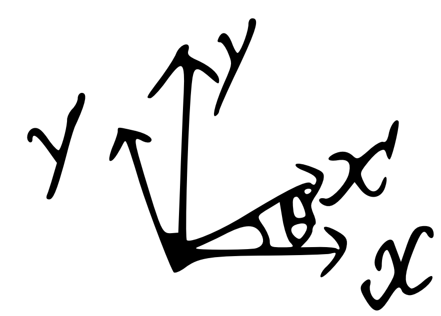
$R \to e$

1\.



$\therefore R_{o\to w} = \begin{pmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{pmatrix}$

here x', y' fixed on object

1.3 $x'_w = col(R_{o\to w}, 1) = \begin{pmatrix} c\theta \\ s\theta \end{pmatrix}$

$x_o = row(R_{o\to w}, 1) = \begin{pmatrix} c\theta \\ -s\theta \end{pmatrix}$

2\. if $f(-x) = -f(x)$

$\Rightarrow f^{-1}(-y) = -f^{-1}(y)$

~~proof~~: let $x = f^{-1}(-y)$

$\therefore f(x) = -y$ $\therefore y = -f(x) = f(-x)$

$\therefore x = -f^{-1}(y)$

$\therefore f^{-1}(-y) = -f^{-1}(y)$

3\.

3.2 $\because \sin(-x) = -\sin x$

$\therefore \sin^{-1}(-x) = -\sin^{-1} x$

3.3 $\because \tan(-x) = -\tan x$

$\therefore \tan^{-1}(-x) = -\tan^{-1} x$

3.4

$$\theta = \mathrm{atan2}(y, x) := \begin{cases} \tan^{-1}\frac{y}{x}, & x>0, y>0 \\ & \text{or } x>0, y<0 \\ \tan^{-1}\frac{y}{x}+\pi, & x<0, y>0 \\ \tan^{-1}\frac{y}{x}-\pi, & x<0, y<0 \\ \frac{\pi}{2}, & x=0, y>0 \\ -\frac{\pi}{2}, & x=0, y<0 \\ 0, & x>0, y=0 \\ \pi, & x<0, y=0 \\ \pi, & x=-\infty, y>0 \\ -\pi, & x=-\infty, y<0 \end{cases}$$

3.5

$\mathrm{atan2}(-y, x)$

$= -\mathrm{atan2}(y, x)$

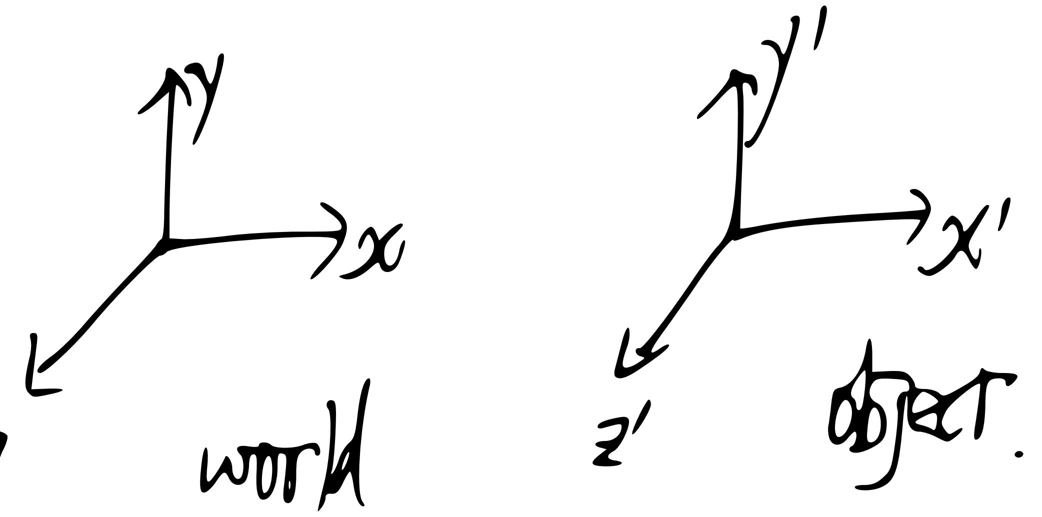
4\. Given $R_{o\to w}$, solve azimuth, pitch, roll, s.t.,

$R_{o\to w} = R(z, \theta_z) R(x, \theta_x) R(y, \theta_y)$

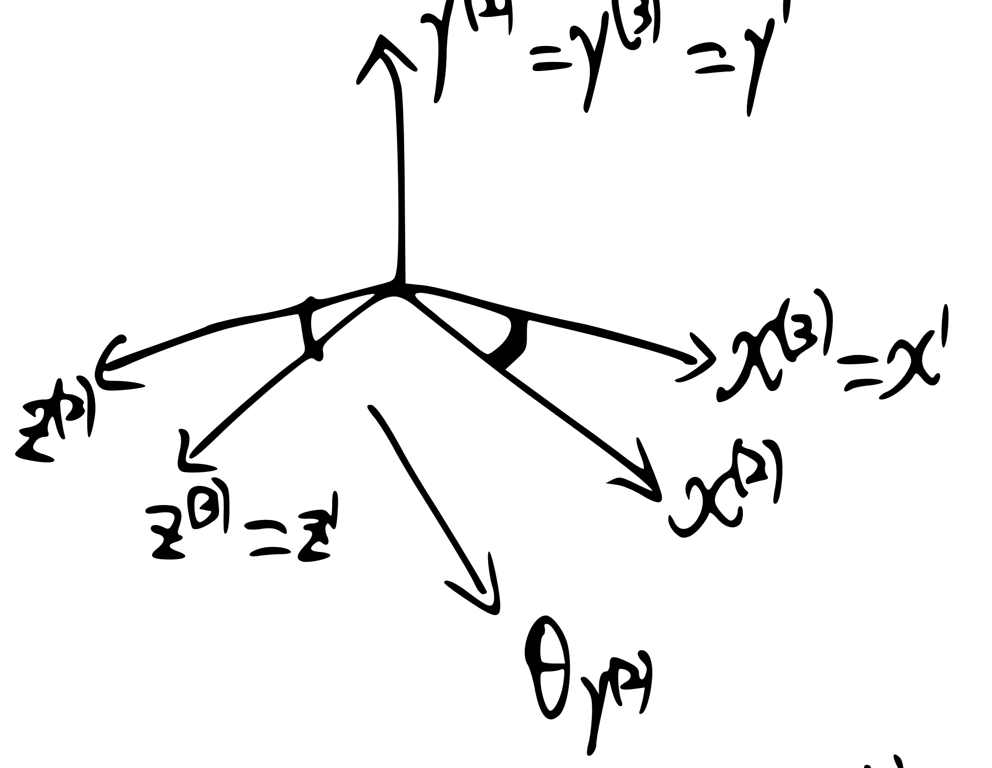
here θ_z = azimuth, θ_x = pitch, θ_y = roll.

solve: ① let $R = R_{o\to w}$

define



④



$$\theta_y = \theta_{y^{(2)}} = \mathrm{atan2}\left(z_{0_2}^{(3)}(1),\ z_{0_2}^{(3)}(3)\right)$$

$$\because \begin{pmatrix} z_{0_2}^{(3)}(3) \\ z_{0_2}^{(3)}(1) \end{pmatrix} = \begin{pmatrix} z_{0_3}^{(2)}(3) \\ -z_{0_3}^{(2)}(1) \end{pmatrix}$$

$$\because z_{0_3}^{(2)} = \begin{pmatrix} z_{0_3}(1) \\ 0 \\ z_{0_3}(3) \end{pmatrix} = \begin{pmatrix} z_0(1) \\ 0 \\ z_0(3) \end{pmatrix}$$

$$\therefore \theta_y = \theta_{y^{(2)}} = \mathrm{atan2}\left(-z_0(1),\ z_0(3)\right)$$

⑤ method 2: (sketch)

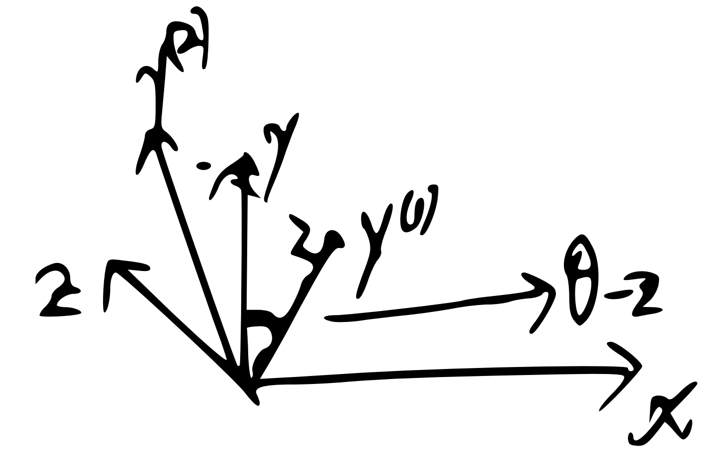
$$\theta_y = \cos^{-1}\left(x^{(1)} \cdot x^{(3)}\right)$$

$$\because x^{(1)} = x^{(2)} = y^{(2)} \times z^{(1)} = y'_w \times z = \begin{vmatrix} i & j & k \\ R_{12} & R_{22} & R_{32} \\ 0 & 0 & 1 \end{vmatrix} = \begin{pmatrix} R_{22} \\ -R_{12} \\ 0 \end{pmatrix},\quad x^{(1)} \leftarrow x^{(1)}/\|x^{(1)}\|,\quad x^{(3)} = x'_w = \begin{pmatrix} R_{11} \\ R_{21} \\ R_{31} \end{pmatrix}$$

$$\therefore \theta_y = \cos^{-1}\left(\frac{R_{11}R_{22} - R_{21}R_{12}}{\sqrt{R_{22}^2 + R_{12}^2}}\right)$$

⑥ $\because y_w^{(2)} = y_w^{(3)} = y'_w = \mathrm{col}(R,2)$, $z = \mathrm{row}(R,3)$, $\therefore \theta_z = -\mathrm{atan2}(R_{12}, R_{22})$, $\theta_x = \sin^{-1} R_{32}$,
$\theta_y = \mathrm{atan2}(-R_{31}, R_{33})$

②

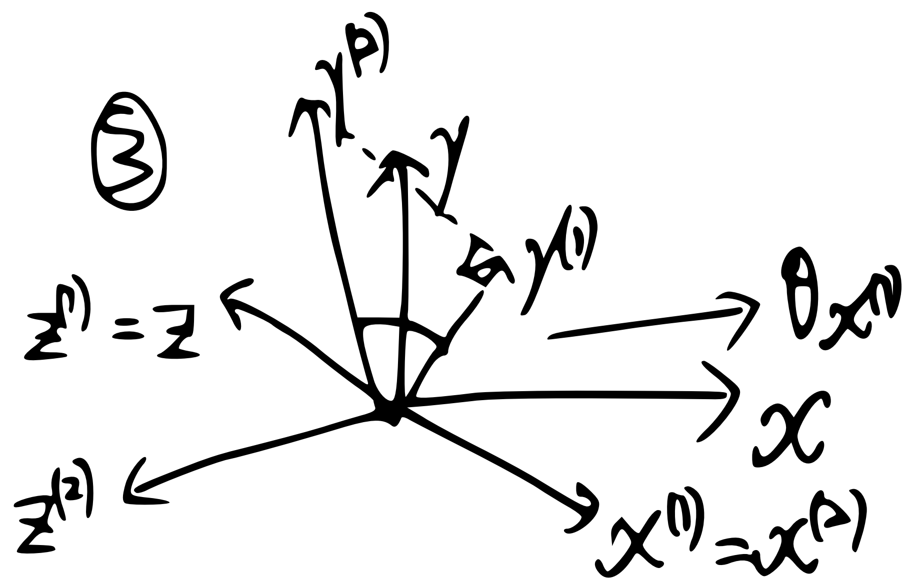


$$\theta_{-z} = \mathrm{atan2}\left(y_w^{(2)}(1),\ y_w^{(2)}(2)\right)$$

here rotation axis is $-z$

$$\therefore \theta_z = -\theta_{-z} = -\mathrm{atan2}\left(y_w^{(2)}(1),\ y_w^{(2)}(2)\right)$$

③



$$\theta_x = \theta_{x^{(1)}} = \sin^{-1}\left(y_w^{(2)}(3)\right)$$