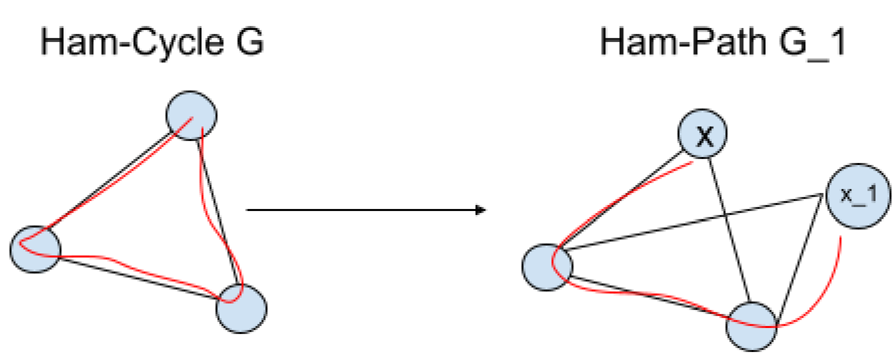
1)
a) **False,** X is at most as hard as Y
b) **False,** we don't know if Y is in NP
c) **False,** X can be reduced to Y, but not the other way around
d) **True,** since X is easier than Y, and since Y is in NP, Y must be NP-complete
e) **False,** X reduces to Y, so Y can be anything harder than X
f) **True,** X reduces to Y, and Y is in P
g) **False,** They can both be NP-complete, thus both in NP

2)
a) **True**, by definition an NP-complete can be reduced to another NP-complete problem in polynomial time.
b) **False,** if P != NP, and there is polynomial time reduction from 3-sat to 2-sat, then 2-sat is NP-complete, but 2-sat is solvable in polynomial time, which means P is a supset of NP, which would mean P = NP, which is a contradiction.
c) **True**, this would make 2-sat an NP-complete problem solvable in polynomial time, thus would satisfy P=NP

3)
Proving HAM-PATH is NP:
We can easily check if a path is a HAM path by simply traversing the supposed path and checking to see if each vertex is visited at least once. This can obviously be done in polynomial time.

Proving HAM-PATH is NP-HARD:
We already know that finding a hamiltonian cycle is NP-complete, thus we can reduce HAM-cycle to HAM-path to prove HAM-path is NP-complete.

Given a graph G, we construct G_1 such that G contains a HAM-cycle if and only if G_1 contains a HAM-path. We do this by choosing a random vertex x in G and adding another vertex x_1, which is a copy of x, to the graph, connecting x_1 to all the same vertices that are adjacent to x. This creates a graph that is a HAMPATH.



G_1 is now a HAMPATH, thus G belongs in HAMCYCLE if and only if G_1 belongs in HAMPATH, thus HAMPATH must be NP-hard.

Therefore HAMPATH is NP-complete since it is in both NP and NP-hard.

4)

Proving 4-color is NP:

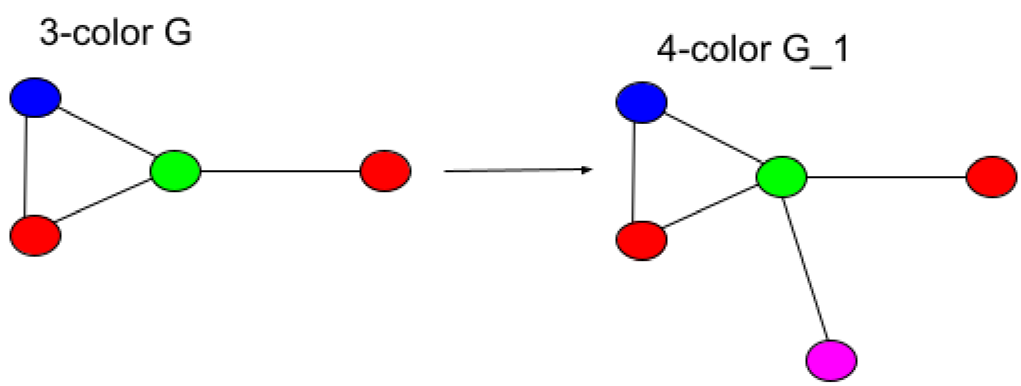
It can be easily checked as to whether a graph is 4-color or not in polynomial time. We can iterate through each edge and check if both ends have the same color or not. If no edge has the same color on both ends, then the graph must belong in 4-color, otherwise it does not. This can obviously be done in polynomial time.

Proving 4-color is NP-hard:

We already know 3-color is NP-complete, thus we can reduce 3-color to 4-color in order to prove that 4-color is NP-complete.

We take a graph G, which belongs to 3-color. We must show that G can be reduced to a new graph G_1 such that G belongs to 3-color if and only if G_1 belongs to 4-color.

We create G_1 by adding a new vertex to G and coloring it with a new color that wasn't already in the graph.



G_1 is now in 4-color. Thus G belongs in 3-color if and only if G_1 belongs in 4-color, thus 4-color must be NP-hard.

Therefore 4-color is NP-complete, since it is in both NP and NP-hard.