CVEN5313: Environmental Fluid Mechanics

Open Channel Flow: Specific Energy
Due Friday, 9/17/10

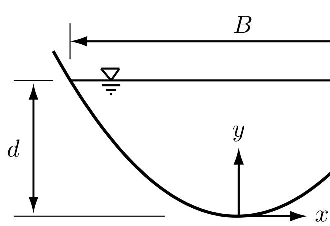

Figure 1: Cross-section of a parabolic channel.

1. The walls of the parabolic channel section shown in Fig. 1 are described by the equation

$$y = ax^2$$

   (a) Show that the free-surface width can be expressed as

$$B = 2\,a^{-\frac{1}{2}}\,d^{\frac{1}{2}}$$

   (b) Show that the cross-sectional area of the flow can be expressed as

$$A = \frac{2}{3}\,B\,d$$

   (c) Show that the critical depth of the flow is (for $\alpha = \cos\theta = 1$)

$$d_c = \left[\frac{27}{32}\frac{a}{g}\right]^{\frac{1}{4}} Q^{\frac{1}{2}}$$

   (d) Nondimensionalize the expression for d_c.

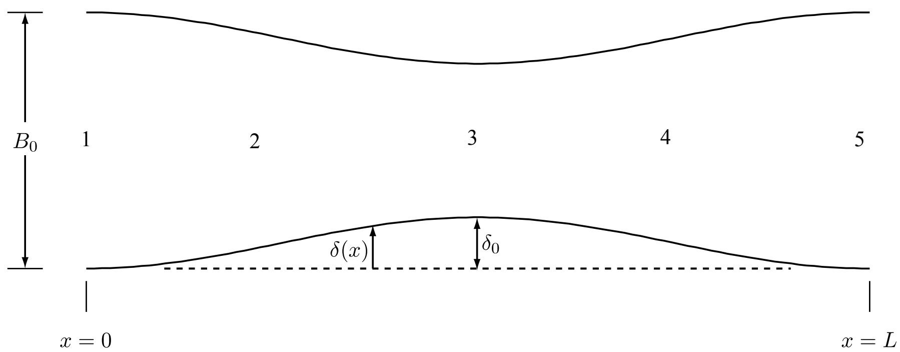

Figure 2: Top view of a width contraction in a rectangular channel.

2. Water flows through a horizontal open channel with rectangular cross section. The side walls form a symmetric contration as shown in Fig. 2. The shape of the lateral contraction is

$$\delta(x) = \frac{\delta_0}{2}\left[1 - \cos\left(2\pi \frac{x}{L}\right)\right]$$

The initial channel width is $B_0 = 1.25$ m, and $Q = 0.125$ m^3/s. Stations 1-5 are located at $x = 0,\ L/4,\ L/2,\ 3L/4,$ and L, respectively. The width at the throat of the contraction (station 3, at $x = L/2$) is $B_0 - 2\delta_0$. Assume that there is no energy loss through the contraction, and that the velocity distribution is uniform over the depth and across the contraction ($\alpha = 1$). The flow depth at $x = 0$ is $d_0 = 0.25$ m, and the contraction magnitude is $\delta_0 = 0.3$ m.

   (a) Determine if the incoming flow at $x = 0$ is subcritical or supercritical.

   (b) Compute and plot the flow depth for at least 1000 points along $0 \le x \le L$

   (c) Plot the corresponding dimensional specific energy diagram, labeling Stations 1-5 on the plot to show the progression of the flow through the contraction.

   (d) Plot the corresponding non-dimensional specific energy diagram, labeling Stations 1-5 on the plot to show the progression of the flow through the contraction.

   (e) Determine δ_0^{crit}, the value of δ_0 that would make the flow in the throat critical for this condition.

   (f) Repeat parts (b), (c), and (d) with $\delta = \delta_0^{\text{crit}}$. There are now two possible downstream profiles - plot only the physically realizable one (i.e., to make a smooth profile).

   (g) Explain what would happen to the flow if $\delta > \delta_0^{\text{crit}}$.

3. Repeat the previous problem with $d_0 = 0.06$ m and $\delta_0 = 0.2$ m.

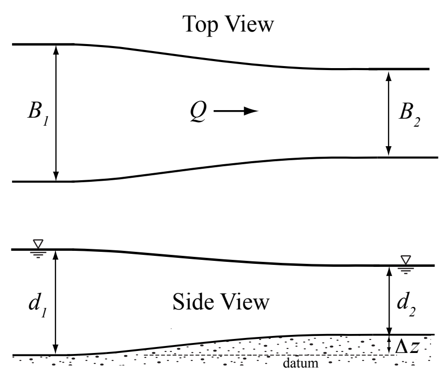

Figure 3: Top and side views of a contracting flume used for flow measurements.

4. A contracting flume (Fig. 3) is used as a gauging station in a rectangular channel. By measuring the upstream and downstream flow depths (d_1 and d_2), the flowrate Q can be determined. Assume that there is no energy loss through the contraction, and that the velocity distribution is uniform over the depth and across the contraction ($\alpha = 1$). Furthermore, you may assume that the incoming flow is subcritical.

   (a) Determine an analytical expression for Q in the form $Q = f(d_1, d_2, B_1, B_2, \Delta z)$

   (b) Let Q_{crit} be the flowrate that produces a critical downstream flow for a given downstream depth. Determine an expression for Q_{crit}, expressed as $Q_{\text{crit}} = f(d_2)$.

   (c) Make a calibration chart for the flume that plots Q vs. d_2 for a given value of d_1. Your chart should include curves for d_1 = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, and 2.0 m. Use the following dimensions for the flume:

$$\begin{aligned} B_1 &= 3 \text{ m} \\ B_2 &= 2 \text{ m} \\ \Delta z &= 0.3 \text{ m} \end{aligned}$$

   Here are a few questions (hints) to consider when making your chart:

   - For a given d_1, what is the maximum possible d_2? Recall that the incoming flow is subcritical.
   - What is the maximum possible downstream Froude number? What constraint does this put on d_2 for a given d_1?

   Also plot Q_{crit} vs. d_2 on your chart.

   (d) Plot dimensional specific energy diagrams for the following cases. In each case, label locations 1 and 2 (i.e., upstream and downstream), and sketch the approximate path taken by the flow between the two points.

      i. $d_1 = 1.8$ m, $d_2 = 1.4$ m
      ii. $d_1 = 1.8$ m, $\mathrm{Fr}_2 = 1$