HW #4 <u>(Lectures 7 and 8, 5333) (Due on September 28)</u>

(1). Assume that a function g(t) is differentiable and satisfies the following functional equation: (5)

$$g(ts)=g(t)g(s)$$

Using differentiability condition, show that the solution of this equation is a power law. The differentiability condition simplifies the derivation compared to the one mentioned under continuity in the lecture notes, where differentiability is not assumed.

Hint: differentiate the functional equation w.r.t. to s, evaluate it at s=1, get an ordinary differential equation in t and solve it.

(1). 1. Start with an equilateral triangle of unit side lengths, and construct a Koch curve on each side. It is called a Koch snowflake. (5)

(a) Show that a Koch snowflake has finite area, but its perimeter has infinite length.

(b) Explain, in what sense does the relationship $A = c(P)^{2/D_B}$ hold for the Koch snow flake, where $\boldsymbol{A}$ is the area, $\boldsymbol{P}$ is the perimeter, $\boldsymbol{c}$ is a constant, and D_B is the box counting dimension of the perimeter.

Note: This question is a mathematical example that exhibits a similar relationship as found between cloud and rain areas and perimeters.