New Mexico Storm Water Infiltration

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Abstract

This report is a response to the National Weather Service's request for data regarding ponding time and infiltration depth in New Mexico. This information will be used to help predict flash flooding in New Mexico. Ponding times and infiltration depths under various storm conditions were determined. Ponding times for a dual-layered soil were analyzed under a range of rainfall rates (10–60 mm/hr). A surface layer of brown fine sandy loam and a subsurface layer of strong brown sandy clay loam was assumed to be the average soil type in the state. The model used to predict the ponding times was based on the two-layer Green-Ampt infiltration equations. The secant and Runge-Kutta methods were used to solve for the cumulative infiltration depth and to estimate infiltration rate. Under heavy rainfall (30 mm/hr) ponding occurred within 1 hr 15 minutes. Under severe rainfall (60 mm/hr) ponding occurred within 30 minutes. A storm of the latter intensity could occur, having implications for flash flooding in the state.

1 Introduction

Flash flooding is a weather concern in New Mexico (NWS 2006). During the thunderstorm season, flash flooding occurs unexpectedly with sometimes devastating consequences in rural and urban areas. The National Weather Service (NWS) is interested in predicting flash floods in areas of New Mexico based on known rainfall rates. Particularly, the NWS is interested in infiltration depth over time and ponding times to estimate how much storm water is absorbed into the ground. Ponding time is defined as the time taken, after rainfall starts, until the rainfall rate equals the infiltration rate. The NWS will use this data to issue more accurate and timely evacuation warnings. This report will:

- Research the storm history of New Mexico.
- Research soil types in the state.
- Research rainfall rates of typical thunderstorms known to cause flash flooding.
- Propose a model for predicting ponding times and infiltration depths based on rainfall rates and soil data.
- Discuss the numerical method used to solve the model and discuss the model predictions.
- Discuss related case studies.
- Make a suggestion as to which rainfall rates would be sufficient to issue a flash flood warning.

2 Literature Review

2.1 Flash Flooding in New Mexico

The thunderstorm season peaks in New Mexico from June through September (Figure 1). Thunderstorms occur most frequently from 12-9 am (0000-0900)(Figure 2)(NWS 2006). During that time, thunderstorms with high rainfall rates can develop quickly causing flash flooding. These thunderstorms are "intense and of short duration, with the period of highest intensity at, or soon after, the beginning of rainfall" (Leopold 1943). This characteristic of New Mexico storms contributes to the likelihood of flash flooding in the state. Other factors that contribute to the distribution and severity of flash floods are (Craft 2004):

- Terrain gradients,
- Soil permeability,
- Soil moisture,
- Drainage basin size and orientation,
- Vegetation, and
- Snow pack.

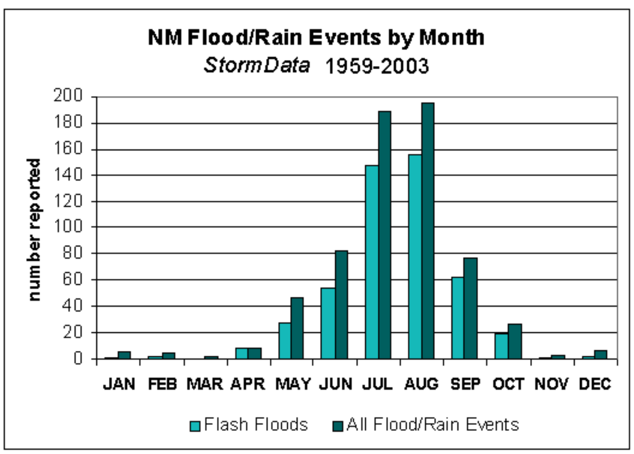

Figure 1: New Mexico flood conditions (NWS 2006).

Flash flooding has caused extensive damage in New Mexico over the past 100 years (NMFMA 2003). Every county in the state has been affected by flooding. The largest recorded storm in the past 100 years occurred on September 20, 1941 near Albuquerque. The highest rainfall rate was recorded at 35 mm/hr (Leopold 1946). The state was largely unprepared and damage was extensive. By today's standards, storm damage was estimated to be hundreds of millions of dollars (NMFMA 2003).

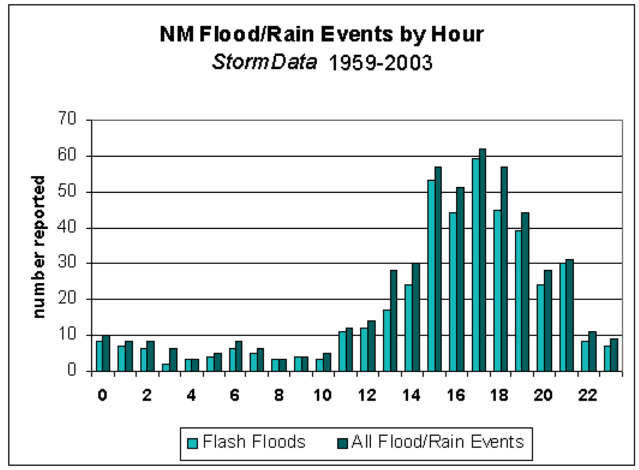

Figure 2: New Mexico flood conditions (NWS 2006).

Flash flooding occurs in New Mexico primarily due to its rocky terrain and dry soil (NWS 2006). To more accurately predict the flash flooding in New Mexico, ponding times and infiltration depths needs to be known. Ponding time is the time necessary for water to start ponding on the surface of a certain soil type under constant rainfall. At the moment of ponding, the rainfall rate equals the infiltration rate. Infiltration depth is the depth to which water has soaked into the soil. The time of ponding depends on parameters associated with soil type and rainfall rates. If rainfall rates of a storm were known, data on ponding times and infiltration depths could be used to predict the time until flash flooding could begin to occur.

2.2 Rainfall

This report will take into account a variety of rainfall rates ranging from light to heavy. Some of the highest known rainfall rates in New Mexico were recorded in the Albuquerque storm of 1941 (Leopold 1946) (NMFMA 2003). For the proposes of this report, rainfall rates ranging from 10–60 mm/hr will be used as a range from light to severe (Table 1). This range is a common range used in infiltration modeling (Govindaraju et al. 2001). These rates are assumed to be provided by the NWS in the event of a storm.

Table 1: Range of rainfall rates (Govindaraju et al. 2001).

Rainfall Type	Rate (mm/hr)
light	10
heavy	30
severe	60

2.3 Soil Data

The majority of soils in New Mexico are classified as aridisols and mollisols (Figure 3)(Figure 4). In hydrologic terms, the surface layer is made up of brown fine sandy loam and the subsoil is strong brown sandy clay loam (NRCS 2003). These soil types exhibit characteristic parameters that will be used in the model. One important assumption is the continuity of soil classification over the area of interest. This assumption is accurate because large areas of the state have been continuously classified. Sandy loam has a greater conductivity than sandy clay loam, so water will absorb more quickly into the surface soil than into the subsoil layer. The soil thickness will be assumed to be 3 cm and 50 cm for the surface and the subsoil layers respectively (NRCS 2003).

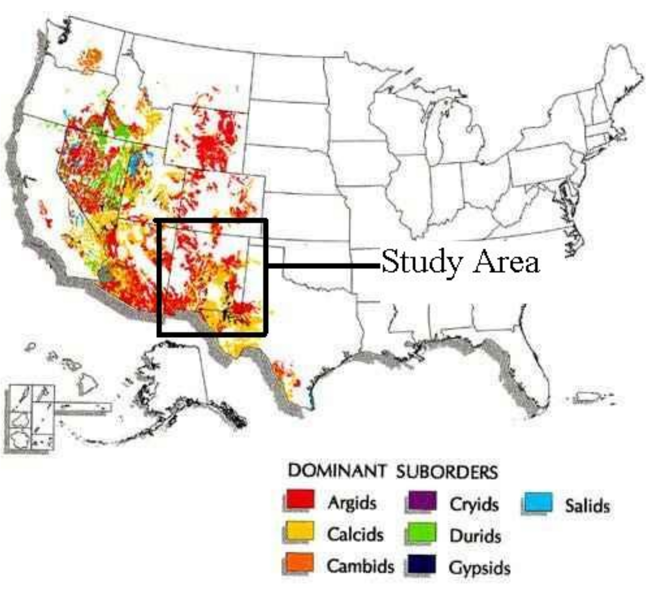

Figure 3: Area covered by aridisol soil categories (NRCS 2003).

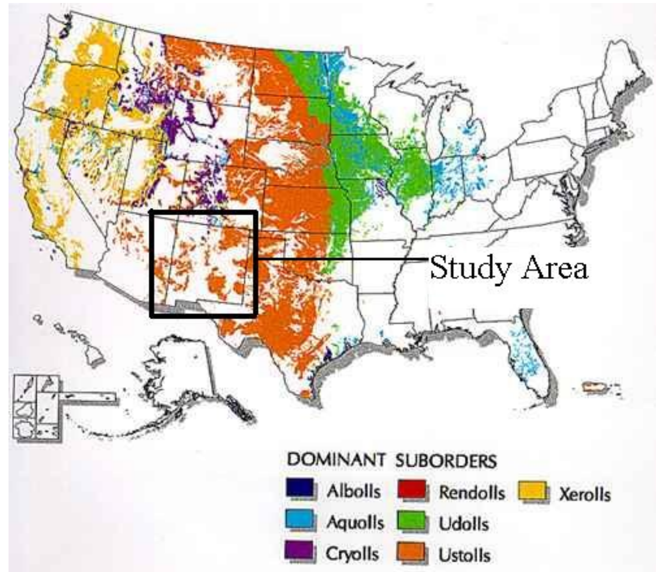

Figure 4: Area covered by mollisol soil categories (NRCS 2003).

2.4 Green-Ampt Equation

The model proposed to predict ponding times and infiltration depth will be based on the Green-Ampt infiltration equation. The basic form of the Green-Ampt equation is (Wallender 2006)

$$\frac{dF}{dt} = K_s \left(\psi_f \frac{\theta_s - \theta_i}{F} + 1 \right)$$

where

$$\begin{aligned} F &= \text{Depth of infiltration (mm)} \\ \psi_f &= \text{Average capillary suction (mm)} \\ K_s &= \text{Saturated hydraulic conductivity (mm/hr)} \\ \theta_s &= \text{Initial moisture deficit } \left(\tfrac{vol.\ air}{vol.\ voids}\right)\text{(dimensionless)} \\ \theta_i &= \text{Saturated moisture content (dimensionless)} \end{aligned}$$

The model parameters can be obtained from tables of standard values for each soil type (Maidment 1993). Some important assumptions are necessary to use the Green-Ampt equation (Wallender 2006):

- The rainfall rate is constant throughout the area of interest for each time step.

- The soil is dry or uniformly wet at the start of rainfall.
- The soil type is uniform throughout the area of interest for each time step.
- At the moment of ponding, the rainfall rate equals the infiltration rate.
- Rain starts instantly and continues indefinitely.
- Infiltration after ponding occurs is negligible.
- The Green-Ampt parameters are not randomly distributed.

2.5 Case Studies

Green-Ampt infiltration models are widely used in the hydrologic sciences. The time of ponding has been used to predict the amount of runoff in grazed and ungrazed fields. In one study, the amount of runoff from the fields was used to predict contaminant transport in the area. The study visually observed ponding within 3-9 minutes under a constant rainfall rate of 60 mm/hr (Fiedler et al. 2002). The Green-Ampt parameters measured on site were indicative of sand loam soil.

In Senegal, a runoff/infiltration study recorded ponding times of ∼17 min under rainfall rates of 70 mm/hr (Ndiaye et al. 2005). The soil type was sandy loam, which is the same as in New Mexico model.

In a one-layer Green-Ampt model ponding times were predicted for sandy loam soil (Diskin and Nazimov 1996) (Table 2). Relative similarities in results are expected for a two-layer model.

Table 2: Predicted and observed ponding times (t_p) (Diskin and Nazimov 1996).

Rainfall Rate	t_p **pred. (hr)**	t_p **obs. (hr)**
38.1 mm/hr	0.22 (13min)	0.19 (11min)
50.8 mm/hr	0.13 (8min)	0.13 (8min)
63.5 mm/hr	0.08 (5min)	0.09 (6min)

This New Mexico report makes the assumption that the Green-Ampt parameters are not randomly distributed. A study of the C-111 basin in southern Florida showed that a nonuniform hydraulic conductivity significantly affects the time of ponding. In the study, ponding occurred faster and runoff continued for a longer time when a variable conductivity was used (James et al. 2003). The hydraulic conductivity "exhibits the maximum variability among all the infiltration parameters" (Govindaraju et al. 2001). This New Mexico report uses different parameter values for each soil layer. This is the only way that soil layers are distinguishable by the model. Results using a homogeneous soil type would be different than using a layered soil model (Luo 2001).

A study of sand ditches in Jordan used to harvest rain investigated wetting depth. The study showed that average wetting depths in sand ranged from 40-100 mm depending on the time of year (Abu-Zreiga et al. 2000). Sand has the highest hydraulic conductivity of any soil type so these values may be expected to be much higher than than those of the New Mexico soil types.

Using the Green-Ampt equation as the basis for a model, the time of ponding can be predicted for the average soil type in New Mexico. Infiltration rates and depths, soil type and the rainfall rate are necessary to estimate the ponding times. Similar models can be used to compare related effects. As with most model studies, various assumptions must be made to use the model.

3 Methodology

In a two-layer model, information about both soil layers must be known. The computation associated with a two-layered soil can be broken into two parts. During the period in which water is infiltrating the top layer of soil, the basic form of the Green-Ampt equation can be used. The differential form of the Green-Ampt equation is (Chow 1988):

$$f_1 = \frac{dF}{dt} = K_s\left(\frac{S\Delta\theta}{F} + 1\right) \quad (1)$$

where

f_1	=	Infiltration rate into surface layer (mm/hr)
F	=	Depth of infiltration (mm)
S	=	Average capillary suction (mm)
K_s	=	Saturated hydraulic conductivity (mm/hr)
$\Delta\theta$	=	Initial moisture deficit(mm/mm)

Equation 1 is a separable ordinary differential equation (ODE). The integrated form is

$$t_s = -\frac{1}{K_1}\left[S_1\Delta\theta_1 \ln\left(1 + \frac{F(t)}{S_1\Delta\theta_1}\right) - F(t)\right] \quad (2)$$

where

$F(t)$	=	Thickness of surface layer (mm)
S_1	=	Average capillary suction of surface layer(mm)
K_1	=	Saturated hydraulic conductivity of surface layer(mm/hr)
$\Delta\theta_1$	=	Initial moisture deficit of surface layer(mm/mm)
t_s	=	Time until surface layer is saturated (hr)

The calculation of t_s is possible because all quantities are known. The value of t_s is important for the second stage of the model. The second stage of the model uses the two layer form of the Green-Ampt equation

$$f_2 = \frac{dL_2}{dt} = \frac{K_1K_2(S_2 + H_1 + L_2)}{H_1K_2 + L_2K_1} \quad (3)$$

where

f_2	=	Infiltration rate in to subsoil layer (mm/hr)
L_2	=	Depth of wetting front starting from bottom of surface layer (mm)
S_2	=	Average capillary suction of subsurface layer(mm)
K_2	=	Saturated hydraulic conductivity of subsurface layer(mm/hr)
H_1	=	Thickness of surface layer (mm)

All the quantities in Equation 4 are known except the depth of the wetting front starting from the bottom of the surface layer (L_2). To obtain the value of L_2 the integrated form of Equation 3 is used

$$t_L = \frac{L_2\Delta\theta_2}{K_2} + \frac{1}{K_1K_2}\left[\Delta\theta_2 H_1 K_2 - \Delta\theta_2 K_1(S_2 + H_1)\right] \ln\left[1 + \frac{L_2}{S_2 + H_1}\right] \quad (4)$$

where

t_L	=	time since saturation of surface layer (hr)

A root finding algorithm was used to find the value of L_2 at every time step. Note that the total time since the start of rainfall is $t = t_s + t_L$. A FORTRAN program employing the secant method is used to calculate the value of L_2 at each time step.

Ponding times can also be determined from this model. At the moment of ponding, $i = f_2$, the rainfall rate (i) equals the infiltration rate (f). Starting at t_L the ponding time is given by

$$L_{2p} = t_{pL} i \quad (5)$$

where

t_{pL}	=	Ponding time of bottom layer (hr)
i	=	Rainfall rate(mm/hr)
L_{2p}	=	Infiltration depth at time of ponding (mm)

Substituting $f_2 = i$ and $L_{2p} = it_{pL}$ into Equation 3 and solving for t_{pL} yields

$$t_{pL} = \frac{K_2(H_1 + S_2) - iH_1}{i(i - K_2)} \quad (6)$$

The total time of ponding is $t_{p\text{total}} = t_s + t_{pL}$. After the ponding time has elapsed not all rainfall is infiltrated. The information of interest is the $t_{p\text{total}}$ and the relationship between t and F.

4 Application

The infiltration depth is dependent on various parameters that are known in the problem (Table 3).

To test the sensitivity of the model, some parameters can be varied (Table 4). The hydraulic conductivity (K_i) is known to exhibit the greatest variability of any Green-Ampt parameters (Govindaraju et al. 2001). It has a range of several orders of magnitude throughout the range of different soil types (Maidment 1993). In infiltration modeling, the hydraulic conductivity is usually estimated as a random variable. To reflect its variation, this report will impose a wider variation on the hydraulic conductivity than the other infiltration parameters. The other Green-Ampt parameters are known to vary but not as significantly as the hydraulic conductivity.

Table 3: Parameters associated with determining infiltration depth for sandy loam soil located in New Mexico (Maidment 1993).

Parameter	**Variable**	**Value**
Thickness of surface layer (mm)	H_1	30 mm
Thickness of subsurface layer (mm)	H_2	500 mm
Average capillary suction of surface layer (mm)	S_1	110.1 mm
Average capillary suction of subsurface layer (mm)	S_2	218.5 mm
Saturated hydraulic conductivity of surface layer (mm/hr)	K_1	21.8 mm/hr
Saturated hydraulic conductivity of subsurface layer (mm/hr)	K_2	3.0 mm/hr
Initial moisture deficit of surface layer	$\Delta\theta_1$	0.358
Initial moisture deficit of subsurface layer	$\Delta\theta_2$	0.250
Rainfall rate rate (mm/hr)	i	10,30,60 mm/hr

Table 4: Variation of parameters for analyzing model sensitivity

Run #	**Variable**	**Initial value**	**New value**	**Variation**
1	H_1	30 mm	15	-50%
2		30 mm	45	50%
3	H_2	500 mm	250	-50%
4		500 mm	750	50%
5	S_1	110.1 mm	55.05	-50%
6		110.1 mm	165.15	50%
7	S_2	218.5 mm	109.08	-50%
8		218.5 mm	327.23	50%
9	K_1	21.8 mm/hr	5.45	-75%
10		21.8 mm/hr	38.15	75%
11	K_2	3.0 mm/hr	.75	-75%
12		3.0 mm/hr	5.25	75%
13	$\Delta\theta_1$	.358	0.179	-50%
14		.358	0.537	50%
15	$\Delta\theta_2$	.250	0.125	-50%
16		.250	0.375	50%

5 Results

For a sandy loam soil in New Mexico, ponding time, infiltration depth, and infiltration rate vary greatly with rainfall rate (Table 5).

Table 5: Time until surface layer saturation.

Rainfall Rate	Ponding Time (hr)
10 mm/hr	10.4 (10hr 24min)
30 mm/hr	1.12 (∼1hr 7min)
60 mm/hr	0.50 (∼30min)

The ponding time in the 60 mm/hr case is much larger than the value in the literature. This may be due to the assumptions of this model. The time until saturation of the surface layer does not vary significantly when rainfall rate is varied (Table 6). This may be due to the qualities and the shallowness of the surface soil.

Table 6: Time until complete surface layer saturation.

Rainfall Rate (mm/hr)	Time (hr)
10	0.35
30	0.35
60	0.34

With a constant rainfall rate of 30 mm/hr (heavy rainfall), ponding occurs in 1.12 hours. After that time, runoff would be expected to occur. The top layer of soil is completely saturated in 0.35 hours (∼21 minutes). After a short period of nonlinear behavior in both layers, the infiltration depth exhibits nearly linear behavior. The graph of cumulative infiltration is discontinuous at the boundary between the two soil types because of the change in soil properties (Figure 6). The discontinuity between soil types is exemplified by the graph of the infiltration rate (Figure 7). Mathematically, this is due to the derivative relationship between infiltration depth and infiltration rate. Infiltration rate decreases as infiltration depth depth increases (Figure 8). The latter two graphs, when i=10 mm/hr, exhibit similar behaviors due to the nearly linear behavior of infiltration depth.

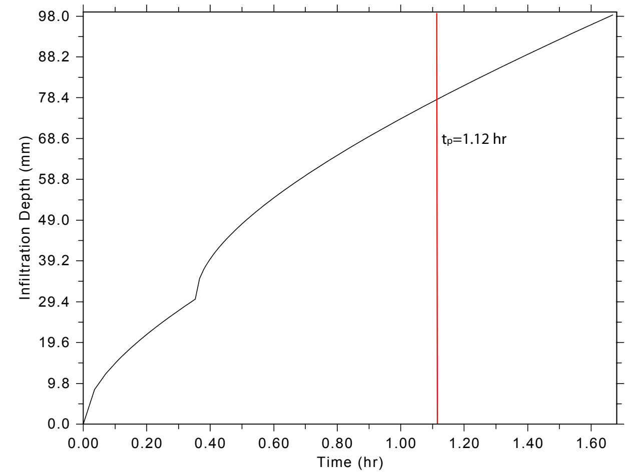

Figure 6: Cumulative infiltration with i=30 mm/hr.

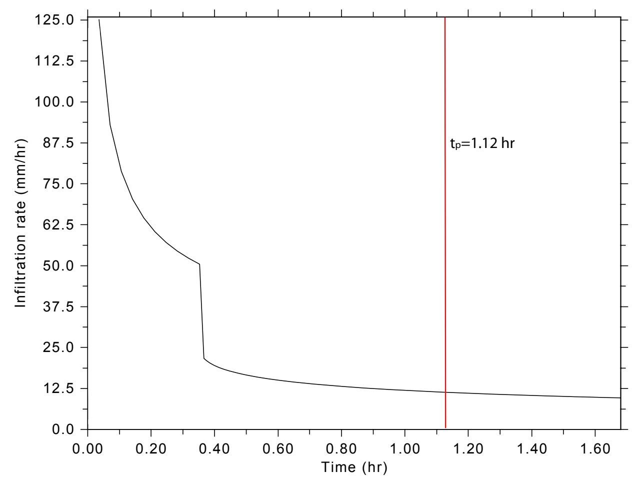

Figure 7: Infiltration rate with i=30 mm/hr.

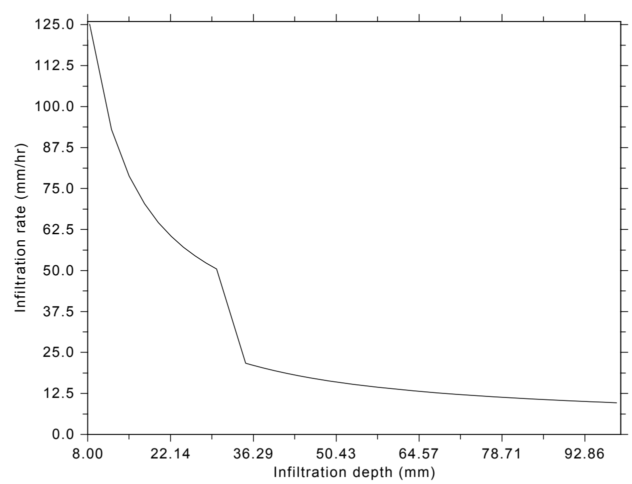

Figure 8: Infiltration rate decreases as wetting depth increases, i=30 mm/hr.

Rainfall at a rate of 30 mm/hr may be a concern for flash flooding. After 1.12 hr, surface runoff will begin to occur. A storm lasting this length of time is less likely to occur in New Mexico, so this situation is therefore less of a concern than a storm of greater intensity. The Albuquerque flooding of 1941 occurred when rainfall intensity was 35 mm/hr.

To test the model a constant rainfall rate of 10 mm/hr was simulated (light rainfall). Under these conditions, ponding occurred after 10.4 hr (10hr 24min). After the surface layer is saturated, the graph of infiltration depth exhibits a long period of nearly linear behavior (Figure 9).

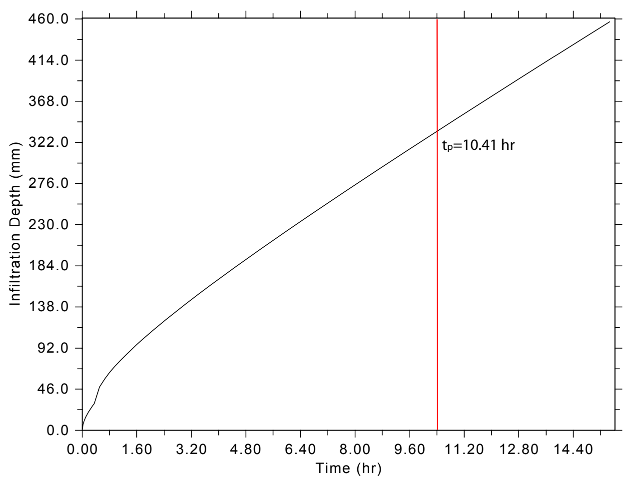

Figure 9: Cumulative infiltration with i=10 mm/hr.

In a one-layer model, the corresponding graphs would be continuous. The graph of the infiltration rate in the one-layer model would approach the rainfall rate because $i = \frac{dF}{dt}$ at the time of ponding. The two layer model graph of infiltration rate exhibits a nearly continuous behavior (Figure 10). In the two layer case the graph of the infiltration rate approaches a value close to the rainfall rate (10 mm/hr). The infiltration rate decreases drastically at first then very slowly as infiltration depth increases (Figure 11).

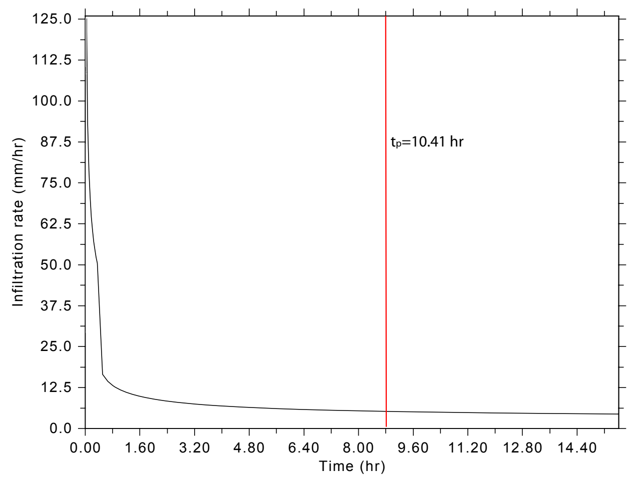

Figure 10: Infiltration rate with i=10 mm/hr.

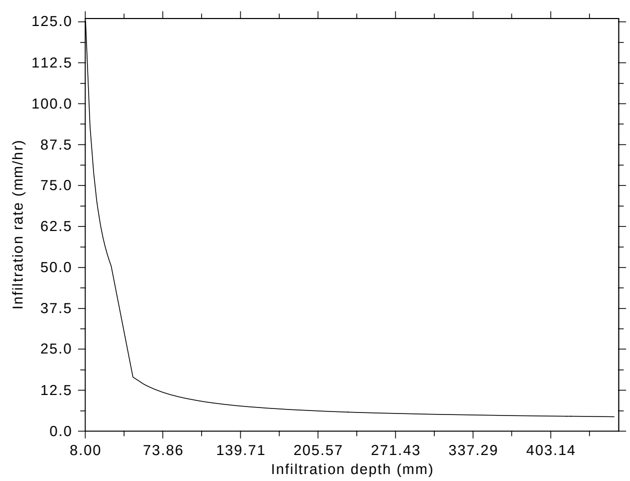

Figure 11: Infiltration rate decreases as wetting depth increases, i=10 mm/hr.

Under severe rainfall conditions of 60 mm/hr ponding occurs in 0.50 hr (∼30min). The time until surface layer saturation is nearly the same as with previous rainfall rates, but ponding occurs much more rapidly under heavy rainfall. The graph of infiltration depth exhibits nonlinear behavior over this short span of time (Figure 12). The surface layer infiltration curve appears to approach the rainfall rate as before (Figure 13). Infiltration rate decreases as infiltration depth increases (Figure 14). The latter two graphs ,when i=60 mm/hr, exhibit similar behavior as seen before due to the relatively linear behavior of the infiltration depth. A New Mexico storm raining 10 mm/hr is unlikely to last for the duration necessary to cause ponding and therefore surface runoff. In the 10 mm/hr case, flash flooding is unlikely to occur.

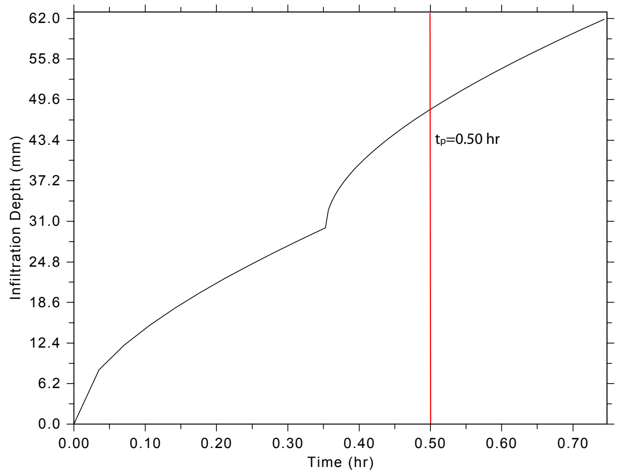

Figure 12: Cumulative infiltration with i=60 mm/hr.

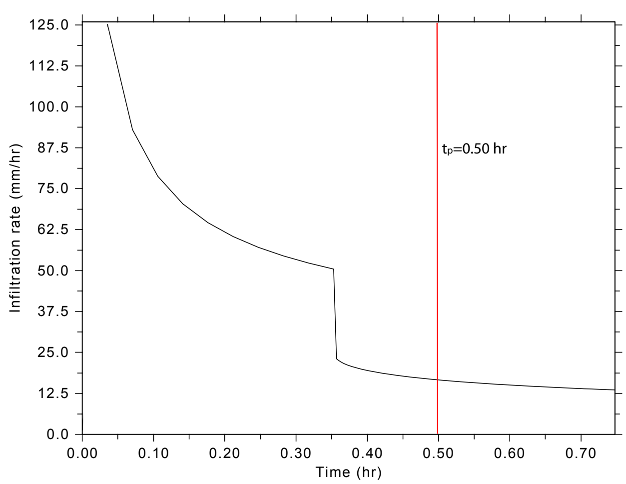

Figure 13: Infiltration rate with i=60 mm/hr.

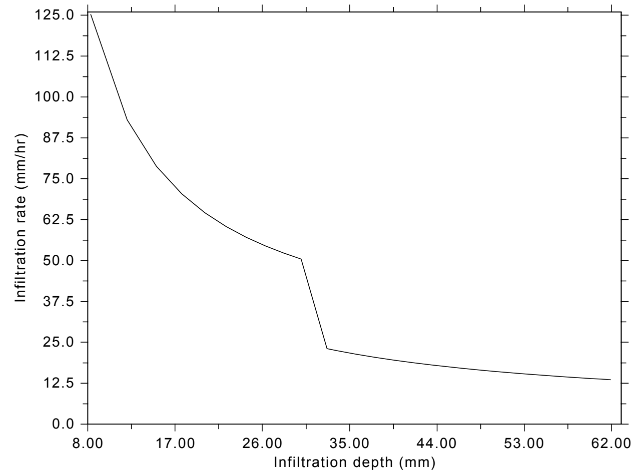

Figure 14: Infiltration rate decreases as wetting depth increases, i=60 mm/hr.

The results from severe rainfall conditions (60 mm/hr) have the most important implications for flash flooding in New Mexico. After approximately 30 minutes, surface runoff will start to occur. The amount of runoff will be the excess water that does not infiltrate. Excess storm water will infiltrate at a known rate. In valleys and ravines, this surface runoff may cause flash flooding. Short storms of high intensity are known to occur in New Mexico. A storm of this intensity (60 mm/hr) and duration is possible.

The sensitivity analysis reveals how the ponding time varies when individual parameters are varied (Table 8). The sensitivity analysis was only carried out when i=60 mm/hr because this case has the most important implications to flash flooding. Infiltration depth is not included in the sensitivity analysis because the NWS is interested in this behavior over time, not at a single point in time.

The most influential parameter is the hydraulic conductivity which caused a 169% variation in the ponding time when increased 75%. This parameter is known to have a wide range of values so this finding is reasonable. The thickness of the surface layer is also a very influential parameter, causing a 62.7% increase in ponding time. The thickness of the subsurface layer (H_2) has no effect because the infiltration depth never exceeds 400 mm at the time of ponding in any case. The

Table 7: Sensitivity analysis

Run #	Variable	New value	% Varied	t_p	Variation
1	H_1	15	-50%	0.274	45.0%
2		45	50%	0.810	62.7%
3	H_2	250	-50%	0.498	0%
4		750	50%	0.498	0%
5	S_1	55.05	-50%	0.685	37.6%
6		165.15	50%	0.410	17.7%
7	S_2	109.08	-50%	0.402	19.3%
8		327.23	50%	0.594	19.3%
9	K_1	5.45	-75%	1.34	169%
10		38.15	75%	0.378	24.1%
11	K_2	.75	-75%	0.390	21.7%
12		5.25	75%	0.618	24.1%
13	$\Delta\theta_1$	0.179	-50%	0.685	37.6%
14		0.537	50%	0.409	17.9%
15	$\Delta\theta_2$	0.125	-50%	0.498	~0%
16		0.375	50%	0.498	~0%

initial moisture deficit of the subsurface layer had a negligible effect on ponding time. The saturated hydraulic conductivity of subsurface layer has the smallest non-negligible effect on the ponding time.

This study did not represent the hydraulic conductivity as a randomly distributed variable. This is a possible source of error in the findings.

The results of this report conform with the values in the literature relatively well. An important note is that this model used no experimental data. If these findings were to be applied to a specific site, specific parameter values such as K_1 and K_2 would need to be measured experimentally. Applying these findings on a large scale would be misleading and most likely incorrect. Based on the physical findings of this study and the recorded rainfall rates from the Albuquerque flooding of 1946, rainfall rates of over 30 mm/hr would be sufficient to issue flash flood warnings in the state of New Mexico. Applying this model on site by site basis is also recommended.

6 Conclusion

- In the heavy rainfall case, ponding occurred after 1.12 hr and the wetting front reached ~80 mm.
- In the severe rainfall case ponding occurred after 0.50 hr and the wetting front reached ~50 mm.
- Flash flooding may occur if a storm with a rainfall rate 60 mm/hr (severe) were to occur.
- Flash flooding is unlikely to occur with a rainfall rate of 10 mm/hr.
- The ponding time is most sensitive to changes in the hydraulic conductivity of the surface layer.
- The initial moisture deficit of the subsurface layer had a negligible effect on ponding time.
- The average capillary suction and the saturated hydraulic conductivity of the subsurface layer had a relatively small effect on the ponding time.
- The graphs of infiltration rate vs. time and depth exhibit similar behavior.
- Rainfall rates of over 30 mm/hr would be sufficient to issue flash flood warnings in the state of New Mexico, but the model should be applied on a site by site basis.

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Appendix A Source Code

```
module prm
  double precision::S1,S2,K1,K2,imd1,imd2,H1,i,ts,tp,t
end module

program grnampt
  use prm
  use dislin
  implicit none
  double precision,allocatable,dimension(:)::L2,Ti,D,y,Rt
  double precision,allocatable,dimension(:,:)::lin
  double precision::tstep1,tstep2,eps1,eps2,depth,s,l,xa,xe,xor,xstep,ya,ye,yor,ystep
  double precision::tstart,tend,hmin,hmax,deltat,h
  integer::j,k,r,count,maxit,npsurf,npsub,numit,npts,neq,exitf
  logical::exitflag
  character(len=200)::legendstring

  interface
    subroutine secant(xold,xolder,maxit,epsi1,epsi2,root,numit,exitflag,f)
      double precision,intent(inout)::xold,xolder
      double precision,intent(in)::epsi1,epsi2
      double precision,intent(out)::root
      integer,intent(in)::maxit
      integer,intent(out)::numit
      logical,intent(out)::exitflag
      interface
        function f(x)
          double precision::x
          double precision::f
        end function f
      end interface
    end subroutine secant
    function depth1(d)
      double precision::depth1
      double precision,intent(in)::d
    end function depth1
    function depth2(d)
      double precision::depth2
      double precision,intent(in)::d
    end function depth2
  end interface


  open(11,file="prm.in")
  open(12,file="depth.dat")
  open(13,file="ode.dat")
  open(14,file="time.dat")
  open(15,file="rate.dat")
  read(11,*)S1,S2,K1,K2,imd1,imd2,H1,i,maxit,npsurf,npsub,npts,eps1,eps2
  allocate(L2(npsurf+npsub+2),Ti(npsurf+npsub+2),lin(npsurf+npsub+2,11),Rt(npsurf+npsub+2))

  write(*,*)"i=",i
    ts=-((1d0/K1)*(S1*imd1*log(1d0+H1/(S1*imd1))-H1))
  write(*,*)"ts=",ts

  tp=ts+(S2+H1-(i*H1)/K1)/(i*(i/K2-1))
```

```
  write(*,*)"tp=",tp
tp=tp+tp/2d0


tstep1=ts/npsurf
tstep2=(tp-ts)/(npsub)
write(*,*)tstep1,"  ",tstep2

tp=dble(int(tp+2d0))
t=0d0
count=0
L2=0d0
Rt=0d0
Ti=0d0
s=3
l=3.2
do j=1,npsurf                !first layer
  t=t+tstep1
  Ti(j+1)=t
  call secant(s,l,maxit,eps1,eps2,depth,numit,exitflag,depth1)
  if(exitflag)then
    L2(j+1)=depth
    Rt(j+1)=K1*((S1*imd1+depth)/depth)
  else
    stop
  end if
end do

t=0
do j=1,npsub               !second layer
  t=t+tstep2
  Ti(j+npsurf+1)=t+ts
  call secant(s,l,maxit,eps1,eps2,depth,numit,exitflag,depth1)
  if(exitflag)then
    L2(j+npsurf+1)=depth+L2(npsurf+1)
    Rt(j+npsurf+1)=(K1*K2*(S2+H1+depth))/(H1*K1+depth*K1)
  else
    stop
  end if
end do

xa=0d0 ! xa is the lower limit of the x-axis.
xe=tp+.000001d0 ! xe is the upper limit of the x-axis.
xor=0 ! xor is the first x-axis label.
xstep=1 ! xstep is the step between x-axis labels.
ya=0.0d0 ! ya is the lower limit of the y-axis.
ye=maxval(l2) ! ye is the upper limit of the y-axis.
yor=0 ! yor is the first y-axis label.
ystep=5d0 ! ystep is the step between y-axis labels.
!Plot data using DISLIN
call metafl("ps") ! or "PS", "EPS", "PDF", "WMF" "BMP"
call setpag("USAL") !"USAL" is US size A landscape, "USAP" is portrait
call scrmod("REVERS") !sets black on white background

call disini() !Initialize dislin
call complx
call name("Time (hr)","X") ! Set label for x-axis
call name("Infiltration Depth (mm)","Y") ! Set label for y-axis
call psfont("Helvetica")
```

```
  call labdig(2,"X")
  call labdig(1,"Y")
  call graf (xa, maxval(Ti), xor, dble(int(maxval(Ti)+1))/(10d0), ya, &
    dble(int(maxval(L2)+1)), yor, dble(int(maxval(l2)))/10d0) ! sets up axis
  call title ! Actually draw the title in over the axis
  call curve(Ti,L2,npsurf+npsub) ! draw the x-y curve
  call disfin ! finish off the plot

  call disini() !Initialize dislin
  call complx
  call name("Time (hr)","X") ! Set label for x-axis
  call name("Infiltration rate (mm/hr)","Y") ! Set label for y-axis
  call psfont("Helvetica")
  call labdig(2,"X")
  call labdig(1,"Y")
  call graf (dble(int(Ti(2))), maxval(Ti), dble(int(Ti(2))), dble(int(maxval(Ti)+1))/(10d0), &
    ya, dble(int(maxval(rt)+1d0)), yor, dble(int(maxval(rt)))/10d0) ! sets up axis
  call title ! Actually draw the title in over the axis
  call curve(Ti,Rt,npsurf+npsub+1) ! draw the x-y curve
  call disfin ! finish off the plot

  call disini() !Initialize dislin
  call complx
  call name("Infiltration depth (mm)","X") ! Set label for x-axis
  call name("Infiltration rate (mm/hr)","Y") ! Set label for y-axis
  call psfont("Helvetica")
  call labdig(2,"X")
  call labdig(1,"Y")
  call graf (dble(int(L2(2))), dble(int(maxval(L2)+1d0)), dble(int(L2(2))), &
    dble(int(maxval(L2)+1))/(7d0), ya, dble(int(maxval(rt)+1d0)), yor, &
    dble(int(maxval(rt)))/10d0) ! sets up axis
  call title ! Actually draw the title in over the axis
  call curve(L2,Rt,npsub+npsurf)!npsurf+npsub) ! draw the x-y curve
  call disfin ! finish off the plot

  do j=1,npsurf+npsub
    write(14,"(f6.3)")Ti(j)
    write(12,"(f8.3)")L2(j)
    write(15,"(f10.5)")Rt(j)
  end do

  do i=1,npsurf+npsub
    lin(i,1:11)=L2(i)
    write(13,"(100f11.5)")(-lin(i,j),j=1,11)
  end do

end program grnampt



function depth1(d)
  use prm
  double precision::depth1
  double precision,intent(in)::d

  depth1=-(1d0/K1)*(S1*imd1*log(1+d/(S1*imd1))-d)-t

end function
```

```
function depth2(d)
  use prm
  double precision::depth2
  double precision,intent(in)::d

  depth2=(d*imd2)/K2+(1d0/(K1*K2))*(imd2*H1*K2-imd2*K1*(S2-H1))*log(1+d/(S2+H1))-t

end function


subroutine secant(xold,xolder,maxit,epsi1,epsi2,root,numit,exitflag,f)
  implicit none
  double precision,intent(inout)::xold,xolder
  double precision,intent(in)::epsi1,epsi2
  double precision,intent(out)::root
  double precision::xnew,fxnew,fxold,fxolder     !local variables
  integer,intent(in)::maxit
  integer,intent(out)::numit
  logical,intent(out)::exitflag
  interface
    function f(x)
      double precision::x
      double precision::f
    end function f
  end interface

  !This is a general rootfinding subroutine that employs the secant method
  !it must be used in conjunction with an external function subprogram that
  !will evaluate the function in question
  !
  !variable list:
  ! local variables:
  !xnew      =updated root estimate
  !fxnew     =function evaluated at updated root estimate
  !fxold     =function evaluated at old root estimate
  !fxolder   =function evaluated at older root estimate
  ! inputs:
  !xold      =old root estimate
  !xolder    =older root estimate
  !epsi1     =stopping criteria for root found, if f(xnew)<epsi1 then root found
  !epsi2     =stopping criteria for slow progress, if abs(xold-xolder)<epsi2) then too slow
  !maxit     =maxumum allowable iterations subroutine will perform until it exits
  ! outputs:
  !root      =value of particular found root
  !numit     =records number of iterations done by subroutine

  numit=0
  fxold=f(xold)
  fxolder=f(xolder)
  do
    xnew=xold-(fxold*((xold-xolder)/(fxold-fxolder)))
              !fxold*((xold-xolder)/(fxold-fxolder)) is root update
    fxnew=f(xnew)
    !write(*,*)"fxnew=",fxnew
    numit=numit+1
    if(abs(fxnew)<epsi1)then
      !write(*,*)"root found"
      root=xnew
```

```
      exitflag=.true.
      return
      exit
    else if(numit>maxit)then
      write(*,*)"No root found (max iterations exceeded), try a better guess."
      exitflag=.false.
      return
      exit
    else if(abs(xold-xolder)<epsi2)then
      write(*,*)"No root found (progress too slow),try better guess."
      exitflag=.false.
      return
    else if ((abs(xnew-xold))>=(abs(xold-xolder)) .and. numit/=1)then
      write(*,"(a38,x,i2,x,a28)")"No root found (solution diverged after",numit,"iterations),&
                              try better guess."
      exitflag=.false.
      return
    end if
    fxolder=fxold    !swap values to reduce number of functional evaluations
    fxold=fxnew
    xolder=xold
    xold=xnew
    !write(*,*)"fxolder=",fxolder
    !write(*,*)"fxold=",fxold
    !write(*,*)"xolder=",xolder
    !write(*,*)"xold=",xold
  end do
end subroutine secant


program grnampt2
  use prm
  use dislin
  implicit none
  double precision,allocatable,dimension(:)::L2,Ti,D,y
  double precision::tstep1,tstep2,eps1,eps2,depth,s,l,xa,xe,xor,xstep,ya,ye,yor,ystep
  double precision::tstart,tend,hmin,hmax,deltat,h,t
  integer::j,k,r,count,maxit,npsurf,npsub,numit,npts,neq,exitf
  logical::exitflag
  character(len=200)::legendstring

  interface
    subroutine rkf(tstart,tend,n,y,h,hmin,hmax,eps1,eps2,f,exitflag)
      double precision,dimension(:),intent(inout)::y
      double precision,intent(in)::hmin,hmax,eps1,eps2
      double precision,intent(inout)::tstart,tend,h
      integer,intent(inout)::exitflag
      integer,intent(in)::n
      interface
        subroutine f(t,y,Dy,exitflag)
          integer,intent(inout)::exitflag
          double precision,intent(in)::t
          double precision,dimension(:),intent(in)::y
          double precision,dimension(:),intent(out)::Dy
        end subroutine f
      end interface
    end subroutine rkf
    subroutine ode(t,y,Dy,exitflag)
      integer,intent(inout)::exitflag
```

```
      double precision,intent(in)::t
      double precision,dimension(:),intent(in)::y
      double precision,dimension(:),intent(out)::Dy
    end subroutine ode
    subroutine ode1(t,y,Dy,exitflag)
      integer,intent(inout)::exitflag
      double precision,intent(in)::t
      double precision,dimension(:),intent(in)::y
      double precision,dimension(:),intent(out)::Dy
    end subroutine ode1
  end interface

  open(11,file="prm.in")
  open(13,file="ode.out")
  read(11,*)S1,S2,K1,K2,imd1,imd2,H1,i,maxit,npsurf,npsub,npts,eps1,eps2
  neq=1
  allocate(y(neq),D(9))


  neq=1
  D=0
  deltat=.1
  hmin=.001
  hmax=100d0
  y(1)=0d0
  do i=1,npts+1
    tstart=0
    tend=0
    do j=1,8
      tstart=tend                         !advance interval
      tend=tend+deltat
      if(y(1)>=30)then
        call rkf(tstart,tend,neq,y,h,hmin,hmax,eps1,eps2,ode,exitf)
      else
        call rkf(tstart,tend,neq,y,h,hmin,hmax,eps1,eps2,ode1,exitf)
      end if
      D(i)=y(1)
    end do
    write(13,"(100f10.5)")(-D(r),r=1,npts)
  end do

end program grnampt2



subroutine ode(t,y,Dy,exitflag)
  use prm
  implicit none
  integer,intent(inout)::exitflag
  double precision,intent(in)::t
  double precision,dimension(:),intent(in)::y
  double precision,dimension(:),intent(out)::Dy

  Dy(1)=-K1*((S1*imd1+y(1))/(y(1)))  !top layer

end subroutine ode
```

```
subroutine ode1(t,y,Dy,exitflag)
  use prm
  implicit none
  integer,intent(inout)::exitflag
  double precision,intent(in)::t
  double precision,dimension(:),intent(in)::y
  double precision,dimension(:),intent(out)::Dy

  Dy(1)=(K1*K2*(S2+H1+y(1)))/(H1*K1+y(1)*K1) !bottom layer

end subroutine ode1


subroutine rkf(tstart,tend,n,y,h,hmin,hmax,eps1,eps2,f,exitflag)
  implicit none
  double precision,dimension(:),intent(inout)::y
  double precision,intent(in)::hmin,hmax,eps1,eps2
  double precision,intent(inout)::tstart,tend,h
  integer,intent(inout)::exitflag
  integer,intent(in)::n
  double precision,dimension(n)::K1,K2,K3,K4,K5,K6,Dy,y4,ysave
  double precision::t,hsave,emax
  double precision,parameter::c1=1d0/5d0,c2=3d0/10d0,c3=3d0/40d0,c4=9d0/40d0,&
            c5=3d0/5d0,c6=3d0/10d0,c7=-9d0/10d0,c8=6d0/5d0,c9=11d0/54d0,&
            c10=5d0/2d0,c11=-70d0/27d0,c12=35d0/27d0,c13=7d0/8d0,&
            c14=1631d0/55296d0,c15=175d0/512d0,c16=575d0/13824d0,c17=44275d0/110592d0,&
            c18=253d0/4096d0,c19=37d0/378d0,c20=250d0/621d0,c21=125d0/594d0,&
            c22=512d0/1771d0,c23=2825d0/27648d0,c24=18575d0/48384d0,&
            c25=13525d0/55296d0,c26=277d0/14336d0,c27=1d0/4d0

  interface
    subroutine f(t,y,Dy,exitflag)
      integer,intent(out)::exitflag
      double precision,intent(in)::t
      double precision,dimension(:),intent(in)::y
      double precision,dimension(:),intent(out)::Dy
    end subroutine f
    subroutine test(t,y,Dy,exitflag)
      integer,intent(inout)::exitflag
      double precision,intent(in)::t
      double precision,dimension(:),intent(in)::y
      double precision,dimension(:),intent(out)::Dy
    end subroutine test
  end interface

  !variable list
  !y=          solution vector
  !h=          step size
  !hmin=       minimum step size
  !hmax=       maximum step size
  !hsave=      save step size for end of interval
  !t=          current time
  !tstart=     start of time interval
  !tend=       end of time interval
  !n=          number of equations
  !Kn=         Runge Kutta constants
  !Dy=         Derivative estimate at current time
  !y4=         fourth order runge-kutta
  !emax=       error between fourth and fifth order estimates
```

```
  !exitflag=   error checking
  !eps1=       minimum allowable error
  !eps2=       maximum allowable error

  t=tstart
  exitflag=0
  if((t+h)>tend)then
    hsave=h
    h=tend-t
  end if
  do
    ysave=y
    call f(t,y,Dy,exitflag)
      if(exitflag/=0)return
      K1=h*Dy
    call f(t+c1*h,y+c1*K1*h,Dy,exitflag)
      if(exitflag/=0)return
      K2=h*Dy
    call f(t+c2*h,y+c3*K1*h+c4*K2*h,Dy,exitflag)
      if(exitflag/=0)return
      K3=h*Dy
    call f(t+c5*h,y+c6*K1*h+c7*K2*h+c8*K3*h,Dy,exitflag)
      if(exitflag/=0)return
      K4=h*Dy
    call f(t+h,y+c9*K1*h+c10*K2*h+c11*K3*h+c12*K4*h,Dy,exitflag)
      if(exitflag/=0)return
      K5=h*Dy
    call f(t+c13*h,y+c14*K1*h+c15*K2*h+c16*K3*h+c17*K4*h+c18*K5*h,Dy,exitflag)
      if(exitflag/=0)return
      K6=h*Dy
    y4=y+(c19*K1+c20*K3+c21*K4+c22*K6)*h
    y=y+(c23*K1+c24*K3+c25*K4+c26*K5+c27*K6)*h
    emax=maxval(abs((y-y4)/y))          !max relative truncation error
    if(emax>eps2 .and. abs(h-hmin)>10d-6)then
      h=h/2                             !large error,reduce step size and try again
      y=ysave
    else
      t=t+h                             !advance time, accept solution
      if(emax>eps2)exitflag=1           ! big error but h=hmin
      if(emax<eps1 .and. h<hmax)then
        h=h*2d0                         !small error increase step size
        if(h>hmax)h=hmax
      end if
      hsave=h                           !save the step size we are on
      if(t>=tend)exit                   !are we done?
      if((t+h)>tend)h=tend-t            !will the next step be beyond the end
    end if
  end do
  h=hsave
  return
end subroutine rkf
```

Appendix B Program Output

```
~depth.dat~
   0.000
```

```
 8.308
12.062
15.068
17.690
20.066
22.268
24.339
26.306
28.188
30.000
32.663
33.799
34.685
35.441
36.115
36.728
37.297
37.832
38.338
38.820
39.281
39.724
40.151
40.565
40.966
41.356
41.736
42.106
42.469
42.823
43.170
43.510
43.844
44.173
44.495
44.813
45.126
45.434
45.737
46.037
46.333
46.624
46.913
47.198
47.479
47.758
48.034
48.306
48.576
48.844
49.109
49.371
49.631
49.889
50.144
50.398
50.649
50.898
51.146
```

```
  51.391
  51.635
  51.877
  52.117
  52.356
  52.593
  52.828
  53.062
  53.295
  53.526
  53.755
  53.983
  54.210
  54.436
  54.660
  54.883
  55.105
  55.325
  55.545
  55.763
  55.980
  56.196
  56.411
  56.625
  56.838
  57.050
  57.261
  57.471
  57.680
  57.888
  58.096
  58.302
  58.507
  58.712
  58.916
  59.119
  59.321
  59.522
  59.723
  59.923
  60.122
  60.320
  60.518
  60.714
  60.911
  61.106
  61.301
  61.495
  61.688
  61.881



~rate.dat~
   0.00000
 125.22205
  93.03911
  78.82426
  70.37300
```

```
64.62127
60.38657
57.10371
54.46405
52.28291
50.44215
23.06842
22.39427
21.89862
21.49539
21.15060
20.84750
20.57505
20.32652
20.09792
19.88580
19.68763
19.50148
19.32580
19.15935
19.00109
18.85019
18.70593
18.56768
18.43493
18.30723
18.18416
18.06539
17.95059
17.83951
17.73188
17.62749
17.52614
17.42764
17.33184
17.23858
17.14772
17.05914
16.97273
16.88837
16.80597
16.72543
16.64668
16.56962
16.49420
16.42034
16.34797
16.27703
16.20748
16.13925
16.07229
16.00656
15.94202
15.87862
15.81632
15.75508
15.69487
15.63566
15.57740
```

```
  15.52008
  15.46367
  15.40812
  15.35343
  15.29956
  15.24649
  15.19419
  15.14265
  15.09184
  15.04174
  14.99234
  14.94362
  14.89555
  14.84812
  14.80131
  14.75512
  14.70951
  14.66449
  14.62003
  14.57612
  14.53275
  14.48990
  14.44756
  14.40573
  14.36439
  14.32353
  14.28313
  14.24320
  14.20371
  14.16466
  14.12605
  14.08785
  14.05007
  14.01269
  13.97570
  13.93910
  13.90289
  13.86704
  13.83157
  13.79645
  13.76168
  13.72726
  13.69317
  13.65942
  13.62600
  13.59290



~time.dat~
 0.000
 0.035
 0.071
 0.106
 0.141
 0.176
 0.212
 0.247
 0.282
```

```
0.318
0.353
0.357
0.361
0.365
0.369
0.373
0.377
0.381
0.384
0.388
0.392
0.396
0.400
0.404
0.408
0.412
0.416
0.420
0.424
0.428
0.432
0.436
0.440
0.444
0.448
0.452
0.456
0.459
0.463
0.467
0.471
0.475
0.479
0.483
0.487
0.491
0.495
0.499
0.503
0.507
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