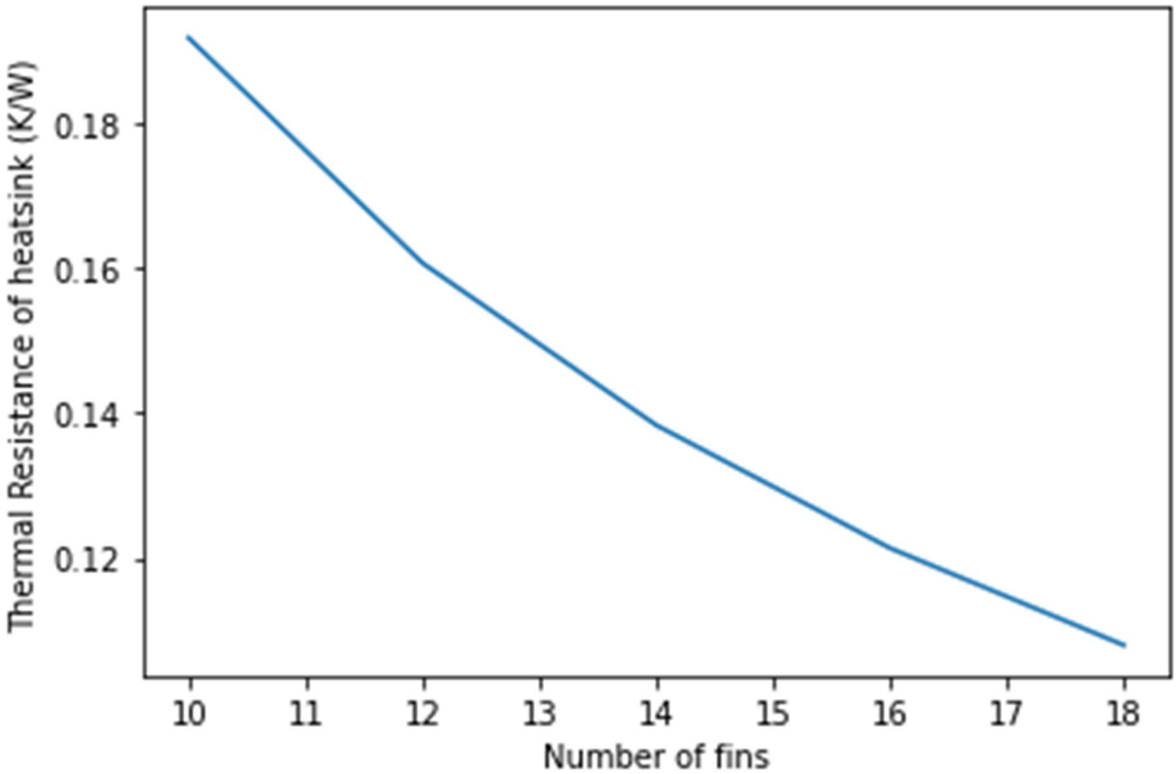


Figure 2. A Graph to show Thermal Resistance of heatsink against Number of fins.

## 2.2 Heat Sink Thermal Equivalent Circuit

Thermal circuits can be modelled in similar ways to electrical circuits. Voltage, current and electrical resistance are analogous to temperature difference, heat flow rate, and thermal resistance, respectively. The equivalent circuit is shown in Figure 3.

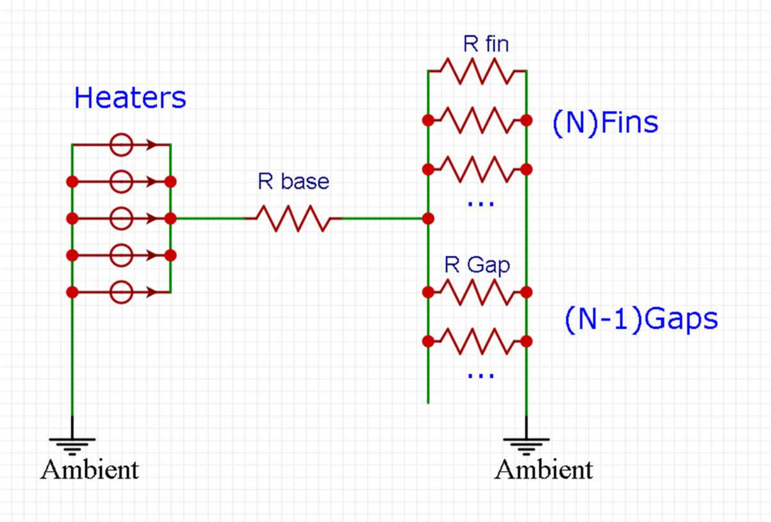


Figure 3. Sketch of equivalent circuit.

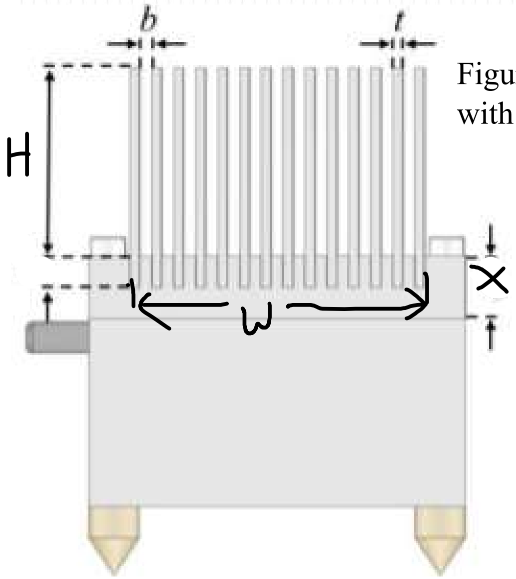


Figure 4. Heat sink with variables shown.

The heaters can be modelled as parallel voltage sources. The voltage is analogous with temperature difference between heaters and air temperature. The base can be modelled as a thermal resistance. It is analogous to electrical resistance which is proportional to length and inversely proportional to area. In the R_sink equation, the base represents the resistance (x/Km*L*W). This is in series with the rest of the circuit so is simply added to equation. The fins and gaps are modelled as parallel resistors. The gaps dissipate heat through convection, which is modelled with newtons law of cooling. Using our electrical-thermal analogy we can represent this resistance by R=(Temp heat source-ambient)/(h*A* Temp heat source-ambient)=1/h*A. Using rules for parallel resistors this gives a total resistance of R=1/h*(N-1)*b*L. The fins are also modelled as parallel resistors. The fin resistance is more complicated as accounts for conduction and convection over differential elements of the fin. Combining the gap and fin resistances gives R=1/((N/Rfin)+(h*(N-1)*b*l))). The current flowing in the circuit is analogous with heat flow rate.

# 3 Experimental Heat Sink Characterisation

## 3.1 Digital Twin of the Experimental Apparatus

### 3.1.1 CAD model

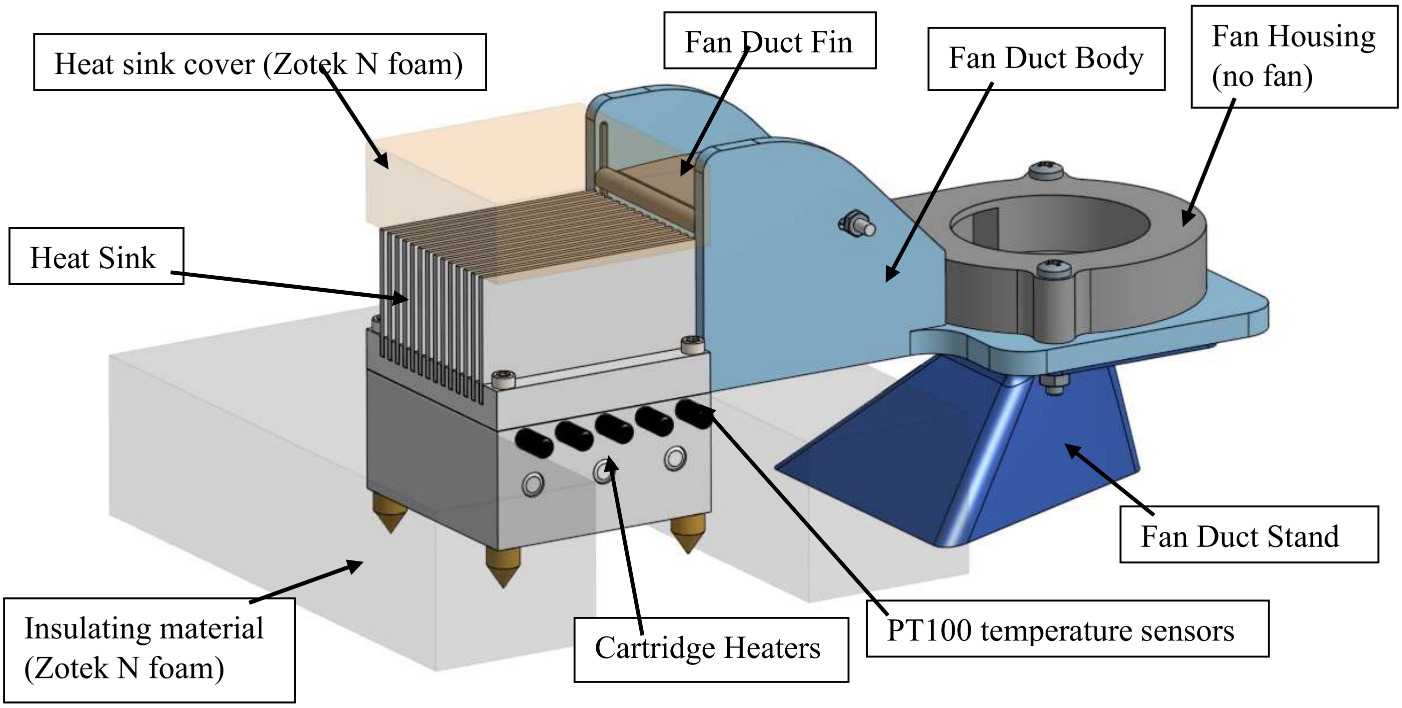


Figure 5. CAD model of the Head Sink Experiment set up.

### 3.1.2 Adjustment Mechanism

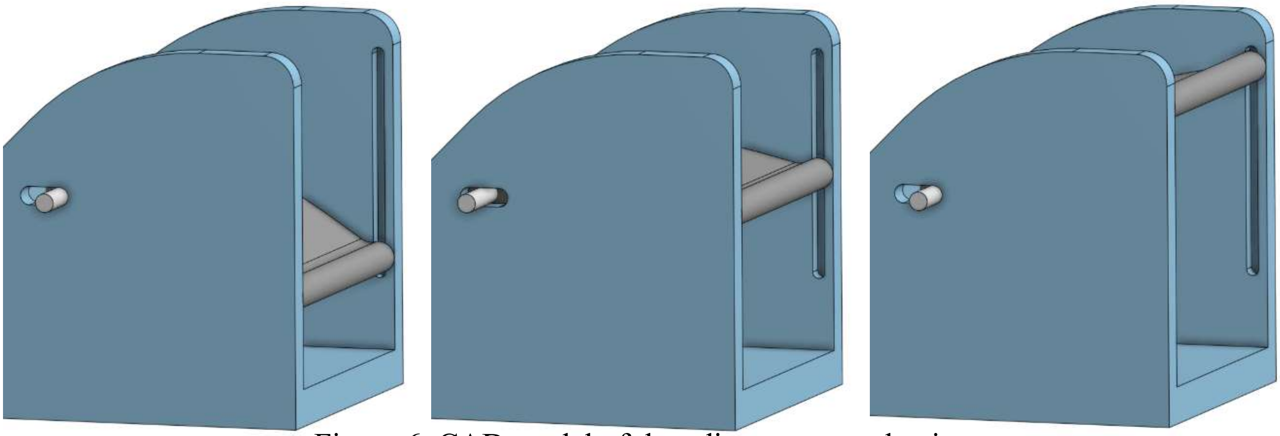

Figure 6. CAD model of the adjustment mechanism.

This mechanism works by having the fin connected to 2 slots so that the fin edge that is nearest to the fan duct is parallel with the edge of the fan duct body. This is good because it means that the fin does not move radially so there is a good seal between the fan duct body and the heat sink no matter what the height of the heat sink is. In Figure 6., the fin is shown at a minimum, in the middle, and at a maximum in terms of height from the ground. In both the minimum and the maximum pictures, the first slot is all the way to the right, whereas in the middle the first slot is as far left as it can be. This shows that the mechanism works for all required heights of heat sink.

## 3.2 Manufacturing Considerations

### 3.2.1 Constraints

| Dimension | Temp Range | Thermostability | Rigidity |
|---|---|---|---|
| $\pm 0.1mm$ | 288.15-373.15 K | Stable under 373K | Wide range |

Table 2. Table to show the Constraints of the fan duct.

### 3.2.2 Cost Modelling

With comparison, the most suitable material is nylon. For estimate and comparison of the price, the 3dprint website[1] is used.

There are three parts required non-standard processing and there are two major way to produce these kinds of parts:

1. Injection moulding: Tolerance $\pm 0.002\ in - \pm 0.008\ in$ [2]
2. SLS 3D Printing: Material: PA2200 Polyamide 12 (Nylon 12)[3] Machine: EOS P110 Series[4] (Tolerance 100um)[5]

The Price VS Unit relation[6] of 3D print & injection moulding for each part is shown in Figures 7 – 9.

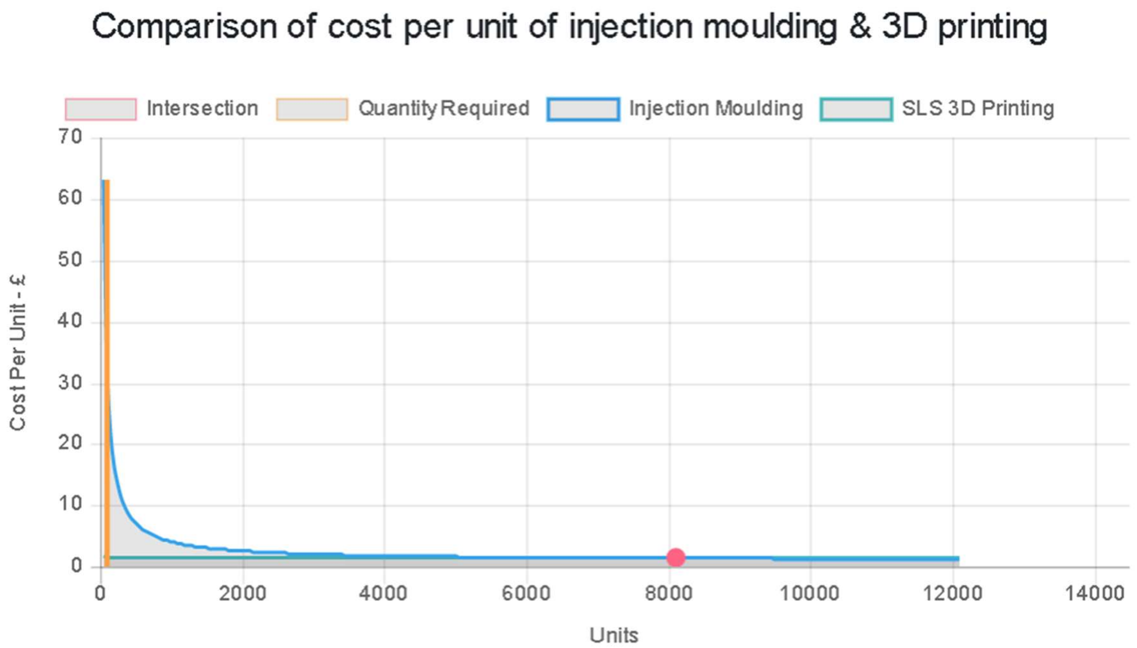


Figure 7. Comparison of cost per unit of injection moulding and 3D printing for the fin.

[1] https://www.3dprint-uk.co.uk/low-volume-production-calculator/
[2] Injection Molding Process, Defects, Plastic (custompartnet.com)
[3] https://www.3dprint-uk.co.uk/material-pa2200-nylon/
[4] https://www.3dprint-uk.co.uk/machines-maximums-and-minimums/
[5] https://www.3dprint-uk.co.uk/machine-accuracy/
[6] https://www.3dprint-uk.co.uk/low-volume-production-calculator/

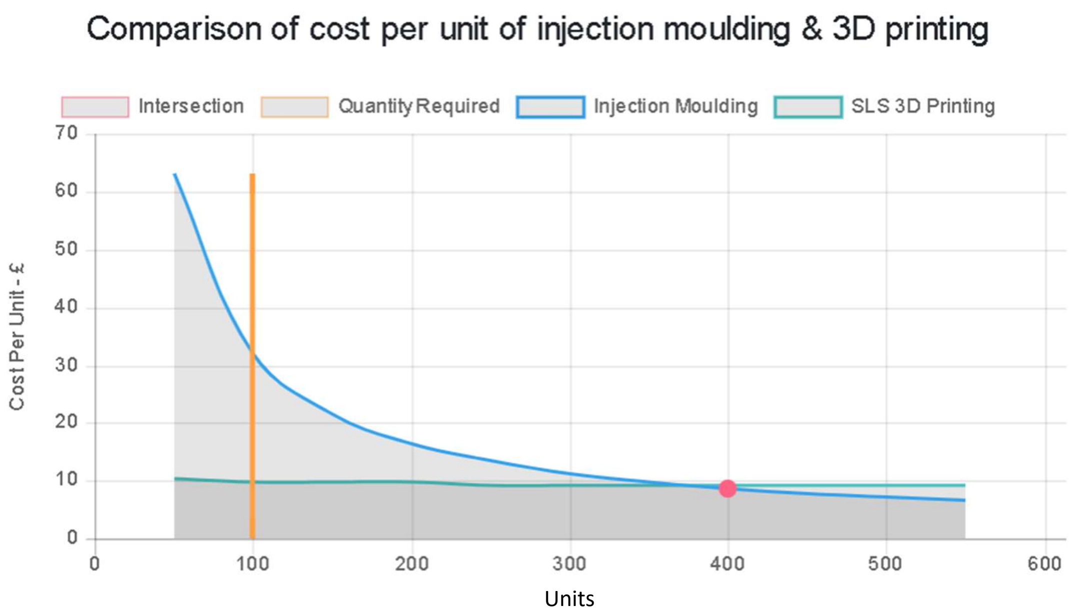


Figure 8. Comparison of cost per unit of injection moulding and 3D printing for the base.

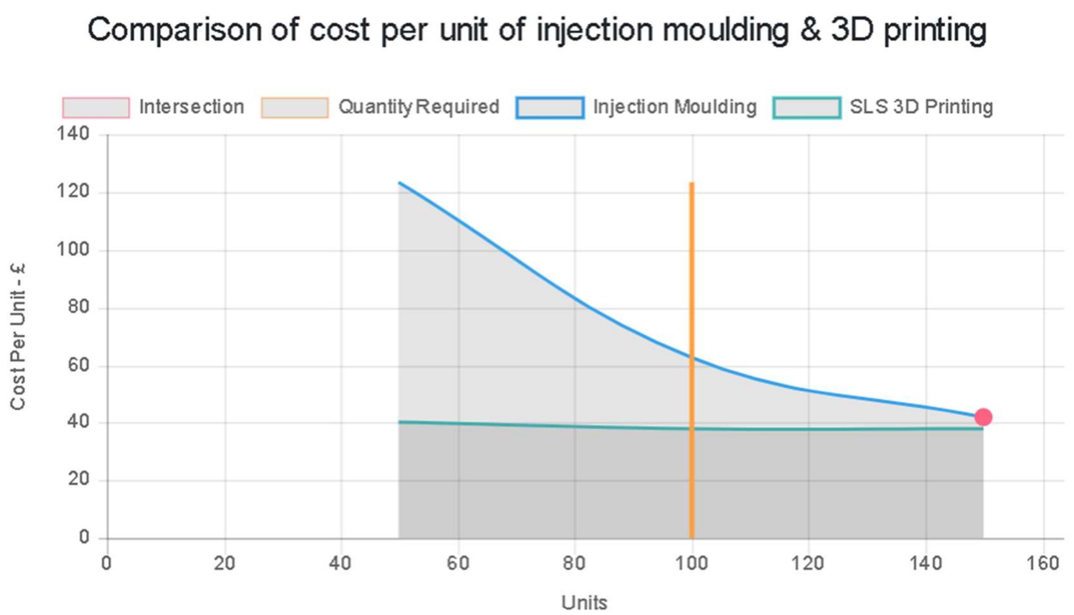


Figure 9. Comparison of cost per unit of injection moulding and 3D printing for the duct.

| | Injection Moulding | | SLS 3D Printing |
|---|---|---|---|
| | Tooling Cost: £ | Unit Cost: £ | Unit cost: £ |
| **Body** | 6100 | 2.16 | 42.12 |
| **Stand** | 4100 | 1.02 | 9.8 |
| **Fin** | 3100 | 1.02 | 1.6 |

Table 3. Cost of each manufacturing process for each part.

### 3.2.3 Cost Comparison

#### 3.2.3.1 Very low volume prototype (1-5)

The best option is choosing SLS 3D Printing for every part, because the work can be completed in only one day and no mould is needed for printing.

| | Quantity | Total price | |
|---|---|---|---|
| | | Injection | 3D |
| Body | 5 | 6110.8 | 210.6 |
| Stand | | 4105.1 | 49 |
| Fin | | 3105.1 | 8 |

Table 4. Total price for very low volume prototype.

#### 3.2.3.2 Quantities of 100s in short time scales (2 weeks time)

From the financial side it is best to choose injection for the "duct" and the "base" and 3D for the fin. But in reality, for injection moulding, it will take a lot of time to design and make the tool, therefore 3D is also the best in this situation because of the short time scale.

| | Quantity | Total price | |
|---|---|---|---|
| | | Injection | 3D |
| Body | 1000 | 8260 | 42120 |
| Stand | | 5120 | 9800 |
| Fin | | 4120 | 1600 |

Table 5. Total price for Quantities of 100s in short time scales.

#### 3.2.3.3 Quantities of 100,000s in medium timescale (6 months to a year)

With large quantities and lots of time, the advantage of choosing injection moulding starts to be seen. Injection moulding will become the best choice for long timescales and large quantities.

| | Quantity | Total price | |
|---|---|---|---|
| | | Injection | 3D |
| Body | 10000 | 27700 | 421200 |
| Stand | | 14300 | 98000 |
| Fin | | 13300 | 16000 |

Table 6. Total price for Quantities of 100,000s in medium timescale.

## 3.3 Description of Experimental Procedure

### 3.3.1 Equipment list:

- 5 Base plates each for a different number of fins (10, 12, 14, 16, 18)
- 18 Fins
- Heater
- Fan
- Simple Fan Duct
- Simple Fan Duct Stand
- Heat sink cover (Zotek N foam)
- Insulating material (Zotek N foam) for adiabatic thermal conditions
- 3 Cartridge Heaters
- Data acquisition system (DAQ) with 8 RTDs (resistance temperature detector) also known as PT100 temperature sensors
- Dual Channel power supply
- Computer
- USB cable

### 3.3.2 Preparation of Equipment

#### 3.3.2.1 Heat Sink

First unscrew the m3 hex screws that connect the base plate and the heater to make sure that the surfaces that connect them together are shiny and polished, this allows them to have good thermal contact when connected. When they have been sufficiently polished (to a shiny surface) you must make sure that the base plate is perpendicular to the temperature sensor holes on the side of the heater. You can then re-screw the m3 hex screws in.

Use the spirit level and the adjustable pins with the stopping nut at the bottom of the heater to make the top surface of the base plate horizontal to the ground. Then you can take the fins that you have and decide which fin fits best in each slot. This means that you maximise thermal contact between each of the fins and the slots as in the model the heat sink is one piece of metal, a completed heat sink is shown in Figure 10.

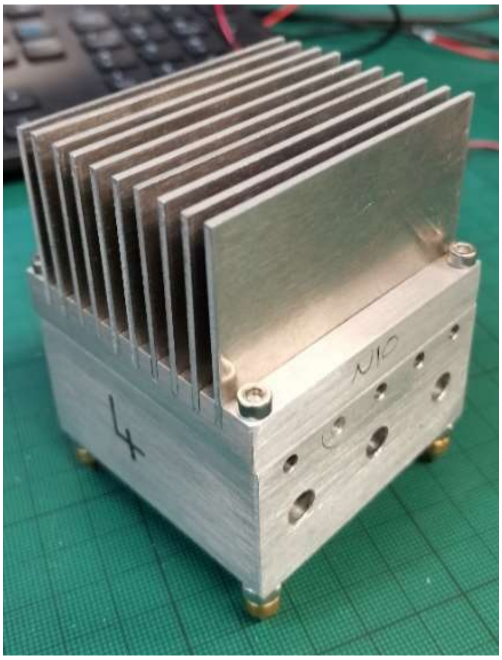


Figure 10. 10-fin heat sink that was prepared in the lab.

After you have completed the heat sink, add the Zotek N foam to 3 sides of the heatsink simulate adiabatic thermal conditions, as shown in Figure 12.

#### 3.3.2.2 Power Supply

Before connecting anything to the power supply you must first connect it to the wall and turn it on to check whether the voltage is too high for any of the equipment because it can cause damage is the voltage is too high. We are going to use Channel 1 to supply power to the fan, and Channel 2 to supply power to the heater.


Figure 11. Front and back of Dual Channel power supply.

#### 3.3.2.3 Fan

The fan is going to be connect through the back of the power supply as the fan uses feral connections. So, with the m3 screwdriver you previously used, press down the orange tab to the right of the green hole that you are inserting the feral connector into (the black wire goes into the negative socket and the red wire goes in the positive socket). You will do this twice for the positive and the negative wires. See Figure 11.

For Channel 1 you need to use this voltage and current: 12.00 Volts and 0.5 Amps

#### 3.3.2.4 Cartridge Heaters

Take the cartridge heaters and connect the 4 mm plugs in series (this means connect all the black plugs to each other and the same with the red ones). Make sure that the power supply is turned off before connecting the plugs as the heaters can become a safety concern when left out of the heater. Connect the plugs to Channel 2 of the power supply so that the red plug goes in the positive red socket and the black plug goes into the negative black socket. Insert the 3 cartridge heaters into the 3 slots that are in the heater. See Figure 11.

For Channel 2 you need to use this voltage and current: 15.00 Volts and 5.5 Amps

#### 3.3.2.5 DAQ

When inserting the PT100 temperature sensors, you can push them in using the sheath covering the connection between the sensor and the cabling, however if there is a problem and you must remove them from the hole then use the m3 screwdriver to push them back out by inserting it through the opposite side of the heater.

Insert them in the following order so that data acquisition is easiest; sensors 0 to 4 are for measuring the temperature of the heater, 5 is for measuring the outlet temperature and 6 to 7 are for measuring the ambient temperature. Figure 12. is a picture of the fan, the cartridge heaters and the PT100 temperature sensors (the 2 ambient are off screen) that were used in the experiment.

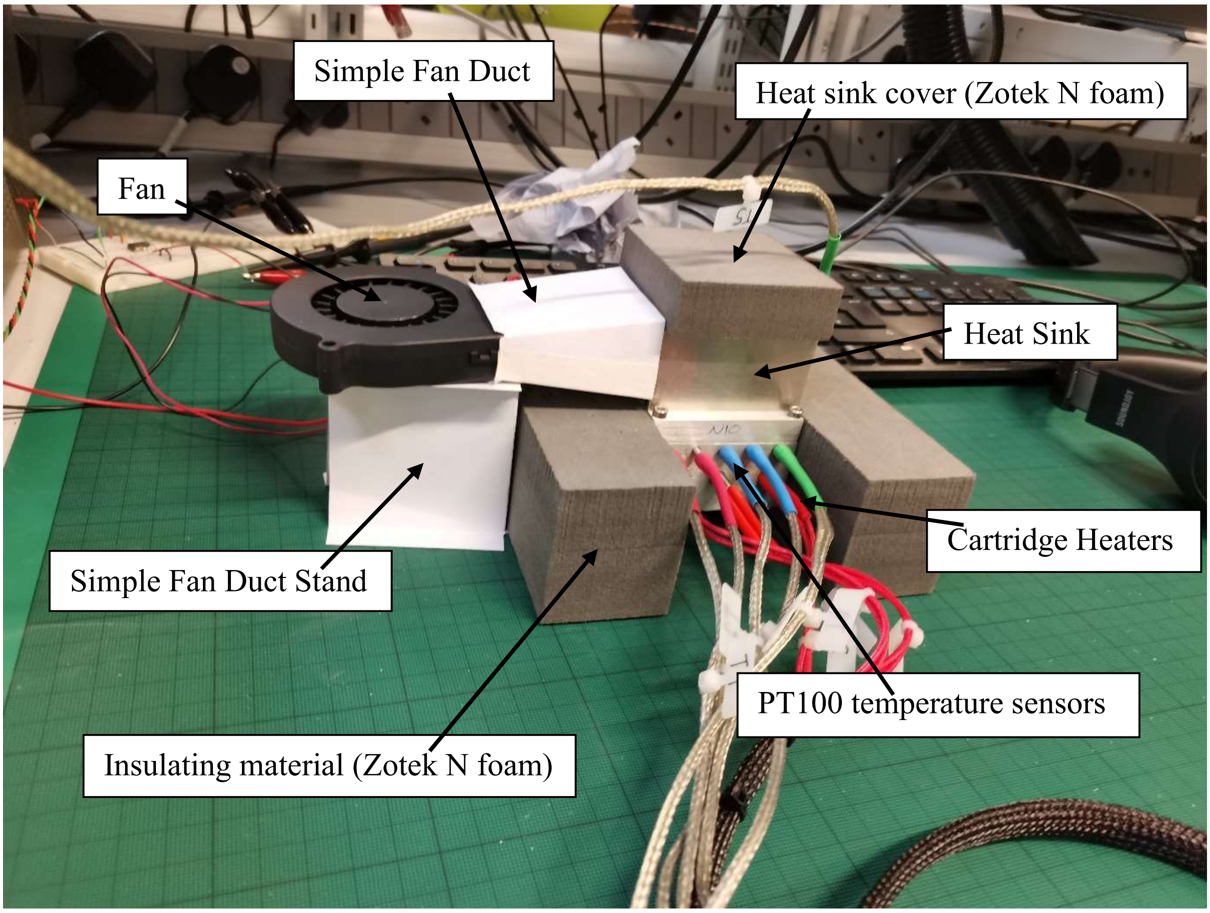


Figure 12. 10-fin heat sink that was prepared in the lab.

### 3.3.3 Data acquisition

Connect the DAQ to the computer using the USB cable. To test and configure the DAQ and the PT100 temperature sensors, use the program InstaCal. All readings should be around room temperature. After making sure everything is set up correctly, use TracerDAQ to record the data by simultaneously pressing the start recording button on TracerDAQ and the button which turns on both Channel 1 and Channel 2 on the power supply.

There will be a graph, created by the TracerDAQ software, that gives a good idea of how well the experiment is going. Change the graph so that it is automatically adjusting the X and Y axis for time and temperature. Once the temperature has stopped increasing (use your own judgement for when this happens) turn off Channel 1 (the heater) and let the fan cool down the heat sink. Eventually, the temperatures will converge on room temperature at which point you can stop recording data and save it as an excel file for analysis. The final graph will be similar in shape to Figure 13.

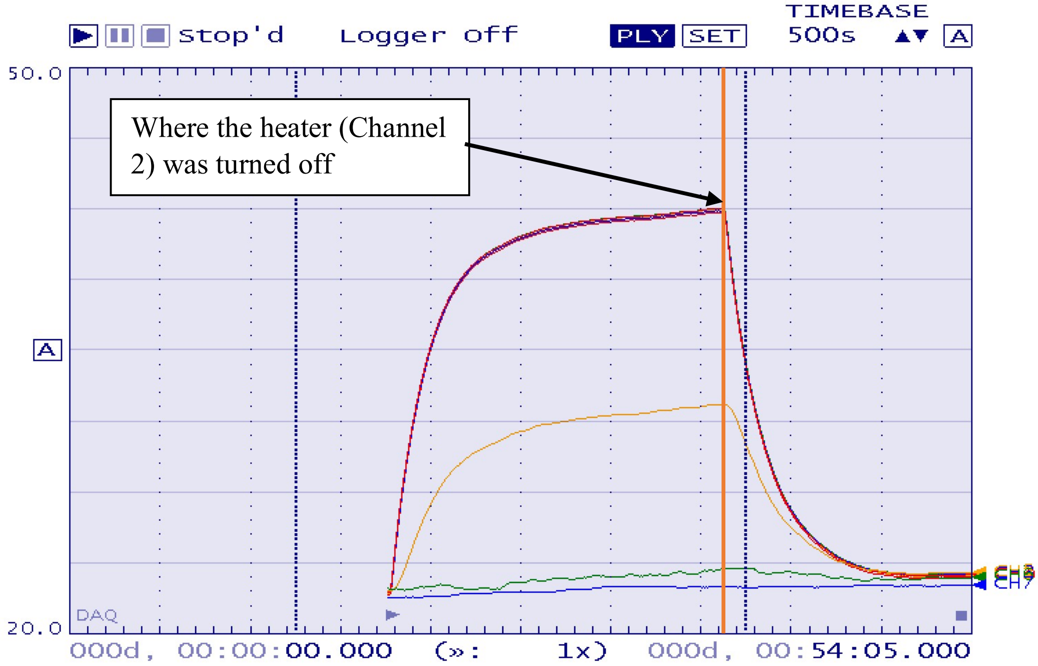

Figure 13. Graph of the temperatures recorded during the experiment.

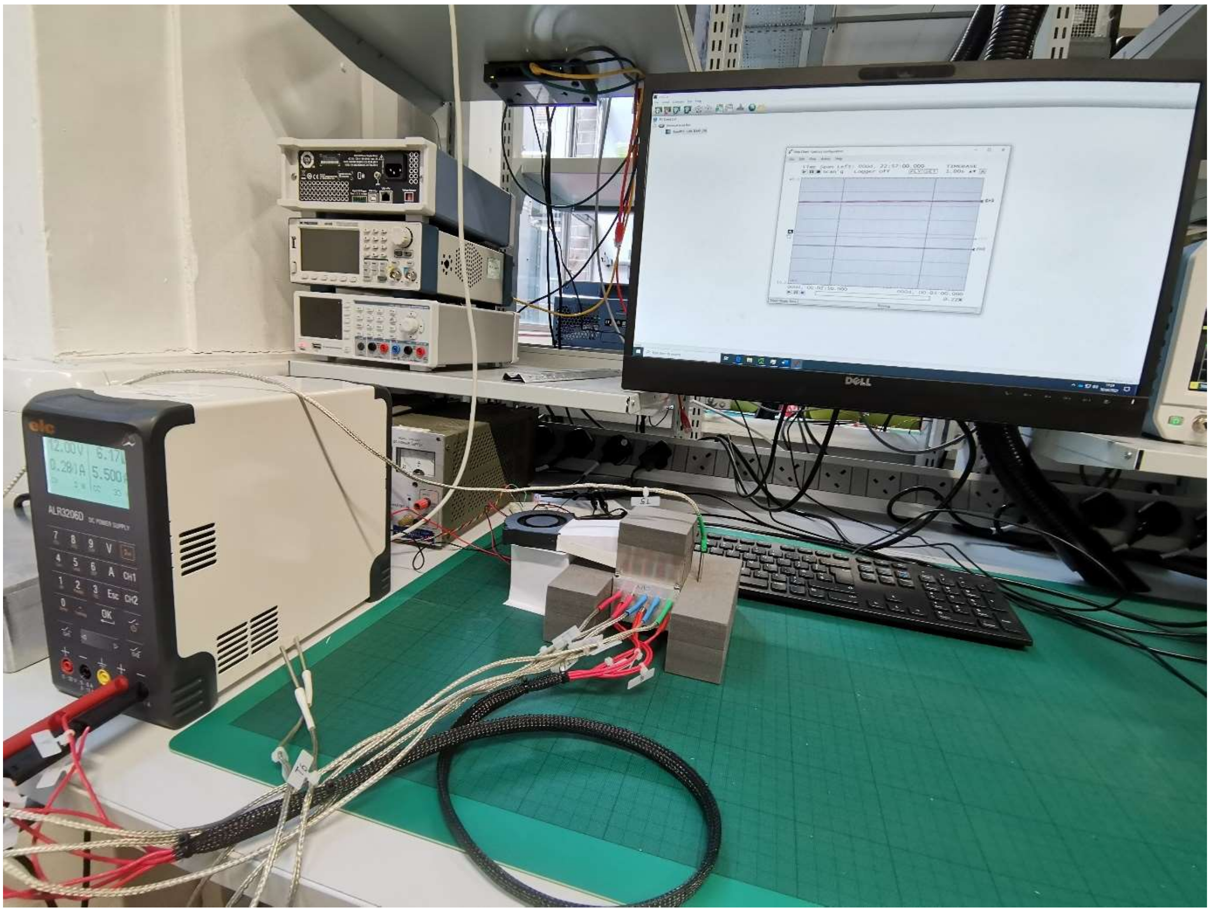
Figure 14. A picture of everything set up correctly, actively acquiring data.

# 4 Results and Discussion

## 4.1 Interpretation of Experimental Data

### 4.1.1 Equation for $R_{sink}$

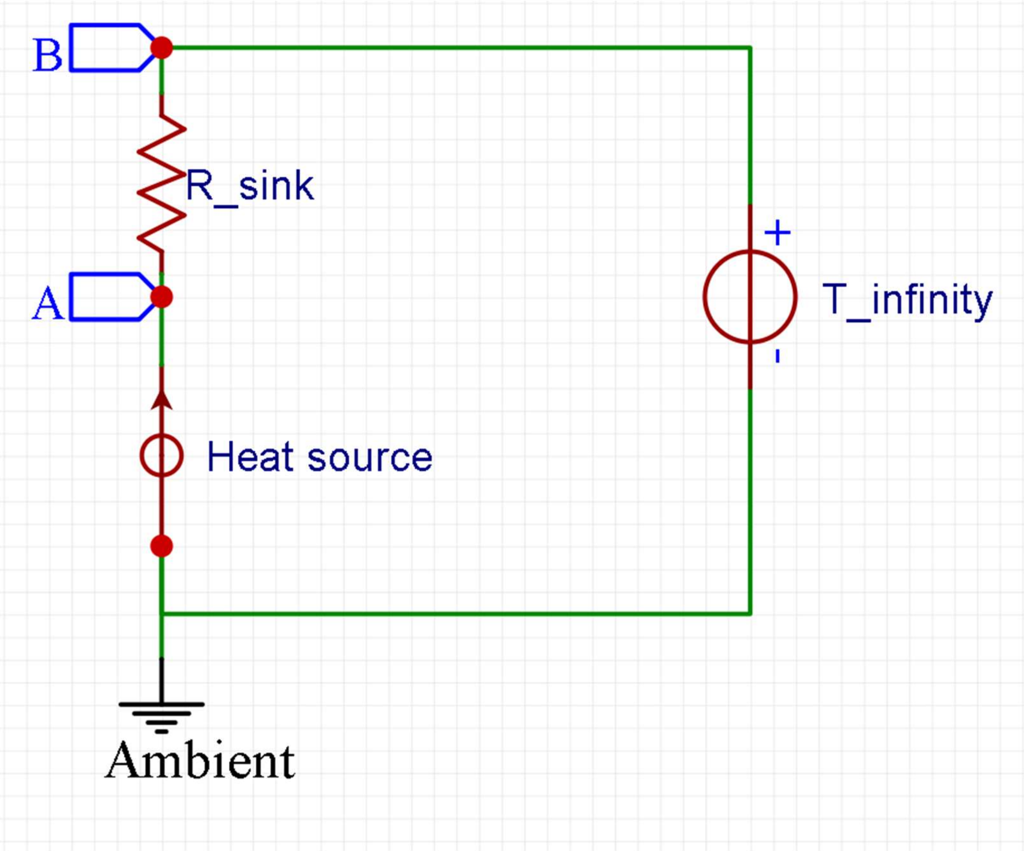


Figure 15. Diagram of equivalent circuit for experimental heat sink.

To calculate $R_{sink}$, Figure 15. must be referenced. This diagram is a model of the heat sink in terms of electrical circuits so that the resistance can be more intuitively calculated. The Heat Source is represented as a current source, the resistance of the heat sink is represented as an electrical resistor, and the ambient temperature is represented as a voltage source. If we wanted to find out the resistance of the resistor using electrical equations, we would use:

$$V = I \times R$$

Rearranged to:

$$R = \frac{V}{I}$$

However, we can say that the electrical resistance (R) is analogous to a thermal resistance ($R_{th}$), the potential difference (V) is analogous to the temperature difference (T) and the current (I) which is measured is coulombs per second is analogous to the rate of heat conduction ($\dot{Q}$) which is measured in Joules per second. Therefore, we can say that the equation for the thermal resistance of the circuit is:

$$R_{th} = \frac{T}{Q^{\cdot}}$$

### 4.1.2 Calculating $R_{sink}$

Now that we have the equation, we can work out $R_{sink}$ for each number of fins by substitution.

To work out the experimental value for $R_{sink}$, I have decided to take the 12 values before the cartridge heaters are turned off and make an average of them to get the steady state value for the temperature of the heat sink. This seems like the best thing to do due to the fluctuations in the steady state value and it should be at a maximum before the cartridge heaters are turned off. As well as this the PT100 temperature sensors have an accuracy of ±0.1°C[7] and the accuracy gets at higher temperatures, so making an average will eliminate some of the errors made by the temperature sensor.

Ambient temperature also needs to be recorded to work out the value for $R_{sink}$, I have decided to again take the 12 values before the cartridge heaters are turned off and make an average of them because it will be the average ambient temperature for the period the steady state values came from.

To calculate temperature difference, we take away the value for Ambient Temperature (B) from Heat Sink Temperature (A) for each fin amount. To substitute in the rate of heat conduction ($\dot{Q}$) we use the table below for each fin amount:

| **Number of Fins** | **Rate of Heat Conduction ($\dot{Q}$)** |
|---|---|
| 10 | 36.9 |
| 12 | 36.9 |
| 14 | 36.9 |
| 16 | 44.9 |
| 18 | 44.9 |

Table 7. Rate of Heat Conduction Values for each number of fins.

This gives us the values for $R_{sink}$ in the table below:

| **Number of Fins** | **$R_{sink}$ (3s.f)** |
|---|---|
| 10 | 0.761 |
| 12 | 0.642 |
| 14 | 0.528 |
| 16 | 0.493 |
| 18 | 0.414 |

Table 8. $R_{sink}$ Values for each number of fins in the experiment.

[7] https://components101.com/sensors/pt100-rtd-temperature-sensor

## 4.2 Comparative Analysis of Experiment and Model Results

### 4.2.1 Creating a Graph

We can create a graph of $R_{sink}$ against Number of Fins for the experimental, using Table 8. and produce a trendline, we can then do the same, using Table 1., for the values of $R_{sink}$ for the model and produce a trendline as well.

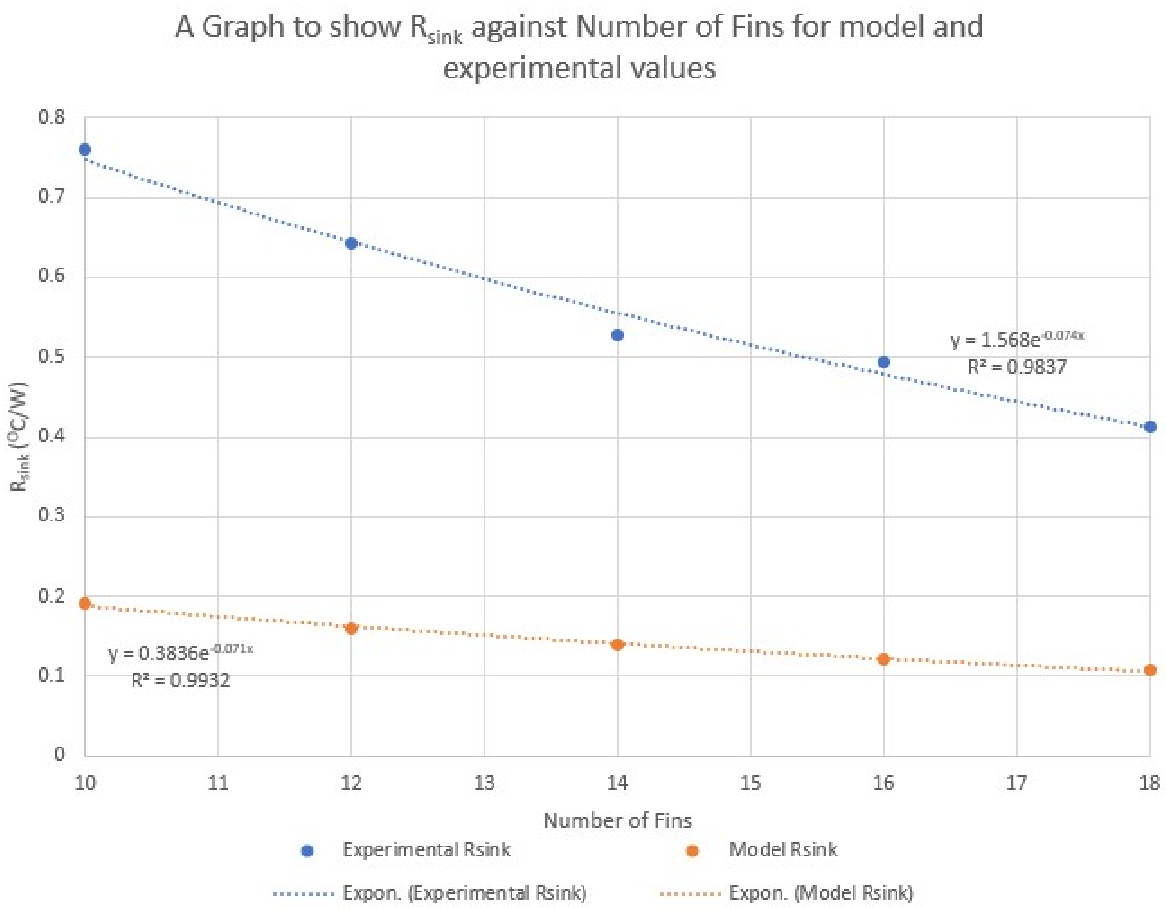


Figure 16. Graph of $R_{sink}$ against number of fins.

### 4.2.2 Comparing the Experimental and the Model Values

There is great disparity between experimental and model, this could be for several reasons that will be thusly mentioned.

There could have been bad tolerances in the connections between fins and the base plate, however these tolerances will never stand up to the model, because the model assumes that the heat sink and base plate are one piece of metal, so there will always be disparity here.

Using Zotek N foam was only simulating adiabatic thermal conditions as heat was still lost to the foam.

The experiments took place at different times in the day, as different times in the day are hotter, this can affect the ambient temperature in the experiment, for example the 16-fin heat sink experiment was started at 11:13:07 in the morning whereas the 18-fin heat sink experiment started at 04:11:11 in the afternoon. The average ambient temperatures for the time when the cartridge heaters were turned off are 22.02°C and 24.10°C respectively, which is a difference of over 2°C.

There could have been an imperfect seal between the heat sink cover and the heat sink top which could have led to air leaking out of the heat sink. As one of the assumptions was that the top of the heat sink was closed this could have caused some disparity.

There could have also been an imperfect seal between the fan duct and the fan, and the fan duct and the heat sink which could have led to air leaking out of either end. As one of the assumptions was that no air was to leak out the system this could have caused some disparity.

When preparing the experiment in the lab, unfortunately one of the cartridge heaters was not fully in during the experiment because they did not fit, leading to deviation from the model as the heating would be different from this cartridge heater.

One side of the heatsink is open, in the model, one of the assumptions made was that the heatsink would have adiabatic boundary conditions, however this is not met because of the open side. We can see how it affected the heat sink with the photos taken during the experiment in Figure 17.

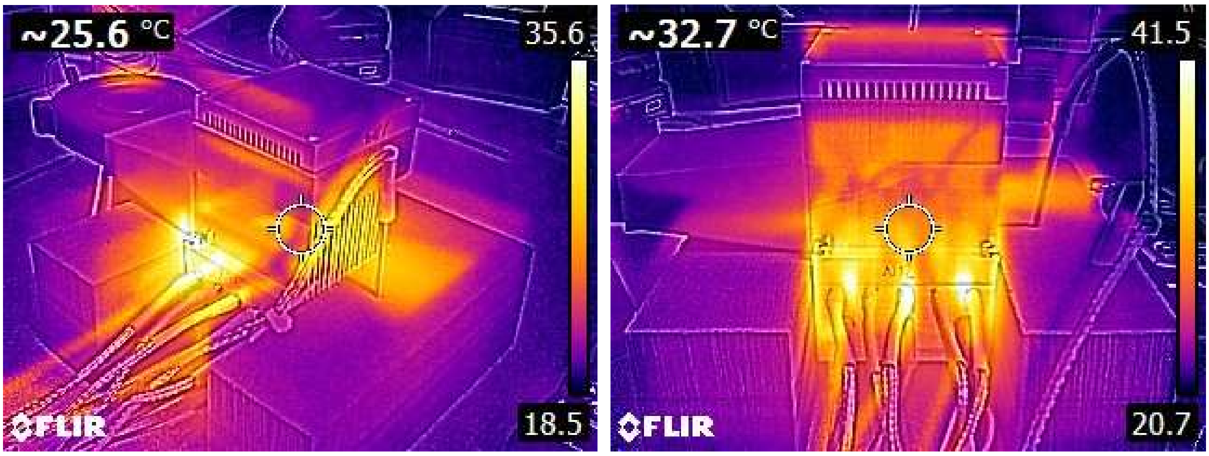


Figure 17. Pictures of the open side of the heat sink using a FLIR thermal imaging camera.

Comparing the data from the experimental data and parametric model suggests the $R_{sink}$ results from the model are wrong by a constant factor. As the predicted exponential trendlines both contain almost the same coefficient for the exponent, therefore, we can say that the disparity is down to the coefficient in front of e. When calculated it is found that for the experimental $R_{sink}$, the value is on average 4.09 (3s.f.) bigger than its model counterpart. This error is likely due to an error in the inputted parameters in the program, or coefficients used in the equations in the program, but we were unable to identify the source. This could be corrected by multiplying R sink by the constant scalar, 4.09, mentioned above.

# 5 Conclusion

## 5.1 Reflection

Across this unit we have learnt how to; use CAD software to construct 3D models, write a report about the planning and the processes that we went through to make a product and how to collect and represent data in an experimental fashion using various software and equipment. In this unit we have also learnt the fundamentals of thermal circuits. By completing tasks T3 and T4, we further practiced our python skills.

## 5.2 Things that we would approach differently

### 5.2.1 Onshape Modelling

When making a 3D model, make different pieces of the part not part of the same construction in Onshape so that each individual part can be assembled and edited later without unneeded hassle. As well as this, communication between team members can be improved by, for example, scheduling a meeting at the same time every week, so that people in different time zones can talk to each other about the project. Furthermore, laying out who is doing what in the project before starting anything so that no one steps on anyone else's toes, is a good idea.

### 5.2.2 Price Predictions

The two main ways of producing these kinds of parts are injection moulding and 3D printing. For these kinds of manufacturing increasing the cubic volume will make the cost increase dramatically. Therefore, some unnecessary portions like, for example, the support plate (supporting the fan) in this design can be replace by an interface. It will reduce the manufacture cost for two process by about 80%.

### 5.2.3 Programming Model

Using python in a single file has the disadvantages listed below:

1. The file is hard to debug (this is the most important)
2. There is low code reusability
3. There is low efficiency (in this case, there are only 5 points that need calculate, but once we include other variables such as H, L, … N variables and take 50 simple point for each, the total time for completion will be $T * 50^N$)
4. Lots of programs are rewritten, which already exist in MATLAB. For a small project, it is ok, but for a bigger project, there will not be enough time to rewrite everything.

Solutions:

1. Using MATLAB with clear multi-file program structure, then we execute the program after compiling it to a .mex file type.
2. Python does the connecting between programs and the high-performance calculation is done by a .dll file generated by the MATLAB coder.