# Linking Long-Term Water Balances and Statistical Scaling to Estimate River Flows along the Drainage Network of Colombia

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**Abstract:** Long-term average river discharges as well as peak and low flows of different return periods are estimated along the entire river network of Colombia, through the conjoint use of the long-term water balance in the river basins and the framework of statistical scaling, taking the average flow field as the scaling variable. Estimation of the long-term water balance considers the spatial variability of hydrologic fields, in which drainage basins are considered the basic hydrological control volumes for estimation. A systematic effort has been made to estimate the long term average precipitation field combining rain gauge measurements with existing handmade expert maps as an input trend for a universal Kriging interpolation technique. Evaluation of estimates for actual and potential long-term evapotranspiration was implemented using diverse methods. Results were tested using the long term water balance equation against 200 streamflow gauging stations. No method for actual evapotranspiration showed significant superiority. Overall, we conclude that the magnitude of errors arises fundamentally from deficiencies in the data and the sparsity of the observations.

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#### Introduction

Colombia, located in northwestern tropical South America exhibits complex hydrological, geomorphological, and climatological features arising from its tropical setting, in addition to: (1) the topographic gradients of the three branches of the Andean Mountains crossing from southwest to northeast; (2) the hydroclimatic and ecological dynamics of the Amazon and Orinoco River basins; (3) the atmospheric circulation patterns over the neighboring

tropical Pacific and Atlantic oceans; and (4) the strong land-atmosphere feedbacks. On top of such natural complexity, additional hurdles impede the advancement of hydrologic engineering: (1) lack of adequate information of relevant hydrologic variables in space and time; (2) poor-quality, limited (in space and time) and expensive data sets sold by the hydrometeorological service; (3) lack of appropriate methodologies to predict hydrologic variables in tropical environments; and (4) prohibitive li-

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censing costs of commercial geographical information systems. Such a situation is the rule throughout the developing world in general. Thereby, prediction of hydrologic processes in ungauged basins ("PUB problem;" http://www.cig.ensmp.fr/iahs/), an important focus of the International Association for Hydrological Sciences (IAHS), becomes an even more challenging task.

This paper represents an effort to overcome most of the aforementioned problems of hydrologic engineering for the case of Colombia. In particular, we aim to estimate mean and extreme river flows (floods and minimum) along the drainage river network of Colombia, using a simple yet powerful methodological approach that links the long-term water balance equation with the methods of regionalization and statistical scaling. These methodologies have been long known in the hydrological literature, but their conjoint usage to tackle the PUB problem at the space-time scales of our interest, constitutes an original contribution of our work, in addition to the newly created hydrological data sets.

The PUB problem for estimating long-term mean and extreme river discharges guided the scientific and practical advancements of hydrology for decades. Estimation of floods and their associated return periods have included the probabilistic flood frequency analysis (Chow 1951, 1964), the so-called regionalization method (Kirby and Moss 1987; Bobee et al. 1993; Hoskings 1990), which involves extrapolation from one catchment to another within a "homogeneous" region, and also the so-called "derived flood frequency" approach (Eagleson 1972), that combines statistical models of rainfall with dynamical modeling of the physical processes underlying runoff generation and routing. A comprehensive account of the suite of computer models of watershed hydrology is reviewed in Singh (1995). The lack of sufficiently long records of river flows and rainfall measurements, and the lack of detailed small scale measurements of parameters and processes prevents their usage, especially in the developing world.

The framework of statistical scaling is based upon the observed scale-invariant behavior of hydrological processes in time and/or space (Sposito 1998), which includes the scaling properties of floods with respect to drainage area (Gupta and Waymire 1990; Smith 1992; Gupta et al. 1994; Gupta and Dawdy 1995; Goodrich et al. 1997; Gupta and Waymire 1998; Schertzer et al. 2002; Ogden and Dawdy 2003). Statistical scaling provides an adequate compromise between the physical description of detailed hydrological processes and a more simplified, larger scale approach having a smaller number of degrees of freedom (Menabde and Sivapalan 2001; Gupta 2004). This approach has been used to study "downstream hydrological processes" (Leopold and Maddock 1953; Jarvis and Woldenberg 1984), with basin area as the scale parameter. The scaling framework also constitutes an adequate methodology to tackle the PUB problem for average and minimum annual river flows (Vogel and Sankarasubramanian 2000; Furey and Gupta 2000). Relevant hydroecological processes exhibit spatial scaling properties in our region of interest, such as mesoscale convective systems (Poveda and Mejía 2004), vegetation activity (NDVI) (Poveda and Salazar 2004), and minimum river flows (Poveda et al. 2002).

Our approach to estimate extreme river flows of different return periods along the river network is based on the classical theory of quantile analysis (Chow 1951), in combination with scaling ideas. There exist clear power laws that relate mean and maximum flows with basin area (Gupta and Waymire 1990; Smith 1992; Gupta and Dawdy 1995; Vogel and Sankarasubramanian 2000). In turn, the long-term water balance equation for river basins is useful to estimate long-term average annual river flows

(Eagleson 1994). Toward that end, we constructed maps of long-term mean annual precipitation and evapotranspiration (actual and potential), using diverse input data sets and Kriging methods for interpolation purposes. The results of this work have been incorporated into an *Interactive Digital Hydrological Atlas of Colombia*, coined as "HidroSIG."

This paper proceeds as follows. The next section describes the data sets followed by the estimation methods. The results, presented next, including the constructed maps of precipitation, evapotranspiration (actual and potential) and runoff, and a discussion of error estimation and validation of results. A brief description of HidroSIG is presented at the end of that section, and the conclusions are drawn in the final section.

#### Data

### Digital Elevation Model and River Network Extraction

Topography is an essential component of this work. We used GTOPO30, a digital elevation model (DEM) from the U.S. Geological Survey. GTOPO30 provides regularly spaced elevations at 30 arc sec (≈1 km). We used the steepest descent method to extract river networks from the topographic DEM, a task requiring high algorithmic efficiency (Band 1986; Garbrecht and Martz 1994). Rigorous quality-control procedures were applied to solve problems within the DEM, to check for consistency with actual river networks, to account for geologic controls, to eliminate errors, and to resolve the appearance of spurious sinks or sources, especially on low-slope terrains and flood plains. Details on these procedures may be found in Ramírez and Vélez (2002).

#### Precipitation

Precipitation data were obtained from Colombian Meteorologic Agency (IDEAM), Empresas Públicas de Medellín (EPM), Corporación del Valle del Cauca (CVC), and Centro Nacional de Investigaciones del Café de Colombia (Cenicafé). A total of 688 rain gauges were used to estimate the average rainfall field, using monthly data for the period 1965-1987. Rain gauges are located mostly along the Andes, but also over central western, and northern Colombia. Long-term data sets over the Colombian portions of the Orinoco and Amazon River Basins are very scarce. To overcome this limitation we digitized and used previous studies and maps developed by the National Water Assessment Study [Estudio Nacional de Aguas, DNP (1984)], and by Oster (1979) and Snow (1976). For the Colombian portion of the Amazon Basin, we used the average precipitation map from the EOS-Amazon Project (INPE-University of Washington; http:// boto.ocean.washington.edu/eos/). Information from neighboring countries was also used to estimate the interpolated rainfall field, including some data from Brazil (LBA-HydroNET; http:// www.lba-hydronet.sr.unh.edu/) and Ecuador (Pourrut 1994).

#### **Temperature**

Given the tropical setting of Colombia, average air temperature (T in Celsius) is strongly likened to altitude above sea level (h in meters) with no major changes throughout the year. Chaves and Jaramillo (1998) regionalized such a relationship for Colombia, using the information of 1,002 gauging stations, as follows:

Andean: T = 29.42 - 0.0061h

Caribbean: T = 27.72 - 0.0055h

Pacific: T = 27.05 - 0.0057h

Eastern Plains and Amazon: T = 27.37 - 0.0057h

The number of stations used to fit these equations was 626, 239, 46, and 91, respectively, with estimated values of  $R^2 \ge 0.9$  in all the cases. The empirical linear dependence between average air temperature and height is in close agreement with the adiabatic saturated lapse rate, typical in tropical regions (Wallace and Hobbs 1977), with minor regional differences associated with air humidity. The aforementioned relations were applied on the DEM to obtain a map of estimated temperature. For evapotranspiration estimation methods that use temperature data at 2 m above ground, we combined the estimated temperature field with information from the global data sets for land-atmosphere models (Sellers et al. 1995; Meeson et al. 1995). Similar relations (not shown) were used to estimate minimum and dew point temperatures.

#### Average Atmospheric Pressure

Data pertaining to surface atmospheric pressure and height at 153 gauging stations were obtained from Eslava (1995) covering the period 1951–1980. The resulting estimated relationship between surface atmospheric pressure, (*P* in hectapascals), and altitude (*h* in meters above sea level), is

$$P = 1009.28 \exp\{-h/8631\}; R^2 = 0.978$$
 (1)

It is worth noting that this equation is in close agreement with the theoretical hydrostatic approximation with a scale height of 8,600 m (Wallace and Hobbs 1977).

#### Radiation

We used monthly data obtained from the surface radiation budget as derived by NASA/Langley Research Center (Darnell et al. 1996) during the period 07/1983-06/1991. The data set includes the following parameters: all-sky surface downward short-wave (SW) and long-wave (LW) radiation, as well as SW and LW net fluxes, clear-sky downward SW and LW fluxes, and cloud fraction, gridded at 1° resolution.

#### Atmospheric Moisture

For this work, data belonging to 45 ground stations from IDEAM were complemented by the large scale maps (1  $^{\circ} \times 1 ^{\circ}$ ) produced by the Global Energy and Water Experiment (GEWEX) (Sellers et al. 1995; Meeson et al. 1995). In general, the observed long term average atmospheric moisture in Colombia shows strong dependence on altitude and distance from the neighboring seas.

# Wind Velocity

Long term records of wind speed and direction are very scarce in Colombia, and are limited to a few stations located in major cities and airports. We complemented those records with the GEWEX Global Data Sets for Land-Atmosphere Models (Sellers et al. 1995; Meeson et al. 1995). The corresponding daily, monthly and yearly average were computed from the original 6 h resolution data.

#### Vegetation Cover

According to Holdridge's pioneering classification (Holdridge 1978), Colombia harbors 26 life zones. Field studies of vegetation and biota, in general, allow the identification of those life zones (IGAC 1998). In view of the lack of hydrologic observations, we can turn around this ecological theory and use life zones as a proxy to estimate evapotranspiration using the rest of the climatic variables. Such map of life zones was digitized to use in the computations and as an independent variable of interest in the constructed database. See details in Vélez et al. (2000).

# Soil Water Holding Capacity

One of the methods to estimate actual from potential evapotranspiration requires quantification of the plant extractable water holding capacity (Vélez et al. 2000). Data for our region of interest was extracted from data sets produced by Dunne and Willmott (1996).

# **Estimation**

# Interpolation of Long-Term Annual Average Precipitation

From long-term point averages, the interpolated long-term average annual precipitation field was estimated using Kriging with drift (Bras and Rodríguez-Iturbe 1984). Kriging is an optimal linear interpolation method that incorporates the spatial correlation structure of the random field to define weighting factors. Topography was chosen as the drift function for interpolating precipitation fields. This decision was made based on the strong orographic forcing that the Andean Mountains impose on tropical rainfall over northern South America at different time scales (Poveda and Mesa 2000; Poveda et al. 2005). Interpolation of rainfall fields is improved by using topographic parameters as auxiliary information (Gómez-Hernández and Cassiraga 2000; Goovaerts 2000; Diodato 2005), especially for long-term annual averages. The direction of maximum isotropy in the precipitation field, was determined to be N30E, coinciding with the main orientation of the Andes across Colombia. We explored seven different models to obtain the interpolated precipitation field, varying the drifting map (topography and previous precipitation maps) and using different options within the Kriging method. Error analysis in the water balance equation closure lead us to define the most accurate precipitation map, as discussed in the results section. For a detailed analysis of different options to develop the long-term average annual precipitation map for Colombia, see Mejía et al. (1999).

### Long-Term Annual Average Evapotranspiration

Long-term annual average fields of actual and potential evapotranspiration were estimated using the methods introduced by Turc (1955, 1962), Coutagne (1954), Thornwaite (1948), Holdridge (1978), Meyer (1942), Penman (1948), Budyko (1974), Morton (1983), and Cenicafé (Chaves and Jaramillo 1998). Except for the latter, all methods are well detailed in the literature and they will not be reviewed here. *Cenicafé* is an empirical equation,  $E_p$ =1,700 exp(-0.0002h), where  $E_p$  represents annual potential evaporation (Penman's) (mm/year), and h represents the height above sea level (m).

For those methods which solely provide estimates of potential evapotranspiration, estimates of actual evapotranspiration E, were obtained using the pioneering formulation proposed by Budyko (1974)

$$E = P \left[ \left( 1 - \cosh \frac{E_p}{P} + \sinh \frac{E_p}{P} \right) \frac{E_p}{P} \tanh \frac{P}{E_p} \right]^{1/2}$$
 (2)

which is based upon a combination of both the annual water and energy balances, with the underlying assumption that potential evapotranspiration can be accurately approximated as the mean annual value of net radiation [see Budyko (1974, p. 338, Fig. 99)].

# Long-Term Mean River Flows

The differential equation for water balance of a drainage basin is given by (Manabe 1969; Schaake 1990)

$$\frac{dS(t)}{dt} = P(t) - E(t) - R(t) \tag{3}$$

where S(t) represents soil and ground water storage as a function of time; P(t) and E(t) represents basin-averaged precipitation and actual evapotranspiration rates in units of depth of per unit time. R(t) represents the total runoff leaving the basin including the streamflow at the basin outlet and the net integrated lateral subsurface runoff. Integrating Eq. (3) over long time scales, say of time length T gives

$$\frac{1}{T}[S(T) - S(0)] = \frac{1}{T} \int_{0}^{T} [P(t) - E(t) - R(t)] dt = \bar{P} - \bar{E} - \bar{R} \quad (4)$$

where the overbars denote time average according to the mean value theorem. As S remains finite as T is increased, the quantity [S(t)-S(0)]/T goes to zero. Thus, the long-term approximation for the water balance gives

$$\bar{R} = \bar{P} - \bar{E} \tag{5}$$

To illustrate the validity of Eq. (5) as an adequate long-term approximation, let us acknowledge that the magnitude of the maximum error is bounded above by

$$|\bar{P} - \bar{E} - \bar{R}| \le \frac{S_{\text{max}}}{n} \le 15 \text{ mm/year}$$
 (6)

where n=number of year (22 in this study, period 1966–1987) used to estimate the long-term averages for  $\bar{P}$ ,  $\bar{E}$ , and  $\bar{R}$ ; and  $\max[S(t)-S(0)]=S_{\max}$  is less than the maximum soil water holding capacity, which we take as 300 mm, a high value according to the Soil Map and Soil Climate Map (USDA-NRCS, Soil Survey Division, World Soil Resources, Washington D.C., http://www.nrcs.usda.gov/technical/worldsoils/). Using this value we obtain that the maximum error for the equation is 15 mm/year corresponding to less than 8% of runoff in arid zones in Colombia (200 mm/year), and less than 2% in wet regions, where runoff is in the order of  $10^3$  mm/year. The validity of our estimation method follows as it yields smaller errors than those associated with  $\bar{R}$ , and therefore with  $\bar{P}$  and  $\bar{E}$ .

Define the mean discharge out of the basin to be  $\bar{Q} = \bar{R}A$ , where A = area of the basin, and assume further that  $\bar{Q}$  is equal to the discharge through the main channel at the outlet. This simplification avoids the difficulty of estimating the net subsurface water budget, and it is almost exact for large basins where groundwater

flow enters the river network as baseflow. In some regions this assumption can lead to some error. The validity of this approximation for the Colombian case will be discussed in the section entitled "Results."

To simplify notation and because of assumed ergodicity, one can replace time averages for expected values, therefore, over bars will be dropped hereafter. The equation

$$Q = A[P - E] \tag{7}$$

is taken as the methodological basis of this study. Long-term river discharge is therefore estimated by integration of the long-term pointwise averages P(x,y) and E(x,y) of precipitation and actual evapotranspiration over the basin, as

$$Q = \int \int_{A} [P(x,y) - E(x,y)] dx dy$$
 (8)

The numerical approximation to Eq. (8) is done over raster maps in HidroSIG via

$$Q \cong \sum_{i,j \in A} (P_{i,j} - E_{i,j}) \Delta_{i,j}$$
 (9)

where  $\Delta_{i,j}$  denotes the area of the (i,j)th pixel in the DEM. For validation purposes, discharge records from more than 200 gauged sites throughout Colombia were compared with estimated long-term river discharges through the water balance equation using different methods to estimate actual evapotranspiration.

# Regionalization of Floods

Our methodological approach to estimate peak flows for different return periods is based on the classical quantile analysis (Chow 1951). This method is used in combination with scaling theories to estimate statistics of annual floods in terms of mean annual river flows. Observations show the existence of power law relations between mean and maximum flows. Besides, both long-term average river flows and annual floods exhibit scaling properties with basin area (Vogel and Sankarasubramanian 2000; Gupta and Waymire 1990; Goodrich et al. 1997; Gupta 2004). Those references contain a theoretical explanation for these observations, both in the framework of simple scaling or multiscaling. Thus, we estimated annual floods,  $Q_{\max}(T_r)$ , for a given return period  $(T_r=1/p)$ , the inverse of the exceedance probability), as (Chow 1951)

$$Q_{\text{max}}(T_r) = \mu_{Q_{\text{max}}} + k(T_r, \gamma)\sigma_{Q_{\text{max}}}$$
 (10)

where  $\mu_{Q_{\text{max}}}$ , and  $\sigma_{Q_{\text{max}}}$ =mean and standard deviation of annual floods and  $k(T_r, \gamma)$ =frequency factor, which is a function of the return period and possibly of other parameters that are generically represented in  $\gamma$ . For different probability distribution functions (PDF) assigned to annual floods, the functional form of k is different (Chow 1951, 1964). After some exploration with various distributions and the standard statistical tests, we chose lognormal distribution, which is a particular case of the family of log-stable distributions whose structure is consistent with the theory of multiscaling (Zolotarev 1986). The frequency factor for the lognormal distribution may be found in Chow (1964, pp. 8–25). We assume a power law relating the statistical parameters of annual floods and mean annual flows, Q, which in turn can be expressed in terms of the long-term water balance equation

$$\mu_{Q_{\text{max}}} = \alpha_{\mu} Q^{\theta_1} = \alpha_{\mu} [A(P - E)]^{\theta_1}$$
 (11)

$$\sigma_{Q_{\text{max}}} = \alpha_{\sigma} Q^{\theta_2} = \alpha_{\sigma} [A(P - E)]^{\theta_2}$$
 (12)

where the *prefactors*  $\alpha_{\mu}$  and  $\alpha_{\sigma}$ , and the *scaling exponents*  $\theta_1$  and  $\theta_2$ , are fitted from observed data. The rationale for using the mean flow as the scaling parameters for the distribution of floods may be appreciated from Eqs. (11) and (12). Basin area alone does not represent the scale of the phenomena in view of the climatic differences, represented by P and E.

To estimate the parameters in Eqs. (11) and (12), a least mean square error analysis was performed between observed and estimated mean and standard deviation of the annual floods within 13 different hydro-climatic regions of Colombia (not shown). The historical information used for this analysis comprised 225 river gauging stations, 65% of which have at least 24 years of record and less than 5% of missing records; also, over 50% of time series are longer than 30 years. Inside each hydroclimatic region, we use the estimated pre-factors and scaling exponents into the righthand side terms of Eqs. (11) and (12). For a specific site which drains subbasins from different hydroclimatic regions, we use the area to weight the prefactors for the corresponding regions. With those estimates, and the integrated maps of precipitation and actual evapotranspiration, we developed maps corresponding to the mean and standard deviation of annual floods using Eqs. (11) and (12). Finally, we can estimate floods associated with different return periods, for any site along the channel network, using the corresponding frequency factors in Eq. (10).

### Regionalization of Low Flows

A similar approach was adopted toward estimating annual (daily) low flows, using the mean and standard deviation of the annual minima fitted by the lognormal distribution. The low flow statistics are related to mean annual discharge as (Poveda et al. 2002)

$$\mu_{Q_{\min}} = \beta_{\mu} Q^{\theta_3} = \beta_{\mu} [A(P-E)]^{\theta_3}$$
 (13)

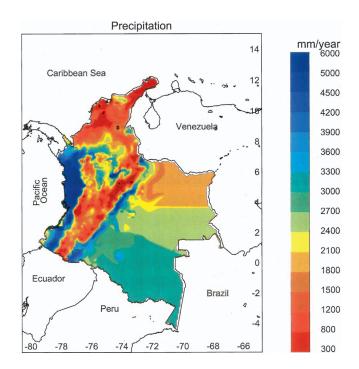
$$\sigma_{Q_{\min}} = \beta_{\sigma} Q^{\theta_4} = \beta_{\sigma} [A(P - E)]^{\theta_4}$$
 (14)

Similarly, the prefactors  $\beta_{\mu}$  and  $\beta_{\sigma},$  and the scaling exponents,  $\theta_3$  and  $\theta_4,$  in Eqs. (13) and (14), were also regionalized according to the same procedure followed for floods. A total of 240 gauging stations having more than 20 years (less restricted than flood data), were used in the analysis. Once the scaling exponents and prefactors of Eqs. (13) and (14) were estimated, we produced maps for the mean and the standard deviation of minimum flows. Such maps and the corresponding equation analogous to Eq. (10) for low flows, allowed estimation for different return periods along the river network.

#### Results

# Long-Term Annual Precipitation

Hydrological fields were estimated with data sets for the period 1966–1987. Fig. 1 shows the estimated long-term average precipitation field for Colombia at 5 arc min resolution. Visual inspection reveals features consistent with characterization of precipitation in Colombia reported in literature: (1) extremely high precipitation occurs along the Pacific coast witnessing one of the rainiest regions of the world (Poveda and Mesa 2000), with 8,000–12,000 mm/year; but also the middle Magdalena river valley and the eastern piedmont of the Andes (5,000–6,000 mm/year), and the Colombian Amazon; (2) the so-



**Fig. 1.** (Color) Long-term average annual precipitation field for Colombia, obtained through interpolation using Kriging with drift (see the text for details)

called "pluviometric optimum" inside the intra-Andean valleys (Hastenrath 1991; Oster 1979), and (3) the dry Guajira peninsula (300–400 mm/year) by the Caribbean Sea. Measures of the variance of estimations from the Kriging method indicate that the largest interpolation errors correspond to regions with scarcer data (not shown). As it was already noticed, the main shortcomings of the estimated precipitation map are the paucity of long-term basic data.

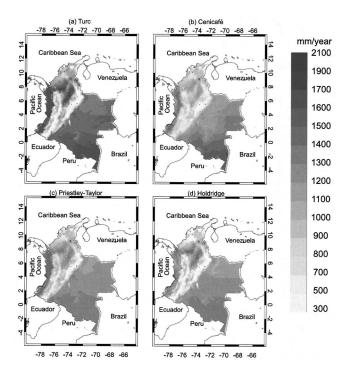
### Long-Term Annual Actual Evapotranspiration

From all the considered methods, only results of water balance using those of Turc, Morton, Cenicafé, Holdridge, and Penman are shown here. These methods were selected because they yielded less errors in closing the water balance equation, and no single method stands out from the other four. From theoretical reasons one could recommend Morton's method. From a practical viewpoint, Turc's methods is more appealing.

*Turc*: Actual evaporation is based on mean temperature and precipitation, both strongly associated with topography in the tropics. Fig. 2(a) shows the derived long-term average actual evapotranspiration map for Colombia.

*Cenicafé*. Estimates of evapotranspiration are based on a simple linear regression between estimates of evapotranspiration (through Penman equation) and elevation above sea level. It weighs the influence of topography, terrain and vegetation complex along the Andes. As Penman estimates potential evaporation, Budyko's Eq. (2) is used to estimate actual evaporation. See map in Fig. 2(b).

*Penman*: Results obtained with this method exhibit an underestimation when compared to estimates of E=P-Q, with observed values of P and Q. This could be due to the fact that most input variables are obtained from very coarse satellite information. The approximation given by Priestley and Taylor (1972), produced much better closure of the water balance equation.



**Fig. 2.** Long-term average annual actual evapotranspiration estimated using the methods by (a) Turc-Budyko; (b) Cenicafé; (c) Priestley-Taylor; and (d) Holdridge

From the obtained potential evapotranspiration map, the actual evapotranspiration map was estimated through Budyko's Eq. (2). Fig. 2(c) shows the map for E using the Priestley-Taylor approximation.

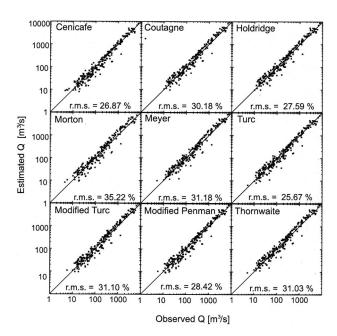
*Holdridge*: This method produced acceptable results under the water balance closure. See map in Fig. 2(d).

Morton: The estimates of both actual and potential evapotranspiration arising from this method tended to overestimate evaporation, perhaps due to very coarse input data for solar radiation and wind velocity. Recent studies have revisited the rationale of this method (Szilagy 2001), which contributed to explaining the so-called evaporation paradox of climate change (Brutsaert and Parlange 1998), and which performs adequately for humid environments (Hobbins et al. 2001) like Colombia's.

The performance of each method used to estimate actual evapotranspiration was compared with estimates from the long-term water balance equation in gauged basins. The results are presented in the following section.

# Long-Term Average Annual Stream Flows

Estimation of average stream flows was done via Eq. (9). For instance, our estimations yield a long-term average annual precipitation of 2,049 mm/year over the basin of the Magdalena River up to the Calamar gauging station near the mouth (259,931 km²), the actual evaporation estimation using Turc's method is 1,131 mm/year, and the long-term river flow estimate is 8,034 m³/s. The measured mean annual flow is 7,593 m³/s, and the relative error in this large basin that drains a very complex terrain, is 5.8%. Water balance estimates were compared to observed long-term river flows at more than 200 gauging stations in Colombia. Fig. 3 shows observed (abscissae) and estimated (ordinates) long-term river flows, using different evapotranspiration methods. Table 1 shows statistics of those errors for the evapo-



**Fig. 3.** Comparison of observed and estimated long-term average river flows at more than 200 river gauging stations in Colombia, using diverse evapotranspiration methods

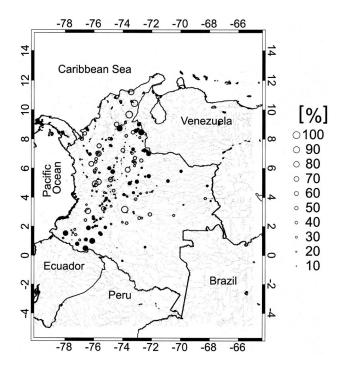
transpiration estimation methods listed in the last section.

An investigation of the geographical distribution of errors indicated no systematic pattern. Fig. 4 shows the spatial distribution of errors for estimated discharges at all gauged sites using Morton's method to estimate actual evapotranspiration. Overall, the magnitude of the measured errors is not small. For some basins one could suspect problems with discharge gauging, or systematic errors in precipitation measurements due to inadequate raingage networks, or both. But errors are significant for a considerable number of basins. For instance, with Turc's method, only 37% of the basins have absolute relative error less than 10%. It is also worth noticing that the probability distribution of the error is positively skewed for all cases, and that the majority of basins exhibit positive errors for most of the methods (65% for Turc's) except for Morton's (47%). One needs to bear in mind the fact that there is no possibility to fit any parameter in any of the methods we use. Nevertheless, for theoretical reasons and since there are basins that do exhibit small errors, we were led to conclude that the errors are mainly due to the low quality and scarceness of the data available. There is an obvious need for improved spatial coverage of hydroclimatological measurements in order to improve the quality of estimations.

Further considerations of the sources of errors are in order. In particular, one needs to consider the possibility of a spurious

**Table 1.** Root Mean Square and Quantiles of the Relative Percent Errors of the Estimated Mean Flows of 200 Colombian Basins for Different Methods Used to Estimate Actual Evapotranspiration

Method	RMSE	$E_{0.10}$	$E_{0.20}$	$E_{0.50}$	$E_{0.80}$	$E_{0.90}$
Cenicafé	23	-22	-13	7	26	50
Holdridge	24	-15	-3	17	40	62
Morton	23	-47	-25	-2	22	33
Turc	23	-21	-10	7	26	45
Penman	28	-7	5	29	56	82

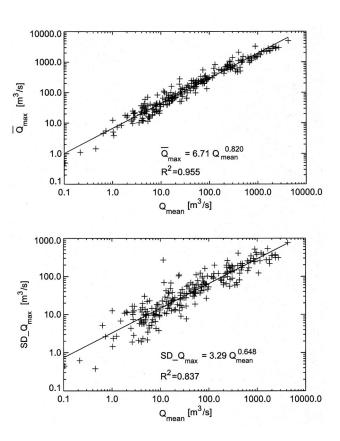


**Fig. 4.** Distribution of errors between observed and estimated long-term river discharges at gauging stations, using Morton's method to estimate actual evapotranspiration

agreement between Q and P-E. This may happen if errors in P and in E do compensate each other. Also, because of the space integration within each basin, there may be cancellation of errors in one or both of the variables, P or E. For the first case, one needs to consider that such errors would be reflected in the energy balance, too. In fact, the available net solar radiation is partitioned into warming (sensible heat) and evapotranspiration (latent heat). Therefore, assuming that good-quality solar radiation data is available, errors in evaporation estimates would be reflected in opposite sign errors in estimated air temperature, especially in the Colombian humid climate, where evaporation is mostly energylimited instead of water-limited. With the kind of data sets available for both solar radiation and surface air temperatures, we do not consider this first possibility to be of first order in our analysis. The second possibility is that spatial compensation of errors will be reflected in the water balance of the subbasins. One cannot discard this possibility altogether but there is no evidence of an overall tendency in the errors either with basin area, altitude or geographical location.

We mentioned the lack of consideration of groundwater runoff as a source of real discordance between P-E and Q. One should notice first that, in the long run, sources should be compensated with sinks. Such sources correspond with the so-called recharge areas from groundwater, which are more important in arid or semiarid climates, but not so in humid climates like Colombia's. One evidence that has some bearing into this issue is the linear dependence of low flows with basin areas, as will be discussed below. This fact indicates that groundwater flows into the river network mostly uniformly in space.

From these analysis, we conclude that the main source of errors in the closure of the water balance resides mainly in the data bases (quality and spatial coverage). Few antecedents on the systematic validation of methods for estimating actual evapotranspiration at this scale appear in the works of Vörösmarty et al.



**Fig. 5.** Regressions between average river flows and statistical parameters of annual floods at 225 stations in Colombia; results are shown for the mean (top) and standard deviation (bottom)

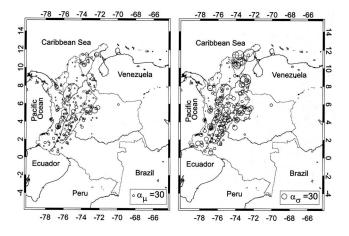
(1998), and Choudhury (1999), contemporaneous with most of the work reported in this paper. Our results are similar to these studies, taking into consideration obvious differences in the amount and quality of the observational database.

#### **Annual Floods**

Estimates of the parameters of Eq. (11) showed little variation among different hydroclimatic regions, with average prefactors  $\alpha_{\mu}{=}6.71,$  and scaling exponents  $\theta_1{=}0.82$  (Fig. 5). A similar analysis was performed to estimate the standard deviation of annual maximum flows. Regression analysis yielded  $\alpha_{\sigma}{=}3.29$  (varying regionally), and highly stable scaling exponent,  $\theta_2{=}0.648.$  Fig. 6 shows the distribution of  $\alpha_{\mu}$  and  $\alpha_{\sigma}$ , from which floods with different return periods are estimated via Eq. (10). For instance, the 100 year flood at the chosen site on the Magdalena River was estimated as 14,197 m³/s, with mean and standard deviation of annual maximum flows estimated as 10,527 and 1,169 m³/s, respectively.

#### Low Annual Flows

Estimation of statistical parameters for Eqs. (13) and (14) produced very similar scaling exponents  $\theta_3 \approx 1$  and  $\theta_4 \approx 1$ , throughout the country. Fig. 7 shows the results for both scaling exponents in gauged basins. These results indicate the existence of simple scaling in the statistics of low flows with basin area, which was confirmed through analysis of linearity in the structure functions of moments up to the fourth order, as reported in Poveda et al. (2002). The prefactors  $\beta_{\mu}$  and  $\beta_{\sigma}$  exhibited remarkable



**Fig. 6.** Maps of  $\alpha_{\mu}$  [left, Eq. (11)] and  $\alpha_{\sigma}$  [right, Eq. (12)], estimated for selected hydrological regions of Colombia

stability among regions (not shown). For instance, using the lognormal distribution, the estimated 50 year return period low flow of the Magdalena river at the aforementioned site is 1,294  $\rm m^3/s$ , whereas mean and standard deviation of annual low flows are 2,539 and 794  $\rm m^3/s$ , respectively.

#### **HidroSIG**

All data sets, methods and results of this work have been incorporated into HidroSIG, the aforementioned geographic informa-

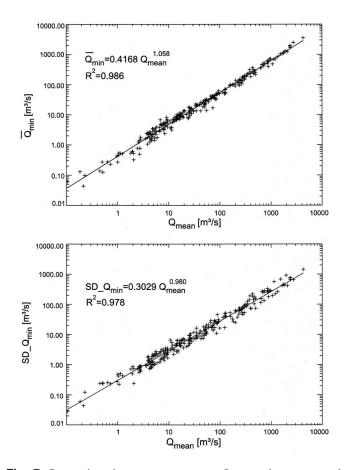


Fig. 7. Regressions between mean streamflows and mean annual minimum flows

tion system. It contains a complete hydroclimatological data base for Colombia, and a series of tools for visualization, interpolation and analysis of spatially distributed hydroclimatic variables, and time series analysis. Additionally, HidroSIG contains modules to estimate, store, and display geomorphological information and parameters from DEMs, including extraction of the stream channel network, river basin areas, relevant geomorphological parameters, etc. HidroSIG has been developed during the last 10 years, and constitutes the hydrological and computational basis of a recently developed tool to study hydrological processes (Mantilla and Gupta 2005), and is freely available. Currently, the Ministry of Mining and Energy of Colombia uses HidroSIG to estimate the hydropower generation capacity of the country. Detailed information can be found in http://cancerbero.unalmed.edu.co/hidrosig/index.php

### **Conclusions**

Estimates of river flows (mean and extremes) along Colombia's main channels were developed through the conjoint use of the long-term water balance equation in river basins, and the framework of statistical scaling of hydrological processes. Both methods are long known in hydrological practice and research, but their combined usage constitutes a contribution of this work. To that end, interpolated maps of long-term mean annual rainfall and evapotranspiration, both actual and potential, were constructed for the country. The method of Kriging with drift was used for interpolation of at-a-station information, and maps from previous studies. We tested the validity of the long-term water balance equation with independent measurements of river discharges at more than 200 river gaging station throughout the country. Our analysis of the estimation errors (20–30% RMSE) concludes that data quality and spatial coverage are the main source of errors, in particular rainfall variability coming from strong topographic controls in a very active tropical convective region. No method for estimating evapotranspiration showed a significant superiority among the methods of Turc, Morton, Cenicafé, Holdridge, and Penman, with some arguments in favor of the first and the second. Again, such inability to discriminate among the different evapotranspiration methods is attributed to low quality of basic information. Results of the water balance to estimate long-term average river flows were used to introduce a methodology that estimates both peak and low flows for different return periods. The method combines the traditional quantile analysis within a scaling framework that relates average and extreme river flows through hydroclimatological parameters. We have developed a hydroclimatic atlas for Colombia, HidroSIG, a GIS that stores, estimates, and deploys all calculations and results from this study. The software and the hydroclimatological data base of Colombia constitute available contributions of this work.

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