CVEN5313: Environmental Fluid Mechanics

Navier-Stokes 1: Conservation of Mass and Momentum
Due 5pm Friday, 11/19/10

1. In lecture, we found the general solution for flow between two horizontal plates to be

$$u_1(x_3) \;=\; -\frac{\lambda}{2\mu}\,{x_3}^2 \;+\; C_1\,x_3 \;+\; C_2$$

For the specific case of Poiseuille-Couette flow (forcing from both pressure and shear) we found that the solution becomes

$$u_1(x_3) \;=\; -\frac{\lambda}{2\mu}\,{x_3}^2 \;+\; \left(\frac{U}{H} + \frac{\lambda H}{2\mu}\right) x_3 \qquad \text{for} \qquad 0 \le x_3 \le H$$

(a) Starting with the general solution, re-solve the problem for the the simpler case where the only applied force is the viscous shear from the moving plate (i.e., λ=0; this is called Couette flow). Express your solution in both dimensional and non-dimensional forms. Make well-labeled sketches of the dimensional and non-dimensional solutions.

(b) Starting with the general solution, re-solve the problem for the the simpler case where the only applied force is the pressure gradient (i.e., U=0; this is called Poiseuille flow). Express your solution in both dimensional and non-dimensional forms. Make well-labeled sketches of the dimensional and non-dimensional solutions.

(c) In lecture, we described the parameter $\mathbb{P} \equiv \dfrac{\lambda H^2}{8\mu U}$ in an ad-hoc manner as a nondimensional measure of the relative strength of the pressure gradient λ and the shear stress due to the upper plate moving at velocity U. Develop a rigorous ratiometric definition for $\mathbb{P}$ that is based on the resulting flow solution rather than the forcing. Hint: Use your Couette flow and Poiseuille flow solutions as a starting point.

(d) The nondimensional form of the P-C flow soultion given in class

$$u_1{}^* \;=\; -4\,\mathbb{P}\,{x_3{}^*}^2 \;+\; (1+4\,\mathbb{P})\,x_3{}^*$$

normalizes the flow velocity by the speed of the plate U (i.e., $u_1{}^* = u_1/U$).

i. Develop an analogous nondimensional solution (i.e., a new definition for $u_1{}^*$) that normalizes the flow velocity by the depth-averaged velocity (i.e., $|V|$, where V is from the open-channel flow notes). Your final answer should be in the form $u_1{}^* = f(x_3{}^*, \mathbb{P})$.
ii. Plot the new solution for $0 \le x_3{}^* \le 1$ for $\mathbb{P} = -100, -2.5, 0, 1, 100$.
iii. What happens if you try to plot the solution for $\mathbb{P} \approx -3/4$? Why?
iv. How does the solution change for larger and larger magnitudes of $\mathbb{P}$ (i.e., $|\mathbb{P}| \gg 100$)? Why?
v. Briefly summarize the relative strengths and weaknesses of the two nondimensionalization schemes.