State Space Variable Definitions and Equations

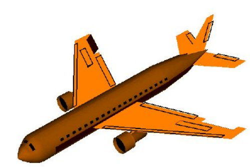
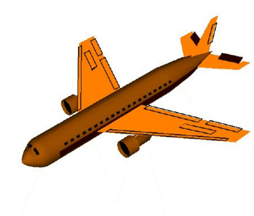
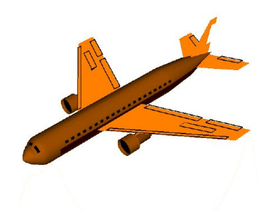
1/20/2020
Revision 3.0

Input Vector, u

Variable	Variable Name	Definition	Relative information / Diagrams
t	Throttle	Control Surface Deflection	Depends on the RC F-16
e	Elevator	Control Surface Deflection	Depends on the RC F-16
a	Aileron	Control Surface Deflection	Depends on the RC F-16
r	Rudder	Control Surface Deflection	Depends on the RC F-16
W_N	Wind (x component)	A derivative of the Wind Velocity Component	Needed to be included in the Input vector for calculation of the vehicle velocity relative to the surrounding air
W_E	Wind (y component)	A derivative of the Wind Velocity Component	Needed to be included in the Input vector for calculation of the vehicle velocity relative to the surrounding air
W_D	Wind (z component)	A derivative of the Wind Velocity Component	Needed to be included in the Input vector for calculation of the vehicle velocity relative to the surrounding air

State Vector, x in the body axis

Variable	Variable Name	Definition	Relative information / Diagram
P_N	Position (x component)	A part of the local geographic system,	Overview on pg 27 of the text

		the north axis is aligned geographically true north	
P_E	Position (y component)	A part of the local geographic system, the East axis is aligned geographically true east	Overview on pg 27 of the text
P_D	Position (z component)	A part of the local geographic system, the Down axis is aligned with the geocentric position vector.	Overview on pg 27 of the text
ϕ (Phi)	Euler Angle (Bank (roll) - x component)	Bank Angle	
θ	Euler Angle (Pitch - y component)	Pitch Angle	
ψ	Euler Angle (Yaw - z component)	Yaw Angle	
U	Velocity in the body (x component)	The velocity of the body relative to the air in the x-direction	
V	Velocity in the body (y component)	The velocity of the body relative to the air in the y-direction	

W	Velocity in the body (z component)	The velocity of the body relative to the air in the z-direction	
P	Angular Velocity (x component)	Euler's Equations of motion angular velocity in the x direction	
Q	Angular Velocity (y component)	Euler's Equations of motion velocity in the y direction	
R	Angular Velocity (z component)	Euler's Equations of motion velocity in the z direction	

Variable	Equation	Relative information/diagram
t	[Lower Limit, Upper Limit] expressed in radians	Related to propulsion effects, we will need to define later: $F = u_i * m * g$; where ui is the index of the input vector
e	[Lower Limit, Upper Limit] expressed in radians	$F = u_i * m * g$
a	[Lower Limit, Upper Limit] expressed in radians	$F = u_i * m * g$
r	[Lower Limit, Upper Limit] expressed in radians	$F = u_i * m * g$
W_N	$V_{w/e}$	Velocity of wind with respect to the earth
W_E	$V_{w/e}$	Velocity of wind with respect to the earth
W_D	$V_{w/e}$	Velocity of wind with respect to the earth

State Vector, x, related equations in the body axis

Variable	Equation	Relative information / Diagram
P_N	$\dot{P}_N = Uc\theta c\psi + V(-c\phi s\psi + s\phi s\theta c\psi) + W(s\phi s\psi + c\phi s\theta c\psi)$	c = cosine s = sine
P_E	$\dot{P}_E = Uc\theta s\psi + V(c\phi c\psi + s\phi s\theta s\psi) + W(-s\phi c\psi + c\phi s\theta s\psi)$	c = cosine s = sine
P_D	$\dot{P}_D = -h,\ h = Us\theta - Vs\phi c\theta - Wc\phi c\theta$	c = cosine s = sine
ϕ (Phi)	$= P + t\theta(Qs\phi + Rc\phi)$	c = cosine s = sine t = tangent
θ	$= Qc\phi - Rs\phi$	c = cosine s = sine
ψ	$= (Qs\phi + Rc\phi)/c\theta$	c = cosine s = sine
U	**U** $= RV - QW - g_d s\theta + (X_A + X_T)/m$	s = sine X_A = Aerodynamic force in the x component X_T = Thrust force in the x component
V	**V** = $-RU + PW + g_d s\phi c\theta + (Y_A + Y_T)/m$	s = sine c = cosine Y_A = Aerodynamic force in the y component Y_T = Thrust force in the y component

W	$\mathbf{W} = QU - PV + g_d c\phi c\theta + (Z_A + Z_T)/m$	s = sine c = cosine Z_A = Aerodynamic force in the z component Z_T = Thrust force in the z component
P	$\Gamma \mathbf{P} = J_{xz}[J_x - J_y + J_z]PQ - [J_z(J_z - J_y) + J_{xz}{}^2]QR + J_z l + J_{xz} n$	J is part of an inertia matrix
Q	$J_y \mathbf{Q} = (J_z - J_X)RP - J_{xz}(P^2 - R^2) + m$	J is part of an inertia matrix (see equation 1.7-9 in the text. It is on pg 38)
R	$\Gamma \mathbf{R} = - J_{xz}(J_x - J_y + J_z)QR + [J_x(J_x - J_y) + J_{xz}{}^2]QR + J_z l + J_{xz} n$	J is part of an inertia matrix (see equation 1.7-9 in the text. It is on pg 38)

Reference: pg 111 of the textbook “Aircraft Control and Simulation” by Brian L Stevens.

Inertia

Note: this is in the stability axis and will need to be transformed into the body axis.

J′x = Jxcos2α + Jzsin2α − Jxz sin 2α

J′y = Jy

J′z = Jxsin2α + Jzcos2α + Jxz sin 2α

J′xz = 1/2(Jx − Jy) sin 2α + Jxz cos 2α

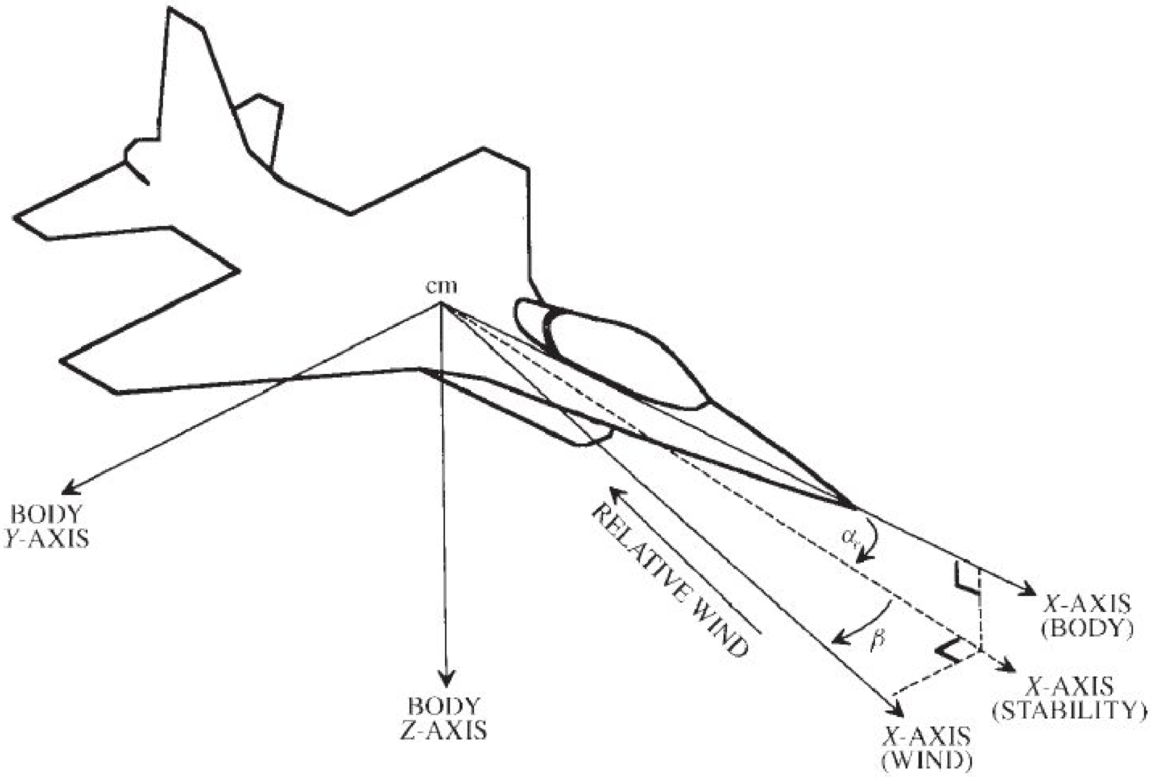


Figure 2.3-1 Definitions of axes and aerodynamic angles.