Probability and Combinatorics Cheat Sheet

Permutation: arrangement in some order

Ordered vs. unordered sampling: In ordered sampling, the order the elements are selected matters (e.g. digits in a phone number or letters in a word). In unordered samples, orders doesn't matter (e.g. elements in a subset or lottery numbers)

Sampling with replacement vs. without replacement: In sampling with replacement, it is allowed to repeat the same element (e.g. numbers on a licenses plate). In sampling without replacement, repetition isn't allowed (e.g. in a lottery drawing, you can't draw the same number multiple times).

Assume we have a set with n elements and we want to draw k elements from the set, then the total number of ways we can do this is given by the following table:

	order matters	order doesn't matter
with replacement	n^k	$\binom{n+k-1}{k}$
without replacement	$\frac{n!}{(n-k)!}$	$\binom{n}{k}$

Cardano's $\mathbb{P}$: If all outcomes are equally likely in set S, then the probability of an event A happening is:

$$\mathbb{P}(A) = \frac{\texttt{number of outcomes favorable to } A}{\texttt{total number of outcomes}} = \frac{|E|}{|S|}$$

where $E \subset S$.

Independence: Independent events A and B are independent if knowing whether A occurred gives no information about whether B occurred. More formally, A and B (which have nonzero probability) are independent if and only if one of the following equivalent statements holds:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B)$$
$$\mathbb{P}(A|B) = \mathbb{P}(A)$$
$$\mathbb{P}(B|A) = \mathbb{P}(B)$$

Conditional Probability: The probability of event A given that event B occurred.

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

Expected Value Assume the probabilities of obtaining amounts a_1, a_2,..,a_n are p_1, p_2,..,p_n, respectively and $\sum_{i=1}^{n} p_i = 1$. Then,

$$\texttt{expected value} = \sum_{i=1}^{n} a_i p_i$$

Bayes' Rule Assume S is partitioned into N events A_1, A_2,...,A_N and $\mathbb{P}(E) > 0$. Then,

$$\mathbb{P}(A|E) = \frac{\mathbb{P}(A) \cdot \mathbb{P}(E|A)}{\mathbb{P}(E)}$$

$$\mathbb{P}(A_j|E) = \frac{\mathbb{P}(A_j) \cdot \mathbb{P}(E|A_j)}{\mathbb{P}(A_1) \cdot \mathbb{P}(E|A_1) + \mathbb{P}(A_2) \cdot \mathbb{P}(E|A_2) + \cdots + \mathbb{P}(A_n) \cdot \mathbb{P}(E|A_n)}$$

$$\mathbb{P}(A|B, C) = \frac{\mathbb{P}(B|A, C) \cdot \mathbb{P}(A|C)}{\mathbb{P}(B|C)}$$