Euler Angle

1. $\begin{array}{c|c} & q_1, q_2, \cdots \\ \hline p_1 & \\ p_2 & \\ \vdots & \end{array} = \left| \begin{array}{c} p_1 \\ p_2 \\ \vdots \end{array} \right.$

permutation matrix

$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = (q_1, q_2, q_3)$

here $p_i = j$, $q_j = i$

1.3 $\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} (p_1\, p_2\, p_3)$

$= \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} q(p_1) \\ q(p_2) \\ q(p_3) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = I$

2. ① $B = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} A (p_1\, p_2\, p_3)$

$\therefore B_{ij} = A_{p_i p_j}$

2. ② $B = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} A (p_1\, p_2\, p_3)$

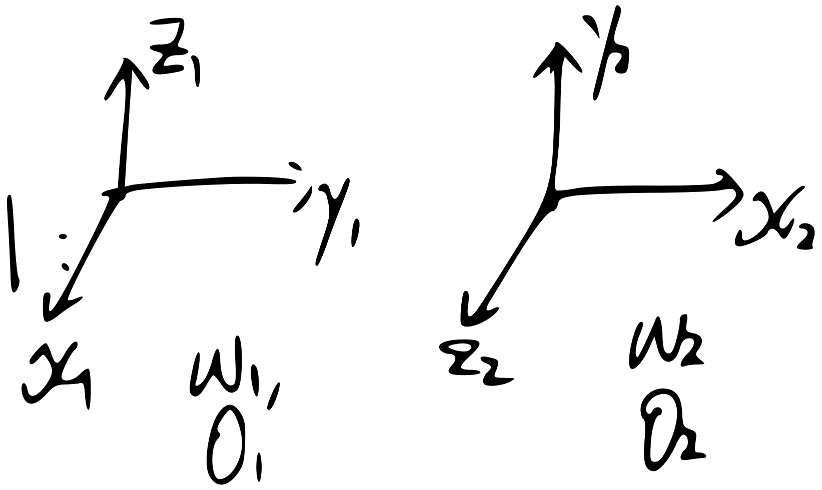
$= (q_1\, q_2\, q_3) A \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$

$\therefore A = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} B (q_1\, q_2\, q_3)$

$\therefore A_{ij} = B_{q_i q_j}$

3.2

method 1 :



$R_1 : O_1 \to w_1$, $R_2 : O_2 \to w_2$

$R_2 : O_2 \to O_1 \to w_1 \to w_2$ $\Rightarrow A_{ij} = B_{q_i q_j}$

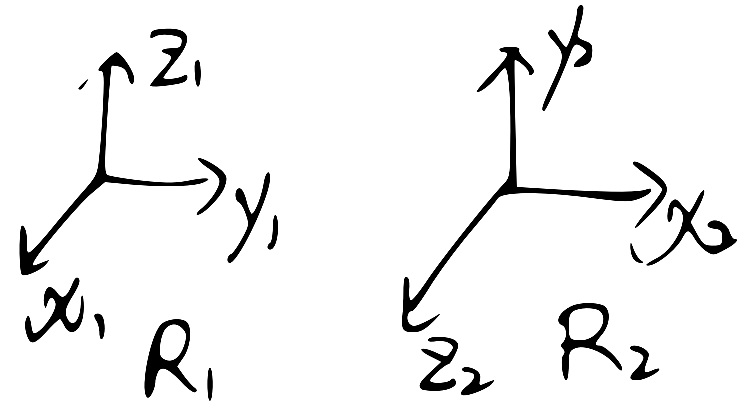
$\therefore R_2 = P_{w_1 \to w_2} R_1 P_{O_2 \to O_1}$ here $(q_1\, q_2\, q_3) = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$

$= P^{-1}_{O_2 \to O_1} R_1 P_{O_2 \to O_1}$ 2.②

3.3 method 2 :

$YZX \to XYZ$

T:



here $x_1 = z_2$
$y_1 = x_2$
$z_1 = y_2$

$\therefore R_2 = B^{-1} R_1 B$

(ItLA, 416, (7))
(V5)

8.2.

$R = R_z R_y R_x$

$= \begin{pmatrix} c_3 & -s_3 & \\ s_3 & c_3 & \\ & & 1 \end{pmatrix} \cdot R_y R_x$

$= \begin{pmatrix} c_2c_3 & s_2s_1c_3 - c_1s_3 & s_2c_1c_3 + s_1s_3 \\ c_2s_3 & s_2s_1s_3 + c_1c_3 & s_2c_1s_3 - s_1c_3 \\ -s_2 & c_2s_1 & c_2c_1 \end{pmatrix}$

$\Rightarrow \theta_1 = \operatorname{atan2}(R_{32}, R_{33})$

$c_2 = \sqrt{R_{11}^2 + R_{21}^2}$

$\theta_2 = \operatorname{atan2}(-R_{31}, c_2)$

$\because R_z = R(R_y R_x)^T$

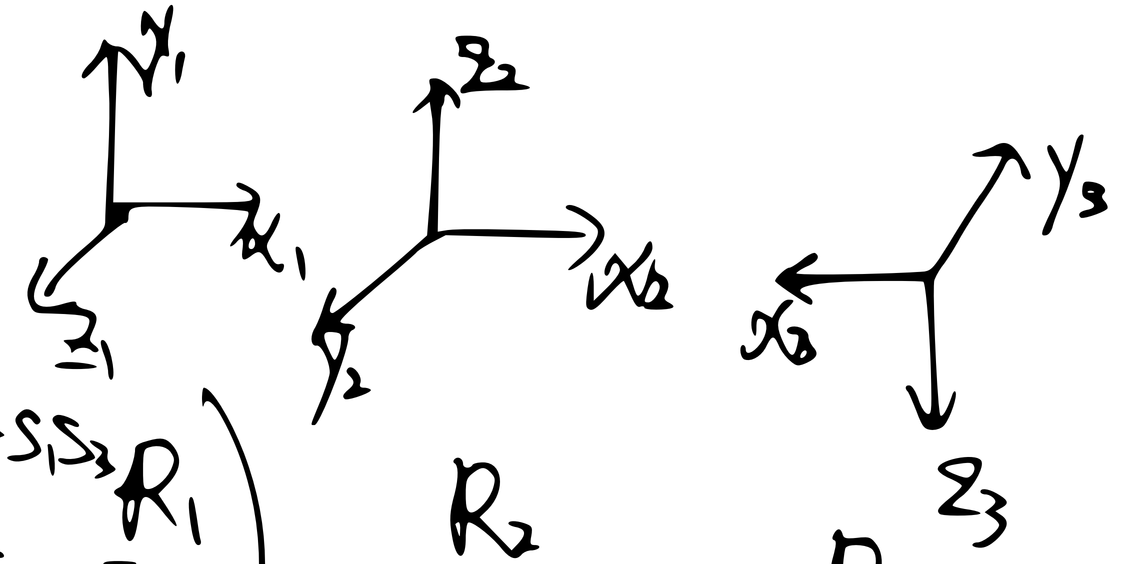
$\therefore \begin{pmatrix} c_3 & -s_3 & \\ s_3 & c_3 & \\ & & 1 \end{pmatrix} = R \begin{pmatrix} c_2 & 0 & -s_2 \\ s_2s_1 & c_1 & c_2s_1 \\ s_2c_1 & -s_1 & c_2c_1 \end{pmatrix}$

$\therefore R_{12}c_1 - R_{13}s_1 = -s_3$

$R_{22}c_1 - R_{23}s_1 = c_3$

$\therefore \theta_3 = \operatorname{atan2}(s_3, c_3)$

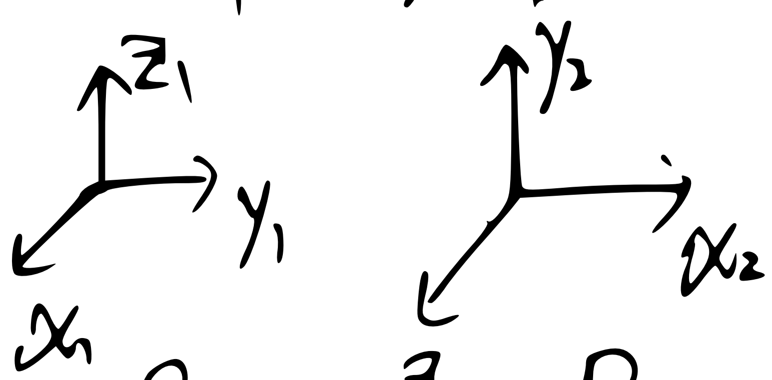
6. $XZY \to XYZ$



R_1 R_2 R_3

$R_3 = R_2$, $R_2 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} R_1 (1\ 3\ 2)$

7. $YZY \to XYX$



R_1 R_2

$\therefore R_2 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} R_1 (2\ 3\ 1)$

$\because R_2 = P^{-1}_{O_2 \to O_1} R_1 P_{O_2 \to O_1}$

$= \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} R_1 (2\ 3\ 1)$

here $P_{O_2 \to O_1} = (2, 3, 1) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

5. $R_1 =$ T in left-handed

$R_2 =$ T in right-handed

$\Rightarrow R_2 = B^{-1} R_1 B = R_1$,

here $B = \begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$