

Problem 3, Part B

```
void f2(int n)
```

```
{
    for(int i=1; i <= n; i++){
        → if( (i % (int)sqrt(n)) == 0){ // constantly evaluated O(1)
            for(int k=0; k < pow(i,3); k++) {
                /* do something that takes O(1) time */ // O(1)
            }
        }
    }
}
```

executes
a certain
amount of times

Iterates through the whole thing
geometric series?

$$T(n) = \sum_{i=1}^n \theta(1) + \sum_i \sum_{i=1}^n \theta(i^p)$$

$$O(n) + \theta\left(\frac{n(n-1)}{2}\right)$$

$$O(n) + O(n) = \theta(n^2)$$

$$\sum_{k=0}^{n-1} k^p = \frac{k^{p+1} - 1}{p+1}$$

$$\frac{k \log(n) - 1}{k-1} = \frac{k \log(n) - 1}{1 - \theta(n)}$$

Problem 3, Part A

```
void f1(int n)
```

```
{
    int i=2;  $\theta(1)$ 
    while(i < n){ // iterating through UNTIL it is greater than  $\theta(n)$ 
        /* do something that takes  $O(1)$  time */  $\theta(1)$ 
        i = i*i; // constantly is called  $\theta(1)$ 
    }
}
```

$$\sum_{i=1}^n \theta(i^p) :$$

$$T(n) = \sum_{i=2}^{n-1} (\theta(1)) + \theta(1)$$

$$= \theta(n) + \theta(1)$$

$$= \theta(n)$$

Problem 3, Part D

```
int f (int n)
```

```
{
    //  $\theta(1)$ 
    int *a = new int [10];
    int size = 10; // executes  $\theta(n)$  times
    for (int i = 0; i < n; i++)
    {
        //  $\theta(1)$  // executes one time
        if (i == size)
        {
            //  $\theta(1)$ 
            int newsize = 3*size/2  $\theta(1)$ 
            int *b = new int [newsize]; // executes  $\theta(\text{size})$ 
            for (int j = 0; j < size; j++) b[j] = a[j]; // times
            delete [] a;  $\theta(1)$ 
            a = b;  $\theta(1)$ 
            size = newsize;  $\theta(1)$ 
        }
        //  $\theta(1)$ 
        a[i] = i*i;
    }
}
```

$$T(n) = 3\theta(1) + \sum_{i=0}^{n-1} (\theta(1)) + 3\theta(1) + \sum_{i=0}^{\text{size}} (\theta(1)) +$$

$$4\theta(1) + \theta(1)$$

$$= \theta(n) + \theta(\text{size}) = \theta(n + \text{size})$$

Problem 3, Part C

```
for(int i=1; i <= n; i++){ //iterates through n times  $\Theta(n)$ 
  for(int k=1; k <= n; k++){ //iterates through n times  $\Theta(n)$ 
    if( A[k] == i){  $\Theta(1)$ 
      for(int m=1; m <= n; m=m+m){ //iterates through  $\Theta(\log(n))$ 
        // do something that takes  $O(1)$  time //  $\Theta(1)$ 
        // Assume the contents of the A[] array are not changed
      }
    }
  }
}
```

halving the time for iteration by
 $m + m$

$$T(n) = \sum_{i=0}^n \sum_{k=1}^n (\Theta(1)) + \sum_{m=1}^n 1 \frac{1}{i}$$

$$T(n) = \Theta(n \times n) \left(\sim (\log(n)) \right) =$$

$$T(n) = \Theta(n^2 \log(n))$$