# CVEN5313: Environmental Fluid Mechanics

Navier Stokes: Kinematics    Due: Friday, 11/5/09

1. Two solid plates in the $x_1 - x_3$ plane are located at $x_2 = -b$ and $x_2 = b$. Flow is driven in the $x_1$-direction between the plates by a pressure gradient. The resulting fluid flow is called a Poiseuille flow, and the velocity field is given by

$$\vec{u} = \left[U_0 \left(1 - \left(\frac{x_2}{b}\right)^2\right), 0, 0\right]$$

   (a) Sketch the physical setup described above; show the shape of the velocity profile in the sketch.

   (b) Calculate and decompose the velocity gradient tensor for this flow. Note that it varies spatially!

   (c) Quantify each of the following kinematic decompositions for this flow (in the given coordinate system).

      i. Linear Strain Rates

      ii. Angular Strain Rates

      iii. Vorticity

   (d) Use an eigenvalue analysis to compute the location of the principal axes, as well as the principal rates of strain. Do the principal axes and principal rates of strain vary spatially, or are they the same everywhere in the flow?

   (e) Sketch the transformation of a square fluid element aligned with the $x_1 - x_2$ axis using three discrete steps using each of the three elemental kinematic motions (linear strain, angular strain, and solid body rotation). You will want to provide separate answers for $x_2 < 0$, $x_2 = 0$, and $x_2 > 0$.

   (f) Repeat part (f) for a fluid element that starts out as a square oriented in the principal axes.

2. View the fluid element animation for a Couette flow.

   (a) Based on your knowledge of the kinematic decomposition for this flow (we performed the eignenvalue analysis on it in lecture), identify straining behaviors in the movie that appear to conflict with the behavior predicted by the flow decomposition. Discuss the blue and red elements separately.

   (b) Explain the cause of the discrepancy between what you expected and what you see in the movie. Hint: If you look at very small times (e.g., the first frame or two), the discrepancies are not evident yet.

3. Consider the unsteady decaying viscous vortex flow described by

$$v_\theta = \frac{\Gamma_0}{2\pi r}\left[1 - \exp\left(\frac{-r^2}{4\nu t}\right)\right]$$

(a) Plot the spatial and temporal evolution of the flow in the $r-\theta$ plane. Choose a range of $r$ and $t$ values that make the behavior clear.

(b) Calculate the vorticity, and plot in the same manner as above.

(c) Calculate the circulation $\Gamma(R,t)$ around a circle of radius $R$ centered at $r = 0$. Then do the following:

i. Plot and explain the behavior of $\Gamma(t)$ for any fixed (finite) $R$.

ii. Plot and explain the behavior of $\Gamma(t)$ as $R \to \infty$.

iii. A scaling problem: Where does the majority of the vortex's circulation (and thus vorticity) reside? Express your answer in terms of the variables given in the problem (no numbers needed in the final answer). How does the size of this region change as time passes?

4. The $x_1$-direction velocity distribution for a narrow two-dimensional jet of incompressible fluid in the $x_1$-$x_2$ plane is given to be

$$u_1 = U{x_1}^{-\frac{1}{3}}\operatorname{sech}^2\left(\alpha x_2 x_1^{-\frac{2}{3}}\right)$$

where $\alpha$ and $U$ are constants, and $x_1 > 0$.

(a) Find a streamfunction $\Psi$ for this flow such that $\Psi = 0$ when $x_2 = 0$. Plot a set of streamlines for the resulting streamfunction.

(b) Calculate $u_2$. Plot $u_2$ vs. $x_2$ for a fixed (positive) value of $x_1$? Explain physically why the plot looks like it does.

(c) If the jet width is defined to be the point at which $u_1(x_1) = e^{-1}u_1(x_1, 0)$, how does the jet width vary with $x_1$? Note: You may omit the constant of proportionality.

(d) For $x_1 > 0$, what is the net flowrate in the $x_1$ direction across any vertical line extending from $x_2 = -\infty$ to $x_2 = +\infty$? Explain why $Q = f(x_1)$.