example

May 19, 2021

```
[1]: from GRTC import *
  from sympy import symbols, sin, init_printing
  init_printing()

# Defining the symbols in the coordinate system
  t, r, theta, phi = symbols('t, r, theta, phi')

#Defining the coordinate system as a list
  coord_sys = [t, r, theta, phi]

# Defining some extra symbols
  G, m, a = symbols('G, m, a')

# Defining the diagonal components of the metric tensor
  diag_comp = [-1, a**(-1), a**2, a * sin(theta)**2]
```

1 Metric Tensor

```
[2]: # Obtaining the metric tensor
mt = MetricTensor(diag_comp, coord_sys)
metric_tensor = mt.get_metrictensor()
metric_tensor
[2]: F 1 0 0 0 7
```

[2]: $\begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & \frac{1}{a} & 0 & 0 \\
0 & 0 & a^2 & 0 \\
0 & 0 & 0 & a\sin^2(\theta)
\end{bmatrix}$

```
[3]: # Default type of the metric tensor
mt.get_metrictensor_type()
```

[3]: 'dd'

```
[4]: # Varying type 'dd' metric tensor to 'ud' mt.vary_metrictensor_type(metric_tensor, 'ud')
```

[4]:

```
[5]: mt.get_metrictensor_type()
 [5]: 'ud'
 [6]: # Varying type 'dd' metric tensor to 'uu'
     mt.vary_metrictensor_type(metric_tensor, 'uu')
[6]: <sub>\(\Gamma-1\)</sub> 0 0
 [7]: mt.get_metrictensor_type()
 [7]: 'uu'
 [8]: # Obtaining the inverse of the metric tensor directly
     mt.get_inverse()
     2 Christoffel Symbol
 [9]: # Obtaining the Christoffel Symbol
     cs = ChristoffelSymbol(diag_comp, coord_sys)
     chris_symbol = cs.get_christoffelsymbol()
     chris_symbol
     0
                                                                    \cos(\theta)
                                                                    \sin(\theta)
[10]: # Default type of the Christoffel Symbol
     cs.get_christoffelsymbol_type()
[10]: 'udd'
[11]: # Varying type 'udd' Christoffel Symbol to 'ddd'
     chris_symbol03 = cs.vary_christoffelsymbol_type(chris_symbol, 'ddd')
     chris_symbol03
```

- [12]: cs.get_christoffelsymbol_type()
- [12]: 'ddd'
- [13]: # Obtaining the non-zero components of the given Christoffel Symbol for type ddd cs.nonzero_christoffelsymbol(chris_symbol03)

 $\Gamma_{\theta\phi\phi} = -a\sin\left(\theta\right)\cos\left(\theta\right)$

 $\Gamma_{\phi\theta\phi} = a\sin\left(\theta\right)\cos\left(\theta\right)$

 $\Gamma_{\phi\phi\theta} = a\sin\left(\theta\right)\cos\left(\theta\right)$

- [14]: # Varying type 'udd' Christoffel Symbol to 'uud'
 chris_symbol21 = cs.vary_christoffelsymbol_type(chris_symbol, 'uud')
 chris_symbol21
- [15]: cs.get_christoffelsymbol_type()
- [15]: 'uud'
- [16]: # Varying type 'udd' Christoffel Symbol to 'uuu'
 chris_symbol30 = cs.vary_christoffelsymbol_type(chris_symbol, 'uuu')
 chris_symbol30
- $\begin{bmatrix}
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix} \quad
 \begin{bmatrix}
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix} \quad
 \begin{bmatrix}
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -\frac{\cos(\theta)}{a^3 \sin^3(\theta)}
 \end{bmatrix} \quad
 \begin{bmatrix}
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & \frac{\cos(\theta)}{a^3 \sin^3(\theta)} & 0
 \end{bmatrix}$
- [17]: cs.get_christoffelsymbol_type()
- [17]: 'uuu'
- [18]: cs.nonzero_christoffelsymbol(chris_symbol30)

$$\Gamma^{\theta\phi\phi} = -\frac{\cos\left(\theta\right)}{a^3\sin^3\left(\theta\right)}$$

$$\Gamma^{\phi\theta\phi} = \frac{\cos\left(\theta\right)}{a^3\sin^3\left(\theta\right)}$$

```
\Gamma^{\phi\phi\theta} = \frac{\cos\left(\theta\right)}{a^3\sin^3\left(\theta\right)}
```

3 Riemann Tensor

```
[19]: # Obtaining the Riemann Tensor
       rt = RiemannTensor(diag_comp, coord_sys)
       riemann_tensor = rt.get_riemanntensor()
       riemann_tensor
[19]: г
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                        0 0 0 0
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                                                  -\frac{\sin^2(\theta)}{}
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                        \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}
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[20]: # Default type of the Riemann Tensor
       rt.get_riemanntensor_type()
[20]: 'uddd'
[21]: # Varying type 'uddd' Riemann Tensor to 'dddd'
       riemann_tensor04 = rt.vary_riemanntensor_type(riemann_tensor, 'dddd')
       riemann_tensor04
[21]:
```

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                                                                               a\sin^2(\theta)
                         \begin{bmatrix} 0 & 0 & 0 & -a\sin^2(\theta) \end{bmatrix}
\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}
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                              0 \quad a \sin^2(\theta)
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```

```
[22]: rt.get_riemanntensor_type()
```

[22]: 'dddd'

```
R_{\theta\theta\phi\phi} = -a\sin^2\left(\theta\right)
```

$$R_{\theta\phi\theta\phi} = a\sin^2\left(\theta\right)$$

$$R_{\phi\theta\phi\theta} = a\sin^2\left(\theta\right)$$

$$R_{\phi\phi\theta\theta} = -a\sin^2\left(\theta\right)$$

4 Ricci Tensor

```
[24]: # Obtaining the Ricci Tensor
rit = RicciTensor(diag_comp, coord_sys)
ricci_tensor = rit.get_riccitensor()
ricci_tensor
```

```
[25]: # Default type of the Ricci Tensor rit.get_riccitensor_type()
```

[25]: 'dd'

```
[26]: # Varying type 'dd' Ricci Tensor to 'uu'
rit.vary_riccitensor_type(ricci_tensor, 'uu')
```

```
[26]: <sub>[0 0 0</sub>
[27]: rit.get_riccitensor_type()
[27]: 'uu'
        Ricci Scalar
[28]: # Obtaining the Ricci Scalar
      rs = RicciScalar(diag_comp, coord_sys)
      ricci_scalar = rs.get_ricciscalar()
      ricci_scalar
[28]: 2
     \overline{a^2}
         Traceless Ricci Tensor
[29]: # Obtaining the Traceless Ricci Tensor
      trt = TracelessRicciTensor(diag_comp, coord_sys)
      traceless_ricci_tensor = trt.get_trclss_riccitensor()
      traceless_ricci_tensor
[29]: [
[30]: # Default type of the Traceless Ricci Tensor
      trt.get_trclss_riccitensor_type()
[30]: 'dd'
[31]: # Varying type 'dd' Traceless Ricci Tensor to 'uu'
      trt.vary_trclss_riccitensor_type(traceless_ricci_tensor, 'uu')
[31]:
```

[32]: trt.get_trclss_riccitensor_type()

[32]: 'uu'

Weyl Tensor

weyl_tensor40

[35]:

```
[33]: # Obtaining the Weyl Tensor
        wyl = WeylTensor(diag_comp, coord_sys)
        weyl_tensor = wyl.get_weyltensor()
        weyl_tensor
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                                                                                        -a\sin^2\left(\theta\right)
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[34]: # Default type of the Weyl Tensor
        wyl.get_weyltensor_type()
[34]: 'dddd'
[35]: # Varying type 'dddd' Weyl Tensor to 'uuuu'
        weyl_tensor40 = wyl.vary_weyltensor_type(weyl_tensor, 'uuuu')
```

 $6a^2$

 $\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array}$

[36]: wyl.get_weyltensor_type()

[36]: 'uuuu'

[37]: wyl.nonzero_weyltensor(weyl_tensor40)

$$C^{trtr} = -\frac{1}{3a}$$

$$C^{trrt} = \frac{1}{3a}$$

$$C^{t\theta t\theta} = \frac{1}{6a^4}$$

$$C^{t\theta\theta t} = -\frac{1}{6a^4}$$

$$C^{t\phi t\phi} = \frac{1}{6a^3 \sin^2(\theta)}$$

$$C^{t\phi\phi t} = -\frac{1}{6a^3\sin^2(\theta)}$$

$$C^{rttr} = \frac{1}{3a}$$

$$C^{rtrt} = -\frac{1}{3a}$$

$$C^{r\theta r\theta} = -\frac{1}{6a^3}$$

$$C^{r\theta\theta r} = \frac{1}{6a^3}$$

$$C^{r\phi r\phi} = -\frac{1}{6a^2\sin^2{(\theta)}}$$

$$C^{r\phi\phi r} = \frac{1}{6a^2\sin^2(\theta)}$$

$$C^{\theta t t \theta} = -\frac{1}{6a^4}$$

$$C^{\theta t \theta t} = \frac{1}{6a^4}$$

$$C^{\theta r r \theta} = \frac{1}{6a^3}$$

$$C^{\theta r \theta r} = -\frac{1}{6a^3}$$

$$C^{\theta \theta \phi \phi} = -\frac{1}{a^5 \sin^2(\theta)}$$

$$C^{\theta \phi \theta \phi} = \frac{1}{3a^5 \sin^2(\theta)}$$

$$C^{\theta \phi \theta \theta} = \frac{2}{3a^5 \sin^2(\theta)}$$

$$C^{\phi t t \phi} = -\frac{1}{6a^3 \sin^2(\theta)}$$

$$C^{\phi t t \phi} = \frac{1}{6a^3 \sin^2(\theta)}$$

$$C^{\phi r r \phi} = \frac{1}{6a^2 \sin^2(\theta)}$$

$$C^{\phi r \phi r} = -\frac{1}{6a^2 \sin^2(\theta)}$$

$$C^{\phi \theta \theta \phi} = \frac{2}{3a^5 \sin^2(\theta)}$$

$$C^{\phi \theta \theta \theta} = \frac{1}{3a^5 \sin^2(\theta)}$$

$$C^{\phi \phi \theta \theta} = -\frac{1}{a^5 \sin^2(\theta)}$$

8 Einstein Tensor

[38]:
$$\begin{bmatrix} \frac{1}{a^2} & 0 & 0 & 0\\ 0 & -\frac{1}{a^3} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

[39]: 'dd'

[41]: 'uu'

9 Kretschmann Scalar

```
[42]: ks = KretschmannScalar(diag_comp, coord_sys) kret_scalar = ks.get_kretschmannscalar() kret_scalar 
[42]: \frac{4}{a^4}
```