

Empirical Asset Pricing: Portfolio Analysis

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1 Portfolio Analysis

1.1 Time-Series Average Returns, Volatilities, and Sharpe Ratios

Tables 1, 2, and 3 report the time-series average excess returns, volatilities, and sharpe ratios, respectively, for each of the 25 portfolios, based on a bivariate sort on value and investment.

Table 1: Average Excess Returns and H-L Spreads

Value (B/M)	Investment (Inv) Quintiles						H-L (Inv)
	Low Inv	2	3	4	High Inv		
Low (Growth)	0.61	0.63	0.64	0.64	0.67	0.06 (0.39)	
2	0.74	0.71	0.48	0.58	0.61	-0.14 (-1.06)	
3	0.78	0.68	0.69	0.65	0.47	-0.31 (-2.22)	
4	0.76	0.65	0.78	0.78	0.62	-0.14 (-1.17)	
High (Value)	1.03	0.89	0.83	0.94	0.61	-0.42 (-3.08)	
H-L (Value)	0.41 (2.26)	0.26 (1.59)	0.19 (1.10)	0.30 (1.62)	-0.06 (-0.30)		—

Note: Right column reports the Investment spread (High - Low Inv) for each Value quintile. Bottom row reports the Value spread (High - Low B/M) for each Investment quintile. Newey-West-adjusted t-statistics are in parentheses below each spread portfolio. Sample begins in July 1963 and ends at October 2025.

Average excess returns for stocks with low investment increase as book-to-market (B/M) increases, with excess returns concentrated around high value firms with low investment rates. One interpretation of this could be that the maturity of value firms reduces the need for constant investment, allowing for reallocation of assets toward other profit-generating projects. Or, perhaps a greater return is demanded as high B/M stocks with low rates of investment pose a higher risk to investors.

Returns are not definitively increasing as we move from low to high investment, except for growth stocks (low B/M) for which we do see returns increasing as investment increases. This may be due to investors demanding a higher return for growth companies that allocate scarce resources (low book values) toward investment.

The value premium is largest and statistically significant for firms with low investment, with an average monthly Value 5-1 return of 0.41% and a Newey-West-adjusted (NW) t-statistic of 2.26 for the sample. For investment quintiles 2-5, we mostly see statistically insignificant value premiums, noting marginal significance for investment quintiles 2 and 4. Economically, the magnitude of the value premium generally declines as investment increases, turning negative for high investment firms. This is a first indication of no excess returns for overinvesting.

Focusing on investment, average high-minus-low (H-L) monthly returns are lowest in magnitude and statistically insignificant for growth firms. Conversely, investment premiums are negative, large in magnitude, and statistically significant for high B/M firms. Within the 3rd and 5th value quintiles, the average monthly investment returns and NW-adjusted t-statistics are -0.31% (-2.22) and -0.42% (-3.08), respectively. This suggests that the investment premium becomes stronger and more negative as value increases.

Table 2: **Excess Return Volatilities**

Value (B/M)	Investment (Inv) Quintiles				
	Low	2	3	4	High
Low (Growth)	5.48	4.65	4.60	4.82	6.17
2	5.07	4.52	4.62	4.85	5.57
3	5.21	4.49	4.50	4.75	5.64
4	5.40	4.62	4.77	5.54	5.56
High (Value)	6.05	5.46	5.69	6.20	6.66

Stock volatility, as shown for each portfolio in Table 2, seems to dip before increasing when we move from low B/M to high B/M, with the highest volatilities primarily observed for portfolios of high B/M stocks across all investment categories (low to high). Interestingly, we see the highest volatility concentrated around high-value, high-investment stocks (south east entries). Perhaps these firms have high B/M ratios because markets penalize their inability to effectively generate earnings despite high asset turnover.

Table 3: Sharpe Ratios

Value (B/M)	Investment (Inv) Quintiles				
	Low	2	3	4	High
Low (Growth)	0.11	0.14	0.14	0.13	0.11
2	0.15	0.16	0.10	0.12	0.11
3	0.15	0.15	0.15	0.14	0.08
4	0.14	0.14	0.16	0.14	0.11
High (Value)	0.17	0.16	0.15	0.15	0.09

Following the previous observations, risk adjusted returns are highest for stocks with high B/M and low investment, though there are some relatively high sharpe ratios for stocks with moderate B/M and moderate investment. This is rather interesting because firm investment can be viewed as reinvesting to grow revenues and deliver higher earnings to investors in the form of dividends, but it seems investors favor moderate investment over excessive, with a tilt toward value stocks that are "undervalued by the market."

1.2 CAPM and Fama-French

1.2.1 CAPM Model Analysis

Among the high-value, low-investment firms, we see large, positive, and significant regressions intercepts, suggesting that the CAPM model systematically underprices these stocks and that they earn higher expected excess returns relative to what the market beta would imply. We can also note marginal significance across investment percentiles within the 3rd quintile of value. Regression intercepts for all high-investment portfolios (across value quintiles) are negative, with marginal significance for firms in the 3rd value quintile. These same high-investment firms have the largest market betas out of all 25 portfolios, as shown in Panel B. Firms that invest too much seem to carry a higher risk premium in this model, and investors may be penalizing these firms for perhaps "careless" corporate investments. Conversely, firms that do not invest enough (left-most column) may be investing too little, as their market betas are higher. Together, we see a "coastal pattern" in Panel B.

Table 4: CAPM Regressions for 25 Portfolios Sorted by Value and Investment

Value (B/M)	Investment (Inv) Quintiles				
	Low	2	3	4	High
<i>Panel A: Intercepts (α)</i>					
Low (Growth)	-0.03 (-0.26)	0.09 (1.08)	0.08 (1.16)	0.06 (0.70)	-0.07 (-0.63)
2	0.17 (1.55)	0.19 (1.95)	-0.08 (-1.00)	-0.02 (-0.20)	-0.07 (-0.77)
3	0.18 (1.53)	0.17 (1.84)	0.17 (1.79)	0.09 (0.97)	-0.20 (-1.87)
4	0.16 (1.24)	0.14 (1.21)	0.25 (2.00)	0.17 (1.12)	-0.00 (-0.01)
High (Value)	0.38 (2.21)	0.30 (2.16)	0.24 (1.56)	0.29 (2.00)	-0.12 (-0.76)
<i>Panel B: Market Betas (β_{Mkt})</i>					
Low (Growth)	1.07	0.91	0.93	0.98	1.25
2	0.96	0.88	0.94	1.00	1.13
3	1.01	0.86	0.87	0.93	1.11
4	1.01	0.85	0.89	1.02	1.04
High (Value)	1.08	0.99	0.99	1.09	1.23

Note: The table reports the results of the CAPM regression $R_{j,t} - R_{f,t} = \alpha_j + \beta_j R_{Mkt-RF,t} + \epsilon_{j,t}$. Panel A shows the intercepts (α) in percent per month with Newey-West-adjusted t-statistics in parentheses. Panel B shows the market betas (β_{Mkt}). The GRS F-statistic for the joint significance of the alphas is 2.139 (p-value = 0.001).

Gibbons, Ross, and Shenkan joint test results on the CAPM alpha reject the null hypothesis and indicate significance at the 1% level. This shows that this model's single factor (market beta) does not capture the risk exposure of excess returns for the 25 portfolios sorted on value and investment. In other words, the constant term, alpha, is picking up return variation that the single factor does not account for, leading to the next analysis.

1.2.2 Fama-French Five-Factor Model Analysis

Table 5 reports the results of the Fama-French Five-Factor Model (FF5). Alphas are notably lower for firms with high B/M ratios and low Inv, and they have low NW-adjusted t-statistics. For example, the alpha of the south-west portfolio (high B/M, low Inv) is 0.06 with a NW-adjusted t-statistic of 0.64, relative to an alpha of 0.38 with a t-statistic of 2.21 under CAPM. This suggests the FF5 model estimates the risk exposures of these stocks better than CAPM. Betas for size, value, and investment are all positive and large in magnitude for this same portfolio. Market beta exhibits the same "coastal pattern" seen under CAPM, with a larger beta for low- and high-investment firms relative to investment quintiles 2-4.

Interestingly, the intercept for growth firms with high investment changes in sign and magnitude. Under CAPM, the alpha of these firms is -0.07 with a t-statistic of -0.63, and under the FF5 model this intercept becomes 0.29 with a t-statistic of 4.48. This is a very high level of significance and a sizeable unaddressed average return, which may indicate that the FF5 factors are leaving some of the risk pricing on the table for these stocks.

Panel F shows the investment factor's loadings, which are negative for all high-investment firms. This adds some clarity as to why the market betas for these firms were higher under CAPM than they are under the FF5 model. Firms that reinvest their earnings at high rates have lower expected excess returns, perhaps because they are not redistributed to investors or the capital is inefficiently allocated, and the Fama-French investment factor seems to capture this. This supports the low time-series average returns for high-investment firms in Table 1.

Profitability betas, shown in Panel E, for all firms with high B/M ratios are negative. This may indicate that these firms' earnings are not reinvested efficiently, either too much or too little, to grow their business and deliver returns to investors.

Table 5: Fama-French Five-Factor Regressions

	Panel A: Intercepts (α)					Panel B: Market (β_{Mkt})				
Value	Investment					Investment				
	Low	2	3	4	High	Low	2	3	4	High
Low (G)	-0.22 (-2.16)	-0.02 (-0.26)	0.01 (0.11)	0.12 (1.72)	0.29 (4.48)	1.14	0.98	0.97	0.97	1.07
2	-0.09 (-0.84)	-0.02 (-0.26)	-0.18 (-2.42)	-0.12 (-1.75)	-0.07 (-0.72)	1.04	0.96	0.97	1.01	1.09
3	-0.08 (-0.88)	-0.05 (-0.72)	-0.06 (-0.87)	-0.07 (-1.08)	-0.23 (-2.03)	1.07	0.96	0.96	0.97	1.08
4	-0.18 (-2.11)	-0.18 (-2.64)	0.01 (0.11)	-0.03 (-0.26)	-0.12 (-1.25)	1.10	0.97	0.96	1.05	1.02
High (V)	0.06 (0.64)	0.01 (0.18)	0.01 (0.14)	0.05 (0.56)	-0.28 (-2.64)	1.12	1.05	0.99	1.10	1.17

	Panel C: Size (SMB)					Panel D: Value (HML)				
Value	Low	2	3	4	High	Low	2	3	4	High
Low (G)	0.15	-0.11	-0.09	-0.11	0.07	-0.48	-0.33	-0.19	-0.26	-0.38
2	0.17	0.04	0.02	0.07	0.15	-0.15	-0.08	0.03	0.08	0.15
3	0.22	0.00	-0.02	0.08	0.16	0.19	0.11	0.30	0.44	0.44
4	0.27	0.07	0.12	0.17	0.20	0.41	0.48	0.53	0.76	0.75
High (V)	0.47	0.21	0.32	0.33	0.49	0.56	0.81	0.94	0.99	0.70

	Panel E: Profitability (RMW)					Panel F: Investment (CMA)				
Value	Low	2	3	4	High	Low	2	3	4	High
Low (G)	0.15	0.25	0.33	0.23	-0.05	0.89	0.45	0.14	-0.10	-0.64
2	0.19	0.17	0.11	0.27	0.13	0.71	0.52	0.16	-0.05	-0.32
3	0.05	0.09	0.21	0.15	0.04	0.45	0.46	0.18	-0.12	-0.44
4	0.07	0.10	0.08	-0.07	0.00	0.45	0.34	0.04	-0.21	-0.49
High (V)	-0.12	-0.06	-0.08	-0.05	-0.10	0.37	-0.01	-0.32	-0.37	-0.29

Note: The table reports results for the Fama-French Five-Factor model for 25 portfolios sorted by Value (B/M) and Investment (Inv). Intercepts (α) are monthly percentages; Newey-West adjusted t-statistics are provided in parentheses. GRS F-stat = 2.38 (p-value = 0.000).

1.2.3 Model Pricing Errors

Table 6: Model Comparison: CAPM vs. Fama-French Five-Factor Model

Model	Avg. Absolute Alpha ($\text{Avg} \alpha_i $)	GRS F-statistic	GRS p-value
CAPM	0.15	2.14	0.001
FF5	0.10	2.38	0.000

Gibbons, Ross, and Shenkan tests strongly reject the joint significance of FF5 alphas at the 0.1% level, even stronger than CAPM, which was rejected at the 1% level. This indicates the factors do not estimate the full risk exposure of each portfolio. Relative to the CAPM model, however, R^2 generally improves in the FF5 model and the magnitude of the pricing error drops from 0.15 to 0.10, supporting FF5 as a stronger model for measuring the risk exposures of expected excess returns.

Appendix

Model Explanatory Power

Table 7: Regression R^2 Comparison: CAPM vs. FF5

Value	Panel A: CAPM R^2					Panel B: FF5 R^2				
	Low	2	3	4	High	Low	2	3	4	High
Low (G)	0.76	0.76	0.81	0.82	0.81	0.82	0.80	0.85	0.88	0.93
2	0.72	0.76	0.82	0.84	0.83	0.77	0.80	0.83	0.86	0.84
3	0.75	0.73	0.74	0.76	0.78	0.82	0.79	0.81	0.83	0.81
4	0.69	0.68	0.70	0.67	0.70	0.84	0.85	0.82	0.81	0.81
High (V)	0.63	0.65	0.60	0.62	0.68	0.83	0.86	0.81	0.81	0.80

Note: This table reports the adjusted R^2 for the CAPM and Fama-French Five-Factor regressions for each of the 25 portfolios sorted by Value and Investment. R^2 increases once controlling for the additional factors in the FF5 model.

Average Market Beta

Table 8: Comparison of Average Market Betas

Model	Average Market Beta ($\bar{\beta}_{Mkt}$)
CAPM	0.9998
FF5	1.0310

As an additional comparative metric, I wanted to analyze the change in the average market beta after including the 4 additional factors in the FF5 model. Interestingly, the average CAPM beta increases once controlling for FF5's additional risk factors, suggesting the CAPM market beta was suppressed by the negative relationships between expected returns and the other factors in the FF5 model, such as the Investment factor, which we observe in Table 5.