

# example1

April 30, 2021

```
[1]: from GR_full_curv import *
from IPython.display import Latex, display
from sympy import *

init_printing()

coord_sys = symbols("t r theta phi") # Coordinate system that we will work on

G, m, c, a = symbols("G, m, c, a") # Defining some extra symbols

diag_comp = [-1, 1, coord_sys[1]**2, coord_sys[1]**2 * sin(coord_sys[2])**2] #_
↪ Defining the diagonal components of the metric tensor
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[2]: cs = ChristoffelSymbol(diag_comp, coord_sys)
chris_symbol = cs.get_christoffelsymbol()
chris_symbol
```

```
[2]: 
$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & \frac{1}{r} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & \frac{1}{r} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -r & 0 \end{bmatrix} & \begin{bmatrix} 0 & \frac{1}{r} & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & -r \sin^2(\theta) \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & -\sin(\theta) \cos(\theta) \end{bmatrix} & \begin{bmatrix} 0 & \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{bmatrix} \end{bmatrix}$$

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```
[3]: cs.vary_type(chris_symbol, 'ddd')
```

```
[3]: 
$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & r & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -r & 0 \end{bmatrix} & \begin{bmatrix} 0 & r & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & -r \sin^2(\theta) \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & -r^2 \sin(\theta) \cos(\theta) \end{bmatrix} & \begin{bmatrix} 0 & r \sin^2(\theta) & r^2 \sin(\theta) \cos(\theta) & 0 \end{bmatrix} \end{bmatrix}$$

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```
[4]: cs.get_type()
```

```
[4]: 'ddd'
```

```
[5]: cs.vary_type(chris_symbol, 'uud')
```

```
[5]: 
$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & \frac{1}{r} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & \frac{1}{r} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -\frac{1}{r} & 0 \end{bmatrix} & \begin{bmatrix} 0 & \frac{1}{r^3} & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & \frac{\cos(\theta)}{r^2 \sin(\theta)} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{r} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & -\frac{\cos(\theta)}{r^2 \sin(\theta)} \end{bmatrix} & \begin{bmatrix} 0 & \frac{1}{r^3 \sin^2(\theta)} & \frac{\cos(\theta)}{r^2 \sin^3(\theta)} & 0 \end{bmatrix} \end{bmatrix}$$

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```
[6]: cs.get_type()
```

```
[6]: 'uud'
```

```
[7]: cs.vary_type(chris_symbol, 'uuu')
```

```
[7]:
```

$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{r^3} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^3 \sin^2(\theta)} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r^3} & 0 \\ 0 & \frac{1}{r^3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\cos(\theta)}{r^4 \sin^3(\theta)} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r^3 \sin^2(\theta)} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{r^4 \sin^3(\theta)} \\ 0 & \frac{1}{r^3 \sin^2(\theta)} & \frac{\cos(\theta)}{r^4 \sin^3(\theta)} & 0 \end{bmatrix} \end{bmatrix}$$

```
[8]: cs.get_type()
```

```
[8]: 'uuu'
```

```
[9]: cs.nonzero_christoffelsymbol(chris_symbol)
```

$$\begin{aligned} \Gamma^r_{\theta\theta} &= -r \\ \Gamma^r_{\phi\phi} &= -r \sin^2(\theta) \\ \Gamma^\theta_{r\theta} &= \frac{1}{r} \\ \Gamma^\theta_{\theta r} &= \frac{1}{r} \\ \Gamma^\theta_{\phi\phi} &= -\sin(\theta) \cos(\theta) \\ \Gamma^\phi_{r\phi} &= \frac{1}{r} \\ \Gamma^\phi_{\theta\phi} &= \frac{\cos(\theta)}{\sin(\theta)} \\ \Gamma^\phi_{\phi r} &= \frac{1}{r} \\ \Gamma^\phi_{\phi\theta} &= \frac{\cos(\theta)}{\sin(\theta)} \end{aligned}$$

```
[10]: cs.get_christoffelsymbol()
```

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[10]:
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$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -r & 0 \\ 0 & 0 & 0 & -r \sin^2(\theta) \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\sin(\theta) \cos(\theta) \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{bmatrix} \end{bmatrix}$$