CVEN5313: Environmental Fluid Mechanics

Navier-Stokes: Index Notation   Due: 5pm Friday 10/14/10

1. Write the following vector expressions in Cartesian Index Notation:
   (a) $\vec{a} \cdot \nabla \vec{a}$
   (b) $(\vec{a} \cdot \nabla)\vec{a}$
   (c) $\vec{a}\,\nabla \vec{a}$
   (d) $\underline{\underline{A}} : \underline{\underline{A}}$
   (e) $\underline{\underline{A}} : \underline{\underline{A}}^T$
   (f) curl grad div$(\vec{a} \times \vec{b})$

2. Show that $\underline{\underline{Q}} : \underline{\underline{R}} = \underline{\underline{R}} : \underline{\underline{Q}} = 0$ if $\underline{\underline{Q}}$ is symmetric and $\underline{\underline{R}}$ is antisymmetric.

3. Show that $A_{ijkl}B_{jklm} = 0$ if $\underline{\underline{A}}$ is symmetric with respect to indices j and k, and $\underline{\underline{B}}$ is antisymemtric with respect to j and k. Note that this is a more general version of problem 2.

4. Show that if $\nabla\rho = 0$ then $\nabla \cdot (\rho \vec{u}\vec{u}) = \rho \vec{u} \cdot \nabla \vec{u} + \rho \vec{u}(\nabla \cdot \vec{u}\,)$

5. Show that $a_i \partial_j a_i = \nabla \left(\frac{1}{2}\vec{a} \cdot \vec{a}\right)$

6. Show that curl (grad ϕ)=0

7. Show that $\vec{a} \times (\vec{b} \times \vec{c}\,) = (\vec{a} \cdot \vec{c}\,)\vec{b} - (\vec{a} \cdot \vec{b}\,)\vec{c}$

8. Show that $(\vec{a} \times \vec{b}\,) \cdot (\vec{c} \times \vec{d}\,) = (\vec{a} \cdot \vec{c}\,)(\vec{b} \cdot \vec{d}\,) - (\vec{a} \cdot \vec{d}\,)(\vec{b} \cdot \vec{c}\,)$

9. Show that $\nabla \cdot (\nabla \times \vec{a}\,) = 0$

10. Show that $\nabla \cdot (\vec{a} \times \vec{b}\,) = \vec{b} \cdot (\nabla \times \vec{a}\,) - \vec{a} \cdot (\nabla \times \vec{b}\,)$

11. Show that $\nabla \times (\vec{a} \times \vec{b}\,) = \vec{a}(\nabla \cdot \vec{b}\,) + \vec{b} \cdot \nabla \vec{a} - \vec{a} \cdot \nabla \vec{b} - \vec{b}(\nabla \cdot \vec{a}\,)$

12. Show that if div $\vec{u} = 0$, div $\vec{v} = 0$, and curl $\vec{w} = 0$ then
$$\nabla \cdot [(\vec{u} \times \vec{v}\,) \times \vec{w}] = \vec{w} \cdot [(\vec{v} \cdot \nabla\,)\vec{u} - (\vec{u} \cdot \nabla\,)\vec{v}]$$

13. Show that $\nabla \times \left[\nabla\left(\frac{1}{2}\vec{a} \cdot \vec{a}\right) - \vec{a} \times (\nabla \times \vec{a}\,)\right] = \nabla \times (\vec{a} \cdot \nabla \vec{a}\,)$

14. Using the identity given in problem 13, show that
$$\nabla \times (\vec{a} \cdot \nabla \vec{a}\,) = \vec{a} \cdot \nabla(\nabla \times \vec{a}\,) + (\nabla \cdot \vec{a}\,)(\nabla \times \vec{a}\,) - (\nabla \times \vec{a}\,) \cdot (\nabla \vec{a})$$