Transformadas de Lagrange

Camargo Badillo Luis Mauricio

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Oscar Gabriel Caballero Martínez
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8. $f(t) = \cos(t)$

Calculamos la transformada de Lagrange de $f(t) = \cos(t)$:

$$\begin{aligned}\mathcal{L}\{\cos(t)\} &= \int_0^\infty e^{-st}\cos(t)\ dt \\ &= \lim_{b\to\infty}\int_0^b e^{-st}\cos(t)\ dt\end{aligned}$$

Integrando por partes, utilizando $u = \cos(t) \implies du = -\sin(t)\ dt$ y $dv = e^{-st}\ dt \implies v = -\frac{1}{s}e^{-st}$, obtenemos:

$$\begin{aligned}&= \lim_{b\to\infty}\left[\frac{-\cos(t)e^{-st}}{s}\bigg|_0^b - \frac{1}{s}\int_0^b e^{-st}\sin(t)\ dt\right] \\ &= \lim_{b\to\infty}\left[-\frac{\cos(b)e^{-sb}}{s} + \frac{\cos(0)e^0}{s}\right] - \lim_{b\to\infty}\left[\frac{1}{s}\int_0^b e^{-st}\sin(t)\ dt\right] \\ &= \frac{1}{s} - \lim_{b\to\infty}\left[\frac{1}{s}\int_0^b e^{-st}\sin(t)\ dt\right] \\ &= \frac{1}{s} - \frac{1}{s}\lim_{b\to\infty}\int_0^b e^{-st}\sin(t)\ dt\end{aligned}$$

Una vez más, integrando por partes con $u = \sin(t) \implies du = \cos(t)\ dt$ y $dv = e^{-st}\ dt \implies v = -\frac{1}{s}e^{-st}$, tenemos:

$$\begin{aligned}&= \frac{1}{s} - \frac{1}{s}\lim_{b\to\infty}\left[\frac{-\sin(t)e^{-st}}{s}\bigg|_0^b + \frac{1}{s}\int_0^b e^{-st}\cos(t)\ dt\right] \\ &= \frac{1}{s} - \frac{1}{s}\lim_{b\to\infty}\left[-\frac{\sin(b)e^{-sb}}{s} + \frac{\sin(0)e^0}{s}\right] + \lim_{b\to\infty}\left[\frac{1}{s}\int_0^b e^{-st}\cos(t)\ dt\right] \\ &= \frac{1}{s} - \frac{1}{s}\lim_{b\to\infty}\left[\frac{1}{s}\int_0^b e^{-st}\cos(t)\ dt\right] \\ &= \frac{1}{s} - \frac{1}{s^2}\lim_{b\to\infty}\int_0^b e^{-st}\cos(t)\ dt\end{aligned}$$

Observemos que:

$$\lim_{b\to\infty}\int_0^b e^{-st}\cos(t)\ dt = \frac{1}{s} - \frac{1}{s^2}\lim_{b\to\infty}\int_0^b e^{-st}\cos(t)\ dt$$

Estableciendo $a = \lim_{b\to\infty} \int_0^b e^{-st}\cos(t)\ dt$, tenemos:

$$\begin{aligned}
a &= \frac{1}{s} - \frac{1}{s^2}a \\
\implies a + \frac{1}{s^2}a &= \frac{1}{s} \\
\implies a\left(\frac{1}{s^2} + 1\right) &= \frac{1}{s} \\
\implies a\left(\frac{1+s^2}{s^2}\right) &= \frac{1}{s} \\
\implies a &= \frac{s^2}{s(1+s^2)} \\
\implies a &= \frac{s}{s^2+1} \\
\implies \lim_{b\to\infty}\int_0^b e^{-st}\cos(t)\ dt &= \frac{s}{s^2+1} \\
\implies \int_0^\infty e^{-st}\cos(t)\ dt &= \frac{s}{s^2+1}
\end{aligned}$$

Por lo tanto, finalmente:

$$\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1}$$

11. $f(t) = e^{4t}$

12. $f(t) = e^{-2t}$

14. $f(t) = \sinh(3t)$

15. $f(t) = \cosh(6t)$

17. $f(t) = \cosh(at)$