is equivalent to setting

$$\mathbf{A}_j = \hat{\mathbf{A}}_j, \qquad \mathbf{b}_j = \hat{\mathbf{b}}_j - \mathbf{A}_j \mathbf{u}_0. \tag{39}$$

We prefer this second approach, since it results in $\mathbf{A}_j$ estimates which are non-negative definite (important for ensuring that the normal equations can be solved stably), and since it better reflects the certainty in a local match.[12]

5 Estimating the focal length

In order to apply our 3D rotation technique, we must first obtain an estimate for the camera's focal length. We can obtain such an estimate from one or more perspective transforms computed using the 8-parameter algorithm. Expanding the $\mathbf{V}_1\mathbf{R}\mathbf{V}_0^{-1}$ formulation, we have

$$\mathbf{M} = \begin{bmatrix} m_0 & m_1 & m_2 \\ m_3 & m_4 & m_5 \\ m_6 & m_7 & 1 \end{bmatrix} \sim \begin{bmatrix} r_{00} & r_{01} & r_{02} f_0 \\ r_{10} & r_{11} & r_{12} f_0 \\ r_{20}/f_1 & r_{21}/f_1 & r_{22} f_0/f_1 \end{bmatrix} \tag{40}$$

where $\mathbf{R} = [r_{ij}]$.

In order to estimate focal lengths f_0 and f_1, we observe that the first two rows (or columns) of $\mathbf{R}$ must have the same norm and be orthogonal (even if the matrix is scaled), i.e.,

$$m_0{}^2 + m_1{}^2 + m_2{}^2/f_0{}^2 = m_3{}^2 + m_4{}^2 + m_5{}^2/f_0{}^2 \tag{41}$$

$$m_0 m_3 + m_1 m_4 + m_2 m_5/f_0{}^2 = 0 \tag{42}$$

and

$$m_0{}^2 + m_3{}^2 + m_6{}^2 f_1{}^2 = m_1{}^2 + m_4{}^2 + m_7{}^2 f_1{}^2 \tag{43}$$

$$m_0 m_1 + m_3 m_4 + m_6 m_7 f_1{}^2 = 0. \tag{44}$$

From this, we can compute the estimates

$$f_0{}^2 = \frac{m_5{}^2 - m_2{}^2}{m_0{}^2 + m_1{}^2 - m_3{}^2 - m_4{}^2} \quad \text{if } m_0{}^2 + m_1{}^2 \neq m_3{}^2 + m_4{}^2$$

[12] An analysis of the relationship between these two approaches can be found in [TH86].