Bitcoin Cryptography:
Hash Functions and Elliptic Curves

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San Francisco, Oct. 26 – Oct. 30, 2015

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Outline

Hash functions

Base 58

Blocks, Transactions, Addresses

Elliptic Curves

ECDSA: Private keys, Public keys, Signatures

Bonus: Encryption

Cryptographic Hash Functions

- data → hash(data)
- Hash value is always the same number of bytes (e.g. 20, or 32)
- Can “easily” compute hash from data – difficulty is O(N)
- Computationally impractical to compute data from its hash
- Changing the data, even slightly, changes the hash
- Impractical to find two pieces of data with the same hash
- Hash value is “random” - all bits are equally possible

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Cryptographic Hash Functions

- md5 ← thoroughly broken
- sha1 ← known to be weak
- sha2 (sha256, sha384, sha512) ← sha256 used by bitcoin
- ripemd160 ← used by bitcoin

Cryptographic Hash Functions

- Data: “example data to be hashed”
- **sha1**: `c9e38c7dd778cdeffcdff7188c1a4c4d3c767e3b`
- **sha256**: `aa2356b0d9098e8fc3cdfe63a07a83c85533d06ea6270abf50e5ef7dc60620f9`
- **sha512**: `760e5df6338ca1a3218fb9f7e5a2e9d20984c3c707f918feb1231c8715636100ff2d484972db370e46b4f8eaaf3294a95191ad6e9f7ebef2d4ffb5c7bef3a13a`
- **sha256sha256**: `08dbfcea88a9fc1b9a91ca40de59bb0f30e99285ff4f4019f6b4c9509043b00b`
- **ripemd160**: `64988b52845db1905cbf2530a9f60c666c8836f2`
- **sha256ripemd160**: `200fbb165d5d9e31ad72ca7fa31618f5dfd4bfd7`

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Transactions and Blocks

- Transaction ID: sha256sha256 hash of the transaction – then always reversed when displayed
- Block ID: sha256sha256 of the block – then always reversed when displayed
- Hash of genesis block:
  `6fe28c0ab6f1b372c1a6a246ae63f74f931e8365e15a089c68d6190000000000`
- ID of genesis block:
  `000000000019d6689c085ae165831e934ff763ae46a2a6c172b3f1b60a8ce26f`

Base 58

- Base 10: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Base 2 (binary): 0, 1
- Base 16 (hex): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f
- Base 64: 0 – 9, a – z, A – Z, and: ' ( ) , - . / : ?
- Base 58: similar to base 64, but only alphanumeric, and no confusingly similar characters like uppercase i and lowercase L. Set: 123456789ABCDEFGHJKLMNPQRSTUVWXYZabcdefghijkmnopqrstuvwxyz

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Base 58 Check

- What if someone is copying a Base 58 value by hand? Would be nice to have a checksum to check for errors ...
- Data + hash(data) → Base58
- Base58Check Encode: Find the first four bytes of sha256sha256(data), and append to data, then Base58 encode
- Base58Check Decode: Base58 decode, then parse data as everything but the last four bytes, and confirm that the last four bytes equal the first four bytes of sha256sha256(data)

Addresses

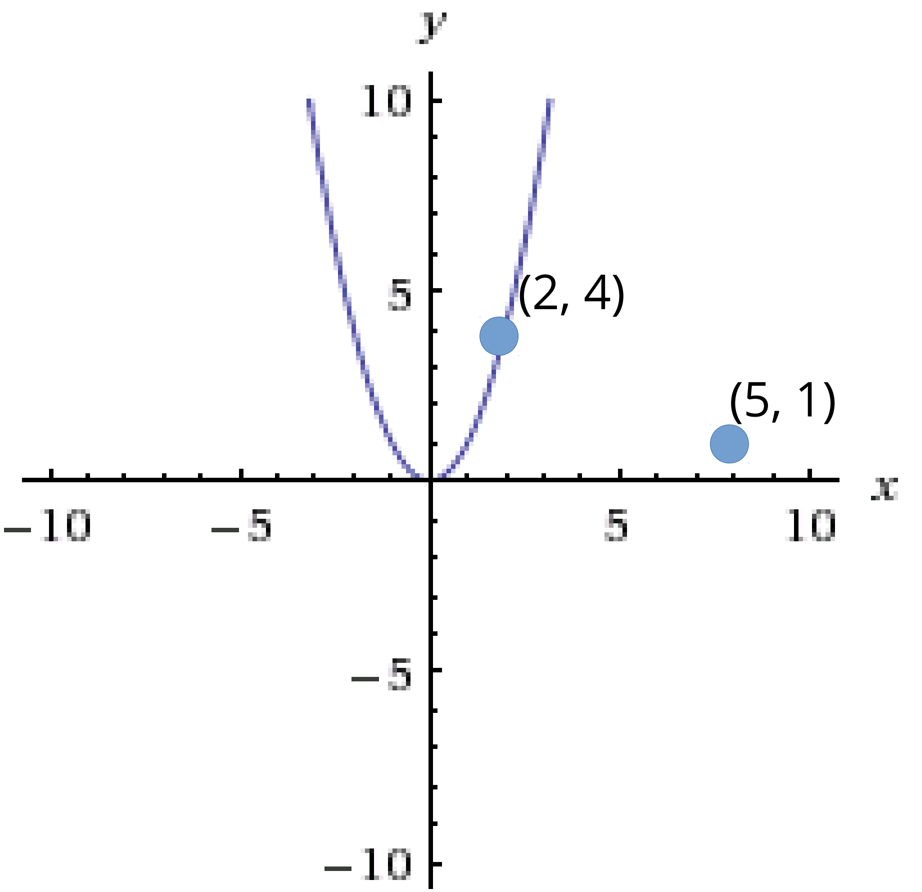
- An address is the sha256ripemd160 hash of a public key (usually a compressed public key)
- When displayed visually, addresses are displayed in base58check notation. They are also prepended with a 0x00 byte (if they are pubkeyhash addresses), or with a 0x05 (if they are a p2sh address)
- It so happens that when encoded this way, normal addresses always start with a “1” and p2sh addresses always start with a “3”
- Example address: 1343UaYEDYpbHQH2KF24mxTJ9GGXbzrGX9
- Example p2sh: 3GENUmnERtWfQct4NegomUqqtZuzYGPwBS

Some Context on ECDSA

- Bitcoin transactions involve “signing”
  - Technically, transactions don't need to involve signatures, but they almost always do
- ECDSA: Elliptic Curve Digital Signature Algorithm
  - ECDSA can: create a signature from data and privkey
  - ECDSA can: validate a signature from data and pubkey
- Signatures and public keys occur in transactions (private keys are kept private, of course)
- Bitcoin's core protocol does not use encryption
  - ECDSA is “cryptography”, but not “encryption”
- **ECDSA** is based on mathematics of **Elliptic Curves**

Basic Math: Numbers, Points, Eqns

- A number: 5
- A point: (5, 1)
- An equation: $y = x^2$
- A solution:
  - $y = 4, x = 2$
  - Or, (2, 4)



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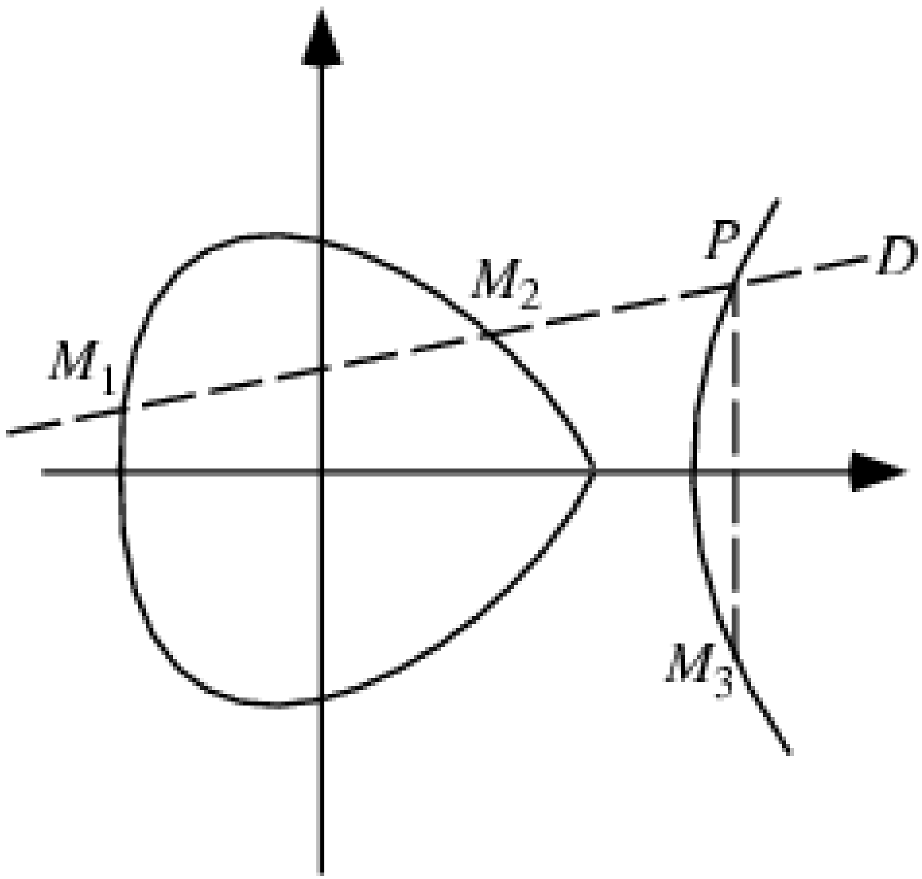
Elliptic Curves

Points on an EC are solutions to this eqn:

$$y^2 = x^3 + ax + b$$

$$M_3 = M_1 + M_2$$

Image Credit: Wolfram



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Elliptic Curves

- Can define “addition” and “multiplication” of points to get a **field**
- A field is a set of points: $X = (x, y)$
  - With addition operation: $(x_1, y_1) + (x_2, y_2) \rightarrow (x_3, y_3)$
  - Multiplication defined as: $2 * (x, y) \rightarrow (x, y) + (x, y)$
  - Commutative: $(x_1, y_1) + (x_2, y_2) = (x_2, y_2) + (x_1, y_1)$
  - Associative: $\mathbf{(}(x_1, y_1) + (x_2, y_2)\mathbf{)} + (x_3, y_3) = (x_1, y_1) + \mathbf{(}(x_2, y_2) + (x_3, y_3)\mathbf{)}$

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Elliptic Curve Cryptography

- Cryptography you can do with elliptic curves:
  - Sign and verify data
  - Encrypt and decrypt data
- EC Crypto is based on **finite fields** – not continuous curves as you might think
  - The same equations and operations apply – but cannot be visualized as a curve
- Confusingly, finite fields do not look like curves

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Elliptic Curve Cryptography

- Mathematically possible to subtract:
- If A + B = C, then B = C – A
- …but this is computationally impracticable: equivalent to solving Discrete Logarithm Problem
- A = 2 B = B + B …
- If you know “A” and “2”, you can only find B from brute force – try every value until you get it.
- Quantum computers can solve discrete log problem – problem for EC crypto!

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secp256k1

- The curve used by bitcoin – specified by curve and finite field parameters: (a, b, p, n, G, h)
- $y^2 = x^3 + ax + b$ (as always for elliptic curves)
- $a = 0, b = 7$
- $p = 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 – 1$
- n = 115792089237316195423570985008687907852837564279074904382605163141518161494337
- G = (55066263022277343669578718895168534326250603453777594175500187360389116729240, 32670510020758816978083085130507043184471273380659243275938904335757337482424)
- h = 1
- N: “order of the curve” = “largest private key”
- G: “base point of the curve”

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Private Keys and Public Keys

- A private key is a number in the range (0, N)
- A public key is a private key times G
- Public Key = Private Key * Base Point
- P = pG
- If p = 5, then P = 5G, or:
  - P = 5 * (x, y)
    - P = (21505829891763648114329055987619236494102133314575206970830385799158076338148, 98003708678762621233683240503080860129026887322874138805529884920309963580118)
- Never use “5” as a private key! Generate a random number in range (0, N)

Private Keys and Public Keys

- A randomly generated private key and corresponding public key
- p = 79166932645790398746161737314012976057631666974820812284863365219350761232064
- Bitcoin base 58 check format: L35wVgKWPyfeMzNUWQR3D2MtjCUzMyrvFpN9R44NTZbhcCea6kyS
- P = (14940432162225245106778043682370687268864377313694688834137709913105567990260, 22312549033088701518791571940340055302090435747572883935510928095745029494859)
- DER compressed hex format: 032107fc24b3569f2feda301c7efc07f56fc8ab6870e61f03dcd93dbd9088a41f4

```
fullnode> var privkey = Privkey().fromRandom()
fullnode> privkey.bn.toString(10)
'79166932645790398746161737314012976057631666974820812284863365219350761232064'
fullnode> privkey.toString()
'L35wVgKWPyfeMzNUWQR3D2MtjCUzMyrvFpN9R44NTZbhcCea6kyS'
fullnode> privkey.toString()
fullnode> var pubkey = Pubkey().fromPrivkey(privkey)
fullnode> pubkey.point.getX().toString(10)
'14940432162225245106778043682370687268864377313694688834137709913105567990260'
fullnode> pubkey.point.getY().toString(10)
'22312549033088701518791571940340055302090435747572883935510928095745029494859'
fullnode> pubkey.toHex()
'032107fc24b3569f2feda301c7efc07f56fc8ab6870e61f03dcd93dbd9088a41f4'
```

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ECDSA

- Given private key and data, need to produce a signature
- Given signature, data, and public key, need to confirm that signature is for corresponding private key
- ECDSA: Elliptic Curve Digital Signature Algorithm
- An ECDSA signature consists of two numbers, r and s. Although a pair of numbers, it is not a point.

ECDSA - Sign

1. Calculate $e = \mathrm{HASH}(m)$, where HASH is a cryptographic hash function, such as SHA-2.
2. Let z be the L_n leftmost bits of e, where L_n is the bit length of the group order n.
3. Select a **cryptographically secure random** integer k from $[1, n-1]$.
4. Calculate the curve point $(x_1, y_1) = k \times G$.
5. Calculate $r = x_1 \bmod n$. If $r = 0$, go back to step 3.
6. Calculate $s = k^{-1}(z + rd_A) \bmod n$. If $s = 0$, go back to step 3.
7. The signature is the pair (r, s).

- Pseudocode credit: Wikipedia
- (Caveat: Not all details here – do not rely on for implementation)

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ECDSA - Verify

1. Check that Q_A is not equal to the identity element O, and its coordinates are otherwise valid
2. Check that Q_A lies on the curve
3. Check that $n \times Q_A = O$

After that, Bob follows these steps:

1. Verify that r and s are integers in $[1, n-1]$. If not, the signature is invalid.
2. Calculate $e = \textrm{HASH}(m)$, where HASH is the same function used in the signature generation.
3. Let z be the L_n leftmost bits of e.
4. Calculate $w = s^{-1} \bmod n$.
5. Calculate $u_1 = zw \bmod n$ and $u_2 = rw \bmod n$.
6. Calculate the curve point $(x_1, y_1) = u_1 \times G + u_2 \times Q_A$.
7. The signature is valid if $r \equiv x_1 \pmod{n}$, invalid otherwise.

- Pseudocode credit: Wikipedia
- (Caveat: Not all details here – do not rely on for implementation)

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ECDSA – Extra Details

- It is possible to derive candidate public keys from a signature – which can be compared to the hash of a public key to see if a signature is valid. Makes possible “**Bitcoin Signed Message**”
- Generating a signature involves a random value “k”. This value needs to be unpredictable, but can be computed deterministically from the data and the private key – standard for this is **RFC 6979**. This is actually safer than a purely random value, in case your entropy pool is broken. Also helps validate the signature algorithm is working correctly by comparing to known test vectors.

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ECDSA - Summary

- ECDSA Sign: Privkey + hash(data) → (r, s)
- ECDSA Verify: (r, s) + hash(data) + Pubkey → valid|invalid
- Computationally impractical to derive privkey from pubkey
- Computationally impractical to derive privkey from signature
- Use RFC 6979 – deterministic k

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