HW# 6 (Lectures 11, 12 , 13, 5333, Oct. 12) (Due Oct. 21, 2010)

1) (a) Draw a mean self-similar network of Strahler order Ω=5 for which $T_1 = 0$, and $T_k = 2^{k-2}\,; k \geq 2$. This is called the Mandelbrot-Vicsek tree. It has been used to understand scaling in peak flows for RF-RO events (Menabde, M. and M. Sivapalan, Advances Water Resources, 24, pp. 1001-1014, 2001). (b) Calculate (count) N_ω^Ω for each different value of $\omega = 1,2,\ldots,\Omega = 5$ and compare them to the values obtained by using the recursive formula $N_\omega^\Omega = 2N_{\omega+1}^\Omega + \sum_{k=1}^{\Omega-\omega} T_k N_{\omega+k}^\Omega$. Plot $R_B(\omega) = \dfrac{N_\omega^\Omega}{N_{\omega+1}^\Omega}$ vs. ω. Describe the apparent behavior of $R_B(\omega)$ w.r.t ω. Does $R_B(\omega)$ appear to be approaching some limiting constant? Comment. (3)

2) Calculate R_B using the above recursion equation by (i) letting $\Omega \to \infty$, and (ii) assuming that Horton law holds in the limit, $N_\omega^\Omega / N_{\omega+1}^\Omega \to R_B$, as $\Omega \to \infty$. (see Eq. (12.9) for a similar example for Tokunaga trees). (3)

3) Calculate the number of topologically distinct binary network configurations for magnitude 4 trees using Eq. (11. 1) from Lecture 11. Draw all these networks. (2)

4) Consider the Hack's relation $L = cA^\beta$ given in Lecture 13. Write it as an area-perimeter relation and compute the fractal dimension of river networks in terms of β. Take, R_B=4.5 and R_C=2.4 from Peckham (WRR, 31(4), 1023-1029,1995), compute the fractal dimension and the Hack exponent using expressions given in Lecture 13. Does it agree with the value of β=0.56 given in Lecture 13? Discuss your reasons for difference between the two values. (2)