Bayesian Statistics

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Statistics and Probability

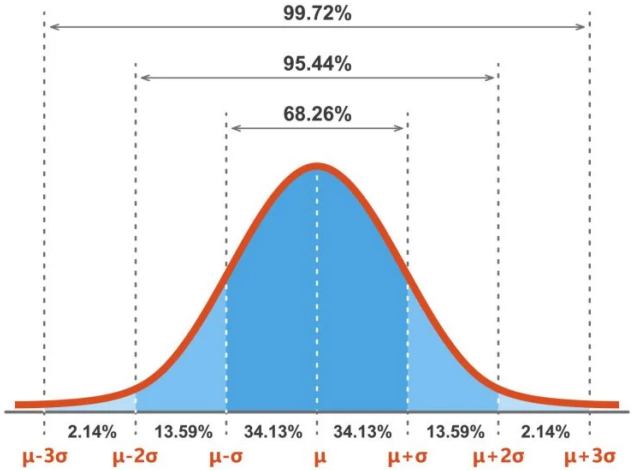
- What is **Statistics**?

  1. Parameters
  2. Hypotheses
  3. Probability

- Two major approaches: **Frequentist** and **Bayesian**

Frequentist vs. Bayesian

Frequentist	**Bayesian**
Hypothesis $\rightarrow$ Data Observed	Data Observed $\rightarrow$ Hypothesis
Parameters are fixed	Parameters are random variables
No belief or uncertainties	Update beliefs with new data



History

Discovery

Reverend Thomas Bayes - An Essay Towards Solving a Problem in the Doctrine of Chances

Proliferation

Found by Bayes' friend Richard Price after his death, who had it published in 1763.

Important Definitions

1. Conditional Probability
   - $P(A|B)$
2. Probability Distribution

X	1	2	3	4	5	6
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

3. Prior Distribution
   - $P(A)$
4. Posterior Distribution
   - $P(A|B)$
5. Likelihood Function
   - $P(B|A)$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ if } P(B) \neq 0$$

where $P(A \cap B)$ is the probability of both A and B.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ if } P(A) \neq 0$$

$$P(A \cap B) = P(B|A)P(A)$$

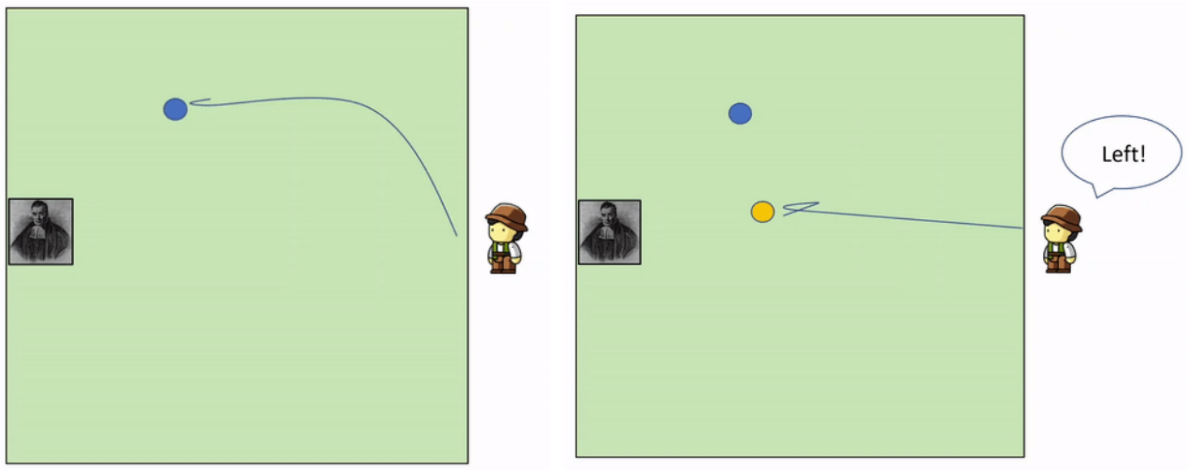
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \text{ if } P(B) \neq 0.$$

Theorem

Bayes' Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

An Example



$P(R_1)$ = likelihood the 1st ball lands on the right

$P(R_2)$ = likelihood the 2nd ball lands to the right of the 1st

Assumptions: $P(R_1) = 0.5$, $P(R_2) = 0.5$, $P(R_2|R_1) = 0.25$

$$P(R_1|R_2) = \frac{P(R_2|R_1)P(R_1)}{P(R_2)} = \frac{0.25(0.5)}{0.5} = 0.25$$

Prominent Uses

- Finance: Predicting lending risks (Bijak et al.).
- Medical Research: New way to analyze drug trials (Wijeysundera et al.).
- Artificial Intelligence: Improving search engines by eliminating bias (Yang et al.).

Thank You!