Chapter 1

Mathematical Induction

The proof technique of Mathematical Induction is powerful and widely used tool

1.1 "Induction"

In daily life, we are exposed to events that strenghthen our conviction of a statement. For example, imagine that, whenever you pay attention to it, the morning bus that goes to the subway always arrives at 10:15 AM at the stop in front of your house. Of course, you would have more conviction toward the statement "The bus always arrives at 10:15 AM at the stop in front of my house" after the 500th time the bus arrives on time as opposed to the 2nd time. This process is induction in action.

Induction, roughly speaking, refers to the process by which our confidence in a statement is supported by the number of verifying examples of that statement. Our confidence increases with the number of examples that support our hypothesis.

The following point is critical. Unless we are able to verify our hypothesis for all (potentially infintite) instances, then we will never achieve complete certainty of our statements. How could we be sure that tomorrow, next week, or next year, our bus will be late?

1.2 Mathematical Induction

In a similar vein to the above, consider the statement: for every $n \in \mathbb{N}, n \geq 4, n^2 < 2^n$. We know that this is true, and one could go about calculating the values for both n^2 and 2^n, for $n = 1$ and then comparing those values to verify the predicate, repeating the process for $n = 2$, then $n = 3, \ldots$ However, the analogous problem presents itself here: what if there was some number $M \in \mathbb{N}$ that is beyond the scope of conceivable human (and computer) discovery[1]? How do we verify the predicate for such an uncomputably large number? And even numbers of that magnitude larger?

[1] For example, Graham's Number

1.3 Proofs by Induction

The structure of a