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2008 Mathematical Contest in Modeling (MCM) Summary Sheet

Pardon My Intrusion

Sea level rise associated with climate change has received extensive media coverage in recent years. Myriad simulations exaggerate the rate of sea level rise and the associated impacts, often projecting rises in excess of 1-2 meters over the next century. In truth, the expected rise is on the order of 0.25 meters in the next five decades [Bindoff et al. 2007]. The impacts of such changes are subtle, and, as we will demonstrate, salty.

In this paper, we identify salt water intrusion as being among the most pressing issues related to sea level rise. Currently, Miami-Dade County obtains over 90% of its drinking water supply from groundwater. To model the temporal changes in salinity we obtained chloride data from 72 USGS monitoring wells spanning 90 kilometers of the greater Miami coastline. We then correlated changes in chloride concentration to changes in sea level using local polynomial regression, and forecasted trends in chloride concentration at each monitoring well over the coming decades. We interpolated between existing wells to obtain isochlors (lines of constant chlorine concentration) at 10-year time steps. The isochlors were used to identify potentially contaminated drinking water wells over the next 50 years. Based on our findings, many of the wells in Miami-Dade County are at risk of saltwater contamination. The highest forecasted rate of intrusion was 100 m/year.

The occurrence of saltwater intrusion presents a host of problems related to fresh water supply, particularly in Miami-Dade county. In response to the potential loss in fresh water supply we have formulated a resource management model to quantify the level of conservation necessary to offset losses. In light of the potential contamination of well fields, we estimate a necessary increase in conservation. Far from comprehensive, our forecast-management framework is an example of how mathematical models can be applied to solve regional problems.

1 Rising Issues

Global climate change and the melting of the polar ice caps are among the causes of rising sea levels. Parts of the world with high urban densities in areas of low elevation will be affected the most. Florida's coastline is densely populated, having 346 people per square mile of coastline.

Florida is particularly susceptible to the effects of sea level rise because of its relatively low elevation and tropical climate. Among the effects are the inundation of the Everglades, loss of barrier islands which serve as a hurricane buffer, and erosion of the coastline. In particular, rising sea levels will likely upset the delicate ecological balance in Florida's wetlands, home to numerous endangered species [Senarath 2005]. Tourism in Florida, the state's largest industry, will likely be impacted as well [Viner and Agnew 1999].

Contamination of coastal freshwater supply is among the most pressing issues facing metropolitan areas. Salt water intrusion, the inland movement of seawater into coastal aquifers, has already occurred along much of the Florida coast. We have chosen to investigate future trends in saltwater intrusion so that residents of Florida may better prepare for losses in fresh water supply.

2 Approaching a Mean Problem

The goal of this paper was to determine the effects of saltwater intrusion on the coast of Florida due to the melting of the polar ice caps, particulary in the Biscayne aquifer, which spans Miami-Dade and Broward Counties. Using chloride concentrations as a surrogate for saltwater we attempted to forecast the extent of saltwater intrusion, and to determine the degree to which existing groundwater pumping wells would be affected. More than ninety percent (>90%) of the fresh water supplied by the Southern Florida Water Management District (SFWMD) comes from groundwater [MDWASD 2007]. Conservation efforts were discussed to determine if the demand for water can be decreased, so that the available freshwater supply can meet the demand.

To evaluate the effect of saltwater intrusion on the Biscayne aquifer, representative data of historical sea levels and chloride concentrations were combined in a statistical model. We used the predicted chloride concentrations to identify pumping wells that may be at risk of contamination (Figure 1). A management model was formulated to determine an optimal conservation policy which would offset the potential losses in fresh water supply.

3 Data

3.1 Sea Level Rise Data

Monthly sea level data at Miami Beach was obtained from the Permanent Service for Mean Sea Level at `http://www.pol.ac.uk/psmsl/datainfo/` [PSMSL 2008]. As proposed by Walton

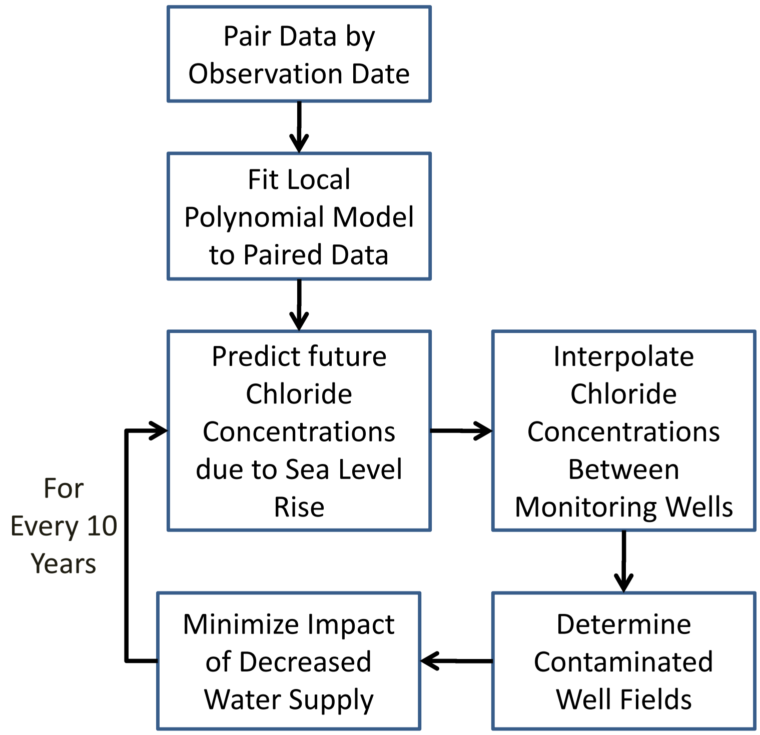


Figure 1: Forecasting and decision making algorithm.

[2007] we assumed Florida sea level rise to be of the form

$$h(t) = a_1 + a_2 t + a_3 t^2 \tag{1}$$

where h is the sea level at Miami Beach, t is time and the a_i's are fitting coefficients to be determined. Using this model for the Miami Beach sea level time series we forecasted until 2058 at 10 year time steps, beginning in 2008. Figure 2 shows the fit, which minimized the sum of the absolute values of the residuals.

3.2 Groundwater Chloride Concentration and Population Data

Chloride ion concentration data was obtained at 72 monitoring wells along a 90 kilometer stretch of coast spanning both Miami-Dade and Broward Counties (Figure 3). The data was obtained from United Stated Geological Survey at `www.sflorida.er.usgs.gov/edl_data/index_qw.html` [USGS 2008]. Population data for Miami-Dade County was obtained from the United States Bureau of the Census [USBC 2007]. We used the forecasted population data as an input to the management model. We are aware of the inherent uncertainty in the data, however, all forecasts are uncertain.

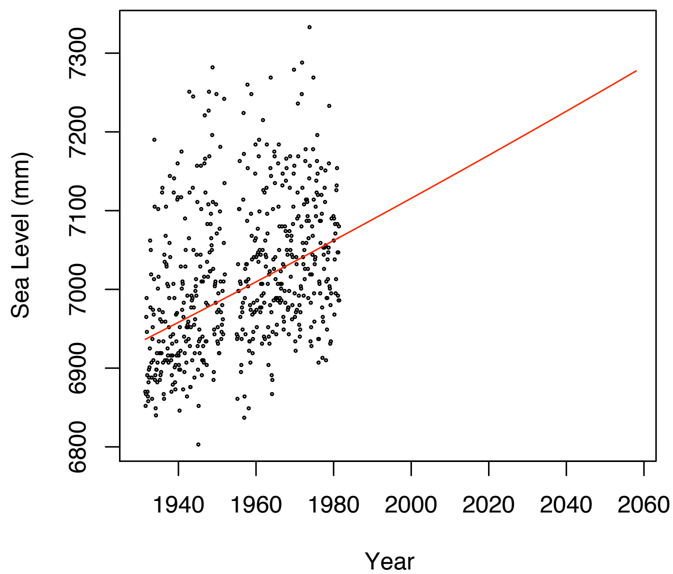


Figure 2: Model fit used for forecasting sea level rise.

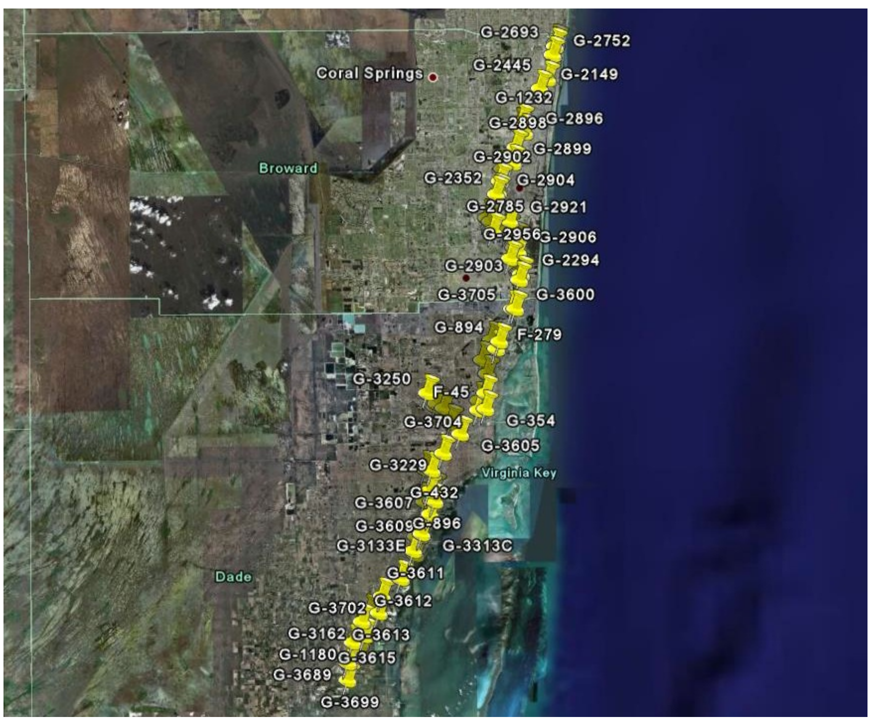


Figure 3: Monitoring well locations that tested for chloride concentrations [Google 2007].

4 Forecasting Chloride Concentrations

4.1 Local Polynomial Regression

In order to forecast saltwater intrusion over a large area we first forecasted the change in chloride concentration at each individual well. We used the nonparametric technique of local polynomial regression to model the change in chloride concentration over time [Loader 2006]. All regression was implemented in R [R 2007]. The model has the form

$$C = f(h) + \varepsilon \tag{2}$$

where C is the chloride concentration, h is the sea level and $\varepsilon \sim N(0,1)$ is the error term. An aspect of the regression which needs to be determined for each well site is the optimal smoothing parameter $\alpha \in (0,1]$, which is the proportion of the data used to fit the local polynomial at every point. An α value closer to zero corresponds to a greater response of the model to local changes in the data. In order to determine the optimal smoothing parameter the generalized cross validation (GCV) statistic was used. The GCV statistic is considered an indication of predictive risk and therefore the optimal value of α is that which minimizes the GCV statistic. The GCV statistic is given by

$$GCV(\alpha) = n\frac{\sum_{i=1}^{n}[C_i - \hat{C}_i(\alpha)]}{[n-\nu(\alpha)]^2} \tag{3}$$

where n is the number of data points, C_i is actual chloride concentration, $\hat{C}_i$ is fitted value and ν is the degrees of freedom of the fit [Regonda et al. 2006].

Some of the advantages of local regression over traditional parametric regression techniques are (1) no functional form of the model is assumed and (2) local regression can capture arbitrary nonlinearities in the data. The USGS chloride data had particularly nonlinear trends at many sites. Figure 4 shows some of the wells which exhibited increasing trends in chloride concentration.

4.2 That's All Well and Good - Combining Multiple Well Predictions

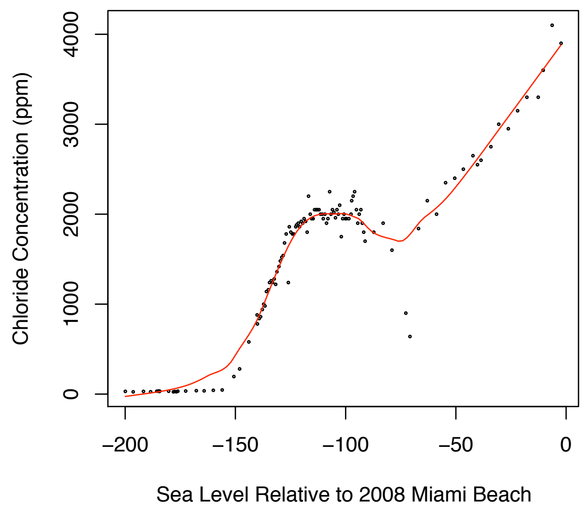
After predictions of chloride concentrations were made for each well site the concentrations were interpolated to a regular grid based on their relative spatial locations. The MATLAB function `griddata` was used for this purpose. The results of this are shown in the results and discussion section.

4.3 Assumptions of Chloride Concentration Correlation

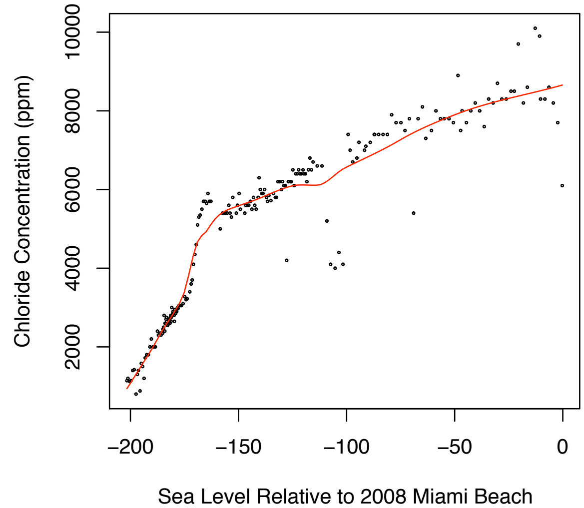
We justified the statistical correlation of chloride concentration and sea level data by assuming that a cause and effect relationship exists. We assumed that sea level rise is the predominant factor affecting saltwater intrusion and that changes in fresh water recharge are negligible. We have omitted corollary effects of climate change , e.g., changes in the frequency of intense storms.

5 A Passion to Ration

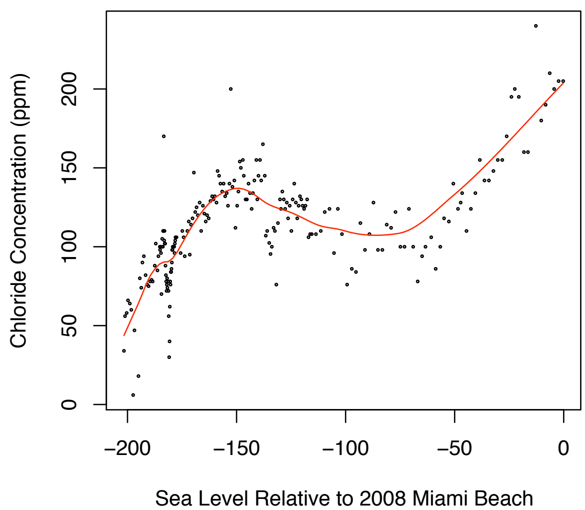
Thus far, we have presented a salt water intrusion forecasting model and offered predictions of the magnitude of intrusion in the coming decades. Although the forecasting model is interesting in its own right, the broader objective is to use it as a tool in the decision making process. In



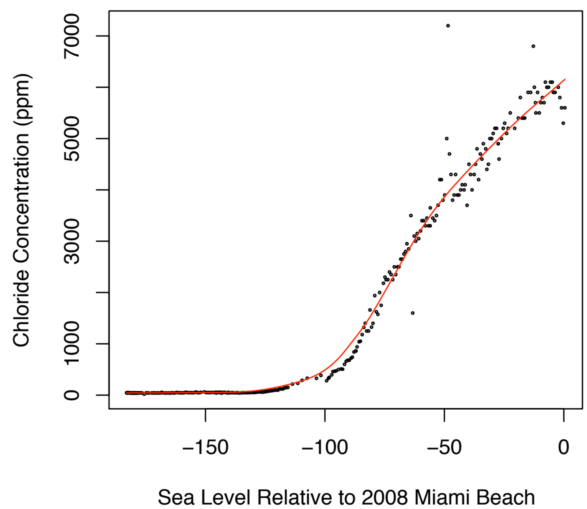
(a) Well G-432, α=0.35



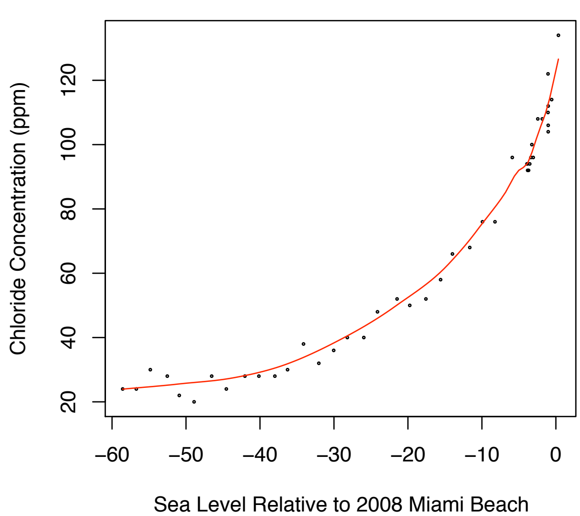
(b) Well G-1435, α=0.30



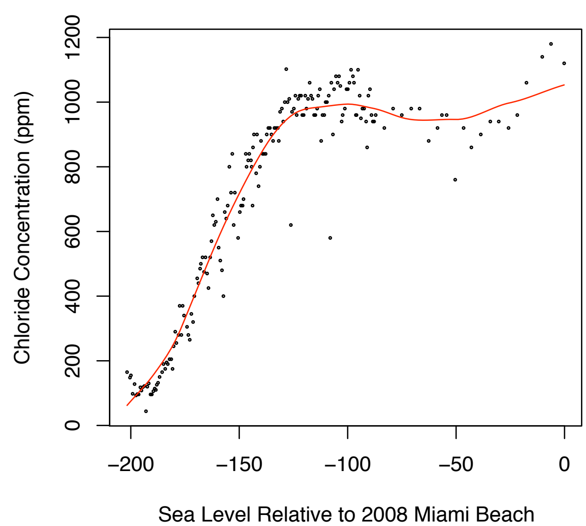
(c) Well G-2294, α=0.30



(d) Well G-2410, α=0.30



(e) Well G-2893, α=0.40



(f) Well G-3162, α=0.30

Figure 4: Local polynomial regression models for wells affected by sea level rise (all sea level units in mm).

light of the evidence of salt water intrusion along the Miami-Dade coastline, we have proposed a plan to mitigate the impact of losses in subsurface freshwater supply.

Namely, we attempted to answer the question: "Is it possible for the residents of Miami-Dade County to offset the loss of freshwater supply by conserving and/or recycling water?" To answer this question, we have formulated a water resource management model. The model was designed to yield optimal management decisions (optimal only within the framework of the simplifying assumptions and estimated parameters).

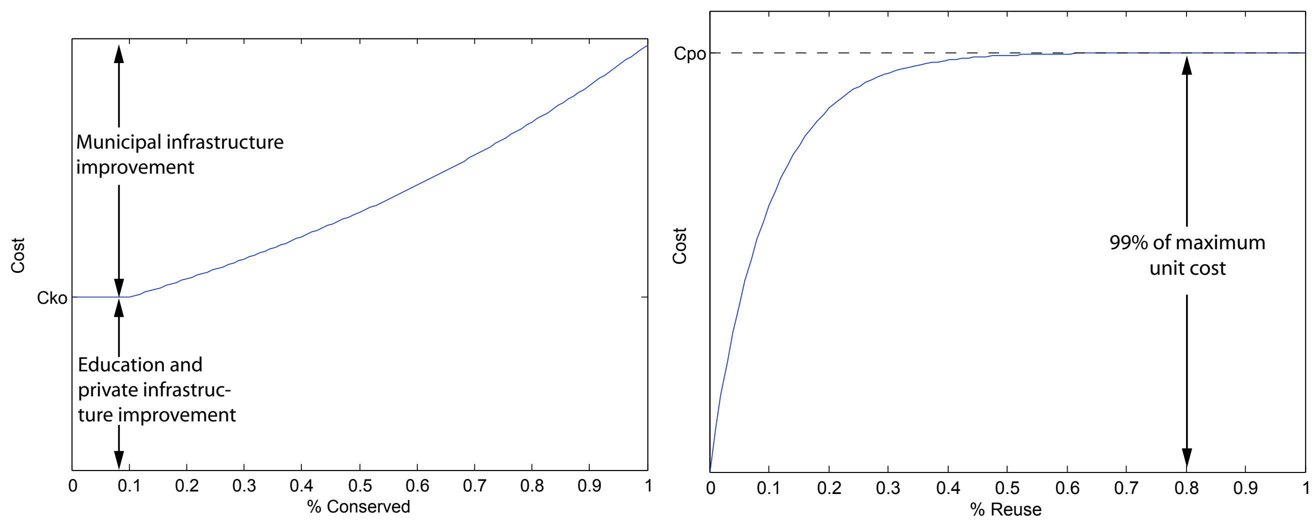
We began by consolidating information on population growth, water demand, and water supply in Miami-Dade County. Forecasted populations at the end of each decade in question were found at the United States Bureau of the Census [USBC 2007]. Population was represented in the model as a discrete function of time (Equation 4). Currently, there are three well fields which supply a majority of the county's water (Figure 8). The combined water supply from these fields is 413.25 million gallons per day (MGD), and the current demand is 154.87 gallons per capita per day (GPCD) [MDWASD 2007]. We have assumed that per capita consumption is independent of the total population, so that the total water demand D is a function of population $P(t)$, per capita demand d, and level of conservation κ (Equation 5). The level of conservation κ was defined as the percent reduction in per capita demand. Assuming negligible consumptive use (i.e., that water 'demanded' is eventually recovered at a waste-water treatment center and subsequently becomes available for reuse), the available water supply S is a function of the current groundwater supply S_0, potential loss in groundwater supply S_{loss}, and fraction of waste-water recycled into the chain of supply r (Equation 6). The daily volume of recycled water is then rD, while the remaining daily volume of waste-water is $(1-r)D$.

$$\begin{aligned} P(t_i) &= P_i \; ; \; i = 1, ..., 5 \quad (4) \\ D &= P(t)(1-\kappa)d \quad (5) \\ S &= S_0 - S_{loss} + rD \quad (6) \end{aligned}$$

Cost functions were developed to asses the cost associated with water supply, conservation, reuse, and waste-water treatment, respectively. The total cost of conservation C_κ was formulated as a function of the reduction in the present per capita demand κ and the volume conserved V_κ (Equation 7, Figure 5(a)). The conservation cost function is based on the following assumptions: (1) There is a baseline (over-head) cost associated with conservation education and minor changes in infrastructure (e.g., replacing toilets, faucets, water heaters, etc.), (2) after a given level of conservation K is reached, the marginal cost associated with further reductions in consumptive use begins to rise (e.g., due to major changes in infrastructure, fixing leaky municipal distribution systems, upgrading industrial equipment, etc.). The cost associated with recycling waste-water was formulated as a function of the fraction r and volume V_r of waste-water recycled (Equation 8, Figure 5(b)). This cost function is based on the assumption of an economy of scale, that is, the marginal cost of recycling waste-water decreases as the volume of recycled water increases. Lastly, we have assumed that the costs associated with water supply C_s and waste-water treatment C_w are proportional to their respective volumetric flows (Equations 9 and 10). (dizzy yet?)

$$
\begin{aligned}
C_\kappa &= C_{\kappa 0}[1+\left\{e^{\alpha_\kappa(\kappa-K)}-1\right\}\mu(\kappa-K)]V_\kappa & (7)\\
C_r &= C_{r0}[1-e^{r\alpha_r}]V_r & (8)\\
C_s &= C_{s0}V_s & (9)\\
C_w &= C_{w0}V_w & (10)\\
\mu(\kappa-K) &= \begin{cases} 0 & \kappa < K \\ 1 & \kappa \geq K \end{cases} & (11)
\end{aligned}
$$

Where $\mu(\kappa-K)$ is the Heaviside step function, $C_{\kappa 0}$ is the initial cost associated with conservation efforts, C_r is a maximum unit cost associated waste-water reuse, C_{s0} and C_{w0} are unit costs associated with supplying potable water and treating waste-water, respectively. Parameters α_κ and α_r are growth rates associated with the the marginal costs of conservation and reuse, respectively.



(a) Conservation cost function (b) Reuse cost function

Figure 5: Cost Functions

The management model is designed to minimize operational costs while satisfying the projected water demand. The decision variables in this model are: (1) the percent reduction in per capita water demand during each decade κ, and (2) the proportion of waste-water to recycle as potable supply r. The management model is given by the following system (Equations 12 and 13):

$$\text{Objective: } \min z = C_\kappa + C_r + C_s + C_w \tag{12}$$

$$\text{Constraint: } S - D > 0 \tag{13}$$

where the costs C_*, supply S, and demand D are ultimately functions of the decision variables κ and r. The solution of equations 12 and 13 is heavily dependant upon the values of the parameters $C_{\kappa 0}$, C_{r0}, C_{s0}, and C_{w0}. We did not have enough information to appropriately evaluate these parameters. Nonetheless, in the spirit of investigation, we assigned reasonable parameter values relative to one another (Table 1). At this point we would like to emphasize that the optimal solution, and therefore our management plan, is limited by the accuracy of the model parameters. The model is meant to serve only as an example of how the issue of water supply might be addressed.

Table 1: Cost Function Parameter Values

Parameter	Value	Description
$C_{\kappa 0}$	0.01	Baseline Conservation Unit Cost
C_{r0}	10	Maximum Unit Cost of Reuse
C_{w0}	10	Unit Cost of Waste Treatment
C_s	1	Unit Cost of Supply

6 Model Results and Discussion

6.1 Pardon My Intrusion - Saltwater Intrusion Forecasts

We forecasted saltwater intrusion at 10 year intervals for the next 50 years. Figure 7(a) is the current state of intrusion. The general trend over the next 50 years is that intrusion increases. Intrusion increases in some areas much more than others. The arm of saltwater which intrudes inland near 40 km on northing axis is a potential threat to the Hialeah-Preston wellfield. The front of this arm moved at a rate of approximately 100 meters/year. The arm of intrusion at 30 km on northing axis is a potential threat to the Alexander Ore Jr. wellfield which is one of the closest fresh water supplies to Miami. The front of this arm moved at approximately 20 m/year. By 2038 everything north of 75 km on northing axis is essentially saturated with sea water (this is actually in Broward County). Figure 8 shows the extent of the saltwater intrusion in 50 years overlayed on a map showing Miami-Dade freshwater well fields.

Our statistical model gave evidence of increased salt water intrusion in the greater Miami area but it is by no means conclusive. We were plagued by the ever-present scientific problem of lack of data. If more data were available a clearer picture of intrusion could be obtained. Some other data related considerations are that (1) different amounts of data are available at each site which make individual well predictions more or less reliable and (2) the error in any 50 year forecast is large and not directly quantifiable.

Table 2: Necessary percent conservation to offset loss of well fields.

Year	2008	2018	2028	2038	2048	2058
Necessary Percent Reduction of Water Usage Per Person						
Loss of South Well Field	0.0	4.0	13.1	20.7	26.9	32.3
Loss of Alexander Orr Field	42.4	48.3	53.1	57.0	60.3	63.1
Loss of Hialeh-Preston Field	41.4	47.4	52.3	56.3	59.7	62.5

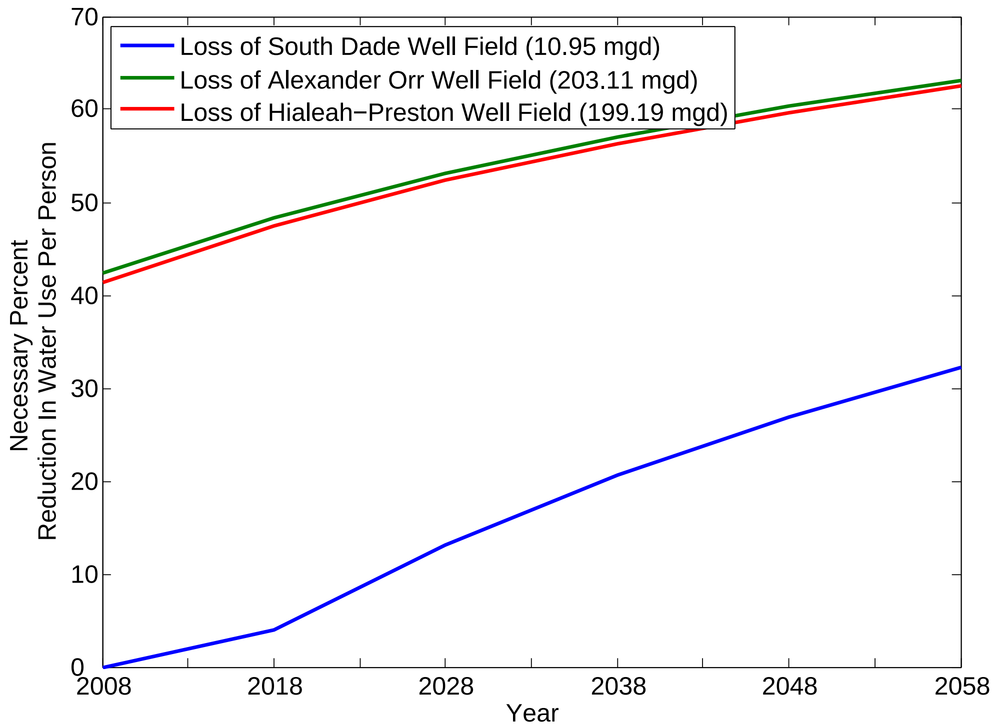
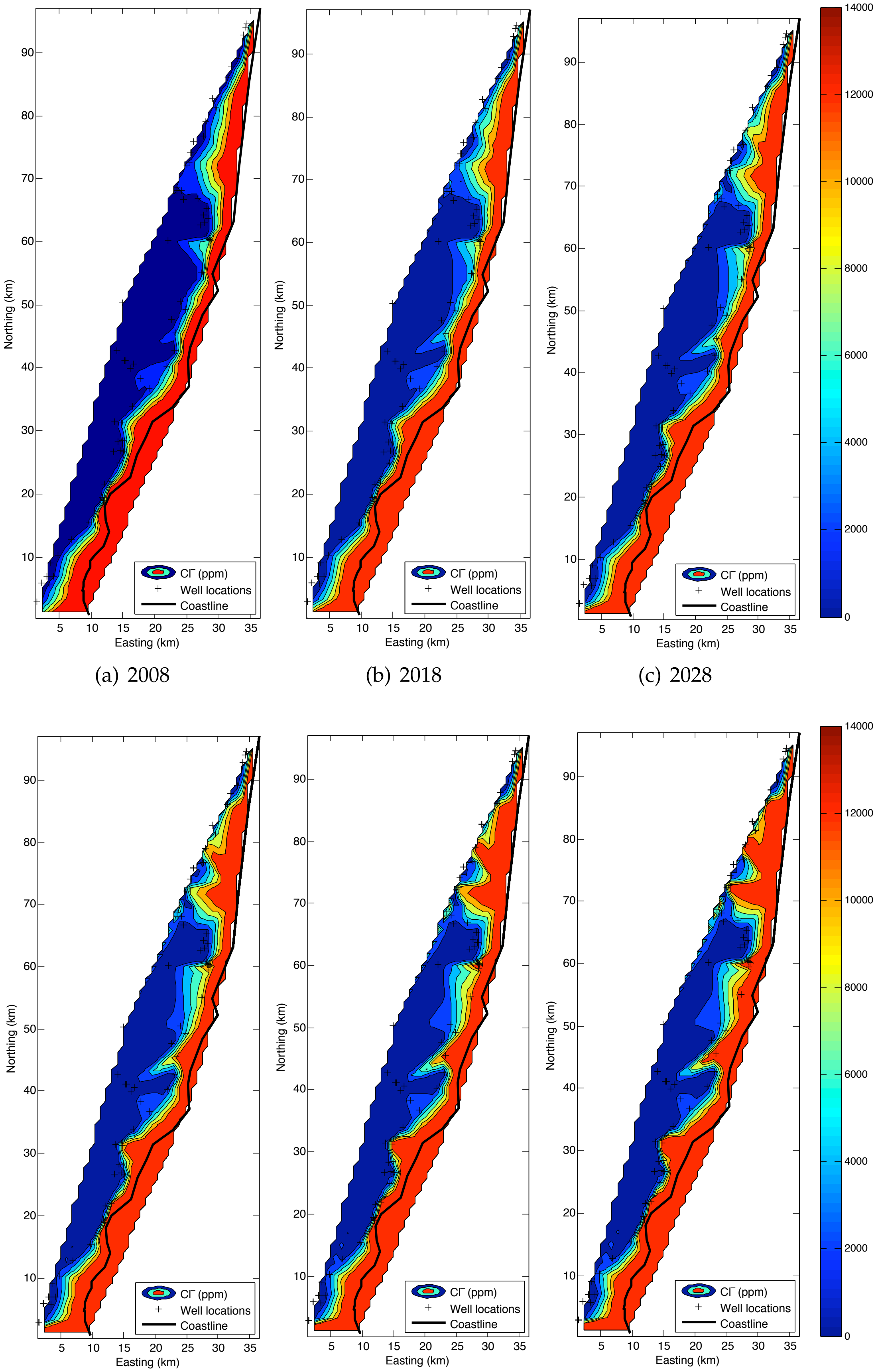


Figure 6: Results of Optimization

6.2 Well, Whad'ya know - Optimization Results

The management model (Equations 12 and 13) was solved using the SOLVER package in Microsoft Excel. Three scenarios were investigated: (1) contamination of the South Dade well field, (2) contamination of the Hialeah-Preston well field, and (3) contamination of the Alexander-Orr well field (Figure 8). In each scenario we have assumed that contamination of the given well field occurs immediately. Figure 6 shows the percent reductions in per capita water usage necessary to offset the loss in supply in the coming decades. (e.g., if the Hialeah-Preston well field were contaminated within the coming year, the loss in supply would be offset by the conservation efforts shown by the red line in Figure 6). Conservation levels must continuously increase because the total water demand increases at a greater rate than the available supply. The rate of increase in water demand is a function of population growth and the level of conservation. The rate of increase in available supply is additionally dependant on the fraction of waste-water recycled. Contamination of either the Hialeah-Preston or Alexander-Orr well fields would require greater conservation efforts to offset the loss in supply as compared to the South Dade well field (the Northern well fields supply a majority of the county's drinking water).

In all three scenarios, the recommended (optimal) fraction r of waste-water to be reused was low (less than 10%). We suspect this value is greatly influenced by the cost parameter C_{r0}, the unit cost associated with reuse. When the value of C_{r0} was reduced relative to the remaining cost parameters, the value of r increased. Hence, the decision to recycle waste-water is dependant upon the cost of treatment. We encourage both conservation and the development of cost-effective treatment technology with the intent of recycling waste-water, so that optimal management decisions would include elements of both practices.



(a) 2008 (b) 2018 (c) 2028

(e) 2038 (f) 2048 (g) 2058

Figure 7: Salt water intrusion predictions at 10 year intervals.

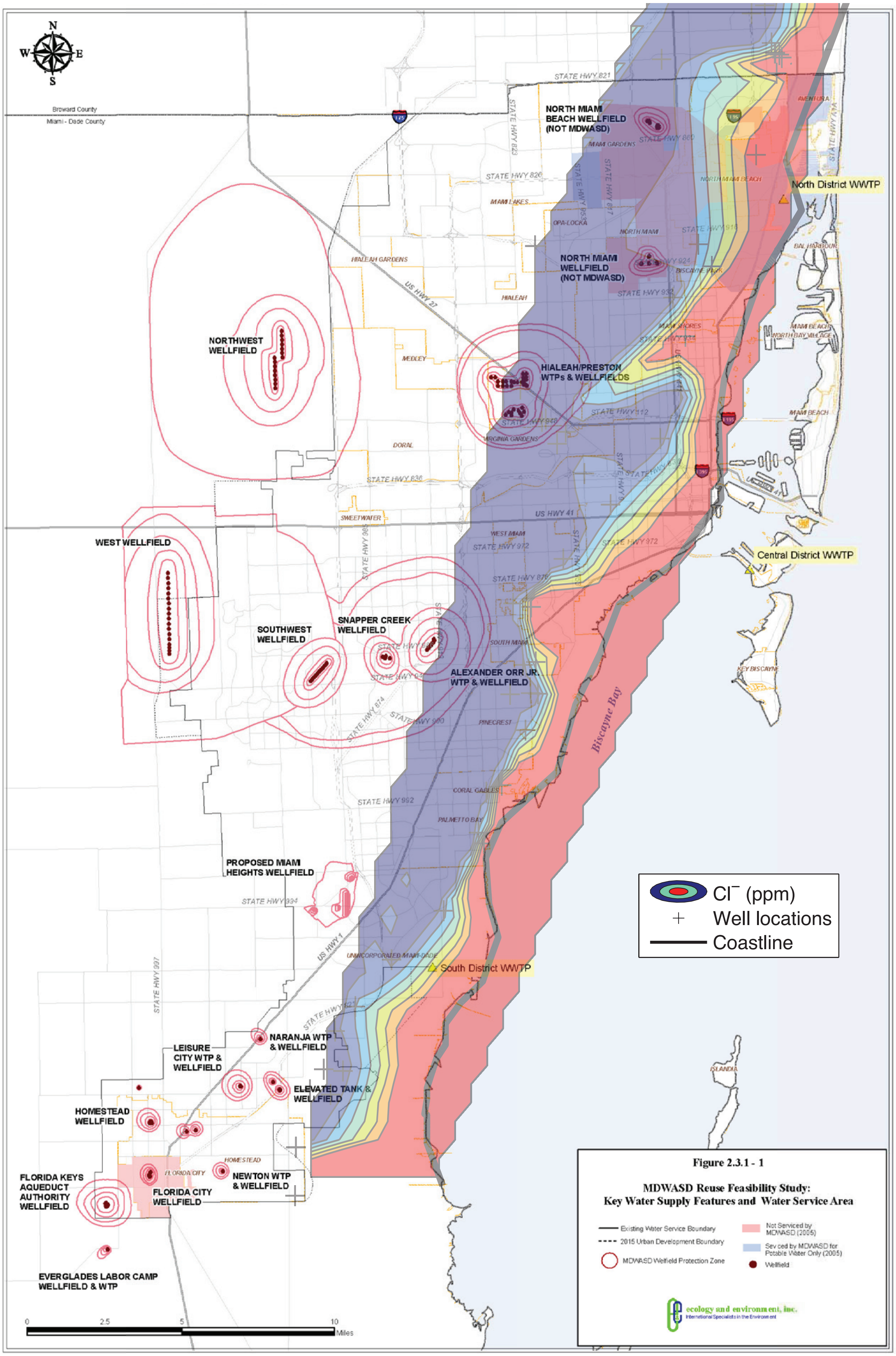


Figure 8: Extent of saltwater intrusion in 2058 [MDWASD 2007].

7 Things We Didn't Do

We have only considered a small piece of the puzzle. We did not address the following issues:

- Various methods of predicting sea level rise.
- Additional predictors of salt water intrusion, e.g., changes in the height of the water table. (We didn't use the water table as a predictor because of the infeasibility of running a groundwater simulation over a 50 year horizon.)
- The possibility that we forecasted erroneous intrusion trends due to limited data.
- The sensitivity of the optimization parameters and the lack of formal justification or literature review related to the cost functions.
- The possibility of recharge, weakness in the population model, outside water supplies, desalination and agricultural/industrial demands in the management model.

We intended to correlate the chloride concentration data with population data, however, both predictors were highly correlated (multicollinear). The population data was assumed to be a surrogate for the water demand. The water demand affects the chloride concentration because pumping from an aquifer results in a cone of depression that changes the hydraulic head gradient, which facilitates saltwater intrusion.

8 Take it with a Grain of Salt

We have developed a statistical model to forecast salt water intrusion in the Biscayne Aquifer along the coast of Southeast Florida. We found a positive correlation between chloride concentration and mean sea level. Based on our findings, several freshwater supply well fields along the coast are at risk of chloride contamination. Contamination of subsurface freshwater supplies, though a subtle issue, is a valid concern in Miami-Dade County. Although we looked at saltwater intrusion as a local problem it is likely to affect many coastal metropolitan areas though further study is needed to support this claim.

In response to the potential loss in freshwater supply we have proposed a management plan. We have quantified the level of conservation necessary to offset the potential loss in freshwater supply. Appropriate levels of conservation (i.e., reduction in per capita daily consumption) and wastewater reclamation were chosen by minimizing the combined operational cost of such practices while satisfying the projected demands.

We stress that our models are limited by the many assumptions we have made; they are further limited by the quantity and quality of our field data. We realize that our approach is one of many, however, we choose to pursue a statistical model because of its generality. The issue of salt water intrusion is not peculiar to Miami-Dade county, rather it is a problem along many if not most coastlines. The strength of our model is the ease with which it can be applied to other locations and used as a tool in the decision making process, taken with a grain of salt, of course.

Appendix: Source Code

Miami Beach Sea Level Forecast

```
    #This Program fits a quadratic function to the sea level data

sl=as.matrix(read.table('miami_beach.txt'))  #Read Sea Level Data File

t=sl[,1]  #time
y=sl[,2]  #Sea Level

source('ssr.r')

p=c(6946,1.08,0.004)    #initial parameter guess
o=optim(p,sav)          #minimize sum of absolute value of residuals
print(o$par)
p=o$par

plot(sl,cex=.25,xlim=c(1930,2058),xlab='Year',ylab='Sea Level (mm)')

t=c(t,1981:2058)
fit=p[1]+p[2]*t+p[3]*t^2
lines(t,fit,col='red')

sav=function(p){
    fit=p[1]+p[2]*t+p[3]*t^2

    resid=y-fit

    sum(abs(resid))
}
```

Well Chloride Concentration Forecast

```
    #This program forecasts chloride concentrations at each well and
    #writes a file containing predictions for each well for 50 years
    #at 10 year timesteps.



filenames=scan('wells.dat',what='character')         #read list of well names
fut=as.matrix(read.table('predict_me.dat'))          #read future predictor values
fut=fut[,1]                                #first column is sea level projections

zs=matrix(0,ncol=length(fut),nrow=length(filenames)) #array to store predictions

plot=T              #plot output
options(warn=-1)    #dont show warning
```

```
    library(locfit)     #load fitting library
    library(DAAG)

for(i in 1:length(filenames)){

    outputfile=paste(filenames[i],'.pred',sep='')  #individual file names

    print(filenames[i],quote=F)
    file=filenames[i]
    x=as.matrix(read.table(file))
    y=x[,5]                         #grab response (cl concentration,y) and
    x=x[,4]                         #predictor (sl,x)

    if(length(x)<15){        #if less than 15 data points at a site, use linear regression
        fit=lm(y~x)
        print(paste('Linear Model with',length(x),'points'),quote=F)
        if(plot){
            plot(x,y,cex=.25,xlab='Sea Level Relative to 2008 Miami Beach (mm)')
            abline(fit,col='red')
            pause()
        }
    }else{
        al=seq(0.3,1,.05)                   #the range of alpha values

        g=gcvplotmod(y~x,deg=1,alpha=al)    #calculate gcv for each alpha value

        theal=al[order(g$values)[1]]        #grab the position of the best alpha
        fit=locfit(y~lp(x,deg=1,nn=theal))  #fit the local regression model
        print(theal)
        if(plot){
            plot(x,y,cex=.25,xlab='Sea Level Relative to 2008 Miami Beach (mm)',
                                            ylab='Chloride Concentration (ppm)')
            lines(fit,col='red')
            pause()
        }
    }

    pred=0
    if(length(x)<15){           #forecast
        a=fit$coefficients[1]
        b=fit$coefficients[2]
        pred=a+b*fut
    }else{
        pred=predict(fit,fut)
    }

    zs[i,]=pred
}
write(t(zs),file='predictions',ncolumns=ncol(zs))
```

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