Math 130, 1.10 Problem 11

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Solve the following initial value problem:

$$y' = 4y - 1 \quad y(0) = 1 \quad h = 0.05 \quad \text{and} \quad y(0.5)$$

Another first order linear ODE. This equation looks seperable:

$$y' = 4y - 1$$
$$\frac{dy}{dx} = 4y - 1$$
$$dy = (4y - 1)dx$$
$$\frac{1}{4y-1}dy = dx$$

Now that we have all the y's on the left and the x's on the right, we can integrate:

$$\int \frac{1}{4y-1}\,dy = \int dx$$

Right side is trivial:

$$\int dx = x + C_1$$

The left side can be solved with substitution:

$$\text{Let} \quad u = 4y - 1 \quad du = 4dy \quad dy = \frac{du}{4}$$

Now do the substitution:

$$\int \frac{1}{4y-1}\,dy = -\int \frac{1}{u}\,\frac{du}{4} = \frac{1}{4}\int \frac{1}{u}\,du = \frac{1}{4}(\ln(u) + C_2) = \frac{1}{4}(\ln(4y-1) + C_2)$$

Now putting both of the integrals together:

$$\frac{1}{4}(\ln(4y-1) + C_2) = x + C_1$$

$$\ln(4y-1)+C_2=4x+4C_1$$
$$\ln(4y-1)=4x+C_3$$
$$4y-1=e^{4x+C_3}$$
$$4y=e^{4x+C_3}+1$$

The general solution is:

$$y=\frac{1}{4}(e^{4x+C_3}+1)$$

Using the initial value $y(0)=1$, we can calculate C_3.

$$y(0)=1=\frac{1}{4}(e^{0+C_3}+1)=\frac{1}{4}(e^{C_3}+1)$$
$$4=e^{C_3}+1$$
$$3=e^{C_3}$$
$$C_3=\ln(3)$$

The particular solution is:

$$y=\frac{1}{4}(e^{4x+\ln(3)}+1)$$