

Maximum Spanning Tree using Prim's Algorithm



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Given <u>undirected weighted graph</u> **G**, the task is to find the **Maximum Spanning Tree** of the Graph using <u>Prim's Algorithm</u>

<u>Prims algorithm</u> is a <u>Greedy algorithm</u> which can be used to find the <u>Minimum Spanning Tree (MST)</u> as well as the **Maximum Spanning Tree** of a <u>Graph</u>.

Examples:

Input: graph[V][V] = {{0, 2, 0, 6, 0}, {2, 0, 3, 8, 5}, {0, 3, 0, 0, 7}, {6, 8, 0, 0, 9}, {0, 5, 7, 9, 0}}

Output:

The total weight of the Maximum Spanning tree is 30.

Edges Weight

3 - 1 8

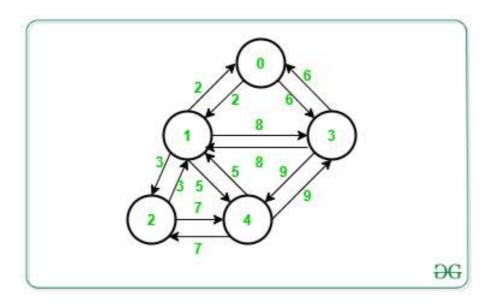
4 - 2 = 7

0 - 3 6

3 - 4 9

Explanation:

Choosing other edges won't result in maximum spanning tree.



Maximum Spanning Tree:

Given an <u>undirected weighted graph</u>, a maximum spanning tree is a spanning tree having maximum weight. It can be easily computed using <u>Prim's algorithm</u>. The goal here is to find the spanning tree with the maximum weight out of all possible spanning trees.

Prim's Algorithm:

<u>Prim's algorithm</u> is a greedy algorithm, which works on the idea that a spanning tree must have all its vertices connected. The algorithm works by building the tree one vertex at a time, from an arbitrary starting vertex, and adding the most expensive possible connection from the tree to another vertex, which will give us the **Maximum Spanning**Tree (MST).

Follow the steps below to solve the problem:

- Initialize a visited array of <u>boolean datatype</u>, to keep track of vertices visited so far. Initialize all the values with **false**.
- Initialize an array weights[], representing the maximum weight to connect that vertex. Initialize all the values with some minimum value.
- Initialize an array parent[], to keep track of the maximum spanning tree.
- Assign some large value, as the weight of the first vertex and parent as **-1**, so that it is picked first and has no parent.
- From all the unvisited vertices, pick a vertex **v** having a maximum weight and mark it as visited.
- Update the weights of all the unvisited adjacent vertices of \mathbf{v} . To update the weights, iterate through all the unvisited neighbors of \mathbf{v} . For every adjacent vertex \mathbf{x} , if the

weight of the edge between \mathbf{v} and \mathbf{x} is greater than the previous value of \mathbf{v} , update the value of \mathbf{v} with that weight.

Below is the implementation of the above algorithm:

C++

```
// C++ program for the above algorithm
#include <bits/stdc++.h>
using namespace std;
#define V 5
// Function to find index of max-weight
// vertex from set of unvisited vertices
int findMaxVertex(bool visited[], int weights[])
{
   // Stores the index of max-weight vertex
    // from set of unvisited vertices
   int index = -1;
   // Stores the maximum weight from
    // the set of unvisited vertices
   int maxW = INT_MIN;
   // Iterate over all possible
    // nodes of a graph
    for (int i = 0; i < V; i++) {
        // If the current node is unvisited
        // and weight of current vertex is
        // greater than maxW
        if (visited[i] == false
            && weights[i] > maxW) {
            // Update maxW
            maxW = weights[i];
            // Update index
            index = i;
        }
    }
   return index;
}
// Utility function to find the maximum
// spanning tree of graph
void printMaximumSpanningTree(int graph[V][V],
                              int parent[])
{
    // Stores total weight of
    // maximum spanning tree
```

```
// of a graph
    int MST = 0;
    // Iterate over all possible nodes
    // of a graph
    for (int i = 1; i < V; i++) {
        // Update MST
        MST += graph[i][parent[i]];
    }
    cout << "Weight of the maximum Spanning-tree "</pre>
         << MST << '\n'
         << '\n';
    cout << "Edges \tWeight\n";</pre>
    // Print the Edges and weight of
    // maximum spanning tree of a graph
    for (int i = 1; i < V; i++) {</pre>
        cout << parent[i] << " - " << i << " \t"
             << graph[i][parent[i]] << " \n";</pre>
    }
}
// Function to find the maximum spanning tree
void maximumSpanningTree(int graph[V][V])
{
    // visited[i]:Check if vertex i
    // is visited or not
    bool visited[V];
    // weights[i]: Stores maximum weight of
    // graph to connect an edge with i
    int weights[V];
    // parent[i]: Stores the parent node
    // of vertex i
    int parent[V];
    // Initialize weights as -INFINITE,
    // and visited of a node as false
    for (int i = 0; i < V; i++) {</pre>
        visited[i] = false;
        weights[i] = INT_MIN;
    }
    // Include 1st vertex in
    // maximum spanning tree
    weights[0] = INT_MAX;
    parent[0] = -1;
    // Search for other (V-1) vertices
    // and build a tree
    for (int i = 0; i < V - 1; i++) {
```

```
// Stores index of max-weight vertex
        // from a set of unvisited vertex
        int maxVertexIndex
            = findMaxVertex(visited, weights);
        // Mark that vertex as visited
        visited[maxVertexIndex] = true;
        // Update adjacent vertices of
        // the current visited vertex
        for (int j = 0; j < V; j++) {
            // If there is an edge between j
            // and current visited vertex and
            // also j is unvisited vertex
            if (graph[j][maxVertexIndex] != 0
                && visited[j] == false) {
                // If graph[v][x] is
                // greater than weight[v]
                if (graph[j][maxVertexIndex] > weights[j]) {
                    // Update weights[j]
                    weights[j] = graph[j][maxVertexIndex];
                    // Update parent[j]
                    parent[j] = maxVertexIndex;
                }
            }
        }
   }
    // Print maximum spanning tree
    printMaximumSpanningTree(graph, parent);
}
// Driver Code
int main()
{
    // Given graph
    int graph[V][V] = { { 0, 2, 0, 6, 0 },
                        { 2, 0, 3, 8, 5 },
                        { 0, 3, 0, 0, 7 },
                        { 6, 8, 0, 0, 9 },
                        { 0, 5, 7, 9, 0 } };
    // Function call
   maximumSpanningTree(graph);
   return 0;
}
```

Java