

example

May 11, 2021

```
[1]: from GR_full_curv import *
from sympy import symbols, sin, init_printing

init_printing()

# Coordinate system that we will work on
coord_sys = symbols('t r theta phi')

# Defining some extra symbols
G, m, a = symbols('G, m, a')

# Defining the diagonal components of the metric tensor
diag_comp = [-1, a**(-1), a**2, a * sin(coord_sys[2])**2]
```

1 Metric Tensor

```
[2]: # Obtaining the metric tensor
mt = MetricTensor(diag_comp, coord_sys)
metric_tensor = mt.get_metrictensor()
metric_tensor
```

```
[2]: 
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{a} & 0 & 0 \\ 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a \sin^2(\theta) \end{bmatrix}$$

```

```
[3]: # Default type of the metric tensor
mt.get_metrictensor_type()
```

```
[3]: 'dd'
```

```
[4]: # Varying type 'dd' metric tensor to 'ud'
mt.vary_metrictensor_type(metric_tensor, 'ud')
```

```
[4]: 
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```

```
[5]: mt.get_metric_tensor_type()
```

```
[5]: 'ud'
```

```
[6]: # Varying type 'dd' metric tensor to 'uu'
mt.vary_metric_tensor_type(metric_tensor, 'uu')
```

```
[6]: 
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & \frac{1}{a^2} & 0 \\ 0 & 0 & 0 & \frac{1}{a \sin^2(\theta)} \end{bmatrix}$$

```

```
[7]: mt.get_metric_tensor_type()
```

```
[7]: 'uu'
```

```
[8]: # Obtaining the inverse of the metric tensor directly
mt.get_inverse()
```

```
[8]: 
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & \frac{1}{a^2} & 0 \\ 0 & 0 & 0 & \frac{1}{a \sin^2(\theta)} \end{bmatrix}$$

```

2 Christoffel Symbol

```
[9]: # Obtaining the Christoffel Symbol
cs = ChristoffelSymbol(diag_comp, coord_sys)
chris_symbol = cs.get_christoffelsymbol()
chris_symbol
```

```
[9]: 
$$\left[ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sin(\theta) \cos(\theta)}{a} \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{bmatrix} \right]$$

```

```
[10]: # Default type of the Christoffel Symbol
cs.get_christoffelsymbol_type()
```

```
[10]: 'udd'
```

```
[11]: # Varying type 'udd' Christoffel Symbol to 'ddd'
cs.vary_christoffelsymbol_type(chris_symbol, 'ddd')
```

```
[11]: 
$$\left[ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a \sin(\theta) \cos(\theta) \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a \sin(\theta) \cos(\theta) \\ 0 & 0 & a \sin(\theta) \cos(\theta) & 0 \end{bmatrix} \right]$$

```

```
[12]: cs.get_christoffelsymbol_type()
```

```
[12]: 'ddd'
```

```
[13]: # Varying type 'udd' Christoffel Symbol to 'uud'
cs.vary_christoffelsymbol_type(chris_symbol, 'uud')
```

```
[13]: 
$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\cos(\theta)}{a^2 \sin(\theta)} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\cos(\theta)}{a^2 \sin(\theta)} \\ 0 & 0 & \frac{\cos(\theta)}{a \sin^3(\theta)} & 0 \end{bmatrix} \end{bmatrix}$$

```

```
[14]: cs.get_christoffelsymbol_type()
```

```
[14]: 'uud'
```

```
[15]: # Varying type 'udd' Christoffel Symbol to 'uuu'
cs.vary_christoffelsymbol_type(chris_symbol, 'uuu')
```

```
[15]: 
$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\cos(\theta)}{a^3 \sin^3(\theta)} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\cos(\theta)}{a^3 \sin^3(\theta)} \\ 0 & 0 & \frac{\cos(\theta)}{a^3 \sin^3(\theta)} & 0 \end{bmatrix} \end{bmatrix}$$

```

```
[16]: cs.get_christoffelsymbol_type()
```

```
[16]: 'uuu'
```

```
[17]: # Obtaining the non-zero components of the given Christoffel Symbol
cs.nonzero_christoffelsymbol(chris_symbol)
```

$$\Gamma_{\phi\phi}^{\theta} = -\frac{\sin(\theta) \cos(\theta)}{a}$$

$$\Gamma_{\theta\phi}^{\phi} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\Gamma_{\phi\theta}^{\phi} = \frac{\cos(\theta)}{\sin(\theta)}$$

3 Riemann Tensor

```
[18]: # Obtaining the Riemann Tensor
rt = RiemannTensor(diag_comp, coord_sys)
riemann_tensor = rt.get_riemanntensor()
riemann_tensor
```

```
[18]:
```

[illegible]

```
[19]: # Default type of the Riemann Tensor
      rt.get_riemantensor_type()
```

```
[19]: 'uddd'
```

```
[20]: # Varying type 'uddd' Riemann Tensor to 'dddd'
      rt.vary_riemantensor_type(riemann_tensor, 'dddd')
```

[illegible]

```
[21]: rt.get_riemantensor_type()
```

```
[21]: 'dddd'
```

4 Ricci Tensor

```
[22]: # Obtaining the Ricci Tensor
rit = RicciTensor(diag_comp, coord_sys)
ricci_tensor = rit.get_riccitensor()
ricci_tensor
```

```
[22]: 
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{\sin^2(\theta)}{a} \end{bmatrix}$$

```

```
[23]: # Default type of the Ricci Tensor
rit.get_riccitensor_type()
```

```
[23]: 'dd'
```

```
[24]: # Varying type 'dd' Ricci Tensor to 'uu'
rit.vary_riccitensor_type(ricci_tensor, 'uu')
```

```
[24]: 
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{a^4} & 0 \\ 0 & 0 & 0 & \frac{1}{a^3 \sin^2(\theta)} \end{bmatrix}$$

```

```
[25]: rit.get_riccitensor_type()
```

```
[25]: 'uu'
```

5 Ricci Scalar

```
[26]: # Obtaining the Ricci Scalar
rs = RicciScalar(diag_comp, coord_sys)
ricci_scalar = rs.get_ricciscalar()
ricci_scalar
```

```
[26]:  $\frac{2}{a^2}$ 
```

6 Traceless Ricci Tensor

```
[27]: # Obtaining the Traceless Ricci Tensor
trt = TracelessRicciTensor(diag_comp, coord_sys)
tracless_ricci_tensor = trt.get_trclss_riccitensor()
tracless_ricci_tensor
```

```
[27]:
```

$$\begin{bmatrix} \frac{1}{2a^2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2a^3} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{\sin^2(\theta)}{2a} \end{bmatrix}$$

```
[28]: # Default type of the Traceless Ricci Tensor
trt.get_trclss_riccitenor_type()
```

```
[28]: 'dd'
```

```
[29]: # Varying type 'dd' Traceless Ricci Tensor to 'uu'
trt.vary_trclss_riccitenor_type(tracless_ricci_tensor, 'uu')
```

```
[29]:
```

$$\begin{bmatrix} \frac{1}{2a^2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2a} & 0 & 0 \\ 0 & 0 & \frac{1}{2a^4} & 0 \\ 0 & 0 & 0 & \frac{1}{2a^3 \sin^2(\theta)} \end{bmatrix}$$

```
[30]: trt.get_trclss_riccitenor_type()
```

```
[30]: 'uu'
```

7 Weyl Tensor

```
[31]: # Obtaining the Weyl Tensor
wyl = WeylTensor(diag_comp, coord_sys)
weyl_tensor = wyl.get_weyltensor()
weyl_tensor
```

```
[31]:
```

$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & -\frac{1}{3a^3} & 0 & 0 \\ \frac{1}{3a^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & \frac{\sin^2(\theta)}{6a} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\sin^2(\theta)}{6a} & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \frac{1}{3a^3} & 0 & 0 \\ -\frac{1}{3a^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{6a} & 0 \\ 0 & \frac{1}{6a} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sin^2(\theta)}{6a^2} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{\sin^2(\theta)}{6a^2} & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6a} & 0 \\ 0 & -\frac{1}{6a} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a \sin^2(\theta) \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{a \sin^2(\theta)}{3} \\ 0 & \frac{2a \sin^2(\theta)}{3} & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & -\frac{\sin^2(\theta)}{6a} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\sin^2(\theta)}{6a} & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sin^2(\theta)}{6a^2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{\sin^2(\theta)}{6a^2} & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2a \sin^2(\theta)}{3} \\ 0 & \frac{a \sin^2(\theta)}{3} & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -a \sin^2(\theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{bmatrix}$$

```
[32]: # Default type of the Weyl Tensor
wyl.get_weyltensor_type()
```

```
[32]: 'dddd'
```

```
[33]: # Varying type 'dddd' Weyl Tensor to 'uudd'
wyl.vary_weyltensor_type(wyl_tensor, 'uudd')
```

```
[33]:
```

$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & \frac{1}{3a^3} & 0 & 0 \\ -\frac{1}{3a^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & -\frac{\sin^2(\theta)}{6a} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\sin^2(\theta)}{6a} & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \frac{1}{3a^2} & 0 & 0 \\ -\frac{1}{3a^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{6} & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sin^2(\theta)}{6a} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{\sin^2(\theta)}{6a} & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & -\frac{1}{6a^2} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{6a^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6a^3} & 0 \\ 0 & -\frac{1}{6a^3} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sin^2(\theta)}{a} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sin^2(\theta)}{3a} \\ 0 & 0 & \frac{2\sin^2(\theta)}{3a} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{6a^2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{6a^2} & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6a^3} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{6a^3} & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & \frac{1}{3} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{bmatrix}$$

```
[34]: wyl.get_weyltensor_type()
```

```
[34]: 'uudd'
```

8 Einstein Tensor

```
[35]: # Obtaining the Einstein Tensor
et = EinsteinTensor(diag_comp, coord_sys)
einstein_tensor = et.get_einsteintensor()
einstein_tensor
```

```
[35]:
```

$$\begin{bmatrix} \frac{1}{a^2} & 0 & 0 & 0 \\ 0 & -\frac{1}{a^3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
[36]: # Default type of the Einstein Tensor
et.get_einsteintensor_type()
```

```
[36]: 'dd'
```

```
[37]: # Varying type 'dd' Einstein Tensor to 'uu'  
et.vary_einsteintensor_type(einstein_tensor, 'uu')
```

```
[37]: 
$$\begin{bmatrix} \frac{1}{a^2} & 0 & 0 & 0 \\ 0 & -\frac{1}{a} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```

```
[38]: et.get_einsteintensor_type()
```

```
[38]: 'uu'
```