Swing Tests for Estimation of Moments of Inertia

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Moments of inertia of the BFF (Body Freedom Flutter) vehicle were estimated via inertia swings which were carried out in the Aeromechanics laboratory of the department. These tests were carried out to estimate pitch (I_{yy}), roll (I_{xx}) and yaw (I_{zz}) moments of inertia. The I_{xz} moment of inertia is assumed small due to lack of a big vertical fin (assuming the standard orientation for body-fixed coordinates), and was estimated using the moment of inertia estimator provided in the aerodynamic analysis software, XFLR-5. Owing to the symmetry of the aircraft, the lateral and vertical coordinates of center of gravity are assumed to lie on their respective center lines of the aircraft body. The final estimated values along with total mass and location of center of gravity are reported in table 1.

Table 1: Mass Properties

Property	Value
Total weight	11.99 lb
C.G. location	23.2585 inches (from nose)
Pitching moment of inertia	1245.83 $lb - in^2$
Rolling moment of inertia	8543.5 $lb - in^2$
Yawing moment of inertia	8118.42 $lb - in^2$
Product of inertia I_{xz} (estimated)	-0.296 $lb - in^2$

Pitch Moment of Inertia

Fig. 1 shows the setup for estimation of pitch moment of inertia.

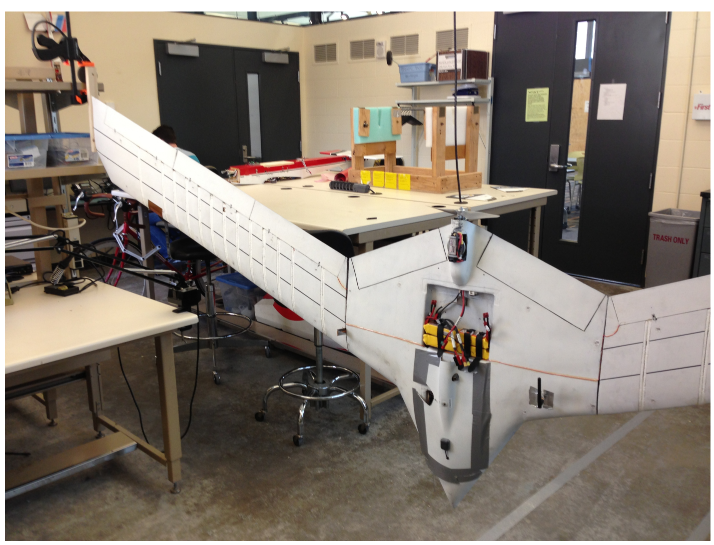

Figure 1: Pitch moment of inertia swing test setup

Setup of the Experiment

Due to limited hard points for suspension of the aircraft, it was suspended from the winglet mounts on wing-tips using wood attachments. The wood attachments were then attached to the axle of the landing gear of a remote-control aircraft. The tires on both ends were firmly clamped at equal heights from the ground, enabling the aircraft to rotate about the axles with very little damping (Fig. 2). This caused the axis of rotation to be 25.79 inches aft of the center of gravity, measured using the CAD model developed from the laser scan of the aircraft. This shift in axis was accounted for in the final calculations using the parallel axis theorem.

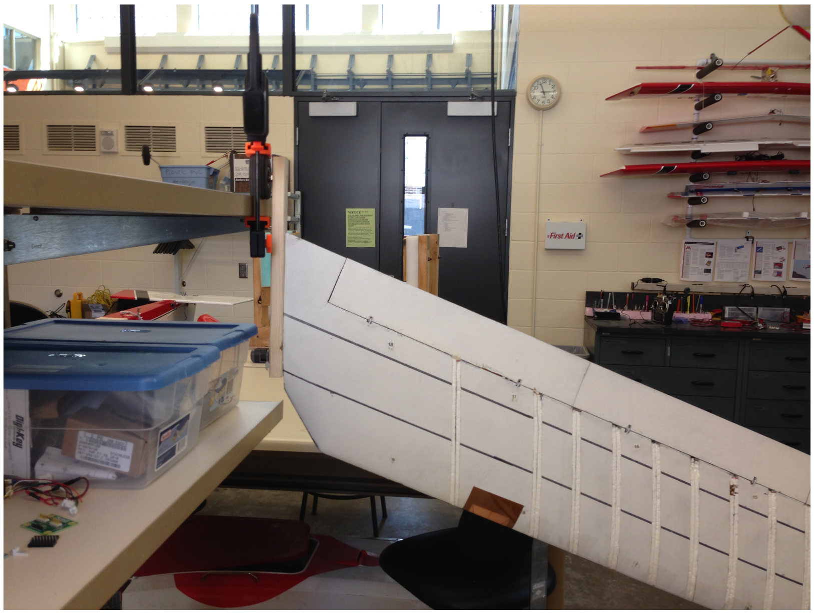

Figure 2: Pitch moment of inertia swing test setup - wing-tip attachement

Readings and calculations

In order to estimate the moment of inertia, the aircraft was approximated as a compound pendulum and its natural frequency of free oscillations was estimated. This was done by measuring time taken for the aircraft to complete 10, 5 and 3 oscillations, in order to account for the light damping in the system. The time period measurements obtained from the swing tests are given in table 2.

The frequencies calculated from these measurements were then averaged to arrive at the final value, 3.598 rad/s. This value of natural frequency was used to estimate the moment of inertia according to equation 1.

$$I = \frac{mgd}{\omega^2} \tag{1}$$

Here d is the distance to the center of gravity from the axis of rotation and ω is the frequency of oscillations in radians/sec (table 3). A more thorough procedure of estimating the moment of inertia was considered, which involved acquiring time history of angular displacement of the aircraft using an IMU, which would then be used to estimate pitch inertia as well as damping using parameter estimation techniques like output-error method. However, since the simplified approach gave acceptable results, this procedure was not employed.

Table 2: Pitch inertia testing data (in sec)

10 Osc	5 Osc	3 Osc
17.49	8.79	5.15
17.49	8.81	5.17
17.55	8.91	5.11
17.46	8.84	5.26
17.48	8.78	5.10
17.38	8.86	5.26
17.46	8.82	5.16
17.48	8.86	5.21
17.39	8.80	5.19
17.57	8.78	5.16

Table 3: Values for equation 1

Property	Value
aircraft weight (mg)	11.99 lb
distance to center of mass (d)	25.79 inches
Frequency (ω)	3.598 rad/s

Observations

1. Since there is very little friction at the tire-axle joint, there is very low frictional damping at the points of suspension. Also, for low angular displacements, the overall damping (aerodynamic and viscous) was observed to be low. The first 10 oscillations were clearly observable inspite of some reduction in angular displacement.

2. The first oscillation observed after releasing the aircraft from an initial displacement also excited the structural modes of the aircraft. This was possibly because the aircraft was suspended from points located on wingtips while the displacement was given by moving the nose. However, the stuctural modes damped out completely within the first oscillation. Hence, the first oscillation was not counted when the oscillation times were measured.

3. Since it was ensured that the points of suspension were at the same height at both the ends, the axis of rotation remained parallel throughout the oscillations to the pitch axis about which the moment of inertia was required.

Roll Moment of Inertia

Setup of the experiment

The bifilar pendulum approach was used for estimating the roll moment of inertia (Fig. 3).
Steel chords (1/16 inch diameter) were used as filars in order to ensure sufficient strength for suspending the aircraft. Due to flexibility of the aircraft, a stand was used to hold the aircraft in the proper orientation and the stand was suspended instead. The swing tests were carried out for the empty stand for its inertia estimation, so as to subtract that value from the estimated inertia of the stand with the aircraft. By measuring the tilt in the suspended stand with and without the aircraft in it using a level (it was zero in both cases), it was verified that the location of center of gravity of the empty stand as well as that of the stand and aircraft together, lay on the same axis of rotation which was within the plane formed by the filars. Owing to the symmetry of the aircraft, its center of

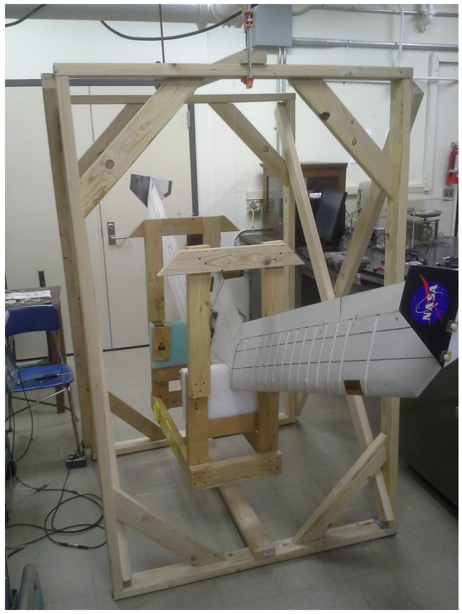

Figure 3: Roll moment of inertia swing test setup

gravity is assumed to lie on the center line. Table 4 lists all the weight and length measurements of the setup.

Table 4: Setup weight and length measurements

Property	Value
Stand weight	33.10 lb
Length of filars (L)	30 inches
Distance between the filars (d)	34.75 inches

Readings and Calculations

The procedure used to estimate the frequencies for the stand with and without the aircraft was similar to the one used for pitch inertia estimation. The final values are 3.284 rad/s and 4.088 rad/s respectively. The data collected from the swing tests is given in table 5. Roll inertia was estimated from the frequency using equation 2.

$$I = \frac{mgd^2}{4L\omega^2} \tag{2}$$

where d is the distance between the filars, L is the length of the filars and ω is the frequency of oscillations.

Equation 2 was obtained from the nonlinear mathematical model of a bifilar pendulum, which was developed from first principles using Lagrange equations (equation 3).

$$I\ddot{\theta} + K_d\dot{\theta}^2 + C\dot{\theta} + \frac{mgd^2 \sin\theta \cos\theta}{4L\sqrt{1 - \left(\frac{d \sin\theta}{2L}\right)^2}} = 0 \tag{3}$$

Table 5: Roll inertia testing data (in seconds)

Stand		Stand + BFF	
10 Osc	5 Osc	10 Osc	5 Osc
15.48	7.67	18.91	9.60
15.71	7.72	19.02	9.73
15.33	7.65	19.05	9.75
15.34	7.68	18.92	9.57
15.34	7.65	19.01	9.77
15.53	7.64	18.95	9.61
15.39	7.70	19.11	9.47
15.34	7.64	19.06	9.51
15.34	7.61	19.07	9.59
15.31	7.67	19.15	9.63

To arrive at the final form of the equation, the pendulum was assumed to undergo small deflections and damping (viscous (C) and aerodynamic (K_d)) was assumed to be zero. As with pitch inertia estimation, the roll inertia can also be estimated using the parameter estimation method if higher degree of accuracy is required. The nonlinear equation for a bifilar pendulum, which has been developed from first principles, can be used for this purpose.

Observations

1. The damping was observed to be low. The stand with and without the aircraft had sufficient angular displacement through to the last oscillation during the experiment, especially when 10 oscillations were taken into account.
2. Overall, the time period of oscillations was slightly higher than desired. Although the readings were taken satisfactorily, it was felt that slightly faster oscillations would help speed up the experiment as well as add to the accuracy of noting the time period of oscillations, since the end of an oscillation would be more abrupt and sharp. This can be resolved in future by using filars of smaller length.
3. The center of gravity of the whole setup was assumed to be on the centerline of the aircraft owing to its symmetry. Although an idealistic assumption, no wobbling was observed during the swing tests, confirming that the center of gravity lay very close to the midpoint between the two filars.

Yaw Moment of Inertia

Setup of the experiment

A different stand was constructed in order to hold the aircraft in the appropriate orientation to conduct the swing tests (Fig. 4).
The hooks for attaching the filar were placed at the center of the sidebars of the stand. However, the center of gravity was found to be slightly ahead of the point of suspension, resulting in a forward tilt of the stand. The tilt was brought to zero (i.e. center of gravity brought in line with the points of suspension) by adding a small weight at the aft of the stand and checking with a level. This approach was preferred over moving the location of points of suspension since it was difficult to estimate the distance by which those points had to be moved for zero tilt. This weight was kept in place when

Figure 4: Yaw moment of inertia swing test setup

the aircraft was mounted on to the stand. The aircraft was placed in the stand and its position was adjusted such that the tilt remained zero with the added weight still in its place, to ensure that the center of gravity af the aircraft coincided with that of the stand and was in line with the points of suspension. Table 6 gives all the weight and length measurements of the setup.

Table 6: Setup weight and length measurements

Property	Value
Stand weight	18.7 lb
Length of filars (L)	30 inches
Distance between the filars (d)	18.5 inches

Readings and Calculations

The procedure for estimating yaw moment of inertia was the same as that for roll moment of inertia. Equation 2 was used for estimating the moment of inertia in this case as well. As in the case of pitch and roll moments of inertia, a simplified approach was used for the estimation, although parameter estimation can be carried out as described earlier if the need arises. The data collected for estimation of frequency oscillations is provided in table 7. The frequency of oscillations for the stand with and without the aircraft are 1.6772 rad/s and 2.298 rad/s respectively.

Observations

1. The distance between the filars was smaller compared to the distance during the roll moment of inertia test. Since length of the filars remained the same, the frequency of oscillations reduced significantly despite the fact that the inertia of the stand itself was smaller than that of the stand used for roll inertia estimation. Shorter filars could be used in the future to increase the frequency and thereby, accuracy in noting the time period.

Table 7: Yaw inertia testing data (in seconds)

Stand		Stand + BFF	
10 Osc	5 Osc	10 Osc	5 Osc
27.31	13.55	37.90	18.43
27.50	13.66	37.50	18.83
27.44	13.68	37.56	18.67
27.41	13.62	37.58	18.66
27.38	13.76	37.51	18.75
27.40	13.75	37.56	18.64
27.40	13.72	37.53	18.72
27.43	13.59	37.49	18.74
27.39	13.56	37.51	18.71
27.41	13.61	37.53	18.63

2. A tilt was observed in the stand when suspended without the aircraft which was adjusted to zero. This required adding a small weight at the rear end of the stand. This adjustment ensured that the center of gravity of the aircraft coincided with that of the stand when placed in it and also lay in the plane defined by the two filars.

3. Just like in the case of roll moment of inertia estimation, no wobbling was observed during the swing tests, confirming that the center of gravity lay very close to half way between the two filars.

Conclusion

The final values of the various moments of inertia estimated through the swing tests are reasonable and the dynamic response of the nonlinear mathematical model constructed using these values matches well with the data already published about the BFF [1]. Major sources of error in estimation are the exclusion of damping effects while measuring time period and the heaviness of the stands themselves, which are used for roll and yaw estimations. The high inertia values of the stands could have possibly affected the overall estimation of moments of inertia of the aircraft itself. However, since the results in [1] are acceptable at this point, these values of moments of inertia are acceptable as well.

References

1. Aditya Kotikalpudi, *'Nonlinear Simulation of Body Freedom Flutter Vehicle'*