

1a.

							Premise	Derived
	A	B	C	D	$A \wedge B$	$C \wedge D$	$A \wedge B \Rightarrow C \wedge D$	$A \wedge B \Rightarrow C$
M0	0	0	0	0	0	0	1	1
M1	0	0	0	1	0	0	1	1
M2	0	0	1	0	0	0	1	1
M3	0	0	1	1	0	1	1	1
M4	0	1	0	0	0	0	1	1
M5	0	1	0	1	0	0	1	1
M6	0	1	1	0	0	0	1	1
M7	0	1	1	1	0	1	1	1
M8	1	0	0	0	0	0	1	1
M9	1	0	0	1	0	0	1	1
M10	1	0	1	0	0	0	1	1
M11	1	0	1	1	0	1	1	1
M12	1	1	0	0	1	0	0	0
M13	1	1	0	1	1	0	0	0
M14	1	1	1	0	1	0	0	1
M15	1	1	1	1	1	1	1	1

1b.

1. $A \wedge B \Rightarrow C \wedge D$
2. $\neg(A \wedge B) \vee (C \wedge D)$ // IE 1
3. $(\neg(A \wedge B) \vee C) \wedge (\neg(A \wedge B) \vee D)$ // Distributivity 2
4. $\neg(A \wedge B) \vee C$ // AE 3
5. $(\neg A \vee \neg B) \vee C$ // DM 4
6. $\neg A \vee \neg B \vee C$
7. $A \wedge B \Rightarrow C$ // Using given Implication Introduction Hint from Horn clause 6

1c.

$$KB = A \wedge B \Rightarrow C \wedge D$$

Convert to CNF

1. $\neg(A \vee B) \vee (C \wedge D)$
2. $(\neg A \vee \neg B) \vee (C \wedge D)$
3. $(\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg B \vee D)$

$$q = A \wedge B \Rightarrow C$$

Convert to CNF

1. $\neg(A \vee B) \vee C$
2. $(\neg A \vee \neg B \vee C)$

$$\neg q = (A \vee B \vee \neg C)$$

Proof:

1. $(\neg A \vee \neg B \vee C)$
2. $(\neg A \vee \neg B \vee D)$
3. $(A \vee B \vee \neg C)$
4. $(\neg B \vee C) \vee (B \vee \neg C)$ // Resolution 1,3
5. $C \vee \neg C$ // Resolution 4
6. \emptyset // Resolution 5, empty clause which means we proved $KB \models q$

2a.

KB:

1. $L1W \Rightarrow \neg C1W$
2. $L1Y \Rightarrow \neg C1Y$
3. $L1B \Rightarrow \neg C1B$
4. $L2W \Rightarrow \neg C2W$
5. $L2Y \Rightarrow \neg C2Y$
6. $L2B \Rightarrow \neg C2B$
7. $L3W \Rightarrow \neg C3W$
8. $L3Y \Rightarrow \neg C3Y$
9. $L3B \Rightarrow \neg C3B$
10. $O1W \Rightarrow \neg C1Y$
11. $O1Y \Rightarrow \neg C1W$

- 12. $O2W \Rightarrow -C2Y$
- 13. $O2Y \Rightarrow -C2W$
- 14. $O3W \Rightarrow -C3Y$
- 15. $O3Y \Rightarrow -C3W$

- 16. $C1W \Rightarrow -C1Y \wedge -C1B$
- 17. $C1Y \Rightarrow -C1W \wedge -C1B$
- 18. $C1B \Rightarrow -C1Y \wedge -C1W$
- 19. $C2W \Rightarrow -C2Y \wedge -C2B$
- 20. $C2Y \Rightarrow -C2W \wedge -C2B$
- 21. $C2B \Rightarrow -C2Y \wedge -C2W$
- 22. $C3W \Rightarrow -C3Y \wedge -C3B$
- 23. $C3Y \Rightarrow -C3W \wedge -C3B$
- 24. $C3B \Rightarrow -C3Y \wedge -C3W$

- 25. $C1W \Rightarrow -C2W \wedge -C3W$
- 26. $C2W \Rightarrow -C1W \wedge -C3W$
- 27. $C3W \Rightarrow -C1W \wedge -C2W$
- 28. $C1Y \Rightarrow -C2Y \wedge -C3Y$
- 29. $C2Y \Rightarrow -C1Y \wedge -C3Y$
- 30. $C3Y \Rightarrow -C1Y \wedge -C2Y$
- 31. $C1B \Rightarrow -C2B \wedge -C3B$
- 32. $C2B \Rightarrow -C1B \wedge -C3B$
- 33. $C3B \Rightarrow -C1B \wedge -C2B$

- 34. $C1W \vee C1Y \vee C1B$
- 35. $C2W \vee C2Y \vee C2B$
- 36. $C3W \vee C3Y \vee C3B$

2b.

Starting with Facts:

- 37. $O1Y$
- 38. $L1W$
- 39. $O2W$
- 40. $L2Y$
- 41. $O3Y$
- 42. $L3B$

Continuing with Natural Deduction

43. $\neg C3B$ // MP 9, 42
44. $\neg C3W$ // MP 15, 41
45. $\neg C3W \wedge \neg C3B$ // AI 44, 43
46. $\neg (C3W \vee C3B)$ // DM 45
47. $C3Y$ // Resolution 36, 46
48. $\neg C1Y \wedge \neg C2Y$ // MP 30, 47
49. $\neg C1Y$ // AE 48
50. $\neg C1W$ // MP 1, 38
51. $\neg C1Y \wedge \neg C1W$ // AI 49, 50
52. $\neg (C1Y \vee C1W)$ // DM 51
53. $C1B$ // Resolution 34, 52
54. $\neg C2B \wedge \neg C3B$ // MP 31, 53
55. $\neg C2B$ // AE 54
56. $\neg C1Y \wedge \neg C2Y$ // MP 30, 47
57. $\neg C2Y \wedge \neg C1Y$ // Commutativity 56
58. $\neg C2Y$ // AE 57
59. $\neg C2Y \wedge \neg C2B$ // AI 58, 55
60. $\neg (C2Y \vee C2B)$ // DM 59
61. $C2W$ // Resolution 35, 60

2c.

KB \Rightarrow CNF:

1. $\neg L1W \vee \neg C1W$
2. $\neg L1Y \vee \neg C1Y$
3. $\neg L1B \vee \neg C1B$
4. $\neg L2W \vee \neg C2W$
5. $\neg L2Y \vee \neg C2Y$
6. $\neg L2B \vee \neg C2B$
7. $\neg L3W \vee \neg C3W$
8. $\neg L3Y \vee \neg C3Y$
9. $\neg L3B \vee \neg C3B$
10. $\neg O1W \vee \neg C1Y$
11. $\neg O1Y \vee \neg C1W$
12. $\neg O2W \vee \neg C2Y$
13. $\neg O2Y \vee \neg C2W$
14. $\neg O3W \vee \neg C3Y$
15. $\neg O3Y \vee \neg C3W$
16. $(\neg C1W \vee \neg C1Y) \wedge (\neg C1W \vee \neg C1B)$

17. $(\neg C1Yv-C1W) \wedge (\neg C1Yv-C1B)$
18. $(\neg C1Bv-C1Y) \wedge (\neg C1Bv-C1W)$
19. $(\neg C2Wv-C2Y) \wedge (\neg C2Wv-C2B)$
20. $(\neg C2Yv-C2W) \wedge (\neg C2Yv-C2B)$
21. $(\neg C2Bv-C2Y) \wedge (\neg C2Bv-C2W)$
22. $(\neg C3Wv-C3Y) \wedge (\neg C3Wv-C3B)$
23. $(\neg C3Yv-C3W) \wedge (\neg C3Yv-C3B)$
24. $(\neg C3Bv-C3Y) \wedge (\neg C3Bv-C3W)$

25. $(\neg C1Wv-C2W) \wedge (\neg C1Wv-C3W)$
26. $(\neg C2Wv-C1W) \wedge (\neg C2Wv-C3W)$
27. $(\neg C3Wv-C1W) \wedge (\neg C3Wv-C2W)$
28. $(\neg C1Yv-C2Y) \wedge (\neg C1Yv-C3Y)$
29. $(\neg C2Yv-C1Y) \wedge (\neg C2Yv-C3Y)$
30. $(\neg C3Yv-C1Y) \wedge (\neg C3Yv-C2Y)$
31. $(\neg C1Bv-C2B) \wedge (\neg C1Bv-C3B)$
32. $(\neg C2Bv-C1B) \wedge (\neg C2Bv-C3B)$
33. $(\neg C3Bv-C1B) \wedge (\neg C3Bv-C2B)$

34. $C1WvC1YvC1B$
35. $C2WvC2YvC2B$
36. $C3WvC3YvC3B$

2d.

Starting with Facts:

37. $O1Y$
38. $L1W$
39. $O2W$
40. $L2Y$
41. $O3Y$
42. $L3B$

$q = C2W$
 $\neg q = \neg C2W$

43. $\neg C2W$

Continuing with Resolution:

44. C2YvC2B // Resolution 35, 43
45. -C2Y // Resolution 12, 39
46. C2B // Resolution 44, 45
47. -C3B // Resolution 9, 42
48. -C3W // Resolution 15, 41
49. -C3B^C3W // AI 47, 48
50. -(C3BvC3W) // DM 49
51. C3Y // Resolution 36, 50
52. -C3YvC1Y // AE 30
53. -C1Y // Resolution 52, 51
54. -C1W // Resolution 1, 38
55. -C1Y^C1W // AI 53, 54
56. -(C1YvC1W) // DM 55
57. C1B // Resolution 34, 56
58. -C1BvC2B // AE 31
59. -C2B // Resolution 58, 57
60. \emptyset // Resolution 46, 59, empty clause which means we proved $KB \models q$

3.

Triggered rules:

1. e
2. m
3. o
4. k
5. j
6. b

List of Inferred Propositions:

1. HaveBike
2. CarRentalOpen
3. IsNotAHoliday
4. CanRentCar
5. CanDriveToWork
6. CanGetToWork

Yes CanGetToWork is among the inferred propositions.

4.

1. CanGetToWork
2. CanBikeToWork
3. HaveBike, WorkCloseToHome, Sunny
4. HaveMountainBike, WorkCloseToHome, Sunny
5. WorkCloseToHome, Sunny
6. Sunny
7. // Backtrack to CanBikeToWork
8. // Backtrack to CanGetToWork
9. CanDriveToWork
10. OwnCar
11. // Backtrack to CanDriveToWork
12. CanRentCar
13. HaveMoney, CarRentalOpen
14. CarRentalOpen
15. HertzOpen
16. // Backtrack to CarRentalOpen
17. AvisOpen
18. ☐ // Success