ROTORDYNAMIC ANALYSIS OF THEORETICAL MODELS AND EXPERIMENTAL SYSTEMS

A Thesis presented to

the Faculty of California Polytechnic State University, San Luis Obispo

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

by

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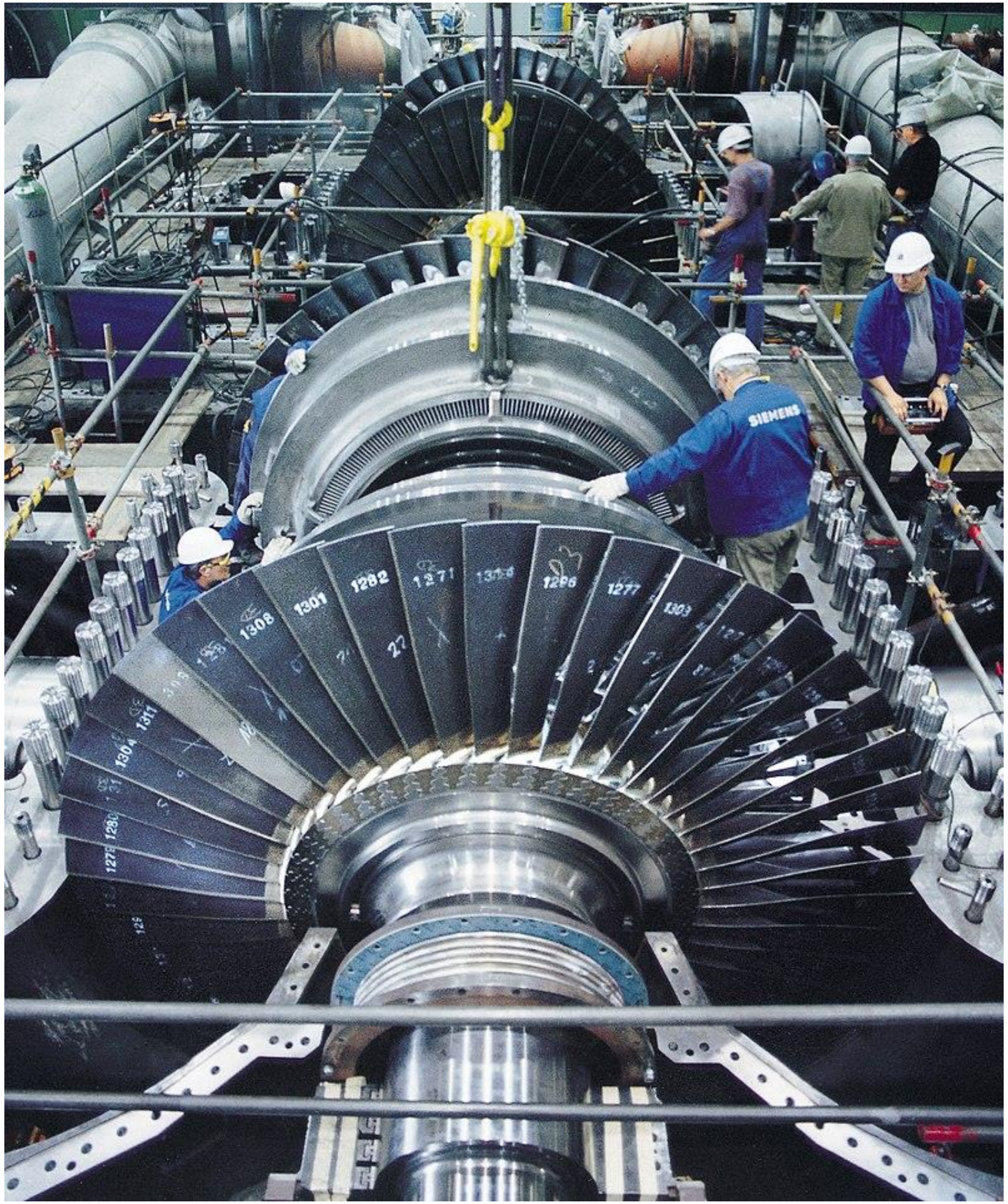
April 2018

- Processing and analysis of vibration signals
- Development of a finite element model for rotordynamics
- Frequency domain processing of rotordynamic models
- Comparison of experimental and model analyses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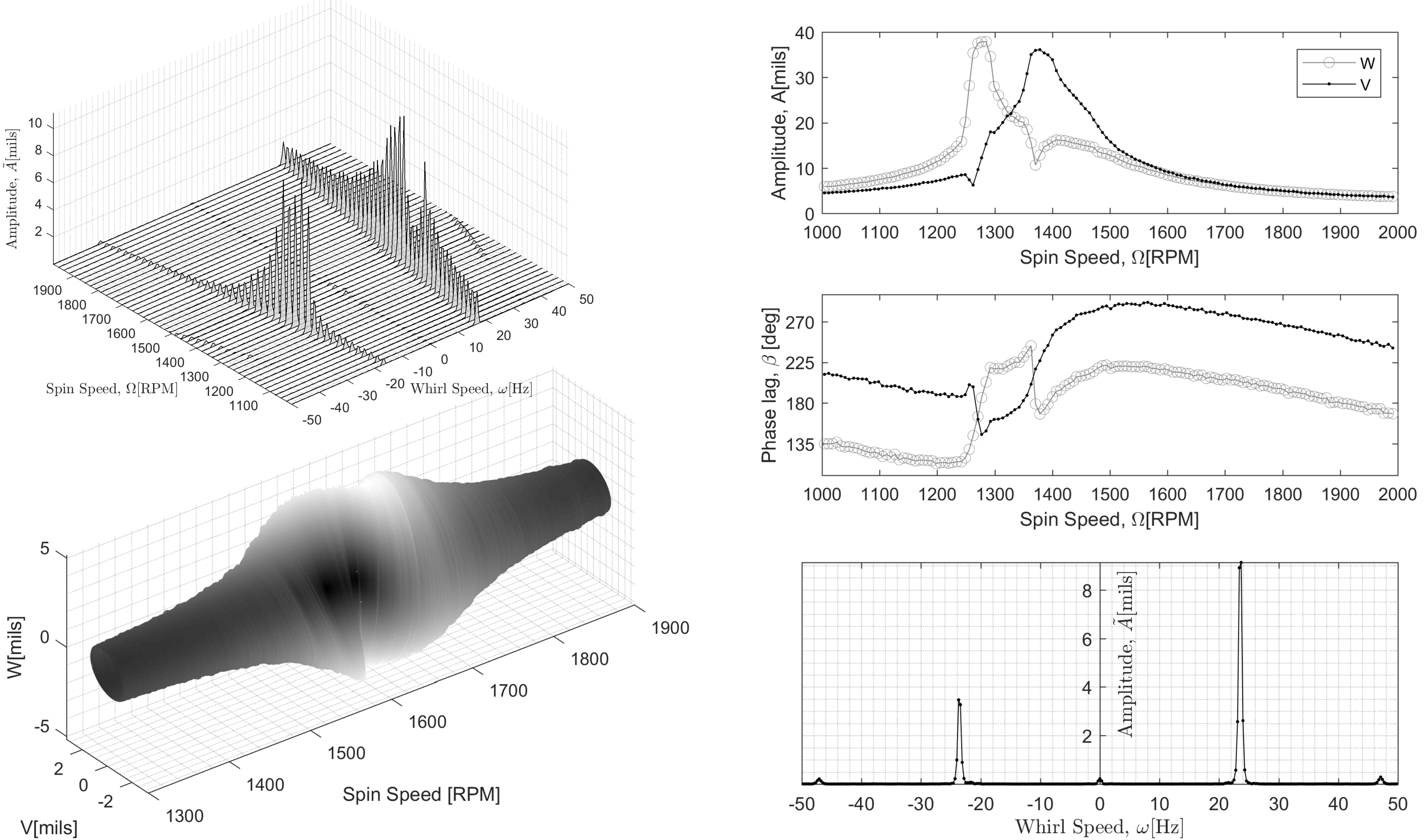
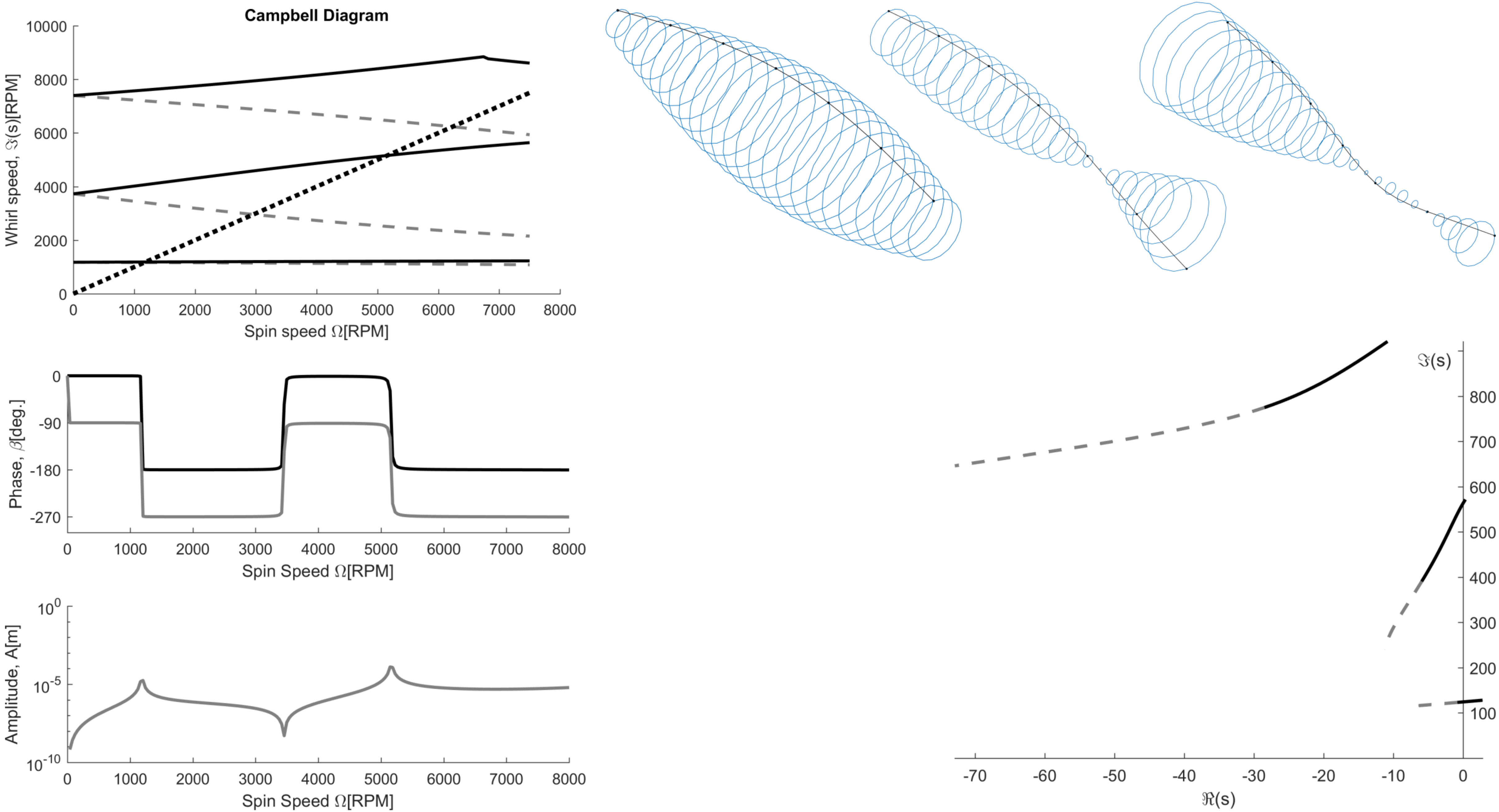
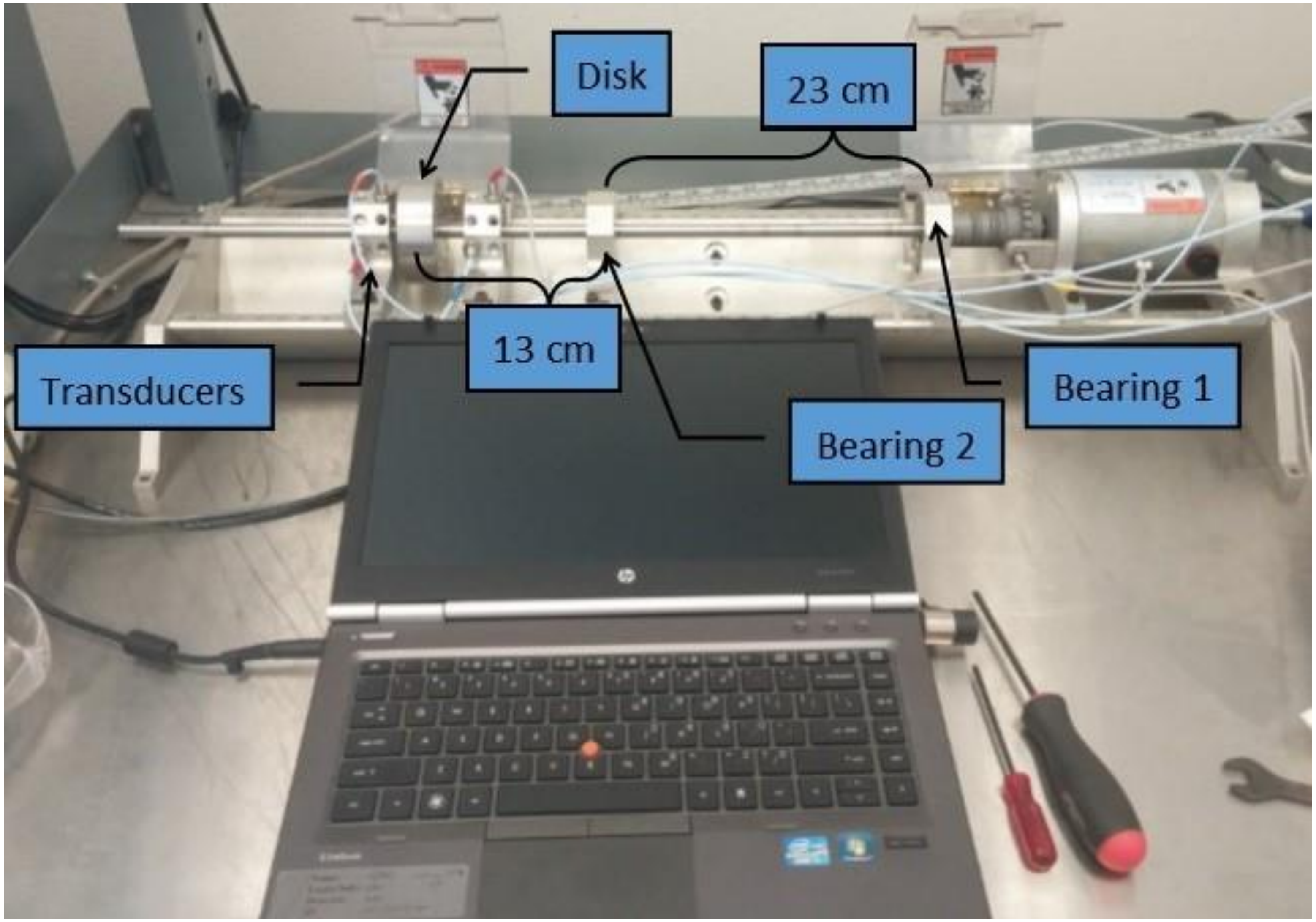


Amplitude, $\tilde{A}$[mils]
Spin Speed, Ω[RPM]
Whirl Speed, ω[Hz]
W[mils]
V[mils]
Spin Speed [RPM]
Amplitude, A[mils]
W
V
Spin Speed, Ω[RPM]
Phase lag, β [deg]
Spin Speed, Ω[RPM]
Amplitude, $\tilde{A}$[mils]
Whirl Speed, ω[Hz]

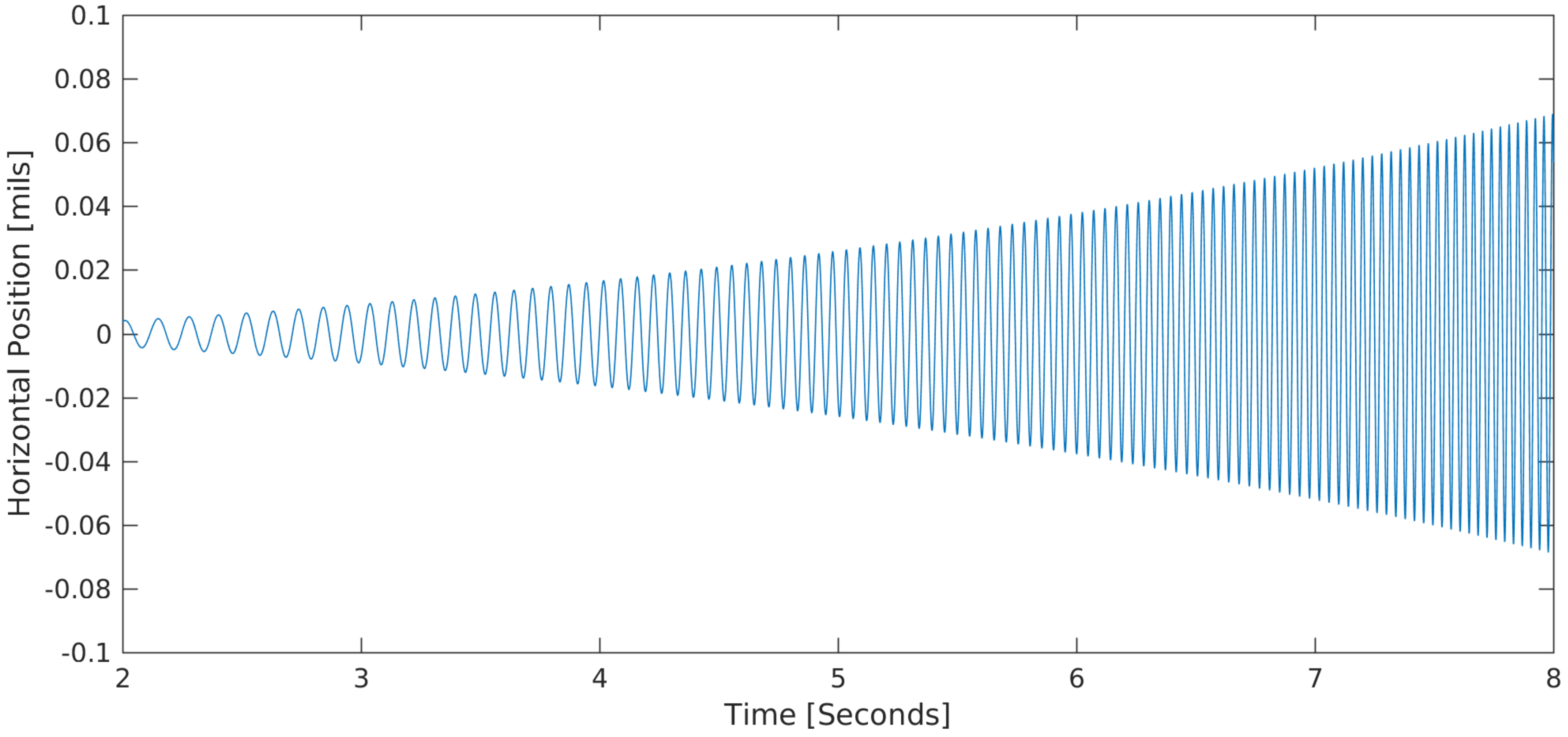


Campbell Diagram
Whirl speed, ℑ(s)[RPM]
10000
8000
6000
4000
2000
0
0
1000
2000
3000
4000
5000
6000
7000
8000
Spin speed Ω[RPM]
Phase, β[deg.]
0
-90
-180
-270
Spin Speed Ω[RPM]
Amplitude, A[m]
10^0
10^{-5}
10^{-10}
Spin Speed Ω[RPM]
ℑ(s)
800
700
600
500
400
300
200
100
-70
-60
-50
-40
-30
-20
-10
0
ℜ(s)

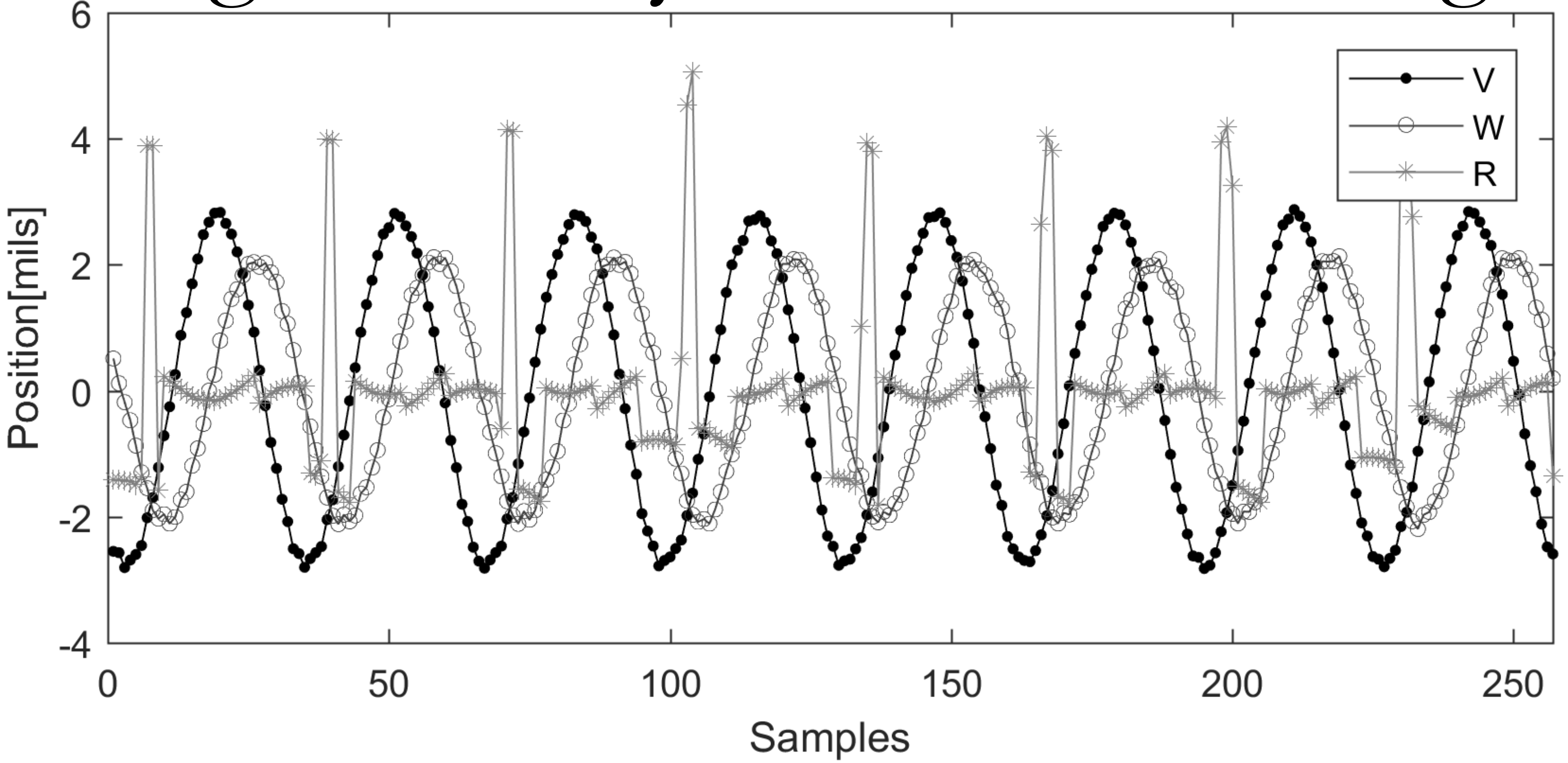
Processing and analysis of vibration signals



Processing and analysis of vibration signals



Processing and analysis of vibration signals



From this set of data we need:

- Average Amplitude[*mils*]
- Phase lag from reference[° *or rad*]
- Frequency domain signal(Amplitude[*mils*], Frequency[*Hz*])

From Experiment or Theory:

- Rotation Speed[*RPM*]

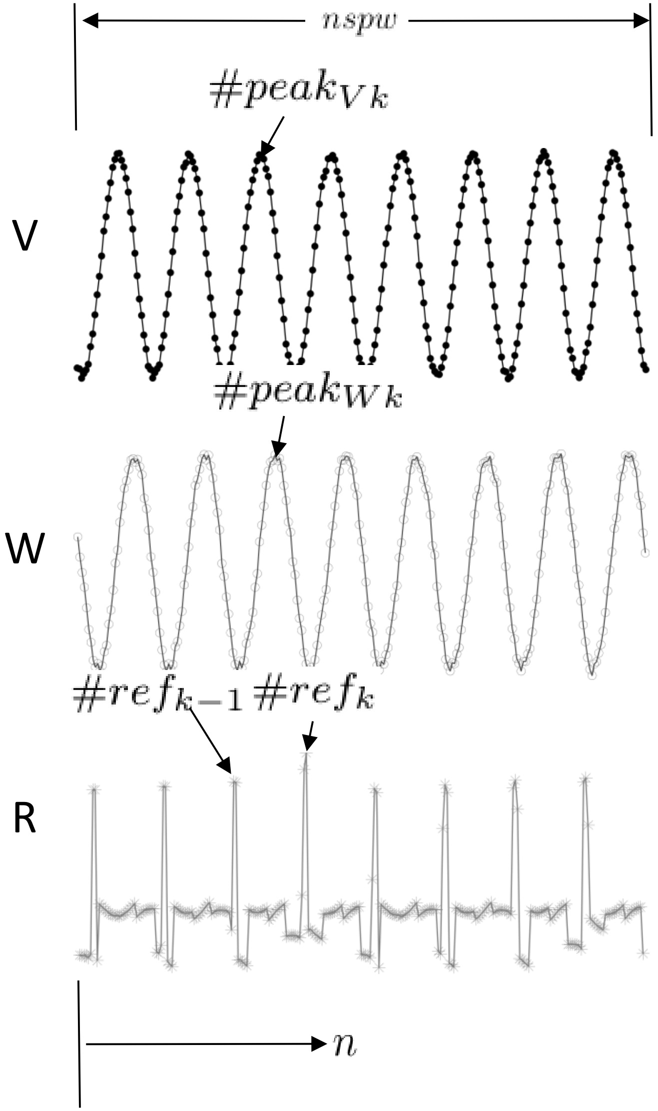
Bode

Polar

Full Spectrum

Cascade

Processing and analysis of vibration signals



Signal Processing

$$A_v(N) = max(V(0:nspw)) - min(V(0:nspw))$$

$$\beta_{Vk} = 2\pi \frac{\#ref_k - \#peak_{Vk}}{\#ref_k - \#ref_{k-1}} \quad \beta_V(N) = avg(\beta_{Vk})$$

$$Z(n) = V(n) + iW(n) \quad \tilde{A}_{\pm}(n) = \frac{\mathrm{fft}(Z(n))}{nspw} \longrightarrow \tilde{A}_{\pm}(N,\omega)$$

$$A_W(N) = max(W(0:nspw)) - min(W(0:nspw))$$

$$\beta_{Wk} = 2\pi \frac{\#ref_k - \#peak_{Wk}}{\#ref_k - \#ref_{k-1}} \quad \beta_W(N) = avg(\beta_{Wk})$$

$$\Omega(N)$$

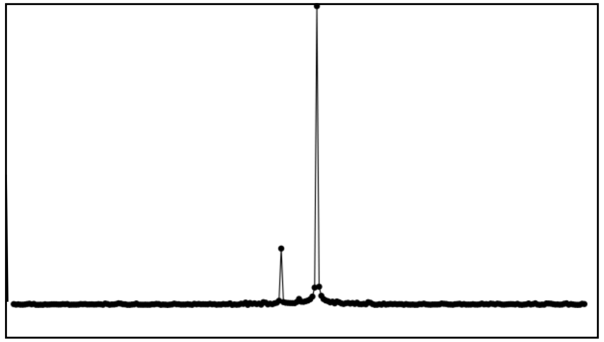
Analysis Variables

$A_V(N)$

$\beta_V(N)$

$\Omega(N)$

$abs(\tilde{A}_{\pm}(N,\omega))$



$A_W(N)$

$\beta_W(N)$

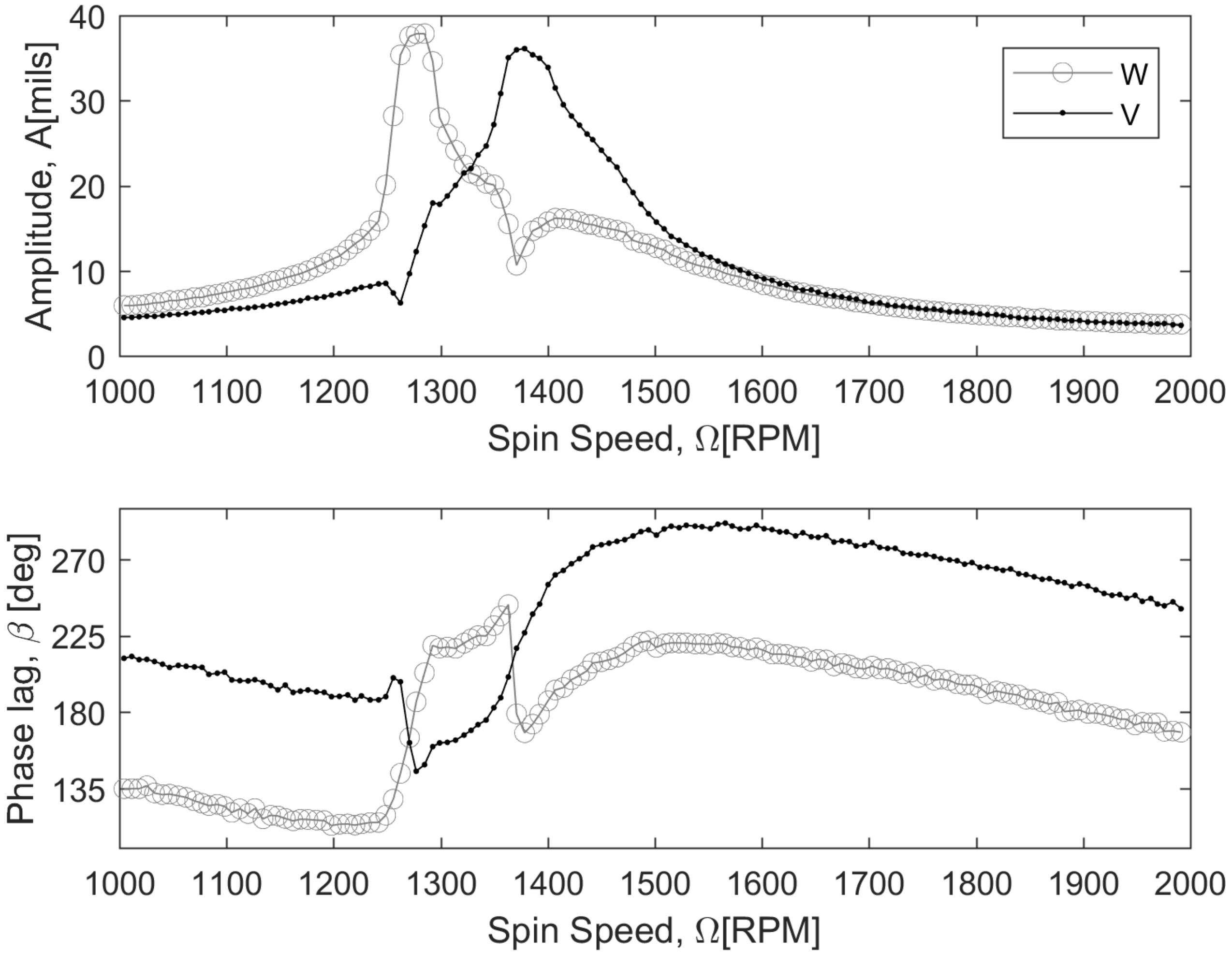
Processing and analysis of vibration signals

Bode Diagram

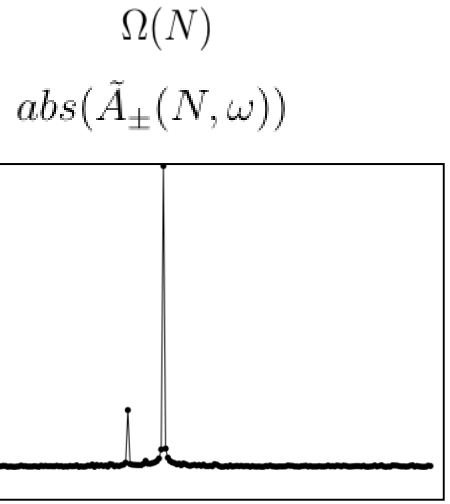
$A_V(N)$
$A_W(N)$

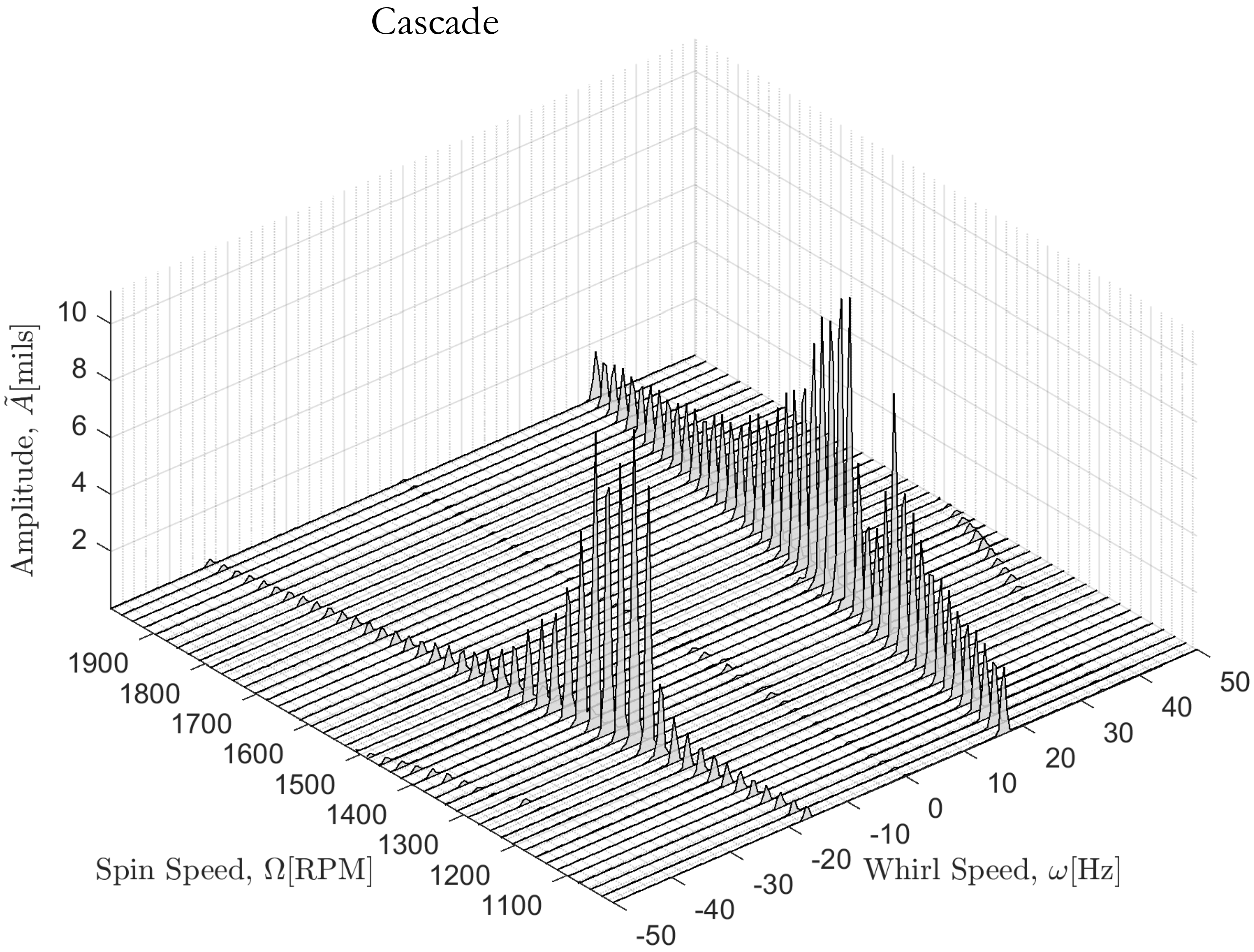
$\Omega(N)$

$\beta_V(N)$
$\beta_W(N)$



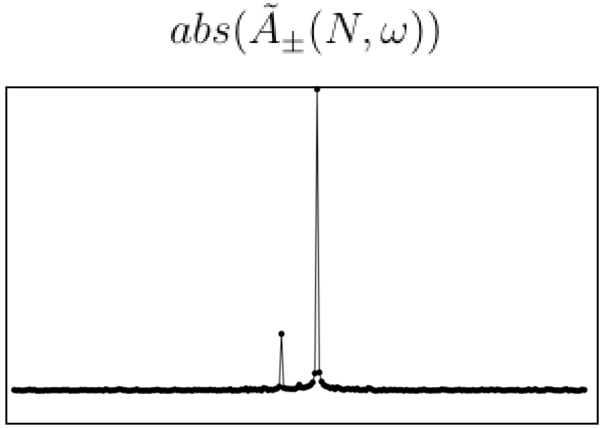
Processing and analysis of vibration signals

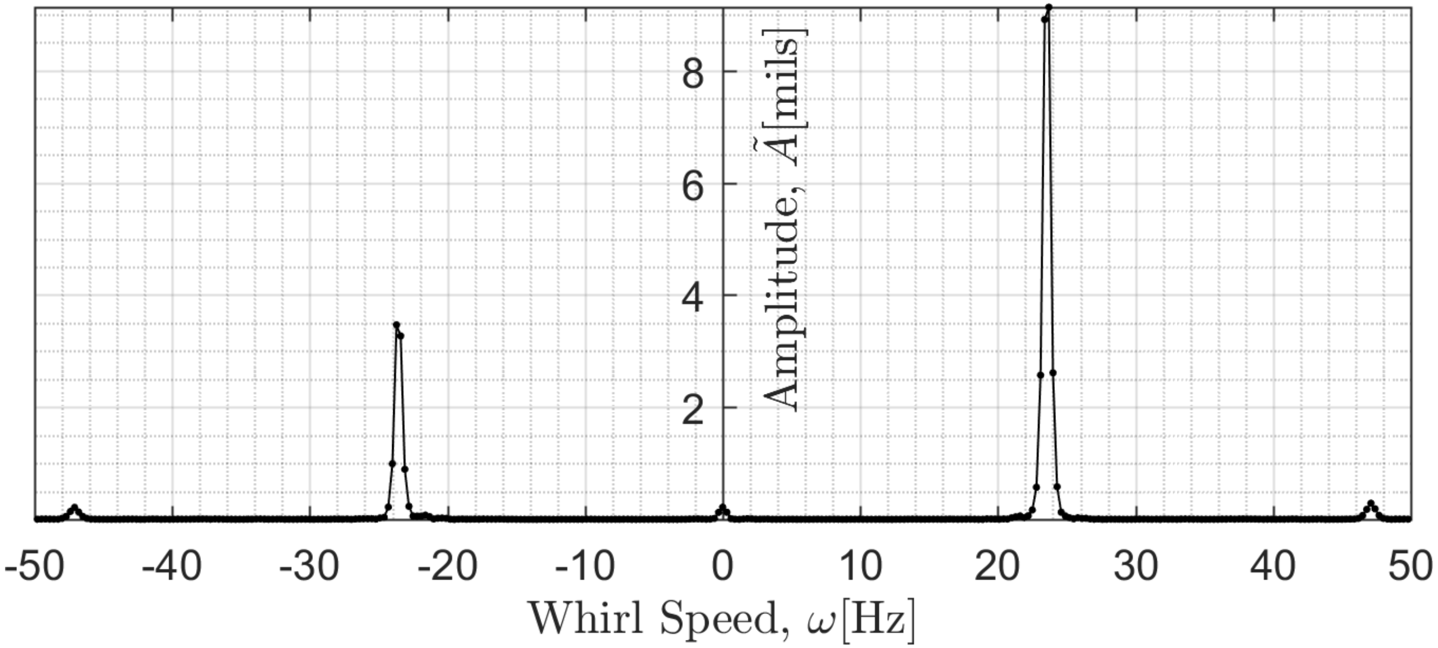




Processing and analysis of vibration signals

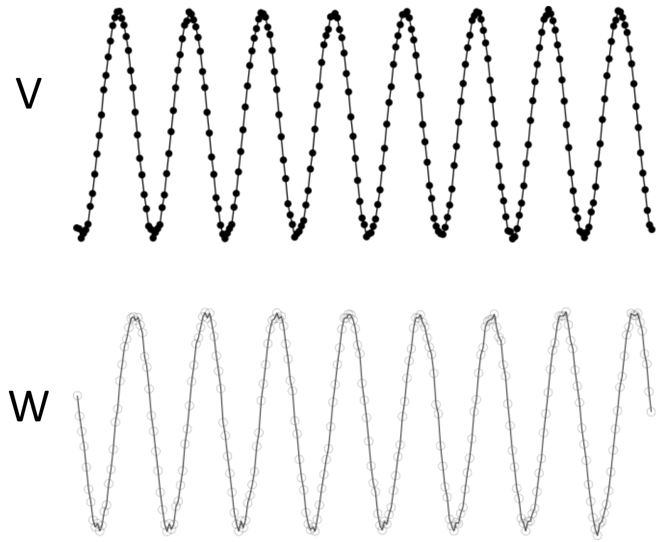
Spectrum

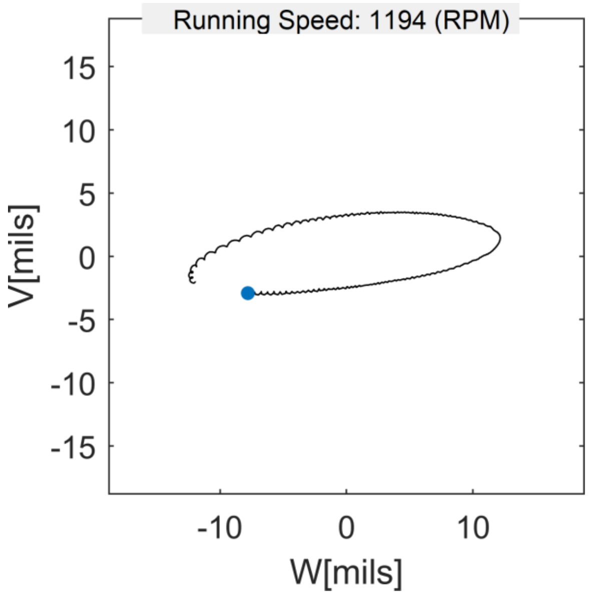




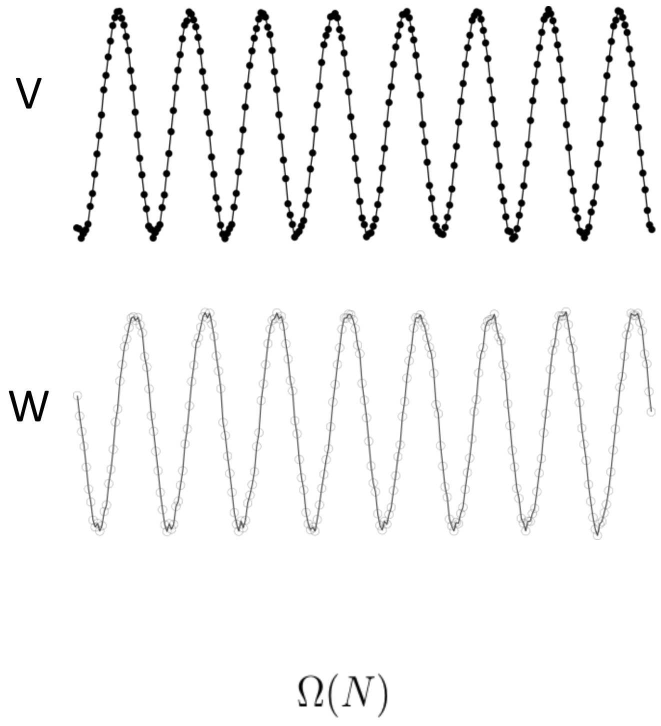
Processing and analysis of vibration signals

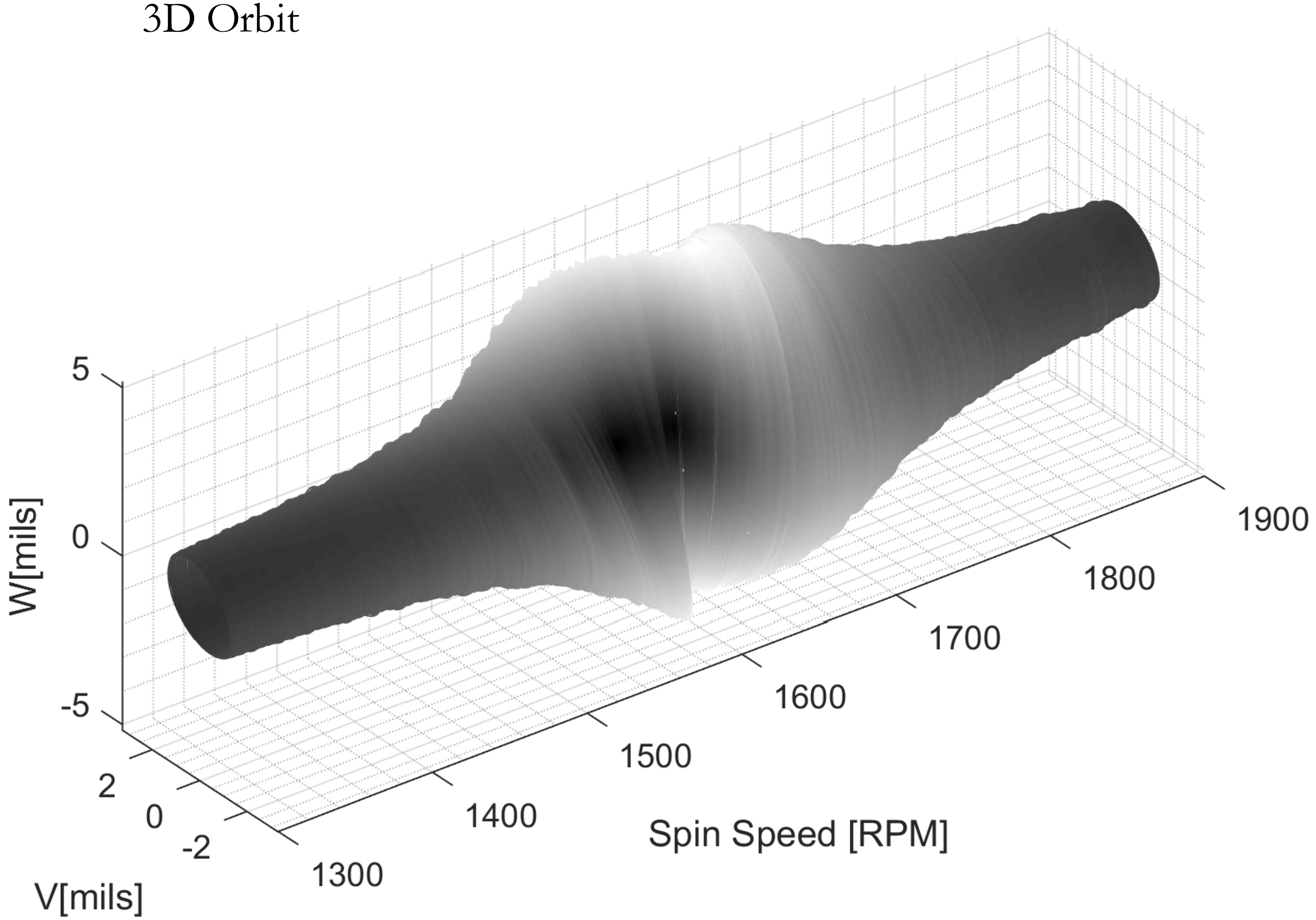
Orbit



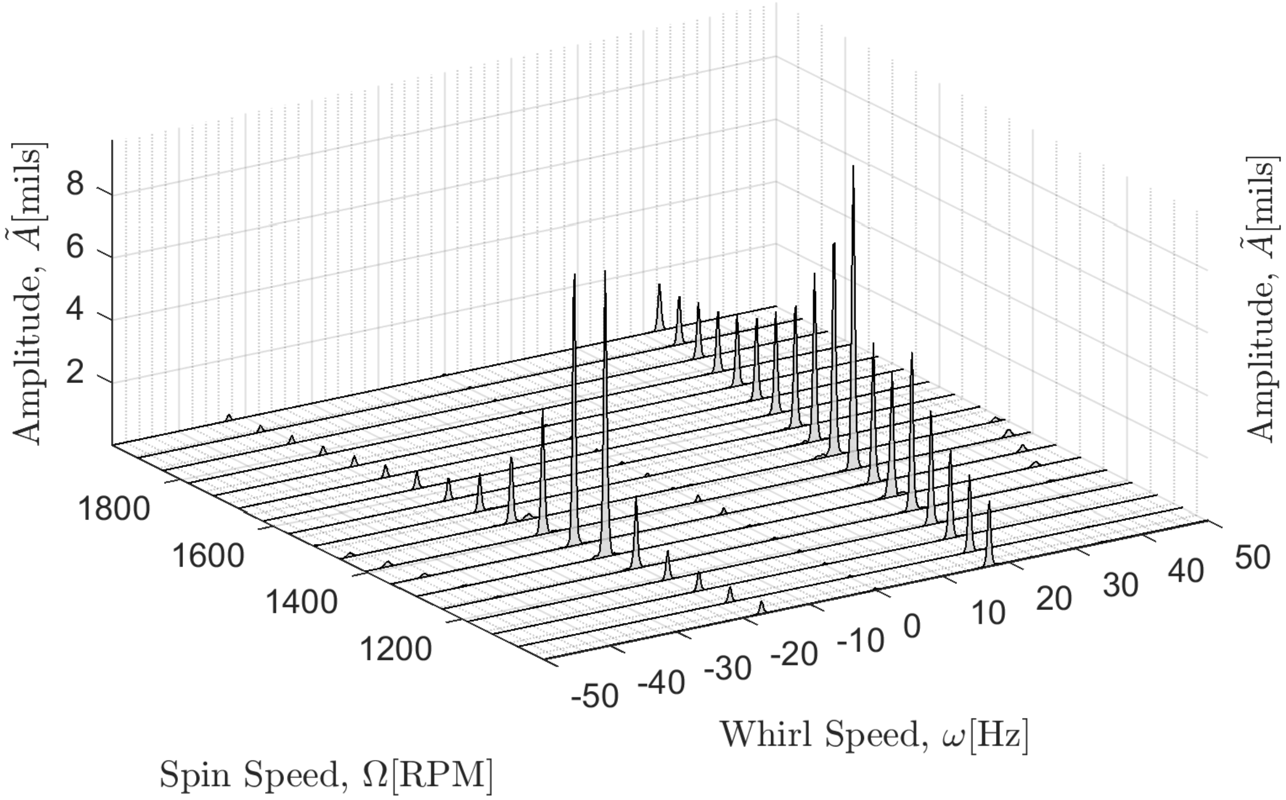


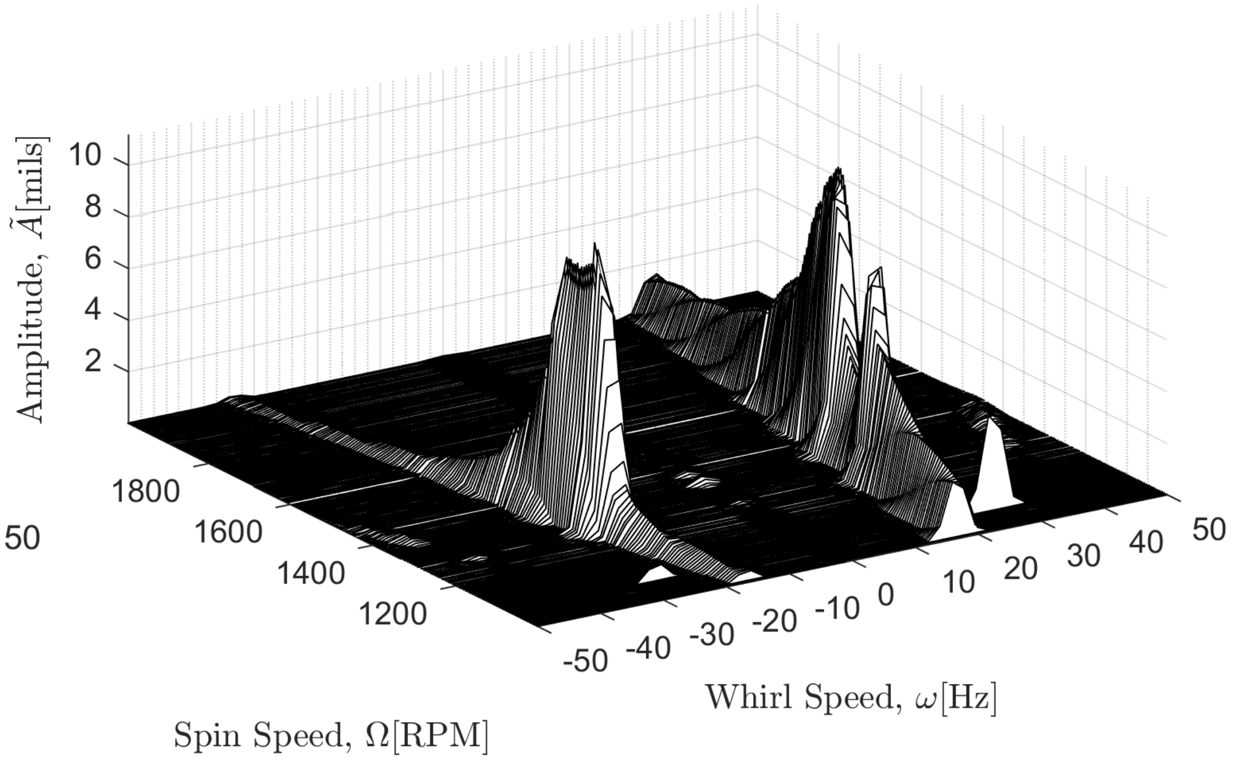
Processing and analysis of vibration signals



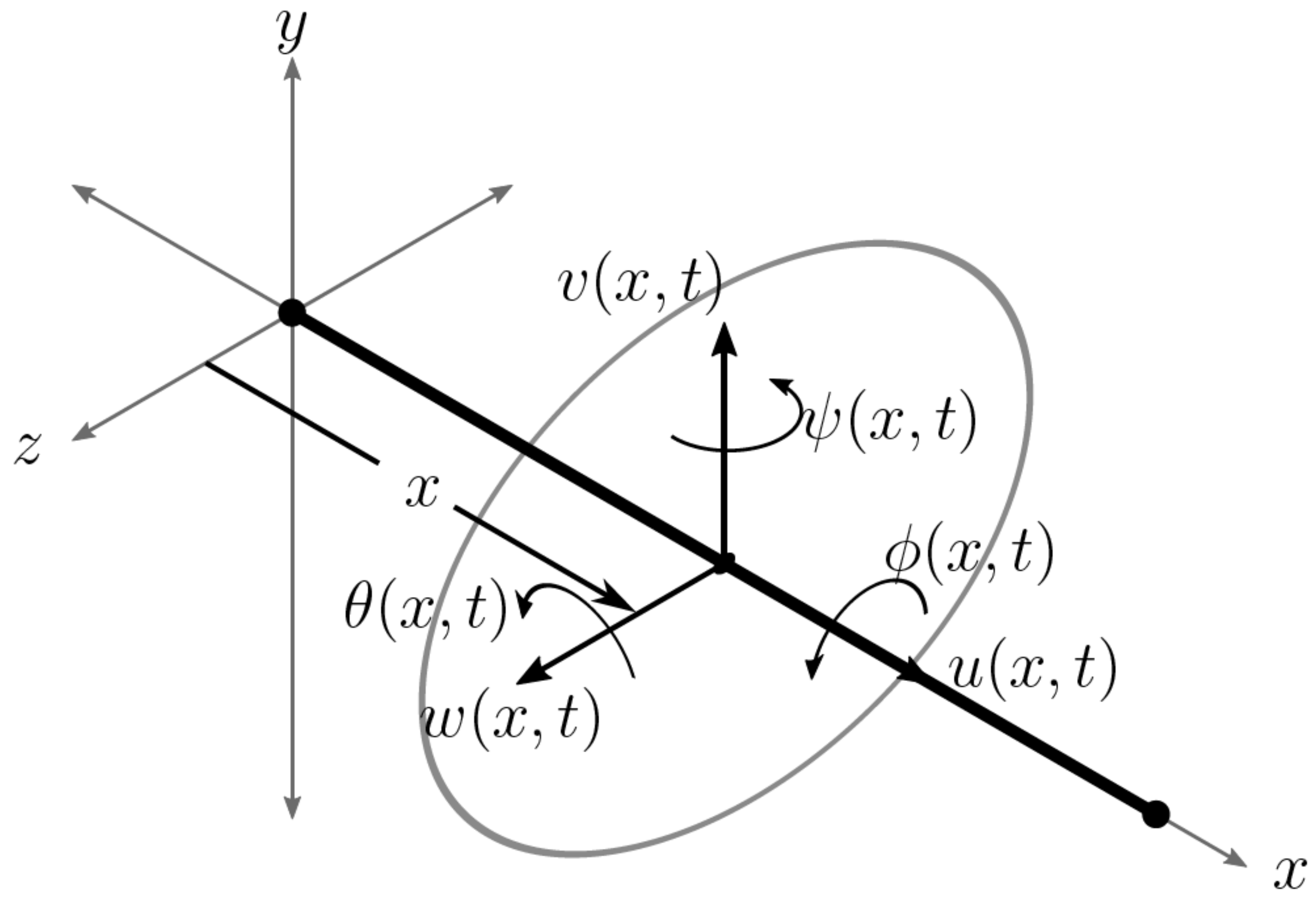


Processing and analysis of vibration signals

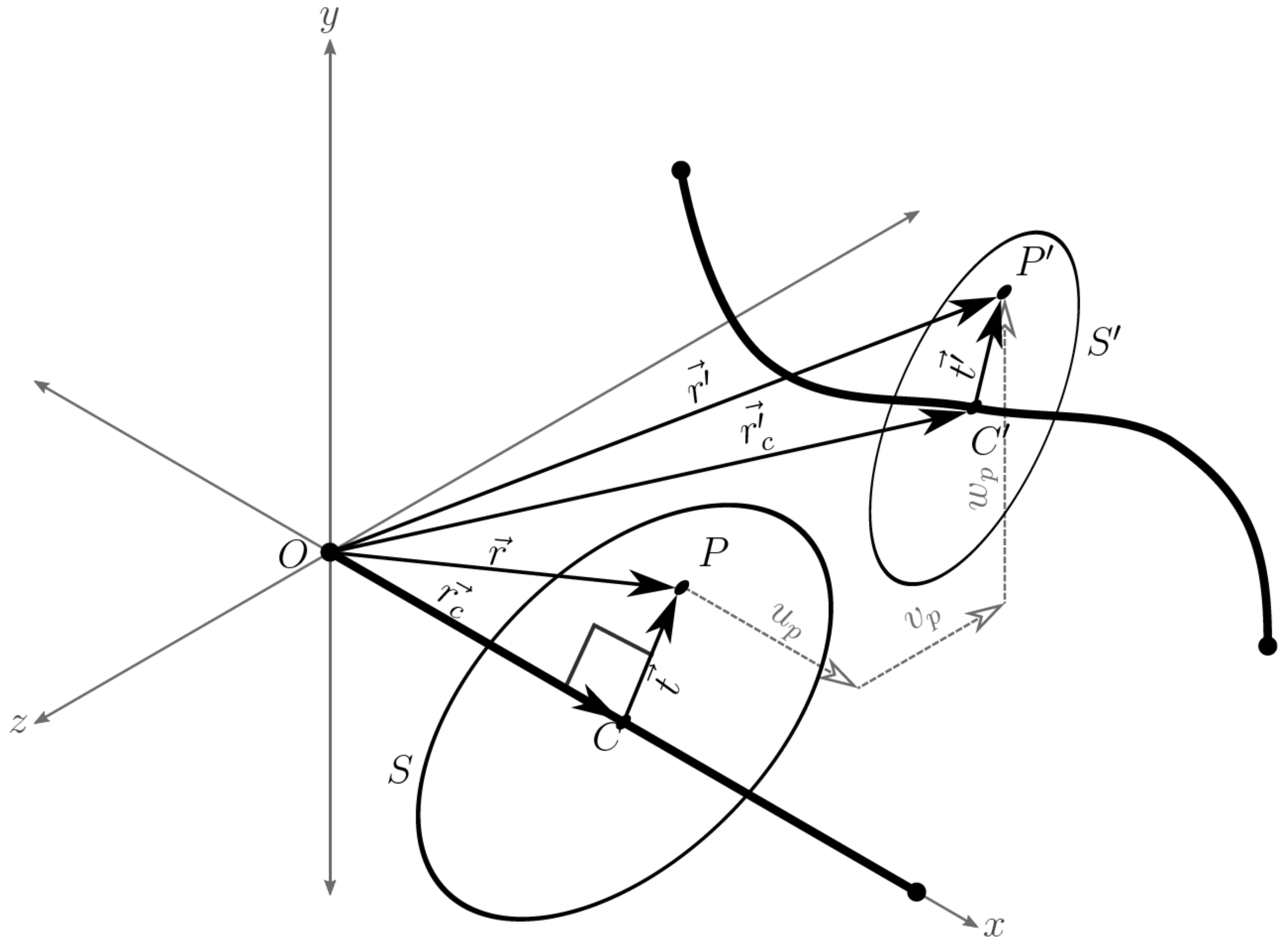




Development of a finite element model for rotordynamics



Development of a finite element model for rotordynamics



$$\vec{u}_p = \begin{Bmatrix} u - \theta y + \psi z \\ v - \phi z \\ w + \phi y \end{Bmatrix}$$

$$\begin{cases} \epsilon_{xx} = u' - \theta' y + \psi' z \\ \epsilon_{xy} = \frac{1}{2}(v' - \phi' z - \theta) \\ \epsilon_{xz} = \frac{1}{2}(w' + \phi' y + \psi) \end{cases}$$

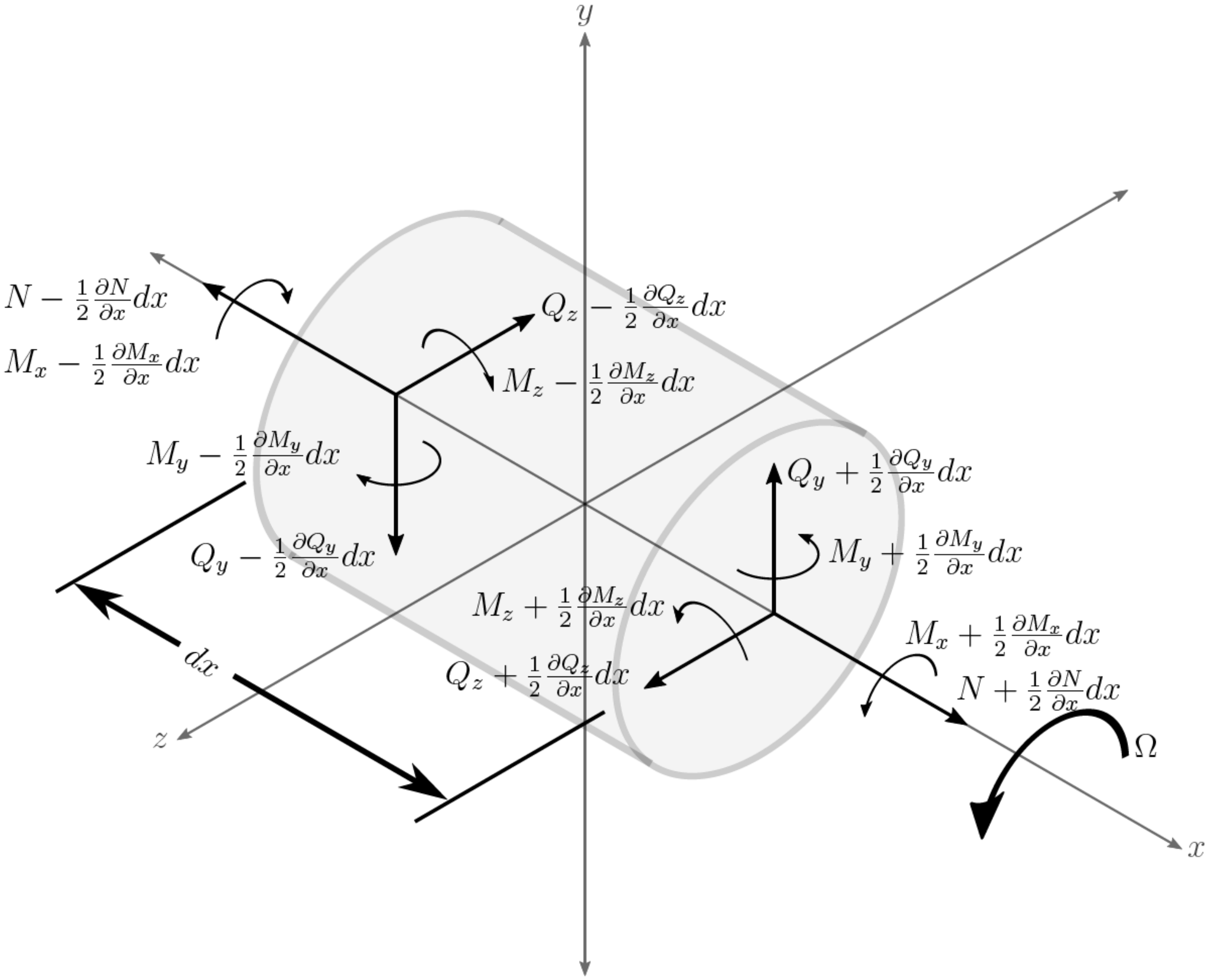
Development of a finite element model for rotordynamics

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} E & 0 & 0 \\ 0 & 2G & 0 \\ 0 & 0 & 2G \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{xy} \\ \epsilon_{xz} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{xz} \end{Bmatrix} = \begin{Bmatrix} E(u' - \theta' y + \psi' z) \\ G(v' - \phi' z - \theta) \\ G(w' + \phi' y + \psi) \end{Bmatrix} = \begin{Bmatrix} E(\varepsilon + \rho_y y + \rho_z z) \\ G(\gamma_y - \varphi z) \\ G(\gamma_z + \varphi y) \end{Bmatrix}$$

$$\begin{cases} \varepsilon = u' \\ \rho_y = -\theta' \\ \rho_z = \psi' \\ \varphi = \phi' \\ \gamma_y = v' - \theta \\ \gamma_z = w' + \psi \end{cases}$$

Development of a finite element model for rotordynamics



$$
\begin{cases}
\dfrac{\partial N}{\partial x} = \rho A \dfrac{\partial^2 u}{\partial x^2} \\
\dfrac{\partial Q_y}{\partial x} = \rho A \dfrac{\partial^2 v}{\partial x^2} \\
\dfrac{\partial Q_z}{\partial x} = \rho A \dfrac{\partial^2 w}{\partial x^2} \\
\dfrac{\partial M_y}{\partial x} - Q_z = \rho I_y \dfrac{\partial^2 \psi}{\partial x^2} + \rho J_x \Omega \dfrac{\partial \theta}{\partial x} \\
\dfrac{\partial M_z}{\partial x} + Q_y = \rho I_z \dfrac{\partial^2 \theta}{\partial x^2} - \rho J_x \Omega \dfrac{\partial \psi}{\partial x} \\
\dfrac{\partial M_x}{\partial x} = \rho J_x \dfrac{\partial^2 \phi}{\partial x^2}
\end{cases}
$$

Development of a finite element model for rotordynamics

$$
\left\{
\begin{aligned}
EAu'' &= \rho A\ddot{u} \\
\kappa GA(v'' - \theta') &= \rho A\ddot{v} \\
\kappa GA(w'' + \psi') &= \rho A\ddot{w} \\
EI_y\psi'' - \kappa GA(w' + \psi) &= \rho I_y\ddot{\psi} + \rho J_x\Omega\dot{\theta} \\
EI_z\theta'' + \kappa GA(v' - \theta) &= \rho I_z\ddot{\theta} - \rho J_x\Omega\dot{\psi} \\
\kappa GJ_x\phi'' &= \rho J_x\ddot{\phi}
\end{aligned}
\right.
\quad\Longrightarrow\quad
\underline{\mathcal{M}}^e\ddot{\vec{\mathbf{u}}} + \Omega\underline{\mathcal{G}}^e\dot{\vec{\mathbf{u}}} - \left(\frac{\partial()}{\partial x}\underline{\mathcal{S}}^e - \underline{\mathcal{P}}^e\underline{\mathcal{S}}^e\right)\vec{\mathbf{u}} = 0
$$

Development of a finite element model for rotordynamics

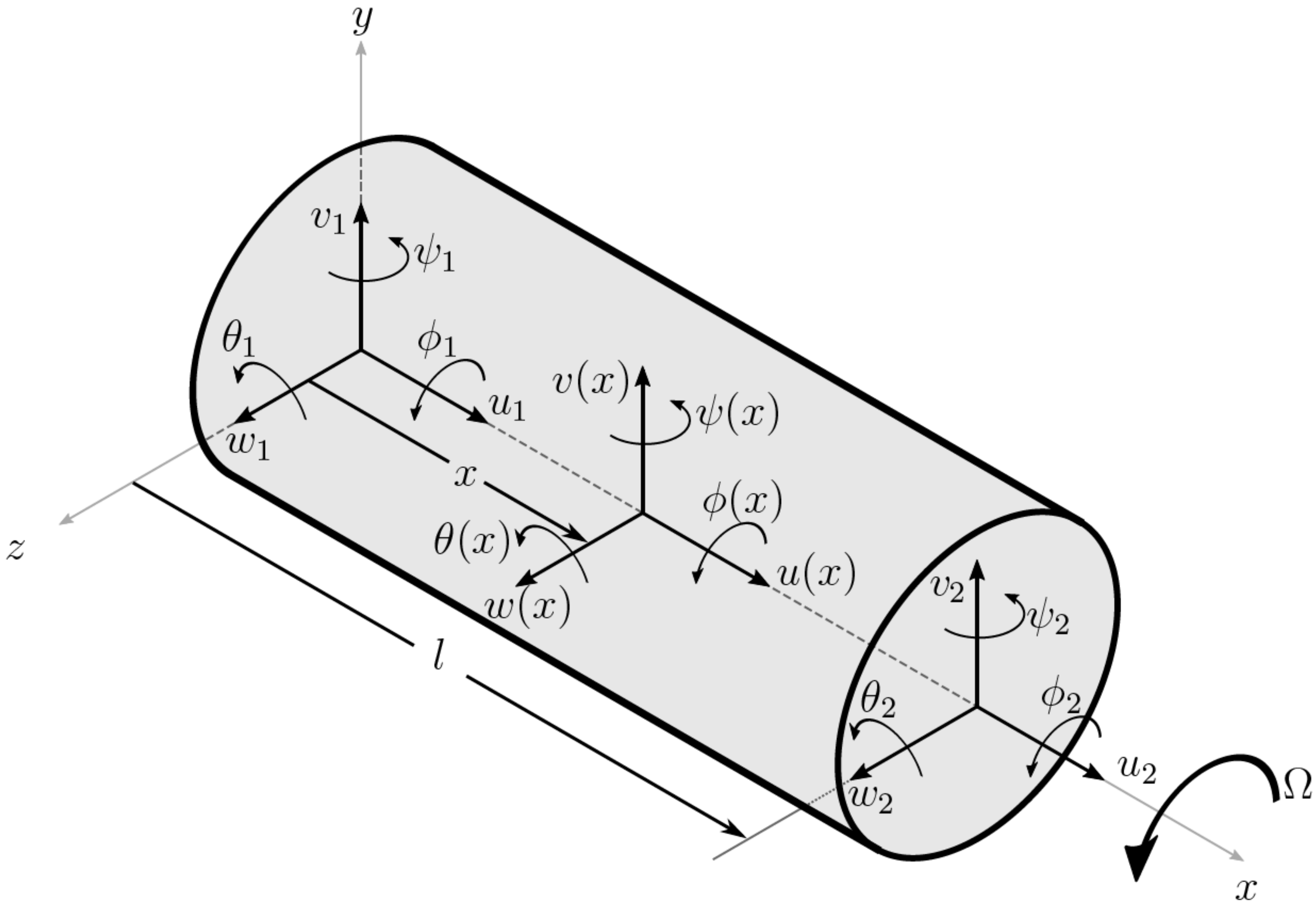
$$
\left\{
\begin{aligned}
&\underline{\mathcal{M}}^e = \begin{bmatrix} \rho A & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho A & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho I_y & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho I_z & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho A & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho J_x \end{bmatrix}
\underline{\mathcal{G}}^e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho J_x & 0 & 0 \\ 0 & 0 & -\rho J_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&\underline{\mathcal{P}}^e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\underline{\mathcal{S}}^e = \begin{bmatrix} \kappa GA\frac{\partial()}{\partial x} & 0 & 0 & -\kappa GA & 0 & 0 \\ 0 & \kappa GA\frac{\partial()}{\partial x} & \kappa GA & 0 & 0 & 0 \\ 0 & 0 & EI_y\frac{\partial()}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & EI_z\frac{\partial()}{\partial x} & 0 & 0 \\ 0 & 0 & 0 & 0 & EA\frac{\partial()}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa GJ_x\frac{\partial()}{\partial x} \end{bmatrix}
\end{aligned}
\right.
$$

Development of a finite element model for rotordynamics

$$\int_0^l \delta\vec{\mathbf{u}}^\intercal \underline{\mathcal{M}}^e \ddot{\vec{\mathbf{u}}} dx + \Omega \int_0^l \delta\vec{\mathbf{u}}^\intercal \underline{\mathcal{G}}^e \dot{\vec{\mathbf{u}}} + \int_0^l \delta\vec{\mathbf{u}}^\intercal \underline{\mathcal{B}}^\intercal \underline{\mathcal{D}}^e \underline{\mathcal{B}} \vec{\mathbf{u}} dx = 0$$

$$\underline{\mathcal{D}}^e = \begin{bmatrix} \kappa GA & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa GA & 0 & 0 & 0 & 0 \\ 0 & 0 & EI_y & 0 & 0 & 0 \\ 0 & 0 & 0 & EI_z & 0 & 0 \\ 0 & 0 & 0 & 0 & EA & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa GJ_x \end{bmatrix} \quad \& \quad \underline{\mathcal{B}} = \begin{bmatrix} \frac{\partial()}{\partial x} & 0 & 0 & \text{-1} & 0 & 0 \\ 0 & \frac{\partial()}{\partial x} & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial()}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial()}{\partial x} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial()}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial()}{\partial x} \end{bmatrix}$$

Development of a finite element model for rotordynamics



Development of a finite element model for rotordynamics

$$\begin{cases} \vec{\mathbf{u}}(x,t) &= \underline{\mathbf{N}}(x)\vec{\mathbf{q}}(t) \\ \dot{\vec{\mathbf{u}}}(x,t) &= \underline{\mathbf{N}}(x)\dot{\vec{\mathbf{q}}}(t) \\ \ddot{\vec{\mathbf{u}}}(x,t) &= \underline{\mathbf{N}}(x)\ddot{\vec{\mathbf{q}}}(t) \\ \delta\vec{\mathbf{u}}(x,t) &= \underline{\mathbf{N}}(x)\delta\vec{\mathbf{q}}(t) \end{cases}$$

$$\vec{\mathbf{u}} \ = \ [v, w, -\psi, \theta, u, \phi]^\mathsf{T} \ \ \& \ \ \vec{\mathbf{q}} \ = \ [v_1, w_1, -\psi_1, \theta_1, v_2, w_2, -\psi_2, \theta_2, u_1, \phi_1, u_2, \phi_2]^\mathsf{T}.$$

Development of a finite element model for rotordynamics

$$\begin{cases} u = c_1 + c_2 x \\ v = c_3 + c_4 x + c_5 x^2 + c_6 x^3 \\ w = c_7 + c_8 x + c_9 x^2 + c_{10} x^3 \\ \phi = c_{11} + c_{12} x \end{cases}$$

$$\begin{cases} \psi = K_y c_{10} - c_8 - 2c_9 x - 3c_{10} x^2 \\ \theta = K_z c_6 + c_4 + 2c_5 x + 3c_6 x^2 \end{cases}$$

$$K_y = \frac{6EI_y}{\kappa GA} \;\&\; K_z = \frac{6EI_z}{\kappa GA}$$

Development of a finite element model for rotordynamics

$$
\begin{Bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \\ w_1 \\ w_2 \\ \psi_1 \\ \psi_2 \\ \theta_1 \\ \theta_2 \\ \phi_1 \\ \phi_2 \end{Bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & l & l^2 & l^3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & l & l^2 & l^3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{-}1 & 0 & \text{-}K_y & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{-}1 & \text{-}2l & \text{-}K_y\text{-}3l^2 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & K_z & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 2l & K_z{+}3l^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & l
\end{bmatrix}
\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{Bmatrix}
$$

Development of a finite element model for rotordynamics

$$\begin{cases} u = N_1 u_1 + N_2 u_2 \\ v = T_{t_1 y} v_1 + T_{t_2 y} v_2 + T_{r_1 y}\theta_1 + T_{r_2 y}\theta_2 \\ w = T_{t_1 z} w_1 + T_{t_2 w} w_2 + T_{r_1 z}\psi_1 + T_{r_2 z}\psi_2 \\ \psi = R_{t_1 z} w_1 + R_{t_2 w} w_2 + R_{r_1 z}\psi_1 + R_{r_2 z}\psi_2 \\ \theta = R_{t_1 y} v_1 + R_{t_2 y} v_2 + R_{r_1 y}\theta_1 + R_{r_2 y}\theta_2 \\ \phi = N_1 \phi_1 + N_2 \phi_2 \end{cases}$$

$$\begin{cases} N_1 = 1 - \zeta & N_2 = \zeta \\ T_{t_1 y,z} = \frac{1}{1+\alpha_{y,z}}(2\zeta^3 - 3\zeta^2 - \alpha_{y,z}\zeta + 1 + \alpha_{y,z}) & T_{t_2 y,z} = \frac{1}{1+\alpha_{y,z}}(-2\zeta^3 + 3\zeta^2 + \alpha_{y,z}\zeta) \\ T_{r_1 y,z} = \frac{l}{1+\alpha_{y,z}}[\zeta^3 - (2 + \frac{1}{2}\alpha_{y,z})\zeta^2 + (1 + \frac{1}{2}\alpha_{y,z})\zeta] & T_{r_2 y,z} = \frac{l}{1+\alpha_{y,z}}[\zeta^3 - (1 - \frac{1}{2}\alpha_{y,z})\zeta^2 - \frac{1}{2}\alpha_{y,z}\zeta] \\ R_{t_1 y,z} = \frac{6/l}{1+\alpha_{y,z}}(\zeta^2 - \zeta) & R_{t_2 y,z} = \frac{6/l}{1+\alpha_{y,z}}(-\zeta^2 + \zeta) \\ R_{r_1 y,z} = \frac{1}{1+\alpha_{y,z}}(3\zeta^2 - (4 + \alpha_{y,z})\zeta + 1 + \alpha_{y,z}) & R_{r_2 y,z} = \frac{1}{1+\alpha_{y,z}}(3\zeta^2 - (2 - \alpha_{y,z})\zeta) \end{cases}$$

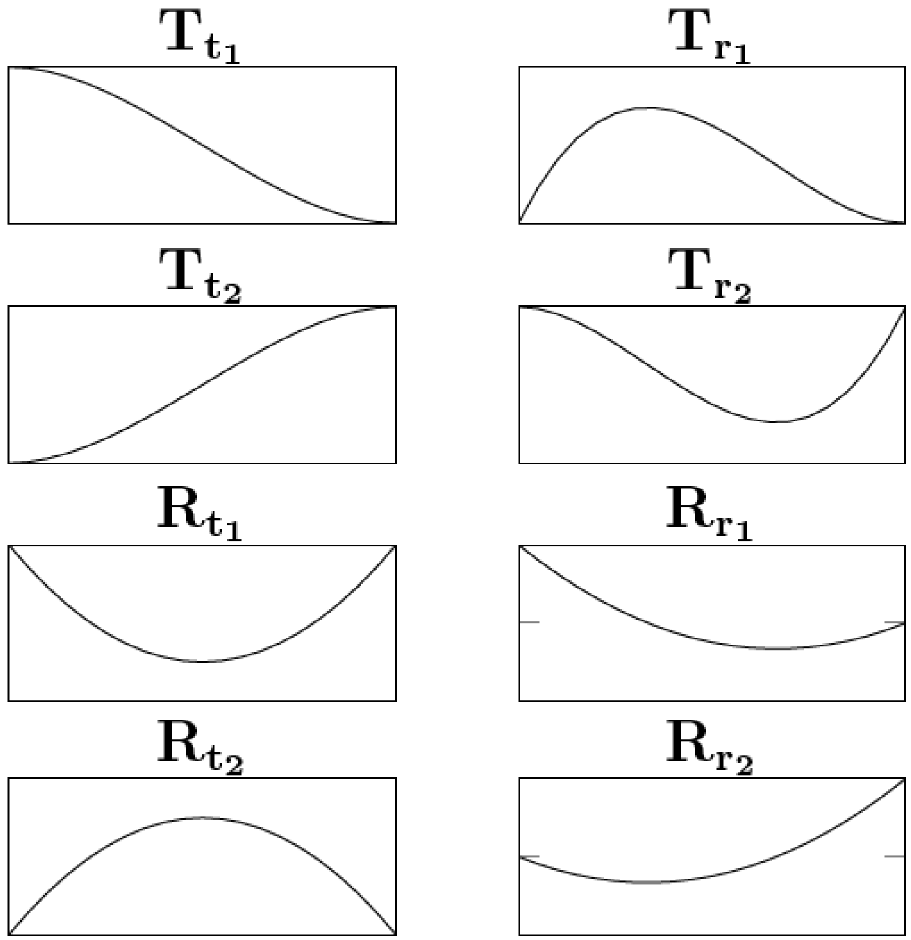
$$\alpha_y = 2K_y/l^2 = \frac{12EI_y}{\kappa GAl^2},\ \alpha_z = 2K_z/l^2 = \frac{12EI_z}{\kappa GAl^2},\ \&\ \zeta = x/l$$

Development of a finite element model for rotordynamics

$$\underline{\mathbf{N}}(x) = \begin{bmatrix} T_{t_1y} & 0 & 0 & T_{r_1y} & T_{t_2y} & 0 & 0 & T_{r_2y} & 0 & 0 & 0 & 0 \\ 0 & T_{t_1z} & T_{r_1z} & 0 & 0 & T_{t_2z} & T_{r_2z} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{t_1z} & R_{r_1z} & 0 & 0 & R_{t_2z} & R_{r_2z} & 0 & 0 & 0 & 0 & 0 \\ R_{t_1y} & 0 & 0 & R_{r_1y} & R_{t_2y} & 0 & 0 & R_{r_2y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_1 & 0 & N_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_1 & 0 & N_2 \end{bmatrix}$$

$$\begin{cases} \vec{\mathbf{u}}(x,t) & = \underline{\mathbf{N}}(x)\vec{\mathbf{q}}(t) \\ \dot{\vec{\mathbf{u}}}(x,t) & = \underline{\mathbf{N}}(x)\dot{\vec{\mathbf{q}}}(t) \\ \ddot{\vec{\mathbf{u}}}(x,t) & = \underline{\mathbf{N}}(x)\ddot{\vec{\mathbf{q}}}(t) \\ \delta\vec{\mathbf{u}}(x,t) & = \underline{\mathbf{N}}(x)\delta\vec{\mathbf{q}}(t) \end{cases}$$

Development of a finite element model for rotordynamics



Development of a finite element model for rotordynamics

$$\int_0^l \underline{\mathbf{N}}^\intercal \underline{\mathcal{M}}^e \underline{\mathbf{N}} dx \ddot{\vec{\mathbf{q}}} + \Omega \int_0^l \underline{\mathbf{N}}^\intercal \underline{\mathcal{G}}^e \underline{\mathbf{N}} dx \dot{\vec{\mathbf{q}}} + \int_0^l \underline{\mathbf{B}}^\intercal \underline{\mathcal{D}}^e \underline{\mathbf{B}} dx \vec{\mathbf{q}} = 0$$

$$\underline{\mathbf{B}} = \underline{\mathcal{B}} \underline{\mathbf{N}}$$

$$\begin{cases} \underline{\mathbf{M}}^e = \int_0^l \underline{\mathbf{N}}^\intercal \underline{\mathcal{M}}^e \underline{\mathbf{N}} dx \\ \underline{\mathbf{G}}^e = \int_0^l \underline{\mathbf{N}}^\intercal \underline{\mathcal{G}}^e \underline{\mathbf{N}} dx \\ \underline{\mathbf{K}}^e = \int_0^l \underline{\mathbf{B}}^\intercal \underline{\mathcal{D}}^e \underline{\mathbf{B}} dx \end{cases}$$

$$\underline{\mathbf{M}}^e \ddot{\vec{\mathbf{q}}} + \Omega \underline{\mathbf{G}}^e \dot{\vec{\mathbf{q}}} + \underline{\mathbf{K}}^e \vec{\mathbf{q}} = 0$$

Development of a finite element model for rotordynamics

$$\sigma_{xx} = E\left\{\frac{\epsilon_{xx}}{\sqrt{1+\eta_h^2}} + \left(\eta_v + \frac{\eta_h}{\omega\sqrt{1+\eta_h^2}}\right)\dot{\epsilon}_{xx}\right\}$$

$$\underline{\mathbf{M}}^e\ddot{\vec{\mathbf{q}}} + (\eta_v\underline{\mathbf{K}}^e + \Omega\underline{\mathbf{G}}^e)\dot{\vec{\mathbf{q}}} + [\eta_a\underline{\mathbf{K}}^e + (\Omega\eta_v + \eta_b)\underline{\mathbf{C}}^e]\vec{\mathbf{q}} = 0$$

$$\underline{\mathbf{C}}^e = \int_0^l \underline{\mathbf{B}}^\intercal\underline{\mathcal{I}}\underline{\mathcal{D}}\underline{\mathbf{B}}dx \qquad \underline{\mathcal{I}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Development of a finite element model for rotordynamics

$$\underline{\mathbf{M}}^d \ddot{\vec{\mathbf{q}}}_k + \Omega \underline{\mathbf{G}}^d \dot{\vec{\mathbf{q}}}_k = \Omega^2 \vec{\mathbf{F}}^d$$

$$\left\{ \begin{array}{l} \underline{\mathbf{M}}^d = \begin{bmatrix} \rho A l & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho A l & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho I_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho I_y & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho A l & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho J_x \end{bmatrix} \underline{\mathbf{G}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho J_x & 0 & 0 \\ 0 & 0 & -\rho J_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \vec{\mathbf{F}}^d = \left\{ \begin{array}{c} \rho A l \epsilon \cos(\Omega t + \delta_\varepsilon) \\ \rho A l \epsilon \sin(\Omega t + \delta_\varepsilon) \\ -\rho (I_y - J_x) \chi \sin(\Omega t) \\ \rho (I_z - J_x) \chi \cos(\Omega t) \\ 0 \\ 0 \end{array} \right\} \end{array} \right.$$

Development of a finite element model for rotordynamics

$$\underline{\mathbf{D}}^b\dot{\vec{\mathbf{q}}}_k + \underline{\mathbf{K}}^b\vec{\mathbf{q}}_k = 0$$

$$\underline{\mathbf{K}}^b = \begin{bmatrix} k_{yy} & k_{yz} & 0 & 0 & 0 & 0 \\ k_{zy} & k_{zz} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \underline{\mathbf{D}}^b = a\underline{\mathbf{K}}^b$$

Development of a finite element model for rotordynamics

$$\vec{\mathbf{s}} = \begin{Bmatrix} \vec{r} \\ \vec{p} \end{Bmatrix} = \begin{Bmatrix} v + iw \\ \theta - i\psi \end{Bmatrix} \qquad \vec{\mathbf{s}} = \underline{\mathbf{N}}^c \vec{\mathbf{q}}^c$$

$$\underline{\mathbf{N}}^c = \begin{bmatrix} T_{t_1 y} & T_{r_1 y} & T_{t_2 y} & T_{r_2 y} \\ R_{t_1} & R_{r_1} & R_{t_2} & R_{r_2} \end{bmatrix} \qquad \underline{\mathbf{M}}^{ec}\ddot{\vec{\mathbf{q}}}^c + (\eta_v \underline{\mathbf{K}}^{ec} - i\Omega \underline{\mathbf{G}}^{ec})\dot{\vec{\mathbf{q}}}^c + [\eta_a \underline{\mathbf{K}}^{ec} - i(\Omega\eta_v + \eta_b)\underline{\mathbf{C}}^{ec}]\vec{\mathbf{q}}^c = 0$$

$$\underline{\mathbf{M}}^{dc}\ddot{\vec{\mathbf{q}}}_k^c - i\Omega \underline{\mathbf{G}}^{dc}\dot{\vec{\mathbf{q}}}_k^c = \Omega^2 \vec{\mathbf{F}}^{dc} \qquad \underline{\mathbf{M}}^{dc} = \begin{bmatrix} \rho A l & 0 \\ 0 & \rho I \end{bmatrix}, \quad \underline{\mathbf{G}}^{dc} = \begin{bmatrix} 0 & 0 \\ 0 & \rho J_x \end{bmatrix}, \quad \vec{\mathbf{F}}^{dc} = \begin{Bmatrix} \rho A l \epsilon e^{i\delta_\epsilon} e^{i\Omega t} \\ \rho (I - J_x)\chi e^{i\Omega t} \end{Bmatrix}$$

$$\underline{\mathbf{D}}^{bc}\dot{\vec{\mathbf{q}}}_k^c + \underline{\mathbf{K}}^{bc}\vec{\mathbf{q}}_k^c = 0 \qquad \underline{\mathbf{K}}^{bc} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}, \quad \underline{\mathbf{D}}^{bc} = a\underline{\mathbf{K}}^{bc}$$

Development of a finite element model for rotordynamics

$$\underline{\mathbf{M}}\ddot{\vec{\mathbf{q}}} + \underline{\mathbf{G}}\dot{\vec{\mathbf{q}}} + \underline{\mathbf{K}}\vec{\mathbf{q}} = \Omega^2\vec{\mathbf{F}}$$

$$\begin{cases} \underline{\mathbf{M}} & = \underline{\mathbf{M}}_G^e + \underline{\mathbf{M}}_G^b + \underline{\mathbf{M}}_G^d \qquad \underline{\mathbf{D}} = \eta_v \underline{\mathbf{K}}_G^e + \Omega \underline{\mathbf{G}}_G^e + \Omega \underline{\mathbf{G}}_G^d + \underline{\mathbf{D}}_G^b \\ \underline{\mathbf{K}} & = \eta_a \underline{\mathbf{K}}_G^e + (\Omega \eta_v + \eta_b)\underline{\mathbf{C}}_G^e + \underline{\mathbf{K}}_G^b \qquad \vec{\mathbf{F}} = \vec{\mathbf{F}}_G^d \end{cases}$$

$$\underline{\mathbf{M}}\ddot{\vec{\mathbf{q}}}^c + \underline{\mathbf{D}}\dot{\vec{\mathbf{q}}}^c + \underline{\mathbf{K}}\vec{\mathbf{q}}^c = \Omega^2\vec{\mathbf{F}}$$

$$\begin{cases} \underline{\mathbf{M}} & = \underline{\mathbf{M}}_G^{ec} + \underline{\mathbf{M}}_G^{dc} + \underline{\mathbf{M}}_G^{bc} \qquad \underline{\mathbf{D}} = \eta_v \underline{\mathbf{K}}_G^{ec} - i\Omega(\underline{\mathbf{G}}_G^{ec} + \underline{\mathbf{G}}_G^{dc}) + \underline{\mathbf{D}}_G^{bc} \\ \underline{\mathbf{K}} & = \eta_a \underline{\mathbf{K}}_G^{ec} - i(\Omega \eta_v + \eta_b)\underline{\mathbf{C}}_G^{ec} + \underline{\mathbf{K}}_G^{bc} \qquad \vec{\mathbf{F}} = \vec{\mathbf{F}}_G^{dc} \end{cases}$$

Frequency domain processing of rotordynamic models

Eigenvalue and eigenvector analysis	$\vec{\mathbf{q}} = e^{st}$		Roots Locus Campbell Mode Shapes
Frequency Response	$\vec{\mathbf{q}} = e^{i\omega t}$	$\vec{\mathbf{q}} = e^{i\Omega t}$	Bode diagram Nyquist Diagram

Frequency domain processing of rotordynamic models

$$\dot{\vec{\mathbf{z}}} = \underline{\mathbf{A}}\vec{\mathbf{z}} + \underline{\mathbf{B}}\vec{\mathbf{w}}, \quad \vec{\mathbf{z}} = \left\{ \begin{array}{c} \dot{\vec{\mathbf{q}}} \\ \vec{\mathbf{q}} \end{array} \right\}$$

$$\underline{\mathbf{A}} = \left[\begin{array}{cc} -\underline{\mathbf{M}}^{-1}\underline{\mathbf{D}} & -\underline{\mathbf{M}}^{-1}\underline{\mathbf{K}} \\ \underline{\mathbf{I}} & \underline{\mathbf{0}} \end{array} \right] \quad \& \quad \underline{\mathbf{B}}\vec{\mathbf{w}} = \Omega^2 \left\{ \begin{array}{c} \underline{\mathbf{M}}^{-1}\vec{\mathbf{F}} \\ 0 \end{array} \right\}$$

$$\vec{\mathbf{z}} = \vec{\boldsymbol{\Theta}} e^{st}$$

Eigenvalue Problem: $\boxed{(\underline{\mathbf{A}} - s)\vec{\boldsymbol{\Theta}} = \underline{\mathbf{0}}}$

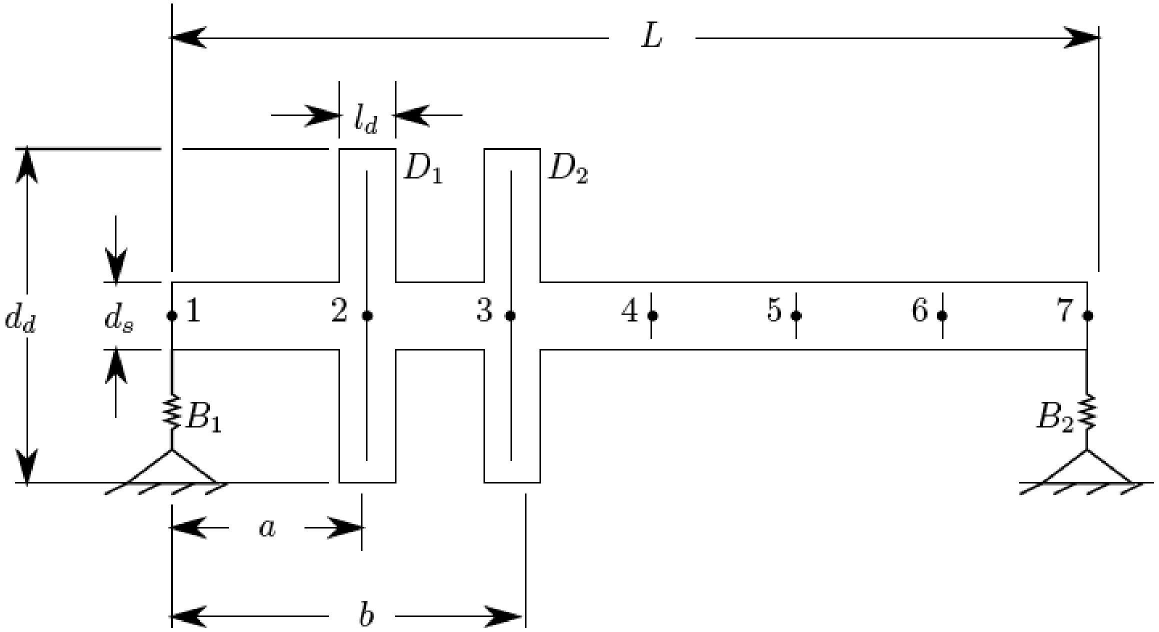
Frequency domain processing of rotordynamic models

$$\vec{\mathbf{q}} = \vec{\mathbf{q}}_0 e^{i\Omega t}$$

Synchronous Frequency Response Function:

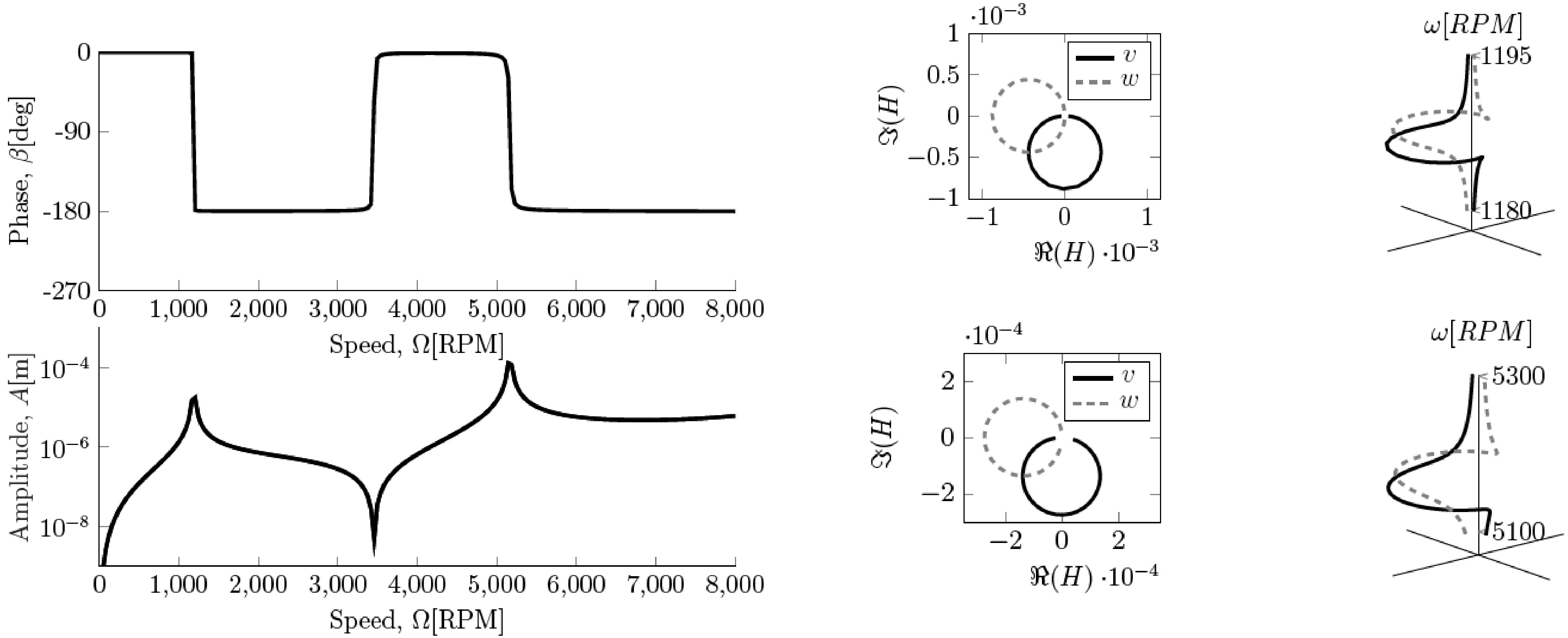
$$\begin{cases} \vec{\mathbf{q}}_0 &= (-\Omega^2\underline{\mathbf{M}} + i\Omega\underline{\mathbf{D}} + \underline{\mathbf{K}})^{-1}\Omega^2\vec{\mathbf{F}} \\ \vec{\mathbf{q}}_0 &= H(\Omega) \end{cases}$$

Frequency domain processing of rotordynamic models

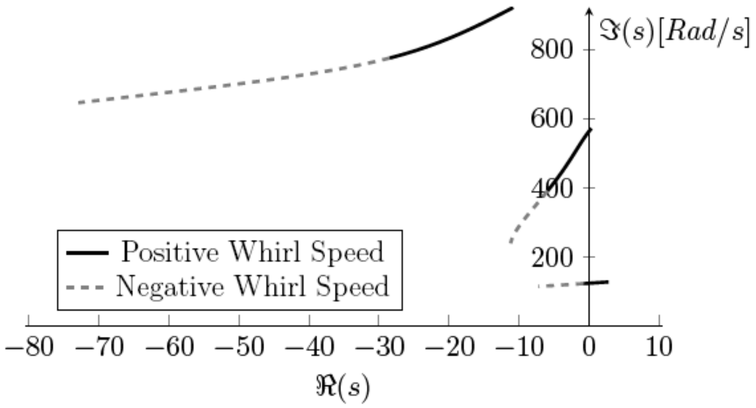


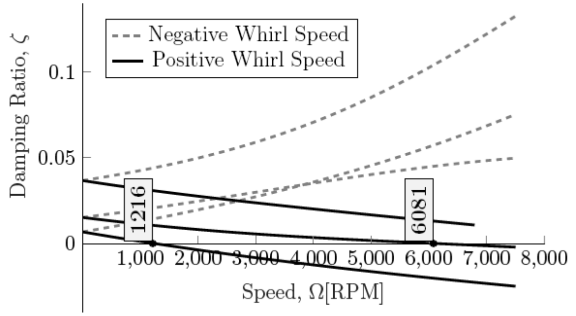
	$\rho[\frac{kg}{m^3}]$	$d_s[m]$	ν	$E[Pa]$	$\eta_v[s]$	η_h
Shaft	7850	0.1	0.3	210×10^9	0.0002	0
	$\rho[\frac{kg}{m^3}]$	$d_d[m]$	$l_d[m]$			
Disks; $\mathbf{D_1}$, $\mathbf{D_2}$	7850	0.6	0.1			
	$k_x[\frac{N}{m}]$	$k_y[\frac{N}{m}]$	$c_x[\frac{Ns}{m}]$	$c_y[\frac{Ns}{m}]$		
Bearings; $\mathbf{B_1}$, $\mathbf{B_2}$	1×10^7	1×10^7	100	100		

Frequency domain processing of rotordynamic models

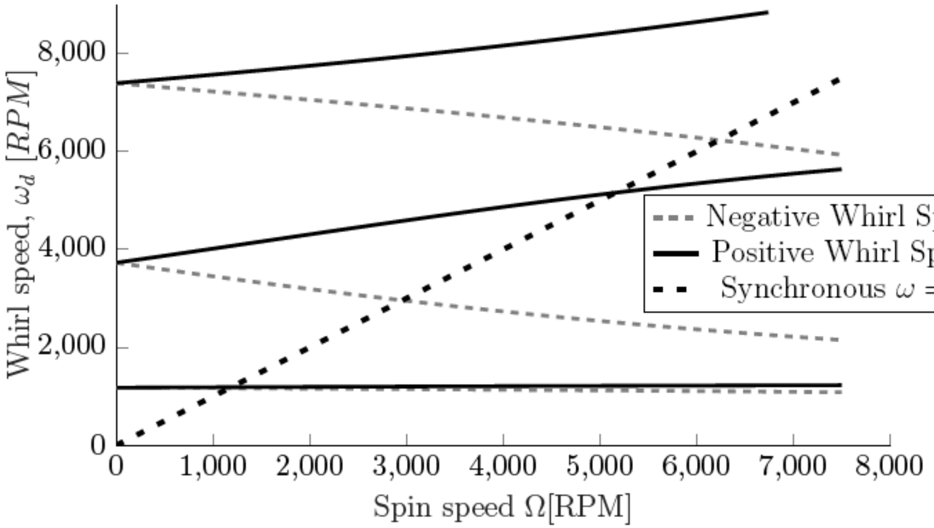


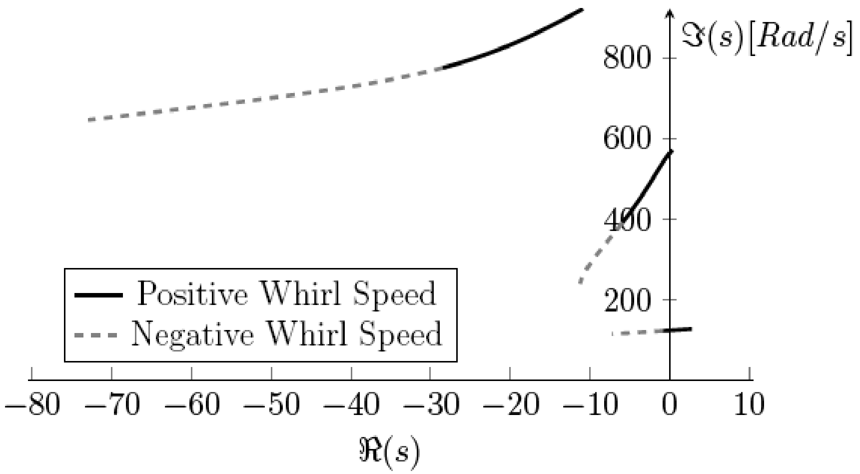
Frequency domain processing of rotordynamic models

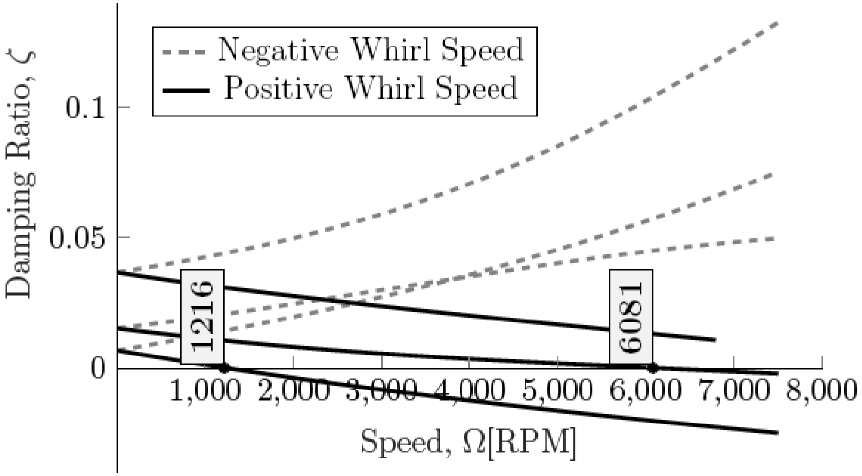


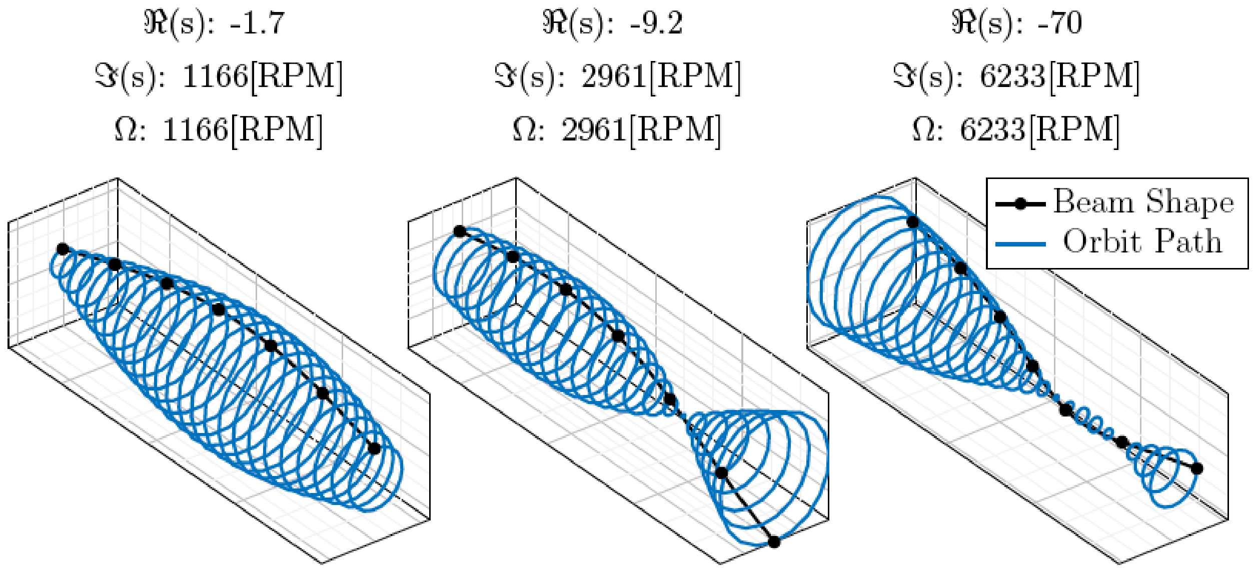


Frequency domain processing of rotordynamic models

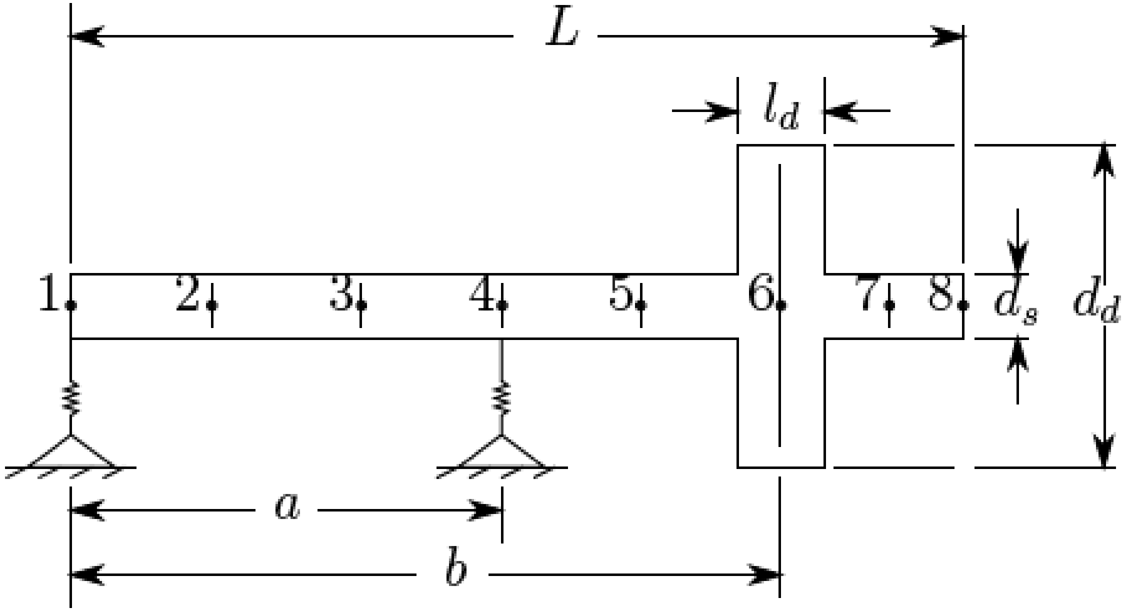




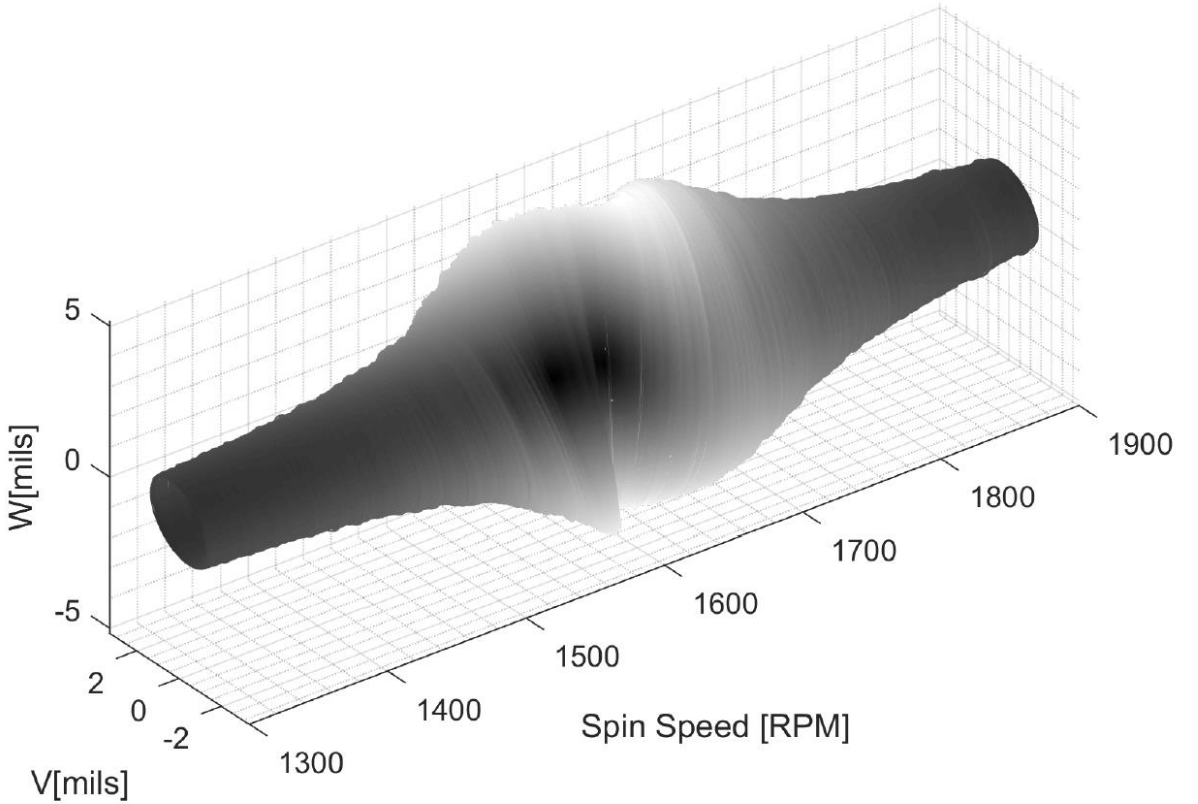




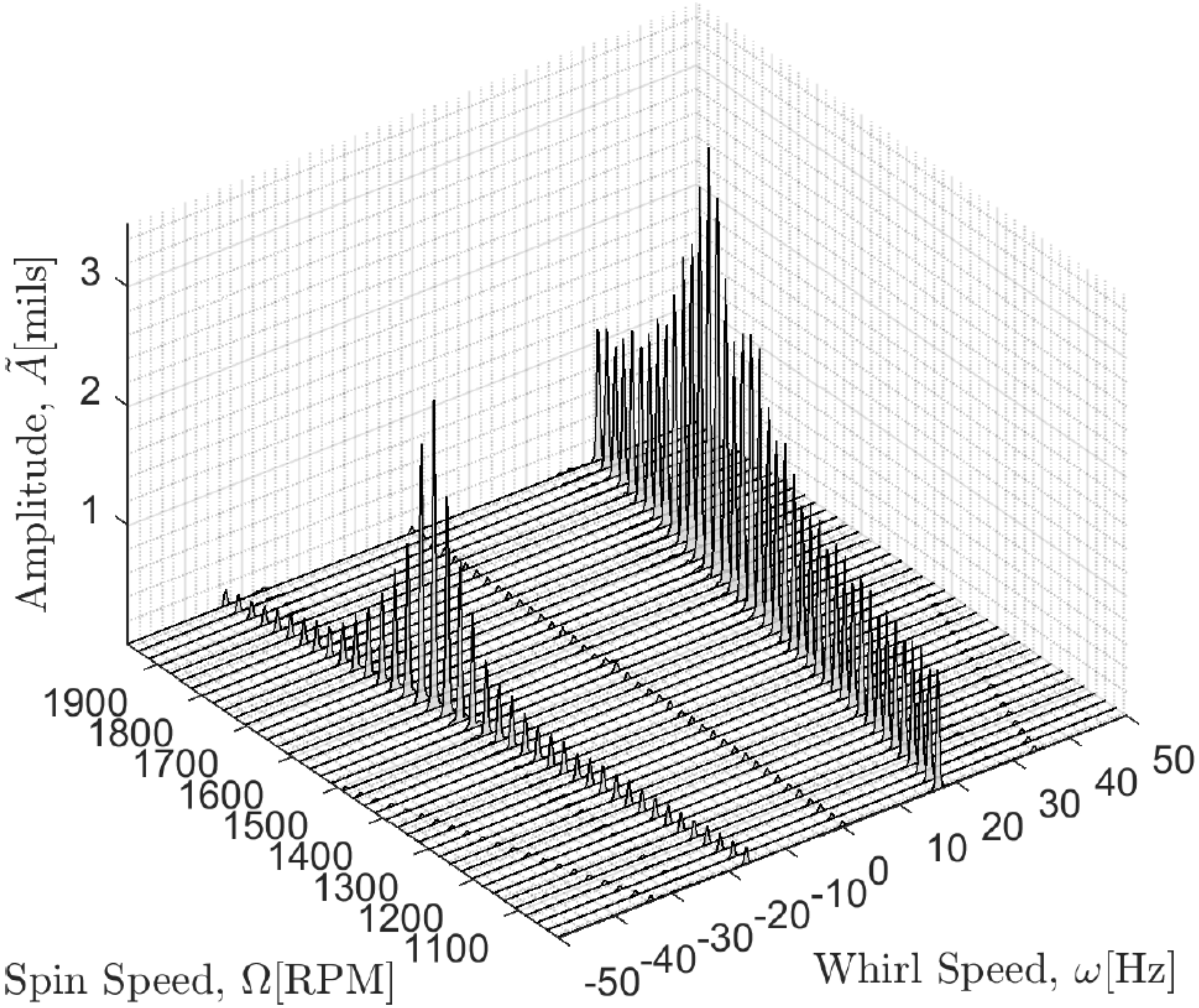
Comparison of experimental and model analyses



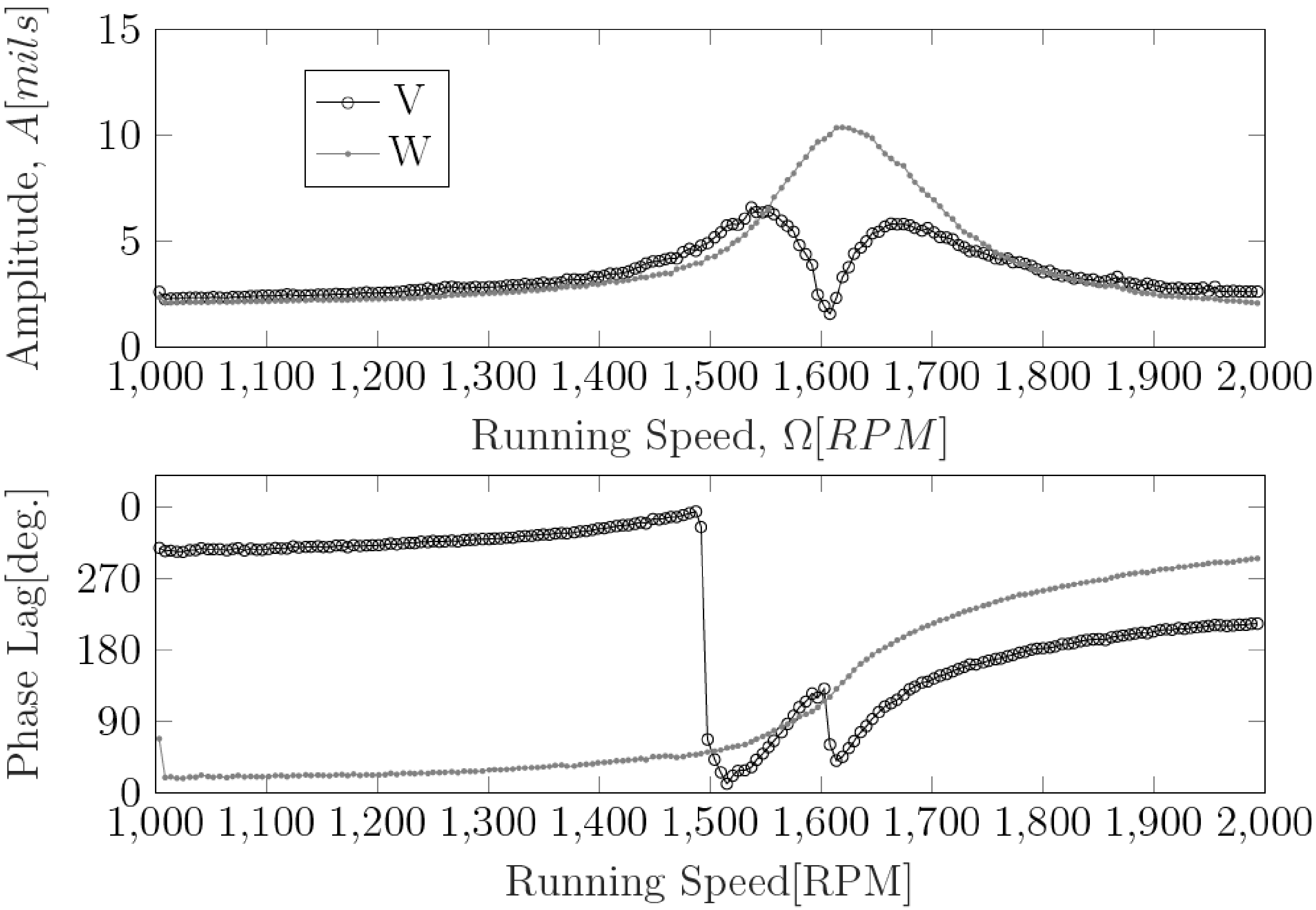
$L[m]$	$a[m]$	$b[m]$	$l_d[m]$	$d_d[m]$	$d_s[m]$
0.5	0.23	0.13	0.025	0.075	0.01



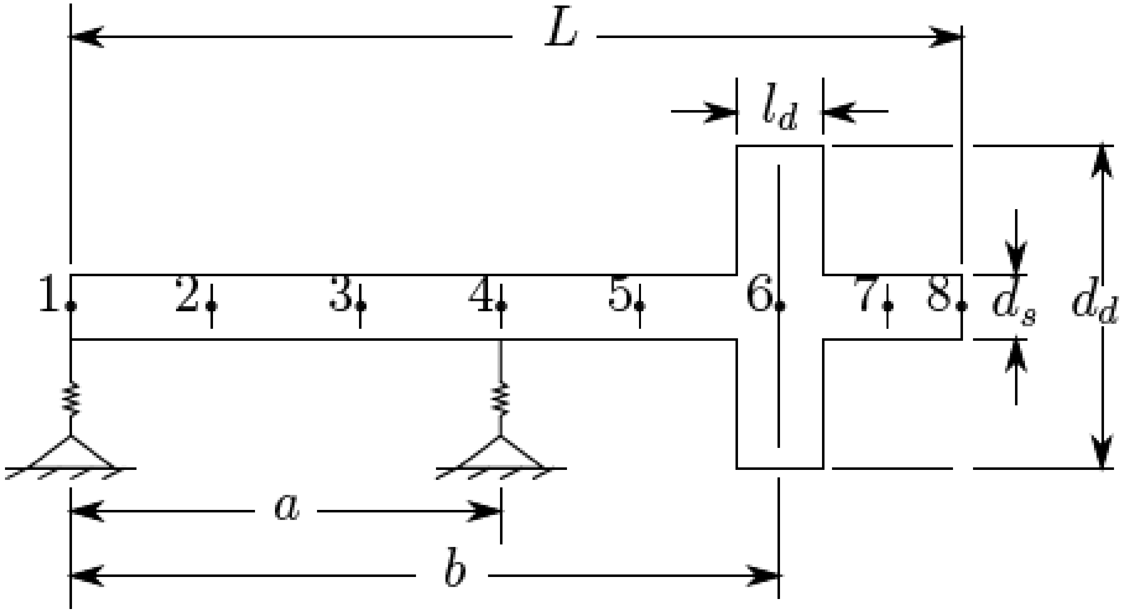
Comparison of experimental and model analyses



Comparison of experimental and model analyses

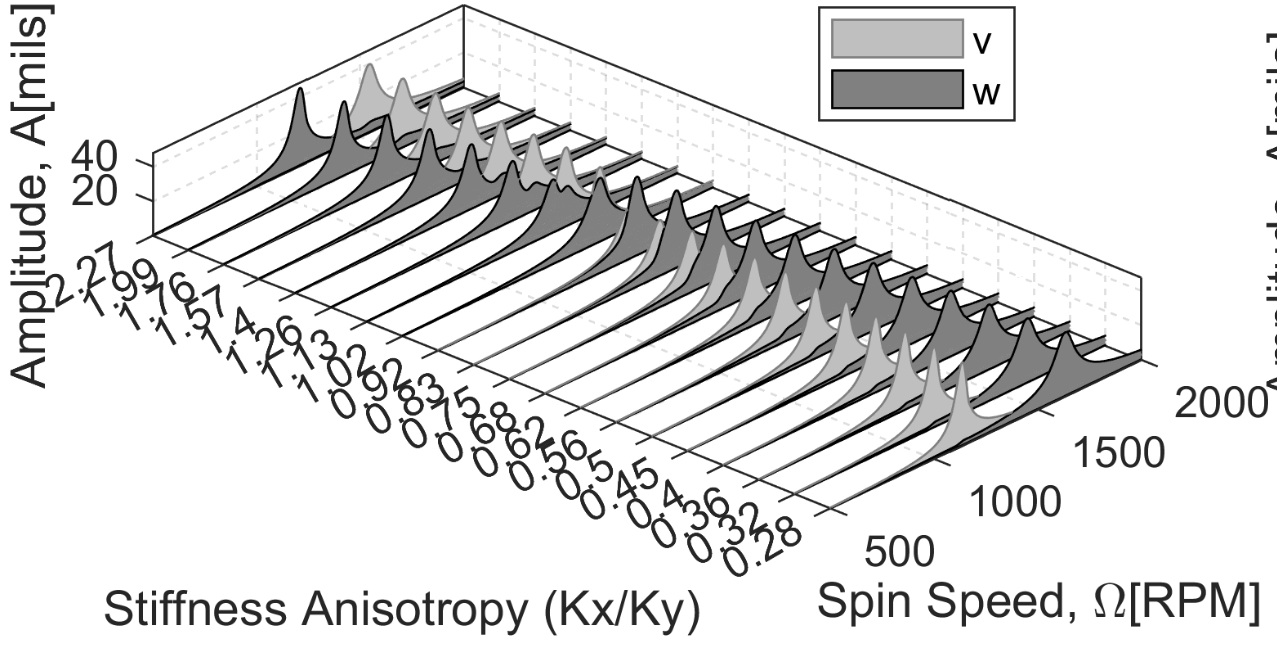


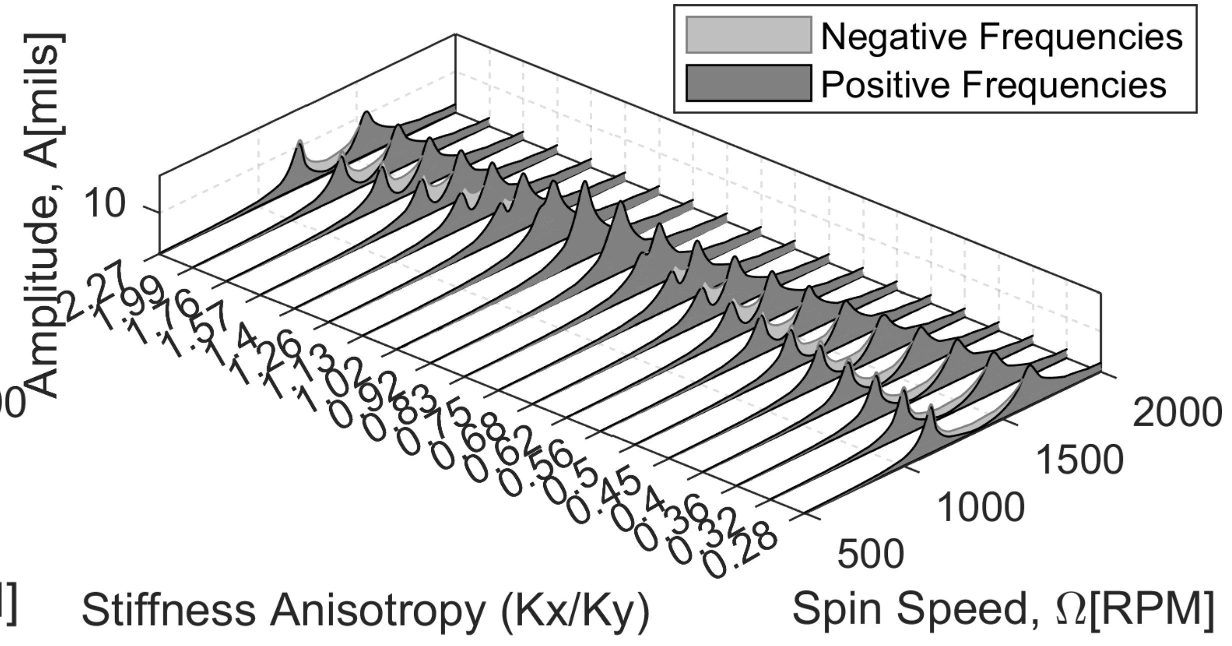
Comparison of experimental and model analyses



	$\rho\left[\frac{kg}{m^3}\right]$	$r[m]$	ν	$E[Pa]$	$\eta_v[s]$	η_h
Shaft	7850	0.005	0.3	210×10^9	0.0002	0
	$\rho\left[\frac{kg}{m^3}\right]$	$r[m]$	$l[m]$			
Disks	7850	0.0375	0.025			
	$k_y\left[\frac{N}{m}\right]$	$k_z\left[\frac{N}{m}\right]$	$c_y\left[\frac{Ns}{m}\right]$	$c_z\left[\frac{Ns}{m}\right]$		

Comparison of experimental and model analyses



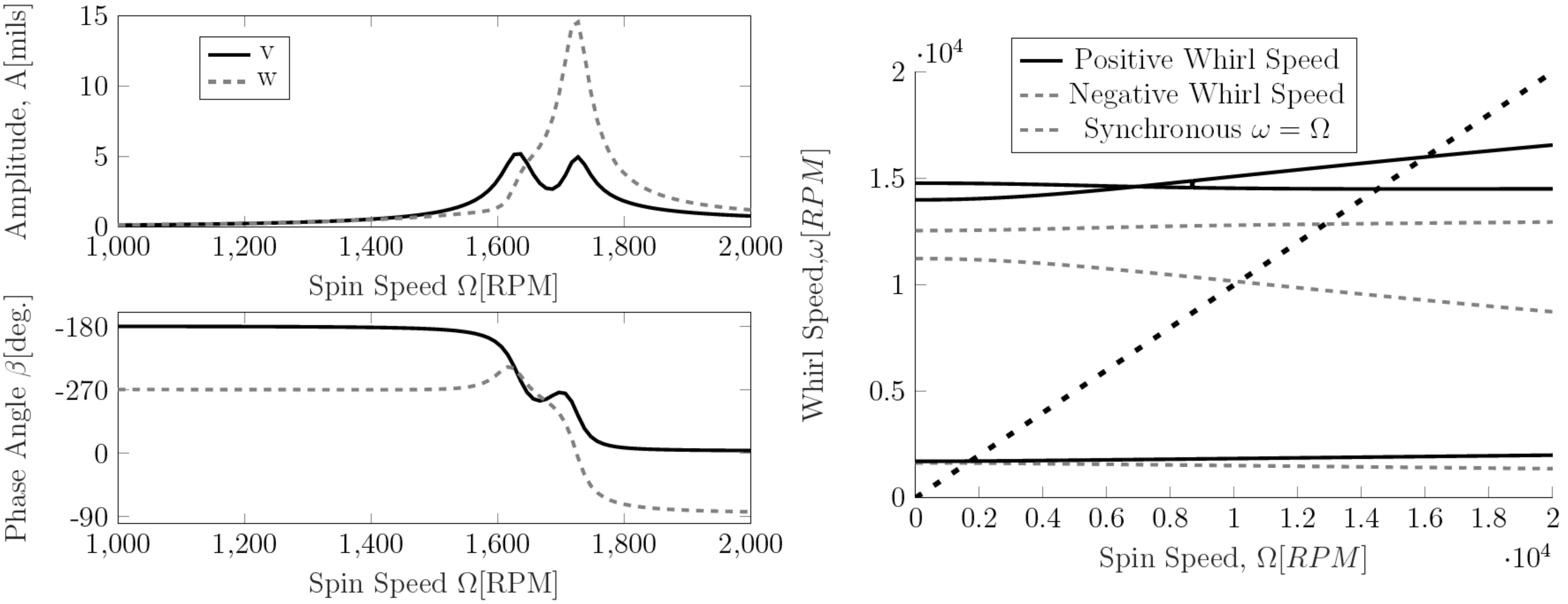


Comparison of experimental and model analyses

	$k_y\left[\frac{N}{m}\right]$	$k_z\left[\frac{N}{m}\right]$	$c_y\left[\frac{Ns}{m}\right]$	$c_z\left[\frac{Ns}{m}\right]$
Bearing A	1.7×10^5	2.2×10^5	68	88
Bearing B	2.04×10^5	2.64×10^5	81.6	105.6

$$\epsilon = 9e-6[m]$$

Comparison of experimental and model analyses

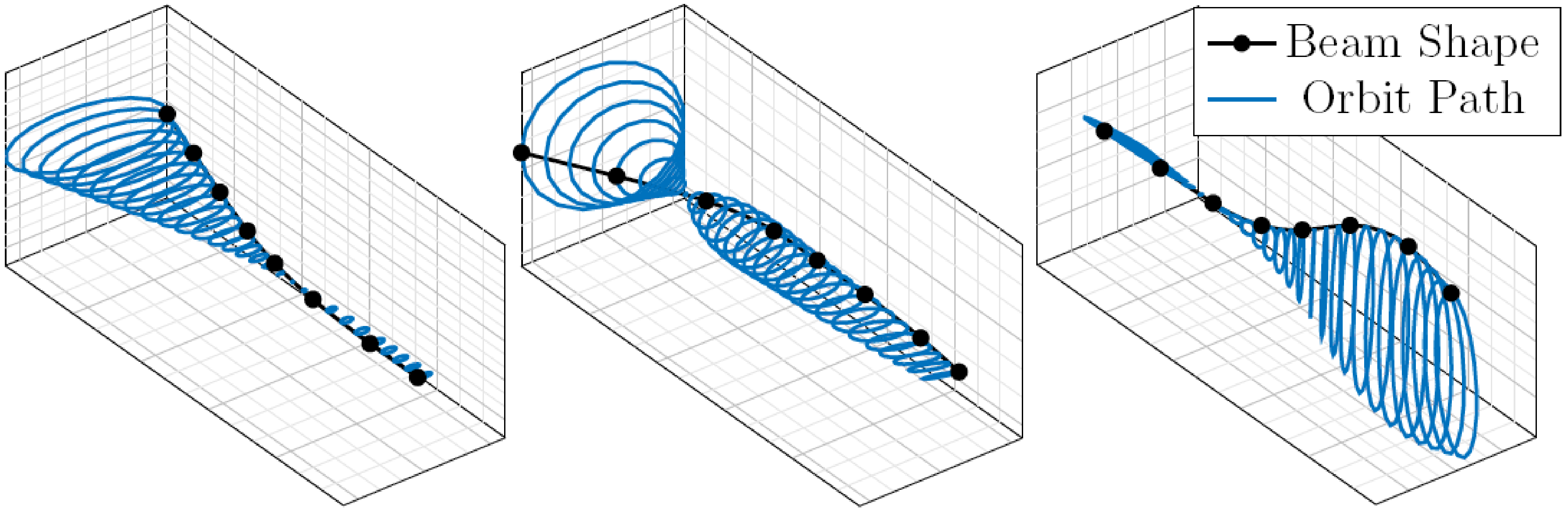


Comparison of experimental and model analyses

$\Re(s)$: -2.6,
$\Im(s)$: 1633[RPM]
Ω: 1633[RPM]

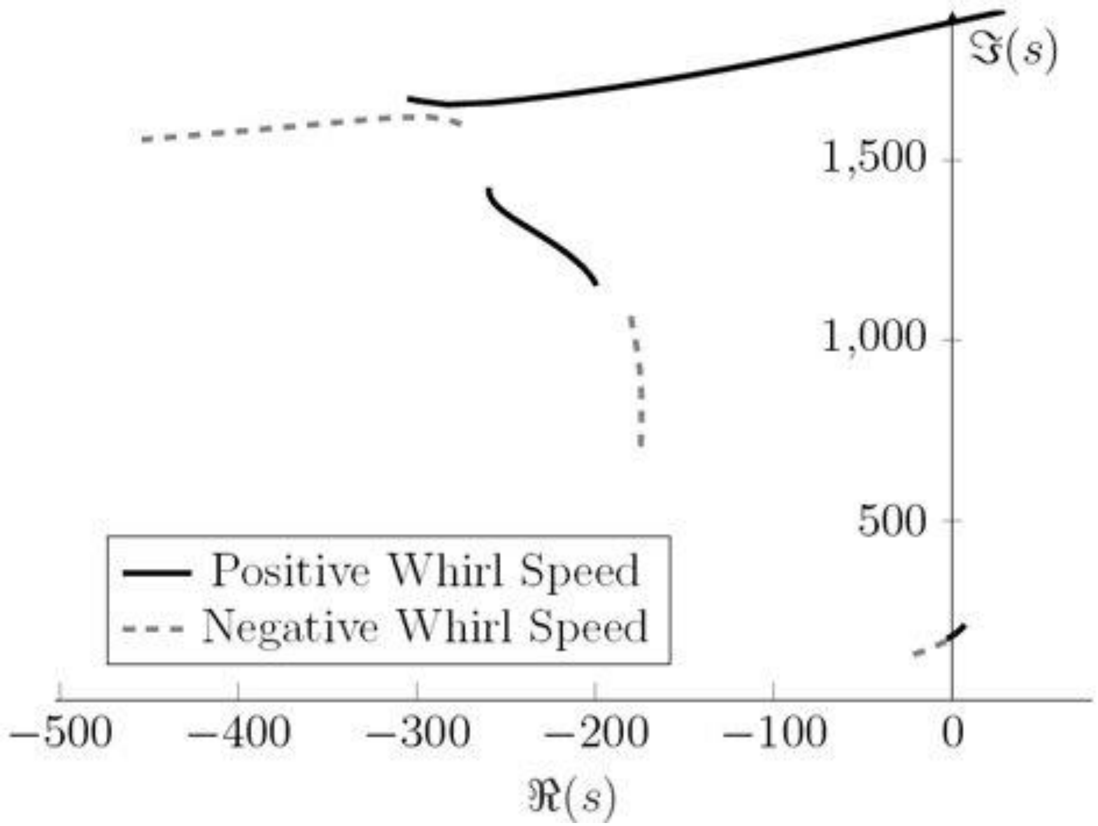
$\Re(s)$: -1.5e+02,
$\Im(s)$: 1.015e+04[RPM]
Ω: 1.015e+04[RPM]

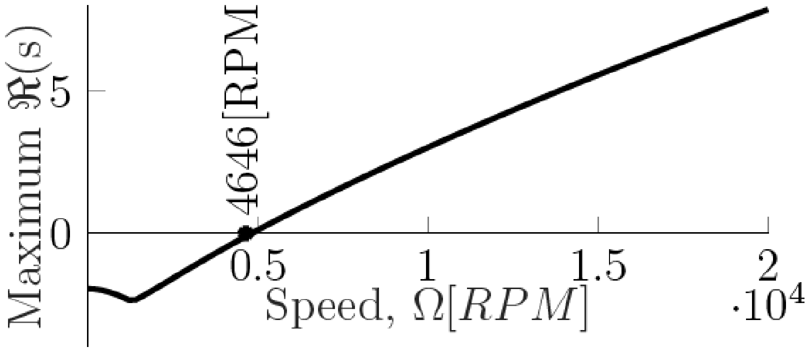
$\Re(s)$:-3.7e+02,
$\Im(s)$: 1.451e+04[RPM]
Ω: 1.451e+04[RPM]



(a) Mode Shape 1.   (b) Mode Shape 2.   (c) Mode Shape 3.

Comparison of experimental and model analyses





Comparison of experimental and model analyses

$$F_y = -(k_g k_i k_p - k_s)v - k_g k_i k_v \dot{v}, \;\; \& \;\; F_z = -(k_g k_i k_p - k_s)w - k_g k_i k_v \dot{w}$$

$k_i = 4K_m \frac{i_0}{g_0^2}$ $\quad$ $k_s = -4K_m \frac{i_0^2}{g_0^3}$ $\quad$ k_g is the power amplifier gain $\quad$ $K_m = k_m \cos(\alpha)$, $\quad$ $k_m = \frac{\mu_0 A_p N^2}{4}$

$$\underline{\mathbf{D}}^m \dot{\vec{\mathbf{q}}}_k + \underline{\mathbf{K}}^m \vec{\mathbf{q}}_k = 0$$

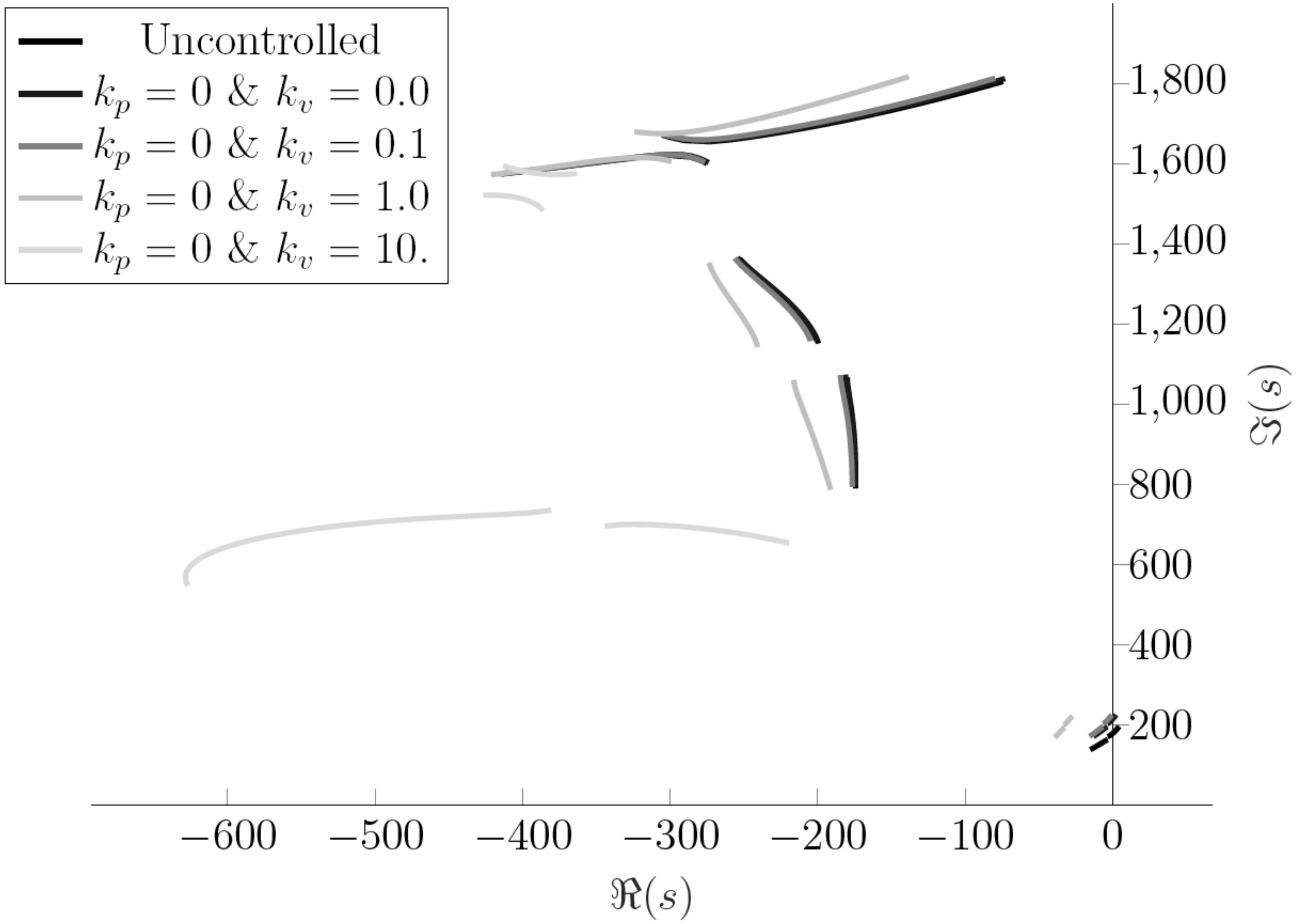
$$\underline{\mathbf{K}}^m = \begin{bmatrix} k_{mag} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{mag} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \underline{\mathbf{D}}^m = \begin{bmatrix} d_{mag} & 0 & 0 & 0 & 0 & 0 \\ 0 & d_{mag} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_{mag} = (k_g k_i k_p - k_s) \qquad d_{mag} = k_g k_i k_v$$

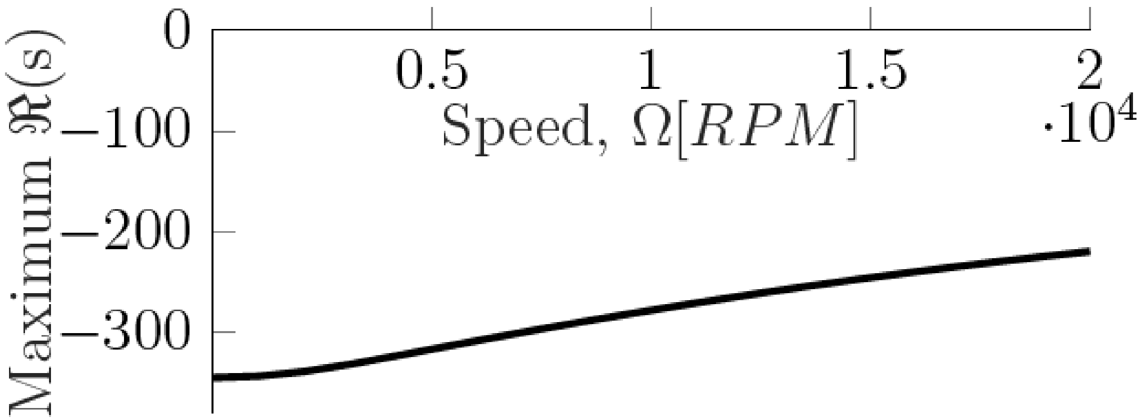
Comparison of experimental and model analyses

$\alpha[rad]$	$g_0[m]$	$i_0[A]$	$k_p[\frac{V}{m}]$	$k_v[\frac{Vs}{m}]$	$k_g[\frac{A}{V}]$	$N[\#]$	$A_p[m^2]$
$\frac{\pi}{8}$	2.5×10^{-3}	0.5	0	10	1	800	$\frac{5}{100*100}$

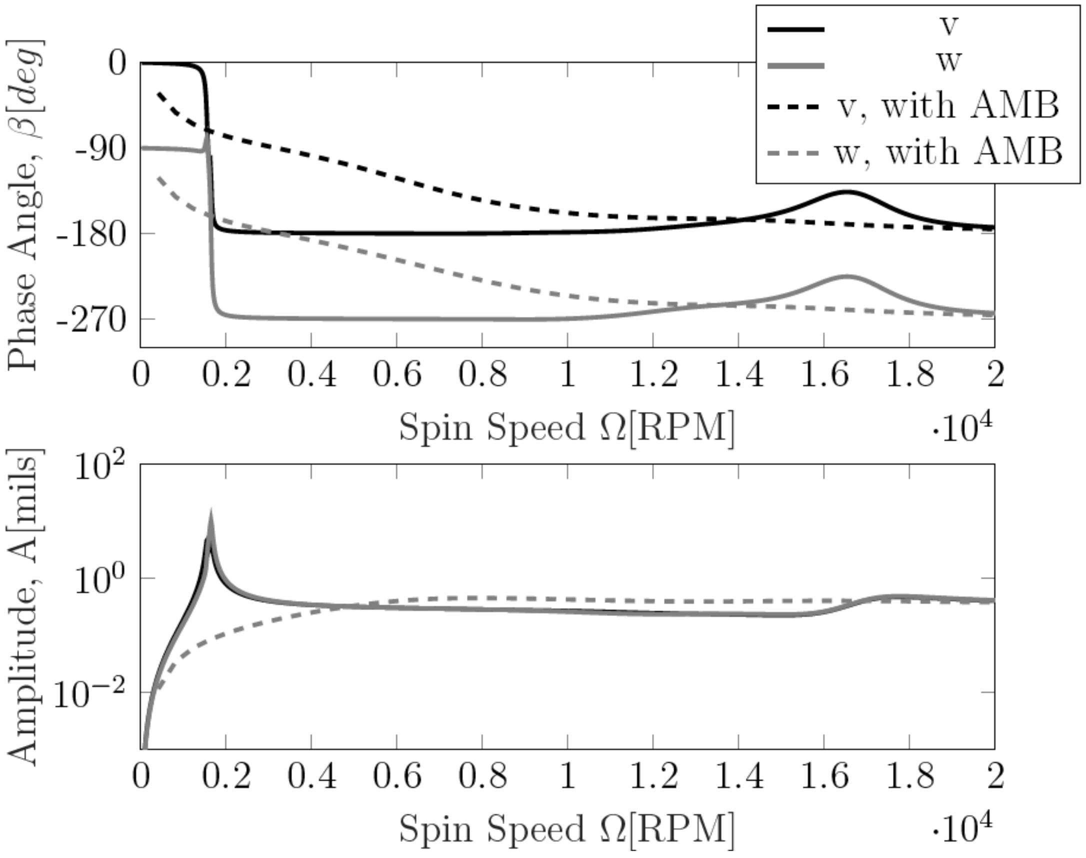
Comparison of experimental and model analyses



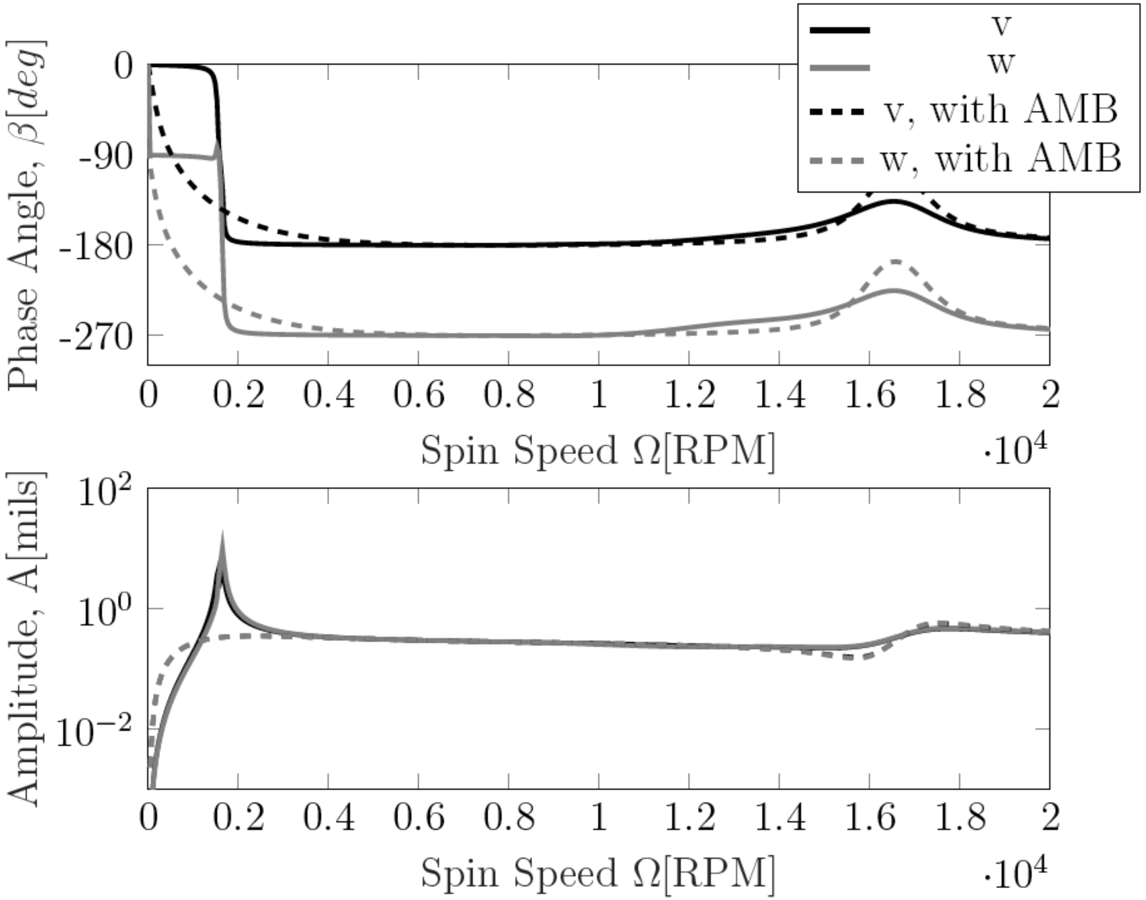
Comparison of experimental and model analyses



Comparison of experimental and model analyses



AMB at node 7



Complete levitation with AMB replacing bearing at node 4

Future Work

Possible future projects based on the continuation of this work will

- further investigate AMB model, and implement an actual experiment on overhung rotor.
- extention of this finite element model to include non linear terms.
- extention of this finite element model to include more detailed disk models for the analysis of turbomachines.
- extention of this finite element model to include rotating damping effects from various fittings, couplings and seals.

Questions?