Lecture 5, September 7, 2010 (Key Points)

Towards a New Physical Approach to Understanding Statistical Power Laws in annual Flood Frequencies

1. We will use Poveda et al. (2007) for this lecture. Their work is based on <u>two key ideas</u>:

   (i) How the mean annual streamflow Q_m is related to annual peak flow and low flow quantiles.

   (ii) Prediction of mean annual streamflow Q_m via water balance involving long-term hydro-climate variables, precipitation and evapotranspiration, P, ET respectively.

Relating Peak Flow Quantiles to Mean Hydro-Climatic Variables:

1. Poveda et al. wrote,

$$Q_{\max}(T_r) = \mu_{Q_{\max}} + \sigma_{Q_{\max}} k(T_r, \gamma) \qquad (5.1)$$

   Here, $Q_{\max}(T_r)$ denotes the annual flood quantile corresponding to return period T_r, $\mu_{Q_{\max}}$ and $\sigma_{Q_{\max}}$ are the mean and standard deviation of annual floods, and $k(T_r, \gamma)$ is a quantile of an assumed distribution of a random variable mean 0 and variance 1, and with probability of exceedance, $p = 1/T_r$.

2. Poveda et al. assumed that $k(T_r, \gamma)$ is the quantile of a log-normal distribution, whose parameters are denoted by γ. This assumption is not quite correct. Why? But it does not impact the rest of their results!

3. Poveda et al. estimated mean streamflows and the mean and standard deviation of peak streamflows throughout Colombia, South America for the period 1966-1987 (22 years).

4. They observed that power laws (scaling) relate $\mu_{Q_{\max}}$ and $\sigma_{Q_{\max}}$ to mean streamflow, Q_m. The figure below shows the empirical power law relationships.

5. Are the results in the figure based on the concept of a basin analysis or a regional analysis? Why?

6. A new predictive equation for annual flood quantiles for any station in Columbia can be written by substituting power law relations from Fig. 1 into eq. (5.1). Replacing Q_m with $P - ET$ using the water balance equation (Lecture 2), Eq. (5.1) can be written as,

$$
\begin{aligned}
Q_{\max}(T_r) &= \mu_{Q_{\max}} + \sigma_{Q_{\max}} k(T_r,\gamma) \\
&= \alpha_\mu Q_m{}^{\theta_1} A^{\theta_1} + \alpha_\sigma Q_m{}^{\theta_2} A^{\theta_2} k(T_r,\gamma) \\
&= \alpha_\mu (P-ET)^{\theta_1} A^{\theta_1} + \alpha_\sigma (P-ET)^{\theta_2} A^{\theta_2} k(T_r,\gamma)
\end{aligned} \tag{5.2}
$$

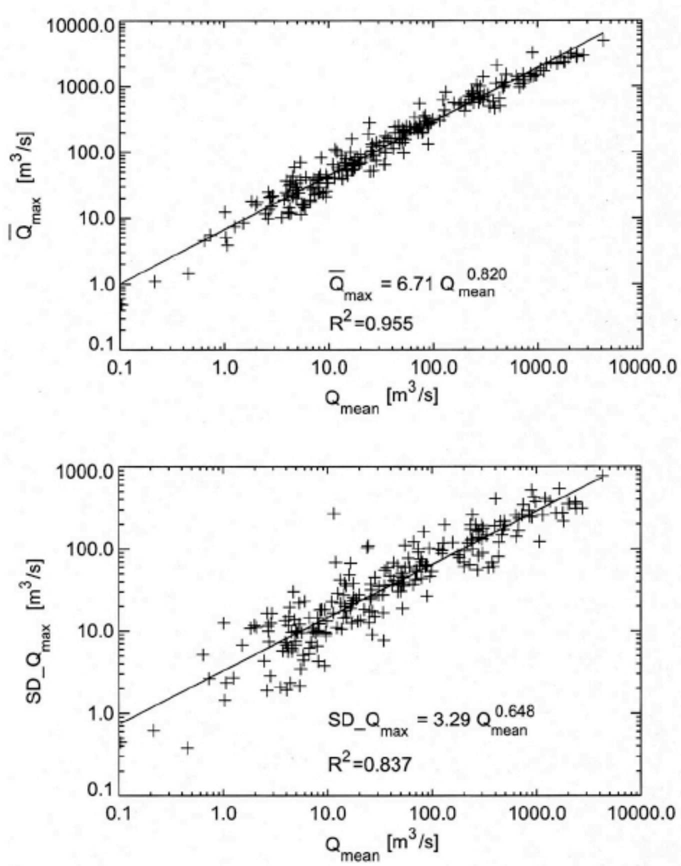

Fig. 5. Regressions between average river flows and statistical parameters of annual floods at 225 stations in Colombia; results are shown for the mean (top) and standard deviation (bottom)

7. Eq. (5.2) predicts peak discharge for any basin throughout Colombia, even though Colombia is divided into 11 non-homogeneous climatic regions.

Power Laws in Low-Flow Quantiles

Power laws (scaling) relations between $\mu_{Q_{\min}}$ and $\sigma_{Q_{\min}}$ to mean streamflow, Q_m are given in Poveda et al. (2007) as shown below in Fig. 7.

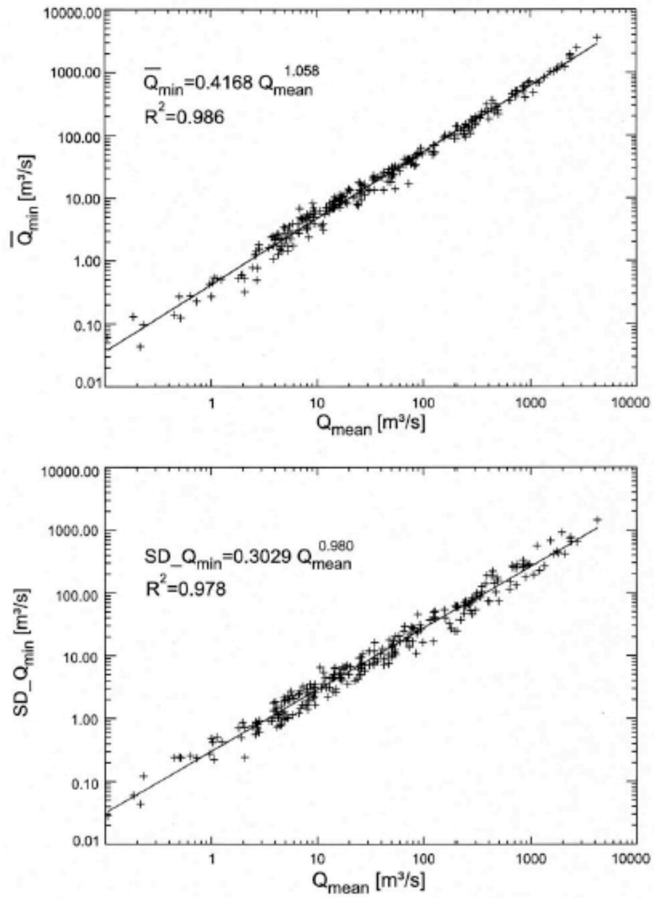

Fig. 7. Regressions between mean streamflows and mean annual minimum flows

Long-term water balance

Poveda et al. computed mean annual flows using long-term water balance as $Q = A(P - ET)$, where A is drainage area, P is mean precipitation and ET is mean evapotranspiration, and compared it with values of observed stream flows from 225 gauging stations covering Colombia (see Lecture 2 for various issues pertaining to the annual water balance).

1. Precipitation data from 688 rain gauges with monthly records were used to estimate the average rainfall field over Colombia for the period 1966-1987. Kriging, a linear interpolation method related to least squares estimation was used to interpolate among gauges. (Kriging is not covered in the course).

Theoretical equations for estimating annual evapotranspiration

A multitude of theoretical expressions have been developed that can be used to compute estimates of annual ET. We will follow Poveda et al. (2007) to illustrate this

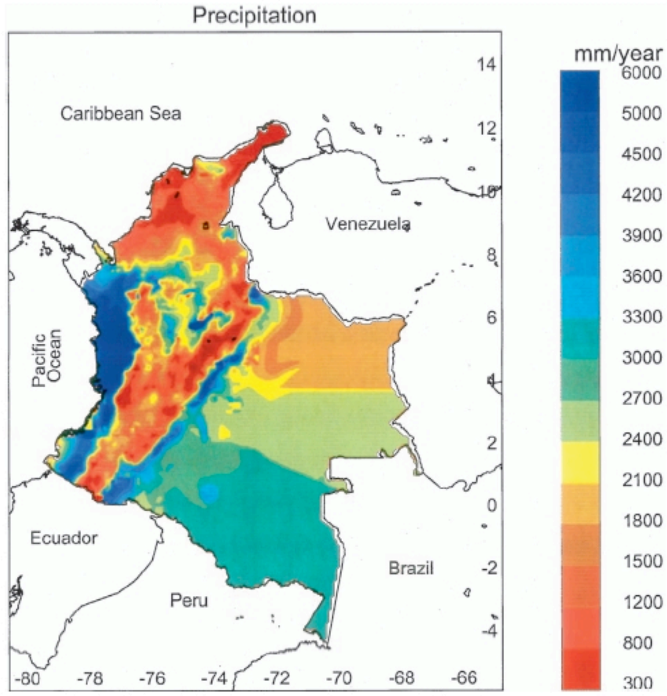

Fig. 1. (Color) Long-term average annual precipitation field for Colombia, obtained through interpolation using Kriging with drift (see the text for details)

fundamental conceptual issue (Brutsaert, 2005 Ch. 4 covers evaporation). Fig. 2 gives estimates of ET from four different equations.

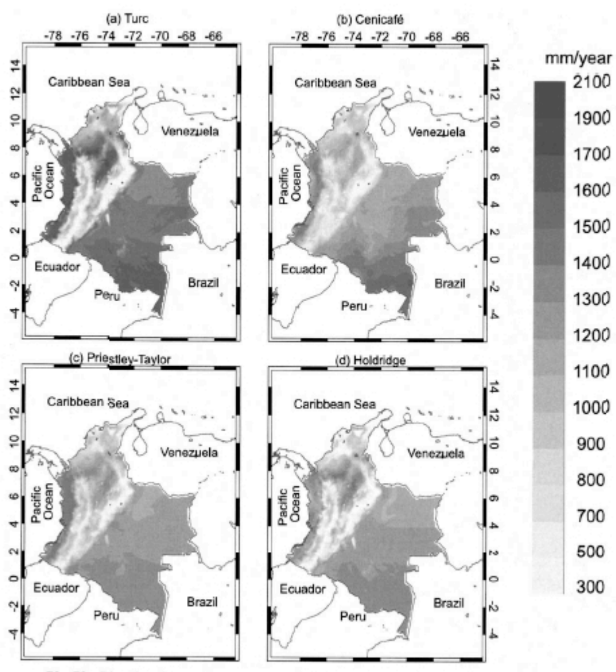

Fig. 2. Long-term average annual actual evapotranspiration estimated using the methods by (a) Turc-Budyko; (b) Cenicafé; (c) Priestley-Taylor; and (d) Holdridge

Poveda et al. (2007) used 9 equations to estimate ET. They used an estimate of P and the water balance equation to calculate 9 different estimates for Q_m, and compared it with the estimates obtained from data. They computed the Root mean square (RMS) as a measure of how well the predicted mean runoff compares with the observed mean runoff using different ET models. The ET estimate from Turc's equation gave the lowest RMS error shown in Fig. 3 below. For example, for Magdelena river basin (259,931 km^2), the long-term river flow estimate is 8.034 m^3/s, which can be compared with the measured long-term mean of 7,593 m^3/s.

Turc's equation gives an expression for annual ET as,

$$\frac{ET}{P} = \frac{1}{\left[1 + (P/E_0)^2\right]^{1/2}} \tag{5.3}$$

Here, E_0 is known as potential evapotranspiration (PET). It is the amount of water that can be evaporated and transpired if water availability is unlimited (for example in case of pan evaporation). Turc estimated PET through an empirical expression, $E_0 = 300 + 25T + 0.05T^3$, where T is the mean annual temperature. Turc equation requires estimates of precipitation and temperature. Temperature fields (degrees C) were estimated from regional linear relationships observed in data for 1002 temperature

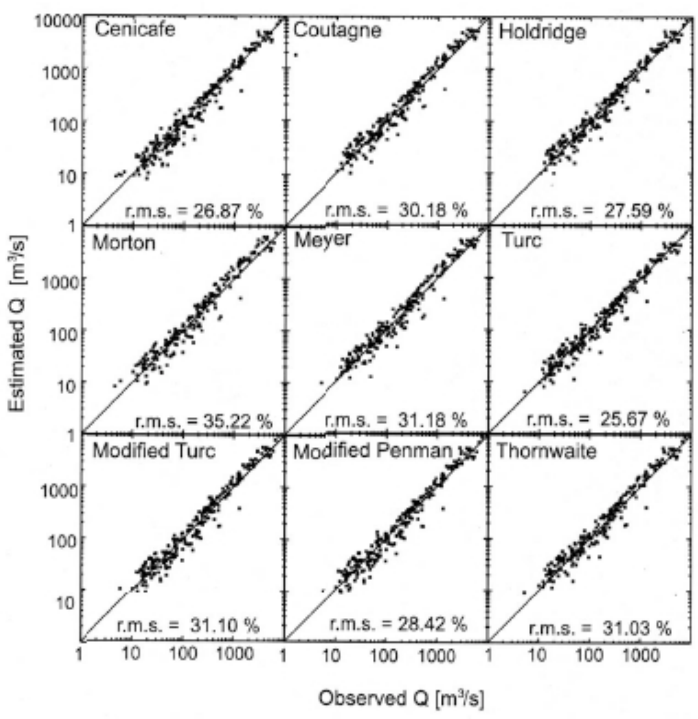

Fig. 3. Comparison of observed and estimated long-term average river flows at more than 200 river gauging stations in Colombia, using diverse evapotranspiration methods

stations. For example, the expression for the Andean region is, $T = 29.42 - 0.0061h$; h is elevation in meters.

Important Questions

1. Can the empirical USGS regional regression equations be used to obtain relations similar to eq. (5.2)?

2. What is the physical basis of power law relations for the mean and standard deviation of annual floods and low flows with respect to long-term mean runoff in Figure 5 and Figure 7?

3. How can the parameters in the power laws ($\alpha_\mu, \theta_1, \sigma_\mu, \theta_2$) be understood in terms of hydrologic processes governing precipitation, evapotranspiration, storages, infiltration, and stream flows?

4. Questions 2 and 3 bring us to the current state of hydrologic science and engineering. It is an important topic for current and future research.

References:

1. Poveda et al. *J. Hydrologic Eng.* (*ASCE*), January/February, 4-13, 2007.