APPM 4600 - HW3 - Cambria C

1) $2x-1=\sin x$

(a) $2x-1-\sin x=0$

$[0,2]$ contains a root r

Intermediate value theorem:
In an interval $[a,b]$, if $c\in[f(a),f(b)]$, then there exists a $w\in[a,b]$ s.t. $f(w)=c$.

In this case w is the x-value of the root and we want $c=0$

$f(a)=f(0)=-1$
$f(b)=f(2)=3-\sin 2\approx 2$

Thus, $c=0$ is in the interval $[-1,2]$
Therefore there is an x-value w that exists such that $f(w)=0$, thus proving the existence of a root.

(b) $f(x)=2x-1-\sin x$
$f'(x)=2-\cos x$

$f'(x)$ is greater than 0 for all x which means the function is strictly increasing for all x. Meaning, there is only one time where $f(x)$ crosses the x-axis, given that there are no sign changes in $f'(x)$.

(c) See code

Final approx: 0.88786221

Iterations: 27

2) (a) root: 5.0000732 $f(\text{root}) = 6.06 \times 10^{-38}$

3) (a) upper bound on # iterations to approximate $x^3 + x - 4 = 0$ in $[1, 4]$ w/ $tol = 10^{-3}$

Solve $\left(\frac{1}{2}\right)^n (b-a) < tol$ for $n = N_{max}$

$\left(\frac{1}{2}\right)^n (4-1) < 10^{-3}$

$\left(\frac{1}{2}\right)^n < \left(\frac{1}{3}\right) \times 10^{-3}$

$n \cdot \ln\left(\frac{1}{2}\right) < \ln\left(\frac{1}{3} \times 10^{-3}\right)$

$\boxed{n < 11.5507 = N_{max}}$ so 12

(b) See code

root approx: 1.378

iterations: 11

The number of iterations is less than our upper bound of Nmax iterations.

2) (b) root: 5.128 $f(\text{root}) = 0$

(c) What's happening is that with the larger floating point arithmetic in the expanded version, it is getting an inaccurate root. But it gets $f(\text{root}) = 0$ due to the subtraction of similar items in the expanded version

4) (a) $x_{n+1} = -16 + 6x_n + \frac{12}{x_n}$, $x_* = 2$

$$g(x) = -16 + 6x + \frac{12}{x}$$
$$g'(x) = 6 - \frac{12}{x^2}$$
$$g'(2) = 6 - \frac{12}{4} = 6 - 3 = 3$$
$$g''(x) = \frac{24}{x^3} \quad g''(2) = \frac{24}{8} = 3 \neq 0$$

Thus it doesn't converge since $|g'(2)| > 1$

(b) $x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n^2}$, $x_* = 3^{1/3}$

$$g'(x) = \frac{2}{3} - \frac{2}{x_n^3}$$
$$g'(3^{1/3}) = \frac{2}{3} - \frac{2}{3} = 0$$
$$g''(x) = \frac{6}{x_n^4} \quad g''(3^{1/3}) = \frac{6}{3^{4/3}} \neq 0$$

Thus it converges w/ $\alpha = 2$.

(c) $x_{n+1} = \frac{12}{1+x_n}$ $\quad x_* = 3$

$$g'(x) = -\frac{12}{(1+x_n)^2} \quad g'(3) = \frac{-12}{4^2} = \frac{-12}{16}$$

$|g'(3)| < 1$ Thus it converges linearly w/ rate $3/4$

5) (a) see code & plot. there are 5 zeros.

(b) $x_{n+1} = -\sin(2x_n) + \frac{5x_n}{4} - \frac{3}{4}$

$g'(x) = |-2\cos(2x) + \frac{5}{4}| \leq 3.25$

$g''(x) = |4\sin(2x)| \leq 4$

roots: 1.732, 3.1618, 4.517, -0.544, -0.898

$-2\cos(2*1.732) + \frac{5}{4} = |3.1461| > 1$

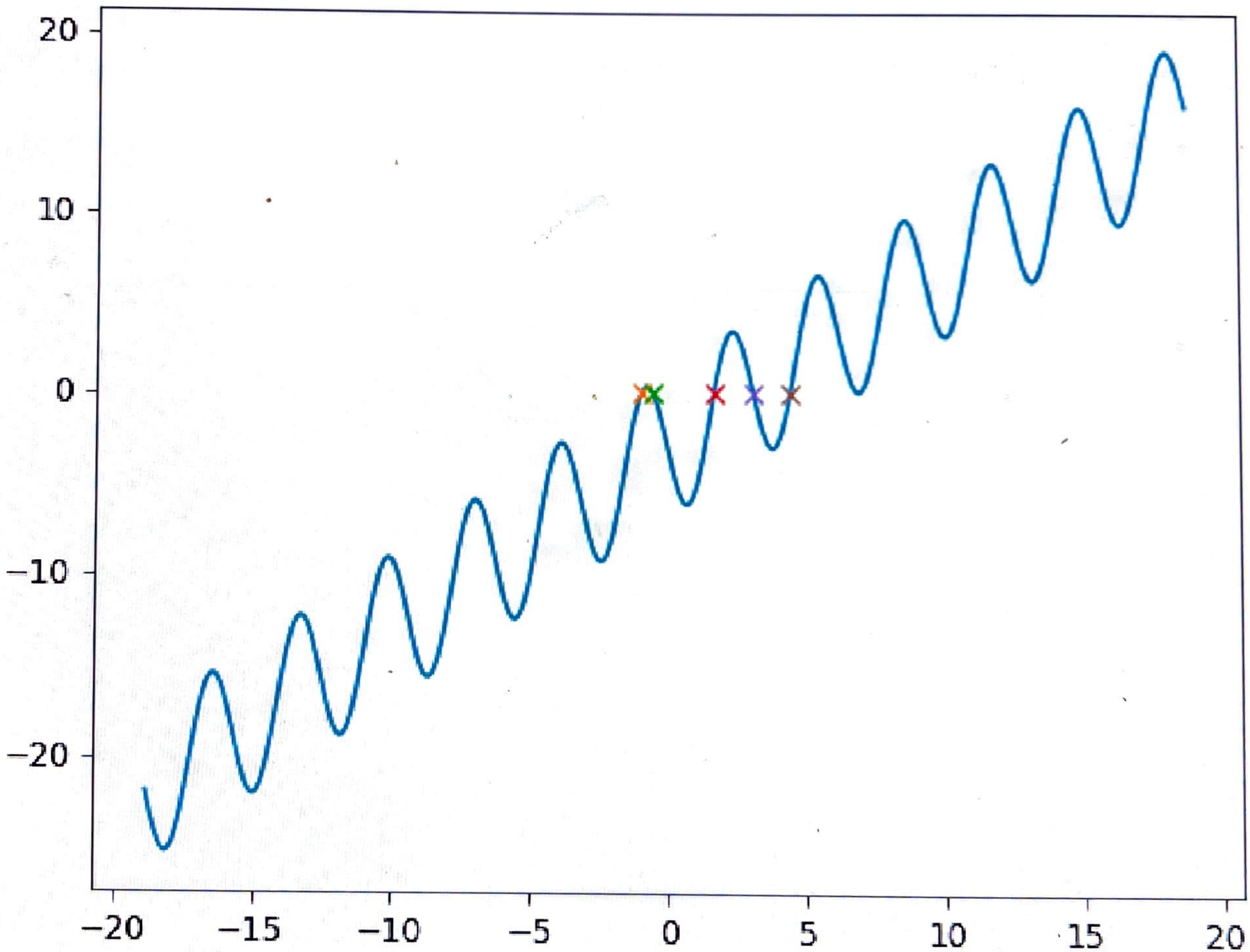
$-2\cos(2*3.1618) + 5/4 = |-0.7483| < 1$

$-2\cos(4.517*2) + 5/4 = |3.099| > 1$

$-2\cos(-0.544*2) + 5/4 = |0.3214| < 1$

$-2\cos(-0.898*2) + 5/4 = |1.696| > 1$

The code only found 2 roots: 3.16 & -0.544. This makes sense because the iteration only converges at those 2 roots out of the 5. When you plug the roots into the derivative, only 2 of them are less than 1 in absolute value, which are the 2 that are found by the computer



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