APPM 4600 - HW 6 - Cambria Chaney

1) (i) $x=1 \quad y=1$

Newton: [-1.81, 0.837]
7 iterations 0.0006s

Lazy: Error - 99 iterations
[nan, nan]

Broyden: [-1.81, 0.837]
12 iterations 0.0007s

In this case both quasi-Newton methods performed worse either with an error or more iterations + time to compute than Newton's method.

(ii) $x=1 \quad y=-1$

Newton: [1.004, -1.729]
5 iterations 0.00066s

Lazy: [1.004, -1.729]
36 iterations 0.0008s

Broyden: [1.004, -1.729]
6 iterations 2.5×10^{-5}s

Lazy Newton had more iterations + time performed worse than Newton's method. Broyden's method took much less time but had one more iteration than Newton's method, so I would say it was fairly equivalent to Newton's method

(iii) $x=0 \quad y=0$

None of the methods here worked because the Jacobian was a singular matrix

2) $X_0 = [0, 0, 1]$

Newton: $10^{-6} = tol$ $\quad [0, 0.1, 1]$

2 iterations, 0.00126s

Steepest Descent: $5\times10^{-2} = tol$

$[-0.02218, 0.0887, 0.995]$

0 iterations

this converged fast due to low tolerance

Steepest Descent: $tol = 10^{-6}$

$[-6.64\times10^{-5}, 9.99\times10^{-2}, 9.99\times10^{-1}]$

4 iterations

Newton w/ $X_0 = [-0.02218, 0.0887, 0.995]$

$[5.814\times10^{-17}, 1.00\times10^{-1}, 1.000\times10^{0}]$

2 iterations 0.00037s

It looks like Newton's methods converged faster than the steepest descent with the same tolerance. It looks like both steepest descent methods are less accurate than Newton's, but when its output combined w/ Newton's method, it was more accurate. (*)

(*) Overall, I think the Newton's method converged the fastest