# Bayesian Statistics

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## Bayesian vs. Frequentist

There are **two** major approaches to statistics: Frequentist and Bayesian

**Frequentist**

- Calculates the probability of observing a set of data given a hypothesis
- Parameters are fixed, but unknown
- Does not factor in belief or uncertainties; purely objective rather than subjective
- Procedures are judged by how well they perform over infinite repetitions of all random samples

**Bayesian**

- Calculates the probability of a correct hypothesis given the data observed
- Parameters are considered to be random variables and therefore described with probabilities
- Probability statements used to describe parameters reflect a "degree of belief" and are therefore subjective rather than objective
- Data is used in iteration to revise beliefs about parameters and create a **posterior distribution**, which incorporates the **prior distribution** and observed data to assign relative weights to each possible parameter

## History

- Reverend Thomas Bayes first wrote up Bayes' Theorem in a paper titled *An Essay Towards Solving a Problem in the Doctrine of Chances*.
- His friend Richard Price discovered the paper after Bayes' death and had it published in 1763 in the *Philosophical Transactions of the Royal Society*.

## Probability Theory

**Definition 1** ***Probability*** *is the likelihood that an event occurs.*

**Definition 2** ***Conditional Probability*** *is the likelihood that an event occurs given that another event has already occurred, denoted as $P(A|B)$ or "The probability of A given B."*

**Definition 3** *The process of determining the probability distribution of an unobserved variable (such as a population) is known as* ***Inferential Statistics****, while it used to be known as* ***Inverse Probability****.*

**Definition 4** *A* ***Prior Distribution*** *is the assumed probability distribution of an event before any data is taken into account.*

**Definition 5** *A* ***Posterior Distribution*** *is an event's calculated probability distribution once data is factored in.*

**Definition 6** *The* ***Likelihood Function*** *is the probability of the observed data given the event occurs, or the inverse of the Posterior Distribution, denoted as $P(B|A)$.*

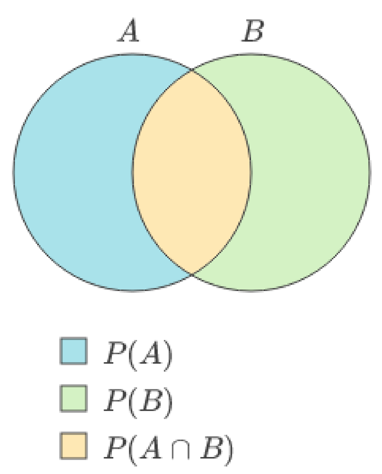
**Definition 7** ***Bayesian Inference*** *is the use of Bayes' Theorem to update the probability of a hypothesis as more data becomes available.*

## Conditional Probability

The formula for **Conditional Probability** is as follows:

$P(A|B) = \dfrac{P(A\cap B)}{P(B)}$, if $P(B) \neq 0$

where $P(A\cap B)$ is the probability of both A and B.




## Bayes' Theorem

Using the inverse probability of $P(A|B)$ and algebraic manipulation yields Bayes' Theorem.

$$P(B|A) = \frac{P(A\cap B)}{P(A)}, \text{ if } P(A) \neq 0$$

Solve for $P(A\cap B)$ and substitute into the formula for $P(A|B)$.

$$P(A\cap B) = P(B|A)P(A)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \text{ if } P(B) \neq 0.$$

## An Example

The following is a simplified version of the thought experiment that Thomas Bayes used to discover his namesake theorem.

While your back is turned, your friend throws a ball on a table behind you, and you must guess if it landed on the left or right side. You assume that the ball has an equal chance of landing anywhere on the table. You ask your friend to throw another ball on the table and tell you if it landed to the left or to the right of the first ball. You then repeat this process, each time gaining more data. With every new piece of data, you are able to more and more confidently guess whether the first ball is on the left or right side of the table.

## With the Numbers

$P(R1) =$ likelihood the 1st ball lands on the right

$P(R2) =$ likelihood the 2nd ball lands to the right of the 1st

$P(R1) = 0.5, P(R2) = 0.5, P(R2|R1) = 0.25$

Any ball is equally likely to land on the left or right side, and if we assume the 1st ball is in the center of the right side, that leaves 75% of the table to its left and 25% to its right. Given this, we can solve for $P(R2|R1)$.

$$P(R1|R2) = \frac{P(R2|R1)P(R1)}{P(R2)}$$

$$P(R1|R2) = \frac{0.25(0.5)}{0.5} = 0.25$$

## Applications

- Bayes' Theorem has been used for calculating loss rates on prospective loans.
- It has also been used to as a new approach for analyzing clinical research results.
- Bayes' Theorem can be used for AI learning models, improving search engines.

## References

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