CVEN5313: Environmental Fluid Mechanics

Open Channel Flow: Conservation of Mass and Momentum
Due 5pm Friday, 09/03/10

1. The velocity profile in the boundary layer of an open-channel flow has no simple closed-form solution. However, good analytical approximations do exist. For laminar flows, the velocity profile is often expressed as

$$v(z) = v_0 \sin\left(\frac{\pi z}{2\delta}\right)$$

where δ is the verticl thickness of the boundary layer, and V_0 is the velocity at $z = \delta$. For turbulent flows, the velocity profile is often expressed as

$$v(z) = v_0 \left(\frac{z}{\delta}\right)^{\frac{1}{N}}$$

where N is an integer that is typically chosen as 6 or 7.

(a) Make a single *non-dimensional* plot showing the laminar and turbulent profiles. For the turbulent case, show both the $N = 6$ and $N = 7$ cases.

(b) Determine analytical expressions for average velocity (V) and the kinetic energy and momentum coefficients (α and β) for the laminar and turbulent profiles (express the answers for the turbulent profile in terms of N). Assume that the boundary layer is fully developed over the depth ($\delta = d$), the channel section is rectangular with width B, and the velocity does not vary across the channel.

2. Figure 1 shows experimentally measured velocity contours in an open-channel flow. I will supply an electronic file containing discretized velocity values at each of the locations indicated by a red dot in the figure. Write a simple computer code to calculate $\overline{V}$, α, and β for the measured flow.

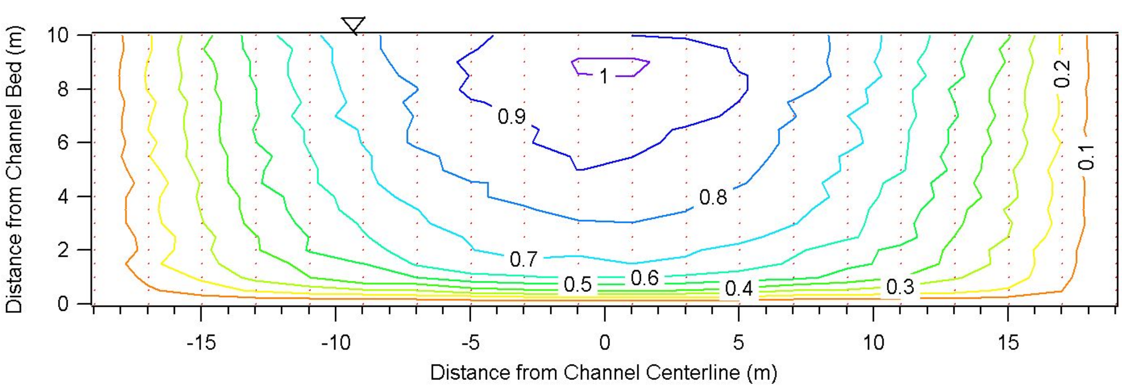

Figure 1: Measured velocity contours (m/s) in a cross section of an open-channel flow.

3. Show that Eq. 3 from the lecture notes

$$\rho\left[\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta V^2 A\right)\right] = -\rho g \cos\theta \frac{\partial z_s}{\partial x} A + \rho g\, S_0 A - \tau_0 P_w$$

can be transformed into Eq. 4:

$$\frac{1}{g}\frac{\partial V}{\partial t} + \frac{\partial H_\beta}{\partial x} = -\frac{\tau_0 P_w}{\rho g A} + (\beta - 1)\frac{V}{gA}\frac{\partial A}{\partial t} - \frac{V^2}{2g}\frac{\partial \beta}{\partial x}$$

where

$$H_\beta = \beta\frac{V^2}{2g} + z_s\cos\theta + z_0 = \text{total head}$$

Hint: Begin by showing (using $Q = VA$ and the continuity equation) that the term in brackets on the LHS of Eq. 10 can be rewritten as

$$A\frac{\partial V}{\partial t} + V\frac{\partial A}{\partial t} - \beta V\frac{\partial A}{\partial t} + A\frac{\partial}{\partial x}\left(\frac{1}{2}\beta V^2\right) + \frac{1}{2}V^2 A\frac{\partial \beta}{\partial x}$$

→ Please write your derivation in a concise, linear manner.