A PRACTICAL METHOD FOR DETERMINATION OF THE MOMENTS OF INERTIA OF UNMANNED AERIAL VEHICLES

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A PRACTICAL MRTHOD FOR DETERMINATION OF THE MOMENTS OF ITERTIA OF UNMANNED AERIAL VEHICLES

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ABSTRACT

Study the dynamics and the rotational flight of an UAV such as spinning, requires consideration of inertial characteristics of the UAV. Moments of inertia are essential characteristics for flight simulation or developing flight control systems for UAVs and an accurate evaluation of moments of inertia is necessary. The aim of this paper is to exhibit the use of pendulum method for the estimation of moments of inertia of an already built UAV, the UH-UAS MK.2, developed by the University of Hertfordshire. The experimental method used here is straightforward and economical to imply on small UAVs such as UH-UAS MK.2. The method is accurate satisfactorily and has the superiority over mathematical method which is difficult to imply and requires extensive information of each individual component of the UAV. The product of inertia is also found in similar manner to moments of inertia, In order to determine the principle axis system. This paper is a preliminary part of an extensive study of flight simulation of the UH-UAS MK.2.

Keywords: UAV, Moments of inertia, Pendulum Method

1 INTRODUCTION

Nowadays UAVs are used widely in civil and military applications. Missions like fire-fighting or detecting radiation levels around nuclear power sites are dangerous and hazardous to be performed by humans, but are obvious applications for UAVs. Increasingly other applications are being considered for economical reasons. As a result of this growth, accurate flight simulation is necessary to accomplish these sophisticated missions.

In investigating the UAV dynamics and for developing a computational model to simulate the UAV's motion, it is necessary to determine moments of inertia accurately. This accuracy is crucial in a study of rotational flight, spinning for instance [3,6,9]. Moments of inertia (and products of inertia) of a body indicate the resistance of the body to rotation.

They depend on the distribution of the mass and bodies with larger moments of inertia, resist more against rotation [7]. There are two approaches to calculate these values; mathematical and experimental. In the theoretical-mathematical method the contribution made by all parts of the UAV has been considered. To calculate the moments of inertia, weight of all individual parts of the UAV and their distances to the UAV Centre of Gravity are required. Due to complexity of these calculations together with bearing in mind the fact that breaking down the already built UAV to its parts is not possible, make this approach impractical. On the other hand, the experimental approach proved itself as a suitable procedure specifically for small UAVs.

2 UH-UAS MK.2

The UH-UAS MK.2 was developed at the University of Hertfordshire to compete in the European Student Competition on Unmanned Aircraft Systems (ESCO-UAS) 2008 competition. The UH-UAS MK.2 is constructed of balsa wood, metal and fiber glass. It is a conventional high-wing UAV driven by a 58 cc engine manufactured by MVVS CANADA. The UAV has a wingspan of 2.6 meter, a length of 2.33 meter and weight of 17 Kg with electronic equipment and a gross weight of 15.78 kg

3 EXPRIMENTAL METHOD FOR MOMENTS INERTIA

The Principle of experimental approach is based on rotating the UAV about X, Y and Z axes and measuring periods of the resultant oscillations. The UAV is mounted as a pendulum and it is displaced from the equilibrium position. After being released, the periods of the oscillations are determined and the moments of inertia are calculated from the weight, period, and dimensions. Moments of inertia can be calculated by the following equation [2,6,9]:

$$I = \frac{W_{UAV+S}L_{UAV+S}}{4\pi^2} P_{UAV+S}^2 - \frac{W_S L_S}{4\pi^2} P_S^2 - \frac{W_{UAV}L_{UAV}^2}{g} \tag{1}$$

Where W is weight, P is the time period and L is distance from axis of rotation. Subscripts "UAV+S", "S" and "UAV" refer to the UAV plus supporting hardware, the supporting hardware alone and the UAV alone, respectively. As the weight and therefore influence of supporting hardware for mounting the UAV is negligible, the equation can be simplified as following equation:

$$I = \frac{W_{UAV}L_{UAV}}{4\pi^2} P_{UAV}^2 - \frac{W_{UAV}L_{UAV}^2}{g} \tag{2}$$

As can be seen from the above equation, the first term is the moment of inertia of the UAV about the axis of oscillation. Thus, the moment of inertia of the UAV about the desired axis can be obtained by using the parallel axis theorem. The procedure is to subtract the additional moment caused by displacement of the CG of the UAV from the axis of oscillation [6]. In industry mainly

“compound” pendulum is used to determine X and Y moments of inertia and a “bifilar torsion type” is used for Z moments of inertia computation. The reasons are complexity of mounting the aircraft vertically and length of the aircraft.
It is proved that the accuracy of the test is depends on the construction of the pendulum, dimensions and the precision of measurements. To obtain more accurate results it is recommended to use shorter string. By using shorter string the moments of inertia of the UAV about its body axis will be a large percentage of the total moment of inertia of the pendulum about the axis of oscillation [6].
Since the pendulum equations are developed based on the assumption $sin\theta = tan\theta = \theta$ (where θ is one-half the angle of oscillation), the angle of oscillation must be small.
To perform the swinging test for obtaining I_y, the UAV is mounted on a rectangular frame as a pendulum in such a way that its CG lies on the centre of the frame as is shown in Figure 1. The rectangular frame made up of 4, I aluminium alloy beams for supporting the UAV. The frame is suspended from a supporting beam in manner that the Y axis of the UAV is parallel to the axis of oscillation and the CG of the UAV is fallen below the beam. The oscillation of the UAV is being recorded in order to calculate the periods of oscillation. Figure 2 shows the configuration of inertia test about Y axis. I_x is obtained in the same way, except for this configuration the X axis of the UAV is set parallel to the axis of oscillation.

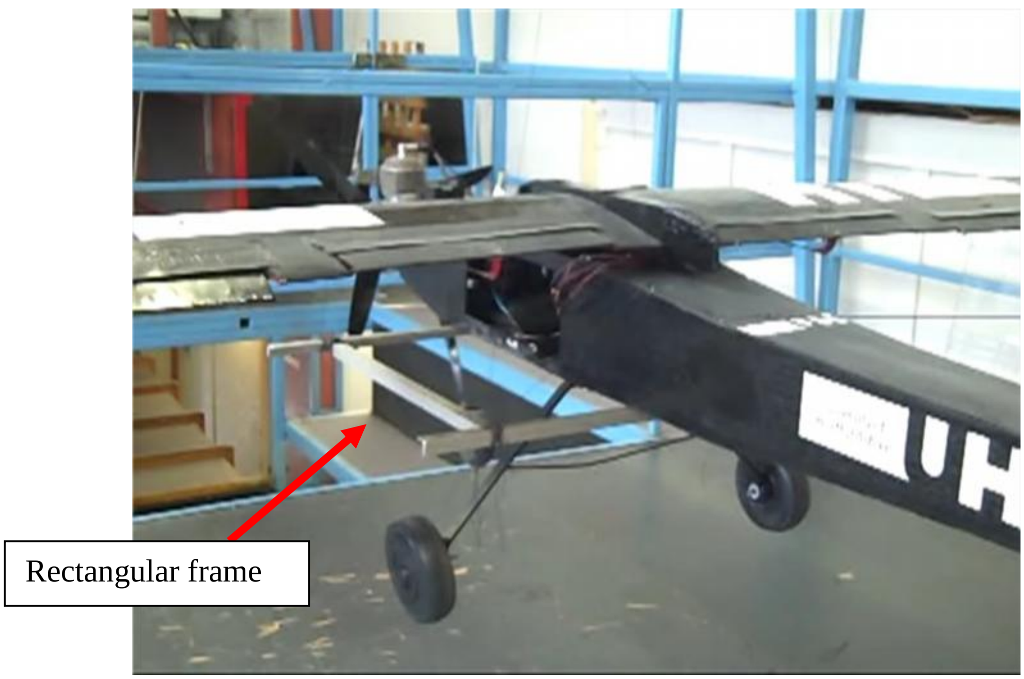

Figure 1: The UAV mounted on the frame

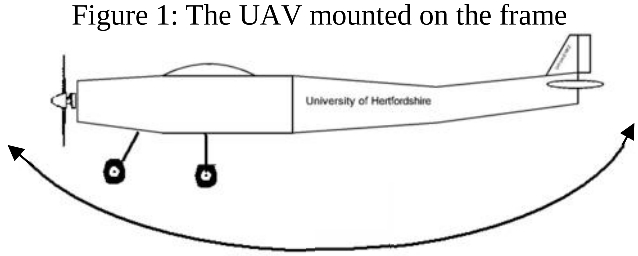

Figure 2 : I_y Inertia test

Moment of inertia about the Z axis is also found in similar manner, but with the changes to the configuration of suspension with no frame. The UAV is hung from tailplane vertically in order to make the Z axis parallel to the axis of oscillation. Figures 3 and 4 show the required configuration for this test.

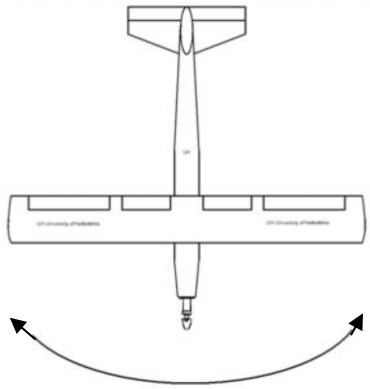

Figure 3 : Inertia test about Z axis

Figure 4 : Inertia test about Z axis

Periods were timed during the swing test for each configuration, and the tests were repeated three times. An average of the time periods used for computing the moments of inertia.
Tables 1, 2 and 3 show the obtained periods for all tests. In the tables the reference length, number of oscillations and the time for oscillations are available.

Test	Ref. Length (m)	No. of oscillations	Time for oscillations (Sec)	Period
1	1.89	17	49.02	2.884
2	1.89	30	86.64	2.888
3	1.89	20	57.7	2.885
Average				2.885

Table 1 : Inertia test for I_y

Test	Ref. Length (m)	No. of oscillations	Time for oscillations (Sec)	Period
1	1.89	37	105.89	2.862
2	1.89	25	71.42	2.857
3	1.89	40	114.32	2.858
Average				2.859

Table 2 : Inertia test for I_x

Test	Ref. Length (m)	No. of oscillations	Time for oscillations (Sec)	Period
1	1.5	4	10.52	2.63
2	1.5	6	15.79	2.64
3	1.5	3	7.92	2.633
Average				2.634

Table 3 : Inertia test for I_z

To test the sensitivity of the method to the length of the string, the test for I_y is repeated with longer string. The Table 4 shows the obtained results for this test.

Test	Ref. Length (m)	No. of oscillations	Time for oscillations (Sec)	Period
1	2.205	25	77.44	3.097
2	2.205	13	40.3	3.1
3	2.205	7	21.56	3.08
Average				3.092

Table 4 : Inertia test for I_y with longer string

Table 5 shows the computed moments of inertia about all desired axes. As can be seen from the table, an increase of 14% in distance from axis of rotation, results in 11% changes in moments of inertia. As mentioned before, pendulum with shorter length of string produces more accurate results. Therefore the moment of inertia obtained with shorter string is used for stability derivatives computations.

Axis of Oscillation	Moments of inertia (Kg m^2)
X	I_x = 4.209
Y	I_y = 5.344
Z	I_z = 5.312
Y	I_y = 5.956(with longer string)

Table 5 : Inertia test result

4 THE PRINCIPLE AXIS SYSTEM

The principle axis is a particular set of body axes which all the product of inertia terms are zero about this axis system. This helps to simplify the equations of motion of the UAV [1,5,8].
In order to determine the principle axis system it is necessary to calculate the products of inertia. The product of inertia I_{xz} is found in similar manner to moments of inertia about the X axis. But the length of front or rear strings is changed to make an inclined axis in the XZ-plane parallel to the axis of oscillation [6].The perfect configuration is to incline the XZ axis 45 degree from the X axis [3].
The angle between the principle axis and the X and the Z axes, τ, is found by following equation [4,6]:

$$tan2\tau = \frac{2\,E}{I_z - I_x} \tag{3}$$

Where $E = \frac{I_x \cos^2\Omega + I_z \sin^2\Omega - I_{xz}}{\sin 2\Omega}$, and Ω is the angle between the X axis and XZ axis. I_z, I_x and I_{xz} are moments of inertia about the Z axis, the X axis and XZ axis, respectively. Therefore the principle moments of inertia about the principle axis are:

$$I_{x\prime} = I_x cos^2\tau - Esin2\tau + I_z sin^2\tau \tag{4}$$

$$I_{z\prime} = I_z cos^2\tau + Esin2\tau + I_x sin^2\tau \tag{5}$$

$$I_{y\prime} = I_y \text{ (Since the XZ is plane of symmetry)} \tag{6}$$

Where $I_{x\prime}$, $I_{z\prime}$ and $I_{y\prime}$ are the moments of inertia about the principle axis.
The UAV is swung in both nose up and nose down attitudes with angle of 16° between X axis and axes and XZ axis. Tables 6 and 7 Show the periods of the resultant oscillations.

Test	Ref. Length (m)	No. of oscillations	Time for oscillations (Sec)	Period
1	1.83	15	42.13	2.808
2	1.83	16	44.82	2.801
3	1.83	30	84.3	2.810
Average				2.806

Table 6 : Inertia test for I_{xz} (Nose Down)

Test	Ref. Length (m)	No. of oscillations	Time for oscillations (Sec)	Period
1	1.85	30	85.06	2.8356
2	1.85	10	28.36	2.836
3	1.85	40	113.44	2.836
Average				2.835

Table 7 : Inertia test for I_{xz} (Nose Up)

Product of inertia for both nose up and nose down are calculated using simplified equation and the parameter E is calculated for both values which ideally must be identical. But due to accuracy of measurements and equipment, there is a difference between the obtained values. Therefore an average of the parameter E is used to determine the principle axis. Table 8 shows the computed value for the parameter E for each configuration.

	I_{xz}	E
Nose up	4.143	0.273
Nose down	4.469	0.322
Average		0.298

Table 8 : computing parameter E

The angle between the principle axis and the X and the Z axes, τ, is found 14.2° by using equation $tan2\tau = \frac{2E}{I_z - I_x}$, The moments of inertia about the principle axis are computed as follow:

$$I_{x'} = 4.127 \tag{7}$$

$$I_{z'} = 5.341 \tag{8}$$

5 CONCLUSION

As demonstrated in this paper using pendulum method for estimation of moments of inertia is very simple and practical for UAVs in comparison to mathematical approach. The Principle of pendulum method based on rotating the UAV about X, Y and Z axes and measuring periods of the resultant oscillations. The effects of string length is presented for I_y and It is proved that the accuracy of the test is depends on the construction of the pendulum, dimensions and the precision of measurements. The results show that the UAV have similar inertial characteristics about Y and Z axes. Small angle of oscillation is used to acquire more precise results. As the approach initially proposed for full scale aircrafts, the formulation are simplified for apparatus required to hung the UAV.

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