Lecture 6, September 9, 2010 (Key Points)

6.1 Energy Balance and Evaporation

Strong linkage between the water cycle and the planetary climate can be illustrated by the estimates of the mean global surface energy budget. Over large areas and over sufficiently long periods (annual or longer), when effects of unsteadiness, melt and thaw, photosynthesis and burning, and lateral advection can be neglected, surface energy balance can be written as (Brutsaert, Ch.1)

$$R_n = L_e E + H \tag{6.1}$$

Here R_n is the specific flux of net incoming short-wave radiation. The unit of measurement is [W Kg^{-1} m^{-2}=J s^{-1} Kg^{-1}m^{-2}]. The major portion of the incoming radiation is absorbed near the surface of the Earth, and it is transformed into internal energy. The subsequent partition of this internal energy includes long-wave radiation back into the atmosphere, upward thermal conduction and convection as sensible heat *H*. EL_e denoting the rate of evaporation *in units of energy is* one of the main processes driving the planetary climate system. L_e is the latent heat of vaporization (Brutsaert, 2005, Ch. 3). *E* also connects the global energy balance with global water balance introduced in Lecture 2. Therefore, understanding evaporation is of fundamental importance to hydrology.

Brutsaert (Table 1.4) illustrates that the net energy is mainly disposed of as evaporation. Over the oceans, the latent heat flux $L_e E$ is on average larger than 90 percent of the net radiation. But even over the land surfaces of the Earth, $L_e E$ is on average still larger than half of R_n. *E* over land also includes transpiration from the plants As a result of the relatively large latent heat of vaporization, evaporation of water involves the transfer and redistributiuon of large amounts of energy under nearly isothermal conditions. Because, even at saturation, air can

Table 1.4 Estimates of mean global heat budget at the earth surface in W m^{-2}

	Land			Oceans			Global		
Reference	R_n	$L_e E$	H	R_n	$L_e E$	H	R_n	$L_e E$	H
Budyko (1974)	65	33	32	109	98	11	96	80	16
Baumgartner and Reichel (1975)	66	37	29	108	92	16	96	76	20
Korzun *et al.* (1978)	65	36	29	121	109	12	105	89	16
Ohmura (2005)	62	36	26	125	110	15	104	85	19

contain only relatively small amounts of water vapor, which can easily be condensed at higher levels, the air can readily be dried out; this release of energy through condensation and subsequent precipitation is the largest single heat source driving the planetary climate system.

6.2 Estimating ET Across Spatial Scales in Annual Water Balance

To understand recent research related to the Turc's equation that gave the best estimates of mean annual evapotranspiration in Poveda et al. (2007) (Lecture 5), we consider an important work on the topic that is based in the classic work of Budyko (1974).

(1) For large enough time scales, the dominant water source for the evaporation is precipitation *P,* and to a good approximation, the only relevant energy source is the net radiation R_n. Budyko (1974) reasoned that

$$E \le \min(P, R_n / L_e) \qquad (6.2)$$

In principle, a negative mean sensible heat flux or the transport of external river water to the region in question could lead to an *E* value that exceeds *P* or R_n / L_e. Such situations, however, are presumably rare.

(2) Under assumptions in item (1), dimensional analysis leads to

$$E / P = \Phi(R_n / PL_e) = \Phi(\phi) \qquad (6.3)$$

Here, $\phi = R_n / PL_e$ is called the "radiative index of dryness". Budyko (1974) developed a semi-empirical functional form for the r.h.s in Eq. (6.3), which is well known in the literature ((Choudhury,1999, Eq. 1). The figure below gives a graphical representation of Budyko equation (Koster and Suarez, 1999).

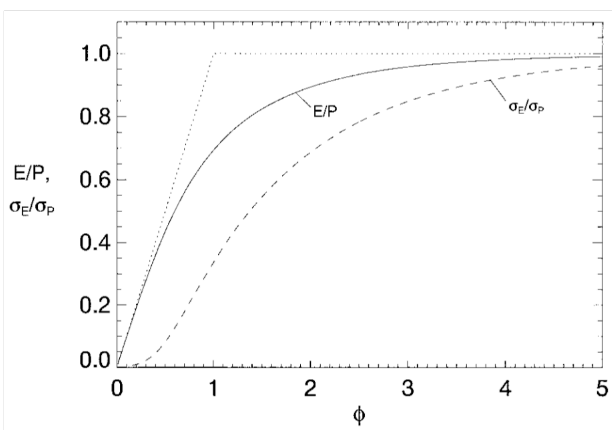

FIG. 1. (a) Graphical Illustration of relationships described in the text. The solid curve represents Budyko's (1958, 1974) semi-empirical relationship between $\overline{E}/\overline{P}$ and the radiative index of dryness ϕ. The dashed curve represents the derived relationship for σ_E/σ_P, as defined in section 3. The dotted straight lines are the asymptotes implied by the inequality in (1).

(3) Choudhury (1999) considered Budyko's equation for ET to understand the effect of spatial variability on annual *ET* over a very wide range of spatial scales. He analyzed two extreme set of spatial scales, field scales ~1 km^2 where ET

measurements were available, and large global river basins ~10^6 km^2, where ET was calculated using a detailed physical model (cited and explained in his paper).

(4) Choudhury found that the following expression gives values of ET within 3% of the original expression that Budyko had considered.

$$\frac{ET}{P} = \frac{1}{\left[1+(PL_e/R_n)^{\alpha}\right]^{1/\alpha}} = \frac{1}{\left[1+(\phi)^{-\alpha}\right]^{1/\alpha}} \tag{6.4}$$

(5) Budyko showed that R_n is proportional to potential evapotranspiration (PET) (denoted as E_0 in Turc's equation) (Budyko, 1974, Fig. 99, p.338). It is defined as evapotranspiration from a saturated land surface. An important point to note is that R_n is measured globally. By contrast, estimation of PET especially at the spatial scales of a field and a river basin, is not a simple issue.

(6) The effects of spatial variability in precipitation and radiation on ET, as one moves up from field scales to medium and large river basin scales, is given in Section 2.3. Let $f(P), f(R_n)$ denote the frequency functions of spatially variable P and R_n respectively. Let the r.h.s of Eq. (6.4) be denoted by, $\Phi(P,R_n)$ (L_e is suppressed for notational convenience as in Choudhury). Then the spatially averaged ET over a region is given by,

$$\langle ET\rangle = \int_0^{\infty}\int_0^{\infty} \Phi(P,R_n) f(P) f(R_n)\, dP dR_n \tag{6.5}$$

(7) Alternatively, if the spatially averaged values, P_0, R_{n0} are used in Eq. (6.4) the resulting expression is denoted as $\langle ET\rangle'$. Choudhury gives a nice analytical explanation to find a relationship between $\langle ET\rangle$ and $\langle ET\rangle'$.

(8) The key idea in item (7) is to expand the function $\Phi(P,R_n)$ in a Taylor series up to the second order term, and evaluate the integrals. It is given by Choudury (1999, Eq. (11)). It shows that $\langle ET\rangle$ is less than $\langle ET\rangle'$ (It is known as the Jenson's inequality in the probability literature).

(9) Choudhury hypothesized that α will be smaller for large river basins $\sim 10^6\ \mathrm{km}^2$ than for field plots $\sim 1\ \mathrm{km}^2$, because variances of P and R_n tend to increase with spatial scale due to spatial variability.

(10) Choudhury (1999) estimated $\alpha = 2.6$ at field scales and $\alpha = 1.8$ for large river basins, which supported his hypothesis in item (8). Further, he noted that if $\alpha = 2.0$ is used to estimate ET (same value as in the Turc's equation), the errors can be expected to be les than 10% over a very wide range of spatial scales of

river basins. For $\alpha = 2.0$, eq. (6.4) becomes *scale invariant* with respect to spatial scales in the sense that the parameter α does not vary form one scale to the next. Scale invariance is a key concept that is being used in modern theories of nonlinear geophysics including hydrology. We will learn a lot about its applications to hydrology in this course.

(HW#3 would serve as a guide to understanding the papers in Lectures 5 and 6 in greater depth)

6.3 Prediction of annual flood statistics in a changing climate

We will briefly illustrate a road map to how annual flood quantiles may be predicted in a changing climate. First step is to use global climate models (GCM) that can predict E/P in a changing climate due to global warming. As an illustrative example, Figure-2 gives such a result from a 20-year GCM simulation (Koster and Suarez 1999).

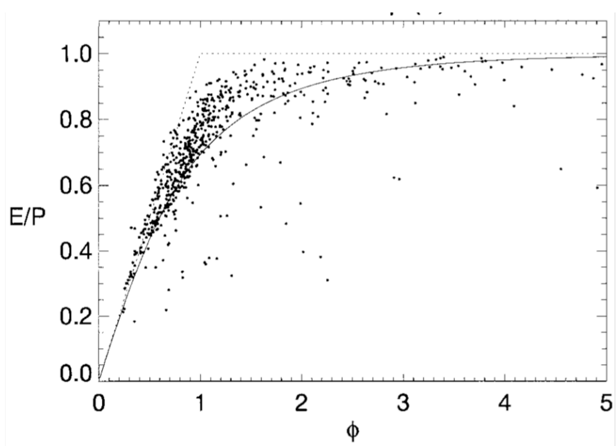

FIG. 2. Plot of $\overline{E}/\overline{P}$ versus ϕ using data from the 20-yr GCM simulation. Each point represents a different nonice land surface grid cell.

The water balance gives estimate of mean annual runoff,

$$Q/P = 1 - E/P \tag{6.6}$$

Finally, a power law relationship between annual flood statistics and annual runoff Q can be used to predict flood statistics in a changing climate using results in Poveda et al. (2007) (see Lecture 5).

Koster and Suarez (1999) also derive an expression for the standard deviations governing interannual variability of *P* and *E* that is based in Eq. (6.3),

$$\sigma_E / \sigma_P = \Phi(\phi) - \phi\Phi'(\phi) \tag{6.7}$$

A plot of σ_E / σ_P is given in Fig. 1. There are many outstanding challenges that remain to be solved before these ideas would be applicable in practice. Further discussion of this topic belongs to CVEN 6333.

References

M. Budyko, *Climate and Life*, Academic press, 1974.
B. Choudhury, *J. Hydrology*, 216, pp. 99-110, 1999.
R. D. Koster and M. Suarez, *J. Climate*, pp. 1911-1917, July 1999.