A NOVEL METHOD FOR DETERMINING DOA FROM FAR-FIELD TDOA OR FDOA

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Abstract. TO DO at end

1. Introduction

Locating a radio-frequency transmitter is a vital step in blah blah applications. Source localization is often performed using measurements of the transmitted signal obtained by several nearby receivers. Specific observable quantities include the time difference of arrival (TDOA) and frequency difference of arrival (FDOA) between receivers. When the distance between the receivers and the transmitter is much greater than the distance between the receivers (far-field case), the angle of arrival (AOA) can be found using the geometric relationship with the TDOA and distance between receivers [1]. Additionally, the problem can be simplified if information about the transmitter is known *a priori*, such as altitude (ALT). This paper introduces a novel approach to calculating the AOA by capitalizing on the interesting geometry of the source-localization problem. This method allows for the calculation of AOA using either TDOA or FDOA measurements and simplifies the problem to the solution of a linear equation.

The FDOA equations are nonlinear and thus have a more complicated geometry than geolocation with TDOA measurements only. While the FDOA measurements are often used as an additional constraint to the TDOA geolocation systems (TDOA/FDOA localization) [3], only a few algorithms have been developed using FDOA alone (us, [4]). There are some special cases where it is desirable to solve for the emitter location using FDOA only. For instance in the case of a narrowband signal with a long pulse duration, the Doppler resolution is higher than the range resolution and it can be difficult to measure the TDOA accurately [2, 4, 5].

As mentioned above, the angle of arrival (AOA), sometimes referred to as the direction of arrival (DOA), is usually calculated using basic trigonometry and TDOA measurements. A nice overview of this relationship is covered in [1]. For a source in the near-field, both the ranges and AOA can be found using this method with a single array of sensors. In the far-field case, only the AOA can be calculated, so more than one set of measurements are needed for source localization. [1]

Add roadmap to paper.

2. Far-field approximation for FDOA

Consider a stationary transmitter located at $\mathbf{x}$. Additionally, there are N receivers located at $\mathbf{x}_1, ..., \mathbf{x}_N$ with velocity $\mathbf{v}_1, ..., \mathbf{v}_N$. The frequency shift of the signal between the emitter and receiver i is,

$$d_i = \mathbf{v}_i^T \cdot \frac{\mathbf{x}_i - \mathbf{x}}{\|\mathbf{x}_i - \mathbf{x}\|}. \tag{1}$$

We now derive a far-field approximation for equation 1.

Assume without loss of generality that the receivers are centered around the origin. We consider the far-field case, where the distance between receivers is much smaller than the distance to the emitter, i.e. $\|\mathbf{x}\| >> \|\mathbf{x}_i\|$, $\forall\, i$ [2]. We approximate $\|\mathbf{x} - \mathbf{x_i}\|$ by expanding:

$$\|\mathbf{x} - \mathbf{x_i}\| \approx \|\mathbf{x}\| - \hat{\mathbf{x}}^T \mathbf{x_i} + \mathcal{O}(\frac{\|\mathbf{x}_i\|}{\|\mathbf{x}\|}).$$

Truncating after the first term above allows for simplification of the factor (in eq. 1):

$$\frac{\mathbf{x}_i - \mathbf{x}}{\|\mathbf{x}_i - \mathbf{x}\|} \approx \frac{\mathbf{x}_i}{\|\mathbf{x}\|} - \frac{\mathbf{x}}{\|\mathbf{x}\|}.$$

Additionally, the far-field assumption implies that the first term will have small magnitude. Thus, $\dfrac{\mathbf{x_i} - \mathbf{x}}{\|\mathbf{x_i} - \mathbf{x}\|}$ is simplified to $\dfrac{-\mathbf{x}}{\|\mathbf{x}\|}$. Equation 1 becomes:

$$d_i \approx -\mathbf{v}_i^T \cdot \hat{\mathbf{x}}, \tag{2}$$

where $\hat{\mathbf{x}} = \frac{\mathbf{x}}{\|\mathbf{x}\|}$, is a unit vector from the centroid of the receivers. The entire system of frequency shifts can be written:

$$\mathbf{d} \approx -\mathbf{V}\hat{\mathbf{x}}, \tag{3}$$

where

$$\mathbf{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_N \end{pmatrix} \qquad \mathbf{V} = \begin{pmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \\ \mathbf{v}_3^T \end{pmatrix}.$$

In practice, the frequency shifts are not observable. Instead the frequency difference of arrival (FDOA) is measured between receivers. The FDOA is equivalent to the difference in frequency shifts,

$$f_{i,j} = d_j - d_i. \tag{4}$$

A system equivalent to equation 3 can be constructed for the FDOA, with the use of a differencing matrix $\mathbf{P}$. The matrix $\mathbf{P}$ has entries of 0 and ± 1 corresponding to the differencing in equation 4. Thus, with the far-field simplification above, the vector of FDOA measurements, $\mathbf{f}$, is equivalent to,

$$\mathbf{f} \approx -\mathbf{P}\mathbf{V}\hat{\mathbf{x}}. \tag{5}$$

The matrix $-\mathbf{PV}$ will be referred to as $\tilde{\mathbf{V}}$ for simplicity.

This far-field simplification reduces the FDOA equations to a linear system. This suggests that feasible FDOA measurements in the far-field case lie on the image of the unit circle transformed by the matrix $\tilde{\mathbf{V}}$. This image is an ellipse with rotation and scaling determined by the singular value decomposition of $\tilde{\mathbf{V}}$. Indeed, this can be confirmed by computing the singular value decomposition of generated far-field FDOA measurements and confirming they lie on the same subspace as $\tilde{\mathbf{V}}$. This relationship can also be demonstrated visually with a plot of generated FDOA measurements (fig. 1).

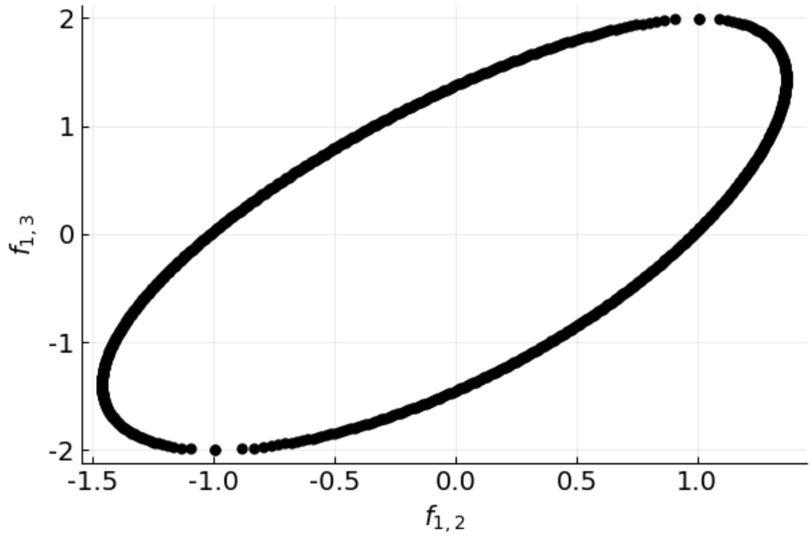

FIGURE 1. Plot of far-field $f_{1,2}$ vs. $f_{1,3}$ for a system of three receivers centered around the origin. Note the image is an ellipse with scaling in the direction of the left-singular vectors of $\tilde{\mathbf{V}}$.

Include symbolic form of ellipse?

2.1. **Calculating direction of arrival (DOA).** The far-field approximated form of the FDOA equations is linear with variable $\hat{\mathbf{x}}$, representing the direction of arrival (DOA) of the signal. Thus, the DOA can be found with a linear solve of equation 5. If there are more FDOA measurements than direction components, we can find the least squares solution to the problem, which is also the pseudo-inverse solution:

$$\hat{\mathbf{x}} \approx (\tilde{\mathbf{V}}^T \tilde{\mathbf{V}})^{-1} \tilde{\mathbf{V}}^T \mathbf{f}. \tag{6}$$

If the receivers move and repeat this process, the intersection of the lines generated by equation 6 will provide an estimate for the location of the emitter. This is the idea behind our source localization algorithm (section 3).

3. Algorithm

Numerical Results

Conclusion

3.1. Acknowledgments.

References

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