Groundwater Hydrology Equations

1. Porosity, n:

$$n = \frac{V_v}{V_T} = \frac{V_T - V_s}{V_T}$$

where V_v is the volume of voids, V_T is the total volume, and V_s is the volume of solids. $(0 < n < 1)$. Also denoted as n_T.

2. Hydraulic head, h:

$$h = \frac{P}{\rho_w g} + z$$

where z is the elevation head, $P/(\rho_w g)$ is the pressure head, P is the fluid pressure, ρ_w is the fluid density, and g is the gravitational constant. The elevation head, z, represents the elevation of the point of interest above the datum ($z = 0$ at the datum); the pressure head represents the height that water will rise above the point of interest.

3. Hydraulic gradient, ∇h:

$$\nabla h = \frac{\partial h}{\partial x}\vec{i} + \frac{\partial h}{\partial y}\vec{j} + \frac{\partial h}{\partial z}\vec{k}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors in the x-, y-, and z-directions, respectively.

4. In two dimensions, if head is measured at three points, and $h_1 = h(x_1, y_1)$, $h_2 = h(x_2, y_2)$, $h_3 = h(x_3, y_3)$, then

$$\begin{aligned} \frac{\partial h}{\partial x} &= \frac{(h_1 - h_2)(y_2 - y_3) - (h_2 - h_3)(y_1 - y_2)}{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)} \\ \frac{\partial h}{\partial y} &= \frac{(h_1 - h_2)(x_2 - x_3) - (h_2 - h_3)(x_1 - x_2)}{(y_1 - y_2)(x_2 - x_3) - (y_2 - y_3)(x_1 - x_2)} \end{aligned}$$

5. Darcy's Law:

$$Q = -KA\frac{dh}{d\ell} = -KA\nabla h$$

where Q is the flow rate, K is the hydraulic conductivity, A is the cross-sectional area **perpendicular to flow**, h is head, ℓ is flow distance, $dh/d\ell$ is the hydraulic gradient between two points in one dimension, and ∇h is the general hydraulic gradient.

6. Darcy's Law in terms of specific discharge:

$$q = -K\frac{dh}{d\ell} = -K\nabla h$$

   where q is the specific discharge or Darcy velocity.

7. Darcy's Law in terms of groundwater velocity:

$$v = -\frac{K}{n}\frac{dh}{d\ell} = -\frac{K}{n}\nabla h$$

   where v is the groundwater velocity or average linear velocity and n is the porosity.

8. Hydraulic conductivity, K:

$$K = \frac{k\rho_w g}{\mu} = \frac{kg}{\nu}$$

   where k is permeability, ρ_w is fluid density, g is gravity, μ is fluid viscosity, and $\nu = \mu/\rho_w$.

9. Average hydraulic conductivity for flow parallel to bedding:

$$\bar{K}_{\parallel} = \frac{\sum_i K_i b_i}{\sum_i b_i}$$

   where K_i is the hydraulic conductivity of layer i and b_i is the thickness of layer i.

10. Average hydraulic conductivity for flow perpendicular to bedding:

$$\bar{K}_{\perp} = \frac{\sum_i b_i}{\sum_i \frac{b_i}{K_i}}$$

   where K_i is the hydraulic conductivity of layer i and b_i is the thickness of layer i.

11. Tangent law:

$$\frac{K_1}{K_2} = \frac{\tan\alpha_1}{\tan\alpha_2}$$

   where α_1 and α_2 are the angles between the line perpendicular to the layering and the flow direction in Layers 1 and 2, respectively.

12. Hydraulic conductivity from a constant-head permeameter:

$$K = \frac{VL}{Aht} = \frac{QL}{Ah}$$

   where V is the volume of water flowing through the permeameter in an amount of time t, L is the length of the sample, A is the cross-sectional area of the permeameter chamber, h is the head drop across the permeameter, and Q is the volumetric flow rate.

13. Hydraulic conductivity from a falling-head permeameter:

$$K = \frac{aL}{A(t_1 - t_0)} \ln\left(\frac{h_o}{h_1}\right) = \frac{2.3aL}{A(t_1 - t_0)} \log_{10}\left(\frac{h_o}{h_1}\right)$$

where a is the cross-sectional area of the tube, A is the cross-sectional area of the permeameter chamber, L is the length of the sample, t_0 and t_1 are the initial and final times, respectively, and h_o and h_1 are the heads in the tubes at times t_0 and t_1, respectively.

14. Anisotropy ratio, K_s:

$$K_s = \frac{K_\parallel}{K_\perp}$$

where $K_\parallel$ is the hydraulic conductivity in the direction parallel to layering and $K_\perp$ is the hydraulic conductivity in the direction perpendicular to layering.

15. Darcy's Law in three-dimensions:

$$\begin{aligned} q_x &= -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} - K_{xz}\frac{\partial h}{\partial z} \\ q_y &= -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} - K_{yz}\frac{\partial h}{\partial z} \\ q_z &= -K_{zx}\frac{\partial h}{\partial x} - K_{zy}\frac{\partial h}{\partial y} - K_{zz}\frac{\partial h}{\partial z} \end{aligned}$$

where q_x, q_y, and q_z are the components of specific discharge in the x-, y-, and z-directions, respectively, and K_{ij} is the hydraulic conductivity in the i-direction due to a gradient in the j-direction, where $i, j = x, y, z$.

16. Darcy's Law in matrix form:

$$\vec{q} = \begin{bmatrix} q_x \\ q_y \\ q_x \end{bmatrix} = -\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{bmatrix}$$

or in short-hand as

$$\vec{q} = -\underline{\underline{K}}\nabla h$$

where

$$\underline{\underline{K}} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

$\underline{\underline{K}}$ is a symmetric matrix (i.e., $K_{xy} = K_{yx}$, $K_{xz} = K_{zx}$, and $K_{yz} = K_{zy}$).

17. Components of $\underline{\underline{K}}$ in two dimensions:

$$
\begin{aligned}
K_{xx} &= K_{\parallel} \cos^2\theta + K_{\perp} \sin^2\theta \\
K_{xy} &= K_{yx} = (K_{\parallel} - K_{\perp}) \sin\theta \cos\theta \\
K_{yy} &= K_{\perp} \cos^2\theta + K_{\parallel} \sin^2\theta
\end{aligned}
$$

18. If the coordinate system is aligned in the principal directions of anisotropy, then

$$
\underline{\underline{K}} = \begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix}
$$

and Darcy's Law becomes

$$
q_x = -K_{xx}\frac{\partial h}{\partial x} \qquad q_y = -K_{yy}\frac{\partial h}{\partial y} \qquad q_z = -K_{zz}\frac{\partial h}{\partial z}
$$

19. If the porous medium is isotropic, $K_{xx} = K_{yy} = K_{zz} = K$ and Darcy's Law becomes

$$
q_x = -K\frac{\partial h}{\partial x} \qquad q_y = -K\frac{\partial h}{\partial y} \qquad q_z = -K\frac{\partial h}{\partial z}
$$

20. Specific yield, S_y:

$$
S_y = \frac{V_d}{A\Delta h}
$$

where V_d is the volume drained, A is the plan area, and Δh is the head drop.

21. Relationship between porosity, n, and specific yield, S_y:

$$
n = S_y + S_r
$$

where S_r is the specific retention.

22. Specific storage, S_s:

$$
S_s = \rho_w g(\beta_p + n_T\beta_w) = \frac{V_d}{V_T\Delta h}
$$

where ρ_w is density of the fluid, g is gravity, β_p is the compressibility of the rock matrix, n_T is porosity, β_w is fluid compressibility, V_d is the volume drained, V_T is the total aquifer volume, and Δh is the head drop.

23. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy:

$$S_s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)$$

24. Flow equation for confined aquifer; isotropic aquifer:

$$S_s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right)$$

25. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy; homogeneous aquifer:

$$S_s\frac{\partial h}{\partial t} = K_{xx}\frac{\partial^2 h}{\partial x^2} + K_{yy}\frac{\partial^2 h}{\partial y^2} + K_{zz}\frac{\partial^2 h}{\partial z^2}$$

26. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy; steady flow:

$$0 = \frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)$$

27. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy; essentially horizontal flow:

$$S\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) \; ,$$

where $S = S_s b$ is the storativity, $T = Kb$ is the transmissivity, and the aquifer thickness is b.

28. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy; essentially horizontal flow and leakage:

$$S\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) + \frac{K'}{b'}(h_a - h) \; ,$$

where K' is the hydraulic conductivity of the aquitard, b' is the aquitard thickness, and h_a is the head in the overlying aquifer.

29. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption):

$$\begin{aligned} S_y\frac{\partial h}{\partial t} \quad &= \quad \frac{\partial}{\partial x}\left(K_{xx}(h-\eta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial x}\left(K_{xy}(h-\eta)\frac{\partial h}{\partial y}\right) + \\ & \qquad \frac{\partial}{\partial y}\left(K_{yx}(h-\eta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}(h-\eta)\frac{\partial h}{\partial y}\right) \end{aligned}$$

where η is the elevation of the aquifer bottom.

30. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption), principal axes aligned with coordinate axes:

$$S_y \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_{xx}(h-\eta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}(h-\eta)\frac{\partial h}{\partial y}\right)$$

31. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption); homogeneous aquifer; principal directions of anisotropy aligned with coordinate axes:

$$S_y \frac{\partial h}{\partial t} = K_{xx}\frac{\partial}{\partial x}\left((h-\eta)\frac{\partial h}{\partial x}\right) + K_{yy}\frac{\partial}{\partial y}\left((h-\eta)\frac{\partial h}{\partial y}\right)$$

32. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption); isotropic aquifer:

$$S_y \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K(h-\eta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K(h-\eta)\frac{\partial h}{\partial y}\right)$$

33. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption); steady flow; principal directions of anisotropy aligned with coordinate axes:

$$0 = \frac{\partial}{\partial x}\left(K_{xx}(h-\eta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}(h-\eta)\frac{\partial h}{\partial y}\right)$$

34. Flow equation for unconfined aquifer with recharge; essentially horizontal flow:

$$S_y \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_{xx}(h-\eta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}(h-\eta)\frac{\partial h}{\partial y}\right) + N$$

where N is the recharge rate.

35. Hvorslev slug test equation:

$$\ln\left(\frac{s_o}{s}\right) = \frac{FK}{\pi R^2}t\ ,$$

where s_o is the drawdown at the time when the slug is removed, s is drawdown in the well at time t, K is the radial hydraulic conductivity, R is the radius of the well, and F is shape factor as shown by Table 12.1.

36. Thiem equation for a confined aquifer:

$$h_2 - h_1 = \frac{Q}{2\pi T}\ln\left(\frac{r_2}{r_1}\right)$$

where h_1 and h_2 are the heads at distances r_1 and r_2 from the well, Q is the pumping rate, and T is the aquifer transmissivity.

37. Thiem equation for an unconfined aquifer:

$$h_2^2 - h_1^2 = \frac{Q}{2\pi K} \ln\left(\frac{r_2}{r_1}\right)$$

where h_1 and h_2 are the heads at distances r_1 and r_2 from the well, Q is the pumping rate, and K is the aquifer hydraulic conductivity.

38. Groundwater flow equation in radial coordinates for a homogeneous, isotropic, confined aquifer with essentially horizontal flow:

$$\frac{S}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right),$$

where S is the storage coefficient, T is transmissivity, h is head, t is time, and r is radial distance.

39. Theis equation:

$$s = h_o - h = \frac{Q}{4\pi T} W(u),$$

where s is drawdown, h_o is initial head, Q is the pumping rate ($Q > 0$ for pumping), T is transmissivity, $u = r^2 S/(4Tt)$, S is storage coefficient, t is time, and $W(u)$ is the well function (exponential integral) given by

$$\begin{aligned} W(u) &= \int_u^\infty \frac{e^{-a}}{a}\, da \\ &= -0.5772 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} \pm \ldots \end{aligned}$$

40. Cooper-Jacob approximation:

$$s = h_o - h = \frac{2.3Q}{4\pi T} \log\left(\frac{2.25Tt}{r^2 S}\right) .$$

41. Hantush Jacob equation for drawdown in a leaky aquifer with aquitard storage:

$$s = h_o - h = \frac{Q}{4\pi T} H(u, \beta),$$

where

$$\beta = \frac{r}{4}\left[\sqrt{\frac{K'S'}{b'TS}} + \sqrt{\frac{K''S''}{b''TS}}\right]$$

b' and b'' are the thicknesses of the upper and lower aquitards, respectively, K' and K'' are the hydraulic conductivities of the upper and lower aquitards, respectively, and S' and S'' are the storage coefficients of the upper and lower aquitards, respectively,

42. Neuman solution for drawdown in a phreatic aquifer:

$$s = h_o - h = \frac{Q}{4\pi T} W(u_A, u_B, \beta) ,$$

where $u_A = r^2 S/(4Tt)$, $u_B = r^2 S_y/(4Tt)$, $\beta = (r^2/b^2)(K_z/K_r)$, b is the initial saturated thickness, K_z is the vertical hydraulic conductivity, and K_r is the radial hydraulic conductivity.

43. Drawdown due to pumping in multiple wells:

$$h_o - h = \sum \frac{Q_i}{4\pi T} W(u_i),$$

where $u_i = r_i^2 S/(4Tt)$.

44. Drawdown due to variable pumping rates in one well:

$$h_o - h = \frac{Q_o}{4\pi T} W(u) + \sum \frac{\Delta Q_i}{4\pi T} W(u_i),$$

where $\Delta Q_i = Q_i - Q_{i-1}$ and

$$u_i = \begin{cases} 0 & t \leq t_i \\ \frac{r^2 S}{4T[t-t_i]} & t > t_i \end{cases}$$

45. Drawdown due to pumping near an impermeable boundary:

$$h_o - h = \frac{Q}{4\pi T} \left[W(u_r) + W(u_i) \right],$$

where $u_r = r_r^2 S/(4Tt)$, $u_i = r_i^2 S/(4Tt)$, r_r is the radial distance from the real well to the point of interest, and r_i is the radial distance from the image well to the point of interest.

46. Drawdown due to pumping near a constant head boundary:

$$h_o - h = \frac{Q}{4\pi T} \left[W(u_r) - W(u_i) \right],$$

where $u_r = r_r^2 S/(4Tt)$, $u_i = r_i^2 S/(4Tt)$, r_r is the radial distance from the real well to the point of interest, and r_i is the radial distance from the image well to the point of interest.

47. Maximum width of a capture zone:

$$y_{\max} = \frac{Q}{2Ti},$$

where i is the hydraulic gradient under ambient conditions.

48. Location of stagnation point in a capture zone:

$$x_o = \frac{-Q}{2\pi Ti}$$

49. Boundary of a capture zone:

$$x = \frac{-y}{\tan(2\pi Tiy/Q)},$$

where tan() is in radians.

50. Location of stagnation points in a hydraulic containment system:

$$x_o = \pm\sqrt{L^2 + QL/(i\pi T)},$$

where $(L, 0)$ and $(-L, 0)$ are the locations of the injection and pumping well, respectively.

51. Maximum width of a hydraulic containment cell:

$$y_{\max} = \frac{-Q}{\pi Ti}\left[\tan^{-1}\left(\frac{y_{\max}}{L}\right) - \frac{\pi}{2}\right]$$

52. Boundary of a hydraulic containment cell:

$$x^2 = L^2\left(1 + \frac{1}{\beta^2}\right) - \left(y - \frac{L}{\beta}\right)^2,$$

where $\beta = \tan(2\pi Tiy/Q)$.

53. Linear sorption isotherm:

$$S = K_d C\ ,$$

where S is the chemical concentration in the sorbed phase [M solute/M solid], K_d is the distribution coefficient [L/g], and C is the aqueous concentration.

54. Freundlich sorption isotherm:

$$S = KC^n\ ,$$

where K is the partition coefficient and n is a constant.

55. Langmuir sorption isotherm:

$$S = \frac{Q^o KC}{1 + KC}\ ,$$

where Q^o is the maximum sorptive capacity (mg/g) and K is a constant (L/mg).

56. Linear regression to estimate slope of straight line passing through the origin:

$$m = \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2} \ ,$$

where $y = mx$ is the equation of the line and N is the number of data pairs.

57. Linear regression to estimate slope and intercept of a straight line:

$$m = \frac{N \sum_{i=1}^{N} x_i y_i - (\sum_{i=1}^{N} x_i)(\sum_{i=1}^{N} y_i)}{N \sum_{i=1}^{N} x_i^2 - (\sum_{i=1}^{N} x_i)^2}$$

$$b = \frac{\sum_{i=1}^{N} y_i - m \sum_{i=1}^{N} x_i}{N} \ , \tag{1}$$

where $y = mx + b$ is the equation of the line and N is the number of data pairs.

58. Advective mass flux:

$$J_{\mathrm{adv}} = qC = nvC$$

where q is specific discharge, C is concentration, n is porosity, and v is groundwater velocity.

59. Diffusive mass flux:

$$J_{\mathrm{dif}} = -n\tau D_m \nabla C,$$

where τ is tortuosity ($\tau \approx n^{1/3}$) and D_m is the molecular diffusion coefficient.

60. Dispersive mass flux (for velocity in x direction only):

$$J_{\mathrm{dis-x}} = -nD_x \frac{\partial C}{\partial x}$$
$$J_{\mathrm{dis-y}} = -nD_y \frac{\partial C}{\partial y}$$
$$J_{\mathrm{dis-z}} = -nD_z \frac{\partial C}{\partial z},$$

where D_x, D_y, and D_z are the dispersion coefficients in the x-, y-, and z-directions, respectively.

61. Dispersion coefficients for flow in the x-direction:

$$\begin{aligned} D_x &= |v_x|\alpha_L \\ D_y &= |v_x|\alpha_{TH} \\ D_z &= |v_x|\alpha_{TV}, \end{aligned}$$

where α_L, α_{TH}, and α_{TV} are the longitudinal, horizontal transverse, and vertical transverse dispersivities, respectively.

62. Dispersive mass flux (general):

$$J_{\text{dis}} = -n\mathbf{D}\nabla C \ ,$$

where $\mathbf{D}$ is the dispersion tensor, given by

$$\mathbf{D} = \begin{bmatrix} D_x & D_{xy} & D_{xz} \\ D_{yx} & D_y & D_{yz} \\ D_{zx} & D_{zy} & D_z \end{bmatrix} \ ,$$

and

$$\begin{aligned}
D_x &= \alpha_L \frac{v_x^2}{|v|} + \alpha_{TH} \frac{v_y^2}{|v|} + \alpha_{TV} \frac{v_z^2}{|v|} \\
D_y &= \alpha_L \frac{v_y^2}{|v|} + \alpha_{TH} \frac{v_x^2}{|v|} + \alpha_{TV} \frac{v_z^2}{|v|} \\
D_z &= \alpha_L \frac{v_z^2}{|v|} + \alpha_{TV} \frac{v_y^2}{|v|} + \alpha_{TV} \frac{v_x^2}{|v|} \\
D_{xy} &= D_{yx} = (\alpha_L - \alpha_{TH}) \frac{v_x v_y}{|v|} \\
D_{xz} &= D_{zx} = (\alpha_L - \alpha_{TV}) \frac{v_x v_z}{|v|} \\
D_{yz} &= D_{zy} = (\alpha_L - \alpha_{TV}) \frac{v_y v_z}{|v|}
\end{aligned}$$

and $|v|$ is the magnitude of the velocity vector.

63. Advection Dispersion Equation (ADE) for groundwater flow in the x direction

$$n\frac{\partial C}{\partial t} + \rho_b \frac{\partial S}{\partial t} = -n\frac{\partial}{\partial x}(v_x C) + n\frac{\partial}{\partial x}\left(D_x \frac{\partial C}{\partial x}\right) + n\frac{\partial}{\partial y}\left(D_y \frac{\partial C}{\partial y}\right) + n\frac{\partial}{\partial z}\left(D_z \frac{\partial C}{\partial z}\right) \ ,$$

where ρ_b is bulk density, C is aqueous concentration, and S is sorbed phase concentration.

64. Solution to the 1-D ADE for an instantaneous point source of a non-sorbing contaminant in an infinite flow field:

$$C(x,t) = \frac{M}{An} \frac{1}{\sqrt{4\pi D_x t}} \exp\left\{ -\frac{(x - x_o - v_x t)^2}{4 D_x t} \right\} \ ,$$

where M is the source mass, x_o is the source location, n is porosity, and A is the cross-sectional area of the one-dimensional aquifer.

65. Relationship between plume size and dispersion coefficient:

$$D_i = \frac{\Lambda_i^2}{11.1t} = \frac{\sigma_i^2}{2t} \ ,$$

where Λ_i is the width of the plume in the i direction between the two points where concentration is $0.5C_{\max}$, $C_{\max}$ is the peak concentration of the plume at time t, D_i is the dispersion coefficient in the i direction, and σ_i is the standard deviation of the contaminant plume in the i direction at time t.

66. Solution to the 2-D ADE for an instantaneous point source of contamination in an infinite flow field:

$$C(x,y,t) = \frac{M}{bn}\frac{1}{4\pi t\sqrt{D_x D_y}} \exp\left\{-\frac{(x-x_o-v_x t)^2}{4D_x t}\right\} \exp\left\{-\frac{(y-y_o)^2}{4D_y t}\right\} ,$$

where M is the source mass, (x_o, y_o) is the source location, n is porosity, b is the aquifer thickness, and D_x and D_y are the dispersion coefficients in the x- and y-directions, respectively. Groundwater is assumed to be flowing in the x direction only.

67. Retardation coefficient:

$$R = 1 + \frac{\rho_b K_d}{n}$$

68. Solution of the 1-D ADE with linear equilibrium sorption for an instantaneous point source of contamination in an infinite flow field:

$$\begin{aligned} C(x,t) &= \frac{M}{An}\frac{1}{\sqrt{4\pi D_x R t}} \exp\left\{-\frac{(R[x-x_o]-v_x t)^2}{4D_x R t}\right\} \\ S(x,t) &= K_d C(x,t) , \end{aligned}$$

where M is the source mass, x_o is the source location, R is the retardation coefficient, and n is porosity.

69. Solution to the 1-D ADE for a continuous source of contamination at $x = 0$ with linear sorption:

$$C(x,t) = \frac{C_o}{2}\left[\operatorname{erfc}\left(\frac{Rx - v_x t}{\sqrt{4D_x R t}}\right) + \exp\left\{\frac{v_x x}{D_x}\right\} \operatorname{erfc}\left(\frac{Rx + v_x t}{\sqrt{4D_x R t}}\right)\right] .$$

70. Volumetric water content, θ (moisture content):

$$\theta = \frac{V_w}{V_T} ,$$

where V_w is the volume of water and V_T is the total volume.

71. Saturation, s:

$$s = \frac{V_w}{V_v} = \frac{\theta}{n} ,$$

where V_w is the volume of water and V_v is the volume of voids.

72. Effective saturation:

$$s_e = \frac{\theta - \theta_r}{n - \theta_r} \ ,$$

where θ is the volumetric water content, θ_r is the residual volumetric water content, and n is porosity.

73. Head in the unsaturated zone:

$$h = z + \psi \ ,$$

where h is total head, ψ is pressure head ($\psi < 0$), and z is elevation head.

74. Brooks-Corey model for effective saturation:

$$s_e = \begin{cases} \left(\frac{\psi_b}{\psi}\right)^{\lambda} & \psi < \psi_b \\ 1 & \psi \geq \psi_b \end{cases}$$

where ψ_b is the bubbling pressure, and λ is the pore size distribution index.

75. van Genuchten model for effective saturation:

$$s_e = \left[1 + (\alpha|\psi|)^{\beta}\right]^{-\gamma} \ ,$$

where α is a parameter ranging from $10^{-3} - 10^{-2}$ cm^{-1}, and β and γ are constants with $\gamma = 1 - 1/\beta$.

76. Brooks-Corey model for specific moisture capacity:

$$c_m = \begin{cases} -(n - \theta_r)\frac{\lambda}{\psi_b}\left(\frac{\psi}{\psi_b}\right)^{-(\lambda+1)} & \psi < \psi_b \\ 0 & \psi \geq \psi_b \end{cases} \ .$$

77. van Genuchten model for specific moisture capacity:

$$c_m = \frac{\alpha\gamma\beta(n - \theta_r)(\alpha|\psi|)^{\beta-1}}{[1 + (\alpha|\psi|)^{\beta}]^{\gamma+1}} \ .$$

78. Relative hydraulic conductivity:

$$K_r(\psi) = \frac{K(\psi)}{K_s} \ ,$$

where K_r is the relative hydraulic conductivity, $K(\psi)$ is the hydraulic conductivity at a pressure head of ψ, and K_s is the saturated hydraulic conductivity.

79. Brooks-Corey model for relative permeability:

$$K_r = (\psi_b/\psi)^{(2+3\lambda)} \ .$$

80. van Genuchten model for relative permeability:

$$K_r = \frac{\left\{1 - (\alpha|\psi|)^{\beta-1}\left[1 + (\alpha|\psi|)^{\beta}\right]^{-\gamma}\right\}^2}{\left[1 + (\alpha|\psi|)^{\beta}\right]^{\gamma/2}} \ .$$

81. Darcy's law for unsaturated flow:

$$q = -K_r(\psi)K_s\left[\frac{\partial\psi}{\partial z} + 1\right] \ ,$$

where q is specific discharge, $K_r(\psi)$ is the relative hydraulic conductivity, K_s is the saturated hydraulic conductivity, ψ is capillary head, and z is distance in the vertical direction.

82. Water velocity in the unsaturated zone:

$$v = \frac{q}{\theta} \ .$$

83. Richards equation:

$$c_m(\psi)\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial z}\left(K_r(\psi)K_s\frac{\partial\psi}{\partial z}\right) + \frac{\partial}{\partial z}\left(K_r(\psi)K_s\right) \ ,$$

where $c_m(\psi)$ is the specific moisture capacity.