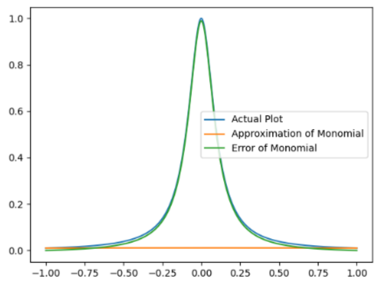
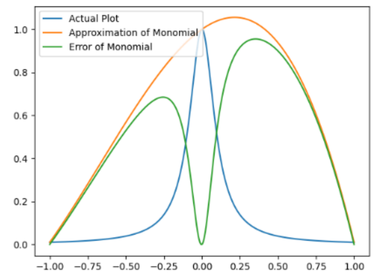
APPM 4600 – Homework 7 – Cambria Chaney

1.) (a) See PDF at the bottom - Monomials

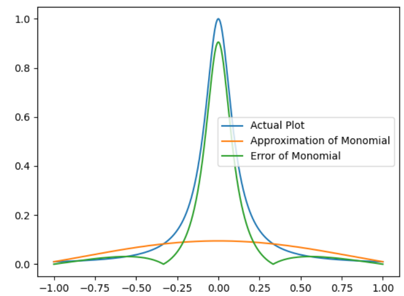
(b) N = 2



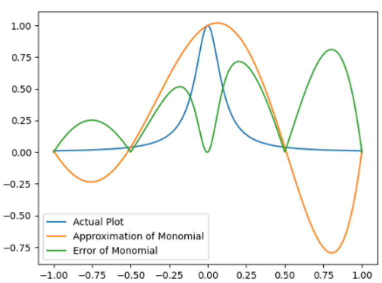
N = 3



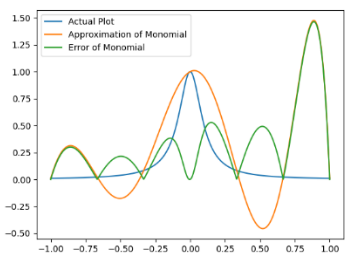
N = 4



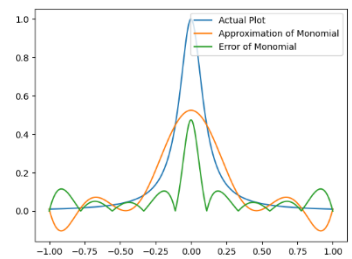
N = 5



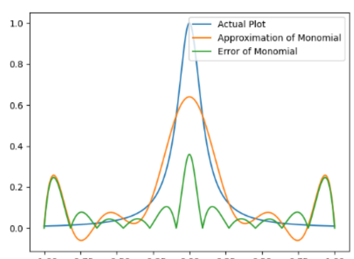
N = 7



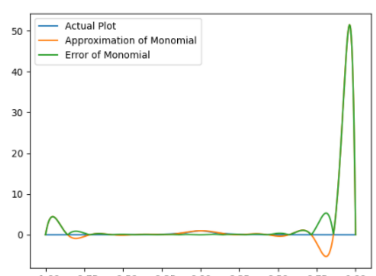
N = 10



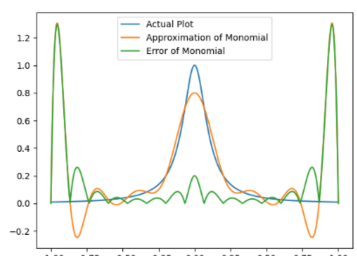
N = 12



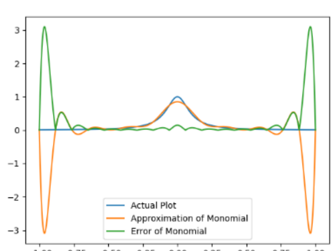
N = 15



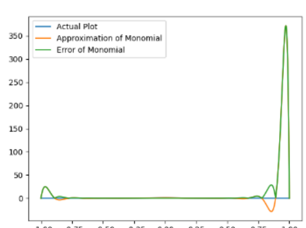
N = 16



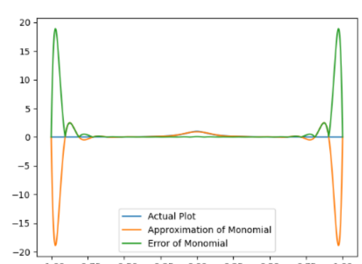
N = 18



N = 19



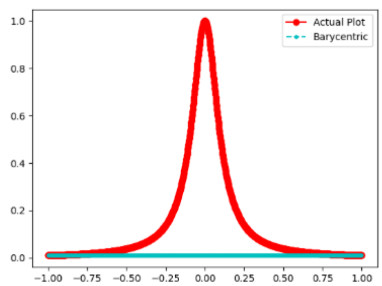
N = 22



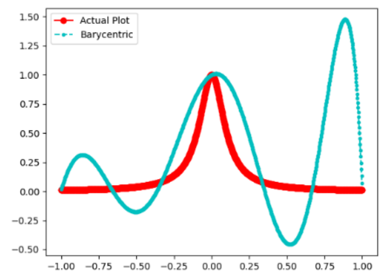
As you can see from the above graphs, as N increases, the polynomial approximation becomes closer to the actual function, anywhere that is not the endpoints. As N increases, the polynomial behaves increasingly worse at the endpoints due to Runge's phenomena.

2.) Barycentric Lagrange

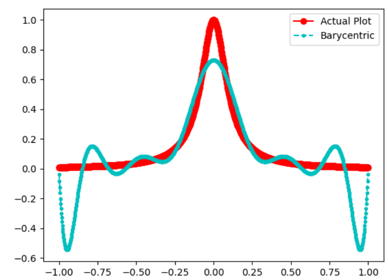
Polynomials: N = 2



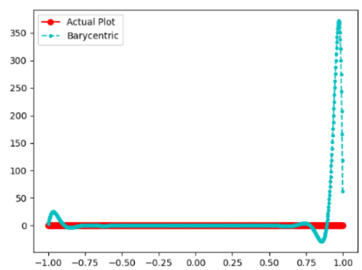
N = 7



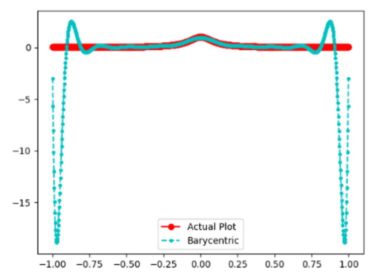
N = 14



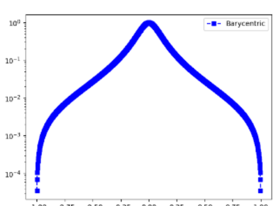
N = 19



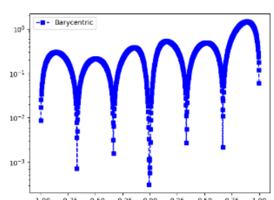
N = 22



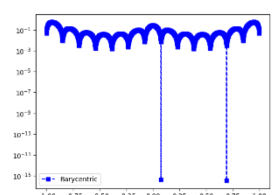
Errors: N = 2



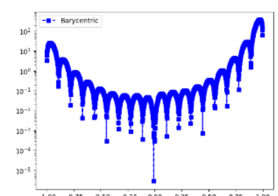
N = 7



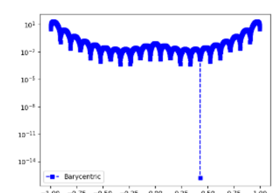
N = 14



N = 19



N = 22

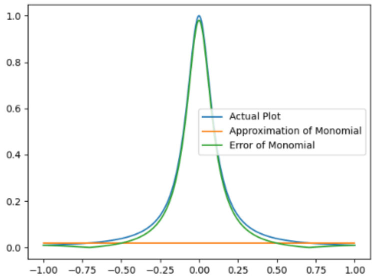


As you can see, you are still getting bad behavior at the endpoints of the polynomial, much like the Monomials method in problem 1, but it is well behaved for large n inside the interval and for small x.

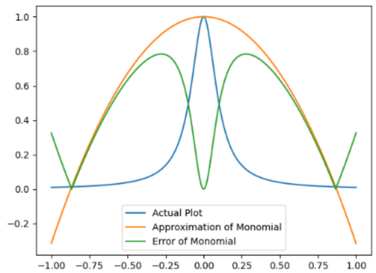
3.) From the graphs below, you can see that the interpolation will not fail now due to the change in the xj, which are now clustered towards the endpoints. If you look closely, the polynomial near the end points does not fail here and as N increases the polynomial gets closer to the actual function.

3.)

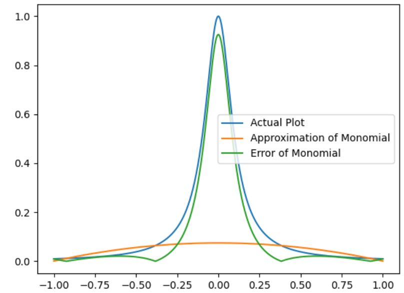
N = 2



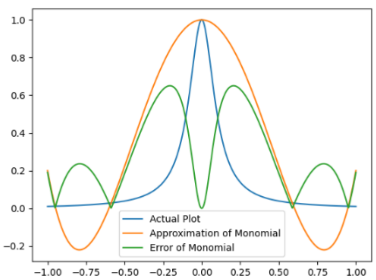
N = 3



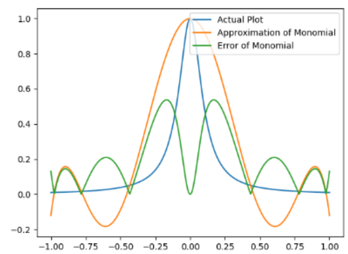
N = 4



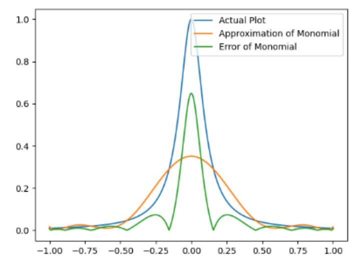
N = 5



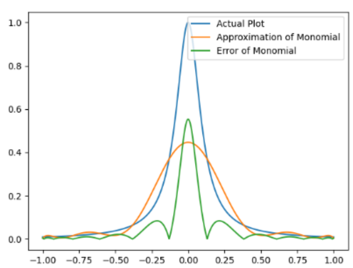
N = 7



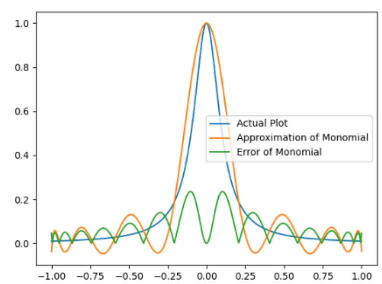
N = 10



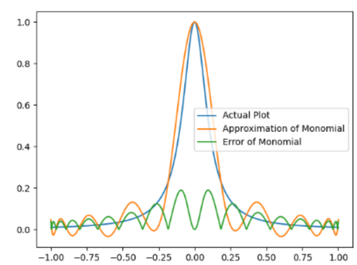
N = 12



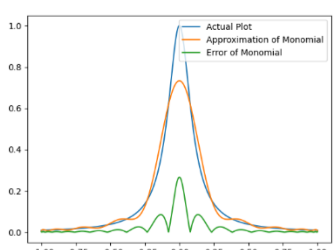
N = 15



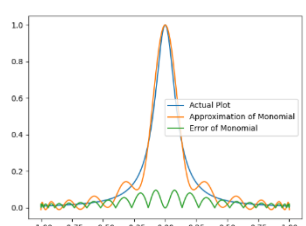
N = 17



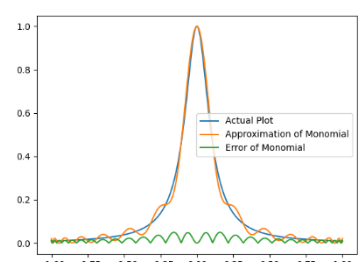
N = 20



N = 23



N = 29



1.) (a)

1) (a) (x_i, y_i) $j=1,\ldots,n$ given

Derive $Vc=y$ & find matrix V

want to construct a polynomial

$P_n(x) = a_0 + a_1 x + \ldots + a_n x^n$

If you plug in x_j for each data point:

$P_n(x_j) = a_0 + a_1 x_j + a_2 x_j^2 + \ldots + a_n x_j^n = f(x_j)$

we have multiple x_j so you must solve a system of equations for the coefficients w/ each $P_n(x_j)$ for $j=0,1,\ldots,n$, which can be written w/ matrices:

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & & x_1^n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix}$$

This derives the system $Vc=y$ where

$$V = \begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & \vdots & \vdots & & \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix}$$