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Problem Chosen

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2007_ Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Type a summary of your results on this page. Do not include
the name of your school, advisor, or team members on this page.

To determine the airplane boarding schedule that minimizes passenger boarding time, a simulation scheme is proposed. A Fortran 90 program has been developed that simulates passenger boarding within a set of assumptions. The model replicates three airplane configurations that can each be simulated for a range of passengers. Algorithms were developed to emulate three popular boarding schedules. The simulation emulates a realistic airplane with passengers boarding, walking down the aisle(s), stopping at their row, delaying any passengers behind them in order to store baggage and let passengers stand who are already sitting in between aisle and the assigned seat, and taking a seat. Assumptions were made that create a uniform passenger profile. Variability was added by randomly assigning seats and randomly assigning delay times.

The Monte Carlo Method was applied to the simulation model. The simulation was executed 1,500 times for each airplane configuration and boarding schedule combination. Total boarding times were determined for the different scenarios. Statistical analysis was completed by determining the sample mean, approximate population mean, 95% confidence intervals about the mean, standard deviation, and variances.

Sensitivity analysis was preformed on the distribution of delay times, time delays associated with passengers standing in order to allow another to sit, and the size of boarding groups. For each of these analyses, the conclusions remained the same.

The Outside In Boarding (OIB) schedule consistently outperformed the Random Boarding Assigned (RBA) and Back to Front Block Boarding (BTF) schedules. Although the OIB performed best, it would be difficult to implement because of passenger desires and needed control measures. The most practical, of the three schedules investigated, is the RBA schedule.

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Executive Summary

To determine the airplane boarding schedule that minimizes boarding time, a simulation was created. A computer program was written, which replicates three boarding schedules (Table 1) and for three airplane layouts (Table 2).

Table 1: Description of Simulated Boarding Schedules (van den Briel, 2006)

Boarding Name	Short Name	Description
Random Boarding Assigned	RBA	All passengers are assigned a seat. The assignment is random. Passengers enter the plane at random.
Back to Front Block Boarding	BTF	All passengers are assigned a seat. A block of rows at the rear of the plane is filled first. When the block is filled, the adjacent block (towards the front of the plane) is filled. The filling of seats within a block is random.
Outside In Boarding	OIB	All passengers are assigned a seat. For planes with one aisle, window seats are filled first. Next, seats adjacent to the windows are filled. This pattern is repeated until all seats are filled. The seating within each boarding group is completely random (e.g. the first passenger to enter the plane might be assigned a window seat on the port side in the 3rd row and the second passenger a window seat on the starboard side in the last row). Similarly, planes with two aisles fill the window seats and the "middle" seats first. The pattern continues to fill from the windows in and from the "middle" seat out until all seats are filled.

Table 2: Seating Arrangements Used in Simulation

Airplane Layouts													
2-3	A	B		D	E	F							
2-5-2	A	B		D	E	F	G	H		J	K		
3-5-3	A	B	C		D	E	F	G	H		I	J	K

The simulation emulates a realistic airplane with:

- passengers boarding;
- passengers walking down the aisle(s);
- passengers stopping at their row and holding up forward movement while stowing carry-on bags; and

- passengers holding up forward movement when other passengers are already seated in the same row and have to get up and step out of the way.

To simplify the simulation, the following assumptions were applied.

- The total amount of time it takes the first class passengers to reach their assigned seats is not a limiting factor.
- First class seating is completed before coach begins to board, and therefore, first class passengers do not interfere with coach boarding.
- No special needs passengers.
- The 'special needs' and first class passengers do not get in the way of any of the other passengers and do not account for any of the seats being simulated.
- Everyone boards the plane in their designated boarding group (i.e. no one is late or breaking the rules).
- The amount of time it takes any single passenger to move one seat position down the aisle is fixed at 1 second.
- Every passenger boarding the plane has carry-on baggage. The amount of time it takes a passenger to stow their bag(s) varies between 10 and 20 seconds.
- The amount of time it takes for any single passenger to get up and let another passenger get to their seat in the same row is fixed at 10 seconds.

The simulation was executed 1,500 times for each airplane configuration and boarding schedule combination. Total boarding times were determined for the different scenarios. Statistical analysis was completed by determining the sample mean, approximate population mean, 95% confidence intervals about the mean, standard deviation, and variances.

Sensitivity analysis was preformed on the distribution of delay times, time delays associated with passengers standing in order to allow another to sit, and the size of boarding groups. For each of these analyses, the conclusions remained the same.

The Outside In Boarding (OIB) schedule consistently outperformed the Random Boarding Assigned (RBA) and Back to Front Block Boarding (BTF) schedules. Although the OIB performed best, it would be difficult to implement because of passenger desires and needed control measures. The most practical, of the three schedules investigated, is the RBA schedule.

1 Introduction

The number of passengers traveling on U.S. airline carriers increased 5% between 2004 and 2005 (DOT, 2006). In total, 738.6 million passengers flew on 890,000 flights during 2005. The amount time that a plane is on the ground between flights is referred to as turnaround time. An airline's revenue is inversely proportional to its turnaround times; airlines only make money while in their planes are in the air (Finney, 2006). According to a November 14, 2006 New York Times article *Loading an Airliner is Rocket Science*, Southwest Airlines would need to increase its airplane fleet by 18 planes at an approximate cost of $972 million if turnaround times were increased by five minutes per flight (Finney, 2006).

Many factors influence boarding time:

- boarding method/schedule applied by airline;
- the needs of passengers (special conditions etc.);
- order in which passengers enter the plane;
- time spent by passengers putting luggage in overhead compartments and taking their seat;
- speed of walking pace through plane;
- airplane schematics; and
- the human factor.

Although all the above factors impact loading, research has found that the boarding schedule is one of the most significant. A boarding schedule describes the method in which an airline chooses to load passengers onto their planes. There are a number of applied methods ranging from completely random (no assigned seats or boarding schedule) to extensively rigid (assigned seats and assigned spot in boarding schedule).

The objective of this project is to evaluate airplane boarding times for different boarding schedules applied to different airplane layouts and configurations. Understanding and improving upon airplane boarding times is important for both the airlines and passengers.

2 Background Information

Many airlines utilize airplanes from the same manufacturers resulting in a limited number of seating arrangements (Table 3). The time it takes passengers to make certain movements around airplanes has been studied (Table 4).

Table 3: Common Seating Arrangements

	Airplane Layouts												
2-3	A	B		D	E	F							
3-3	A	B	C		D	E	F						
2-4-2	A	B		C	D	E	F		G	H			
3-4-3	A	B	C		D	E	F	G		H	I	J	
2-5-2	A	B		D	E	F	G	H		J	K		
3-5-3	A	B	C		D	E	F	G	H		I	J	K

Table 4: Passenger Movement Times (Van Landeghem, 2002)

	Process times (seconds)		
	Min	Modus	Max
Passing one row	1.8	2.4	3
Luggage install time (per piece)	6	9	30
Exit from seat into aisle	3	3.6	4.2

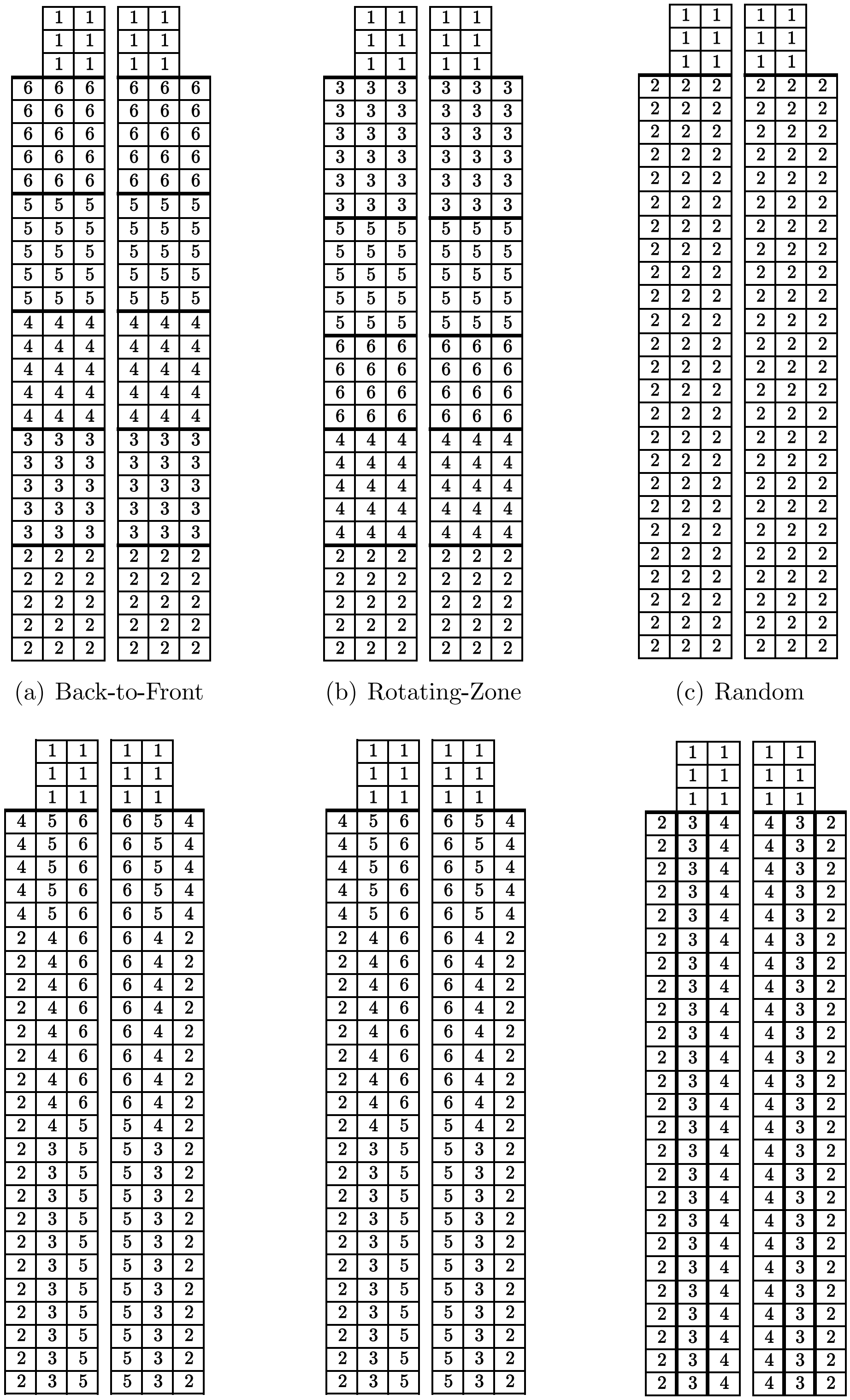
2.1 Airplane Boarding Techniques

The minimization of boarding time has been looked at in the past and seven main boarding schedules (Figure 7) have been recognized as 'optimal.' All schedules divide the passengers into boarding groups (represented by the number in each seat). Note that these boarding schedules include first class as the first boarding group.

2.2 Previous Modeling Approaches

Past studies on airplane boarding schedules have been governed by the critical path method. The critical path method identifies the longest route through a task flow chart. The only way to minimize total time is to minimize the critical path route by reconfiguring the flow chart.

Simulation and optimization have also been used to analyze the airplane boarding problem. Simulation models are used when the impacts of a planning schedule are desired while optimization models are used to identify an optimal schedule (Willis and Finney, 2004).



(a) Back-to-Front (b) Rotating-Zone (c) Random

(d) Block (e) Rev-Pyramid (f) Outside-In

Figure 1: Different Boarding Systems (van den Briel, 2006)

3 Model Development

As discussed in the previous section, there are numerous approaches to evaluating the boarding of passenger airplanes. A simulation approach was applied in this project. Figure 2 details the general logic of the program including:

- initialize static variables;
- generate boarding queue of random seating assignments specific to boarding schedule;
- set random variable bounds for delay times (time spent putting carry on bags away and sitting);
- determine if passengers will need to wait for another passenger in their row to stand so they can assume their seat(if so, they are assigned a penalty); and
- run simulation (see below for simulation logic).

Figure 3 details the logic of the simulation including:

- begin time;
- move first person onto plane;
- if any passengers have a delay time equal to 1, then they are ready to move into their seat;
- decrease all delay times;
- evaluate who can move;
- determine if person waiting to board plane can board (if person at first row, in the appropriate aisle, has a delay time, then waiting person waiting cannot board plane);
- assess if any new delays need to be assigned after latest movement (i.e. the person has reached their row);
- determine if everyone has been seated; and
- report current or final results.

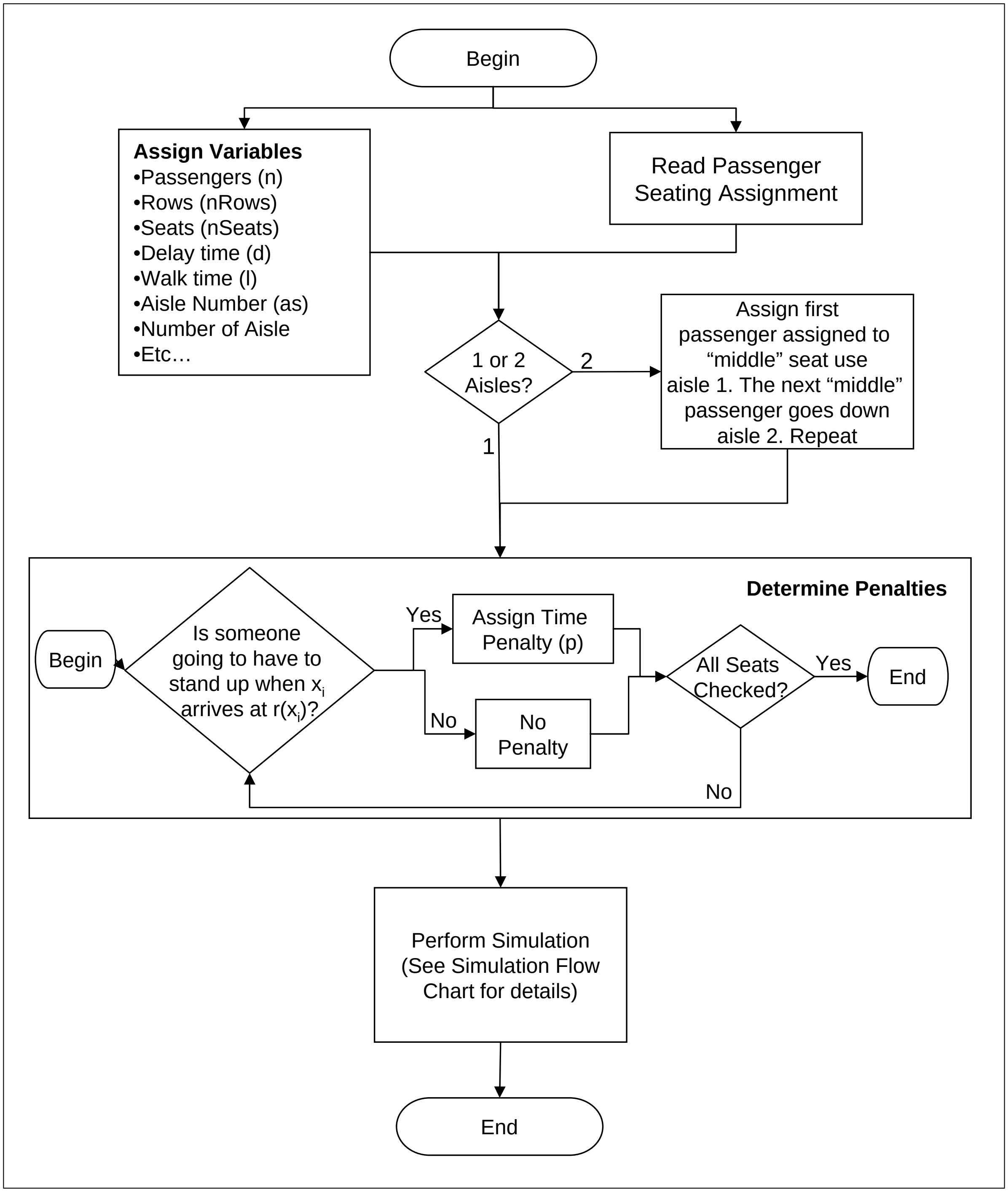


Figure 2: General Flow Chart of Entire Program

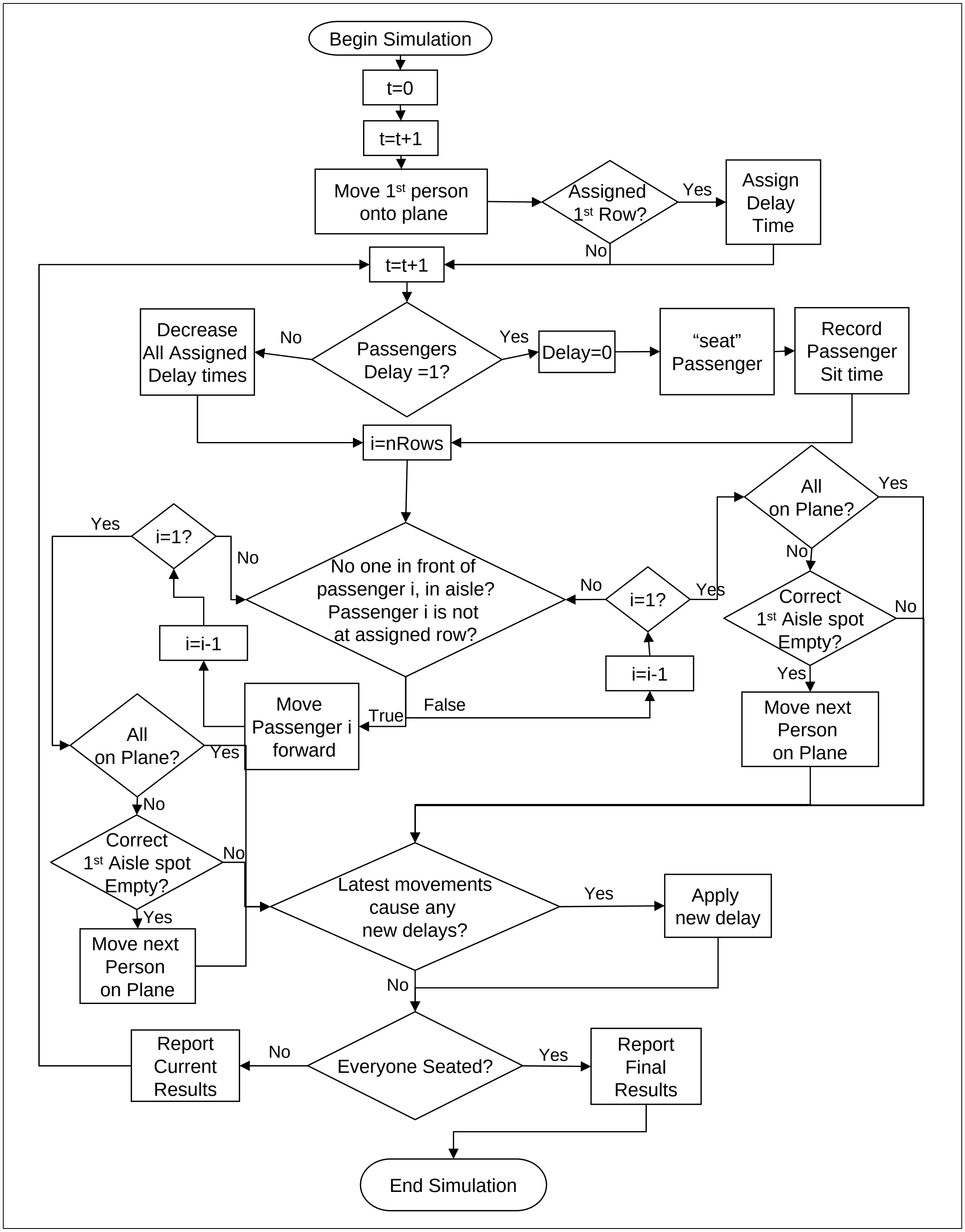


Figure 3: Simulation Flow Chart of Entire Program

The applied simulation incorporates a Monte Carlo approach by creating random values for the seat assignments and the delay times. All passengers are assigned a seat and no two passengers can have the same seat. The boarding queue (the order in which people will enter the plane) is broken into boarding groups delineated by either position on plane or by row of assigned seat. The queueing order of passengers within each boarding group is random.

For example, if a Back to Front Block boarding schedule is applied, the boarding groups are designated by a row number range. However, the passengers in a single group will appear randomly in their boarding group queue. The next boarding group enters the plane only after the previous is completely on. All boarding groups will have a random queue consisting of the people assigned to that group.

The delay times represent the time it takes a passenger to sit from the time they arrive at their row. The portion of the delay times associated with stowing carry-on bags is randomly assigned and uniformly distributed over a set interval.

The random numbers were generated using Fortran's intrinsic random number subroutine. Output was verified by comparing them to realistic values and to other studies' results (Ferrari and Nagel, 2004) (see Figure 5 on page 16 for sample output).

Statistical analysis was performed on the resulting total time data. This included:

- determining the sample mean $\bar{x}$;
- determining the approximate population mean $\mu_{\bar{x}}$;
- determining the 95% confidence interval about $\mu_{\bar{x}}$;
- determining variance $\sigma_{\bar{x}}^2$; and
- determining standard deviation $\sigma_{\bar{x}}$.

The above variables are defined as (Devore, 2000):

$$\bar{x} = \frac{\sum_{i=n}^{n} x_i}{n} \tag{1}$$

$$\mu_{\bar{x}} = \frac{\sum_{i=n}^{n} \bar{x}_i}{n} \tag{2}$$

$$\sigma_{\bar{x}}^2 = \frac{1}{n}\sum_{i=n}^{n}(\bar{x} - \mu_{\bar{x}})^2 \tag{3}$$

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{n}\sum_{i=n}^{n}(\bar{x} - \mu_{\bar{x}})^2} \tag{4}$$

where x_i is the ith observation and
n is the sample size.

4 Model Application

After review of past work, the three loading schedules (Table 5), off of which all others are modeled, were chosen to be simulated. Each of the three loading techniques were simulated on three plane sizes (Table 6).

Table 5: Description of Simulated Boarding Schedules (van den Briel, 2006)

Boarding Name	Short Name	Description
Random Boarding Assigned	RBA	All passengers are assigned a seat. The assignment is random. Passengers enter the plane at random.
Back to Front Block Boarding	BTF	All passengers are assigned a seat. A block of rows at the rear of the plane is filled first. When the block is filled, the adjacent block (towards the front of the plane) is filled. The filling of seats within a block is random.
Outside In Boarding	OIB	All passengers are assigned a seat. For planes with one aisle, window seats are filled first. Next, seats adjacent to the windows are filled. This pattern is repeated until all seats are filled. The seating within each boarding group is completely random (e.g. the first passenger to enter the plane might be assigned a window seat on the port side in the 3rd row and the second passenger a window seat on the starboard side in the last row). Similarly, planes with two aisles fill the window seats and the "middle" seats first. The pattern continues to fill from the windows in and from the "middle" seat out until all seats are filled.

Table 6: Description of Airplane Details

	Plane Size		
Variable	Small	Medium	Large
Seats	100	387	605
Rows	20	43	55
Seats/Row	5	9	11
Aisles	1	2	2

To decrease the amount of variability in the problem, a set of assumptions were made:

- The total amount of time it takes the first class passengers and the ‘special needs’ passengers to reach their assigned seats is fixed for each airplane layout.
- The ‘special needs’ and first class passengers do not get in the way of any of the other passengers and do not account for any of the seats being simulated.
- Everyone boards the plane in their designated boarding group (i.e. no one is late or breaking the rules).

- The amount of time it takes any single passenger to move one seat position down the aisle is fixed.
- Every passenger boarding the plane has carry-on baggage. The amount of time it takes them to stow the bag is uniformly distributed and lies within an interval of size 10 seconds.
- The amount of time it takes for any single passenger to 'get up' to let another passenger get to their seat is fixed.

Table 7: Simulation Parameters

Parameter	Variable	Initial Value (s)
Passenger travel time between rows	L	1
Time to move past seated passenger	p	10
Average time to load bag and sit down	d	15
Minimum possible time to load bag and sit down	ll	d - 5
Maximum possible time to load bag and sit down	ul	d + 5

Table 8: Parameters Varied in Sensitivity Analysis

Parameter	Initial Value (s)	Change in Parameter Value	New Value (s)
p	10	+ 20%	12
		-20%	8
		+40%	14
		-40%	6
d	15	+ 20%	18
		-20%	12
		+40%	21
		-40%	9
Block* (rows)	5	33.3%	3
		50%	10

*Variable in BTF loading schedule. Refers to the number of rows in a boarding group.

5 Results

Sample program output represents the movement of passengers over time as the boarding process proceeds (Figure 4). Due to random variables in the simulation, output of one simulation represents a single stochastic realization of the boarding process. To mitigate the random effects of the boarding process 50 samples were taken from the population and averaged to find a single sample mean ($\bar{X}$). 30 sample means $\left(\{\bar{X_1}, \bar{X_2}, ..., \bar{X_3}0\}\right)$ were calculated for each combination of boarding schedule and plane layout. The average of the 30 sample means ($\mu_{\bar{X}}$) equals the population mean ($\mu_{\bar{X}} = \mu_X$). The sensitivity analysis shows how the parameters affect the resulting average boarding time (Figure 11). All results are reported in seconds(Table 9).

Table 9: Time Conversions

Seconds	Minutes
500	8.33
1,000	16.66
2,000	33.33
3,000	80

The Back to Front boarding took the longest for all airplane layouts (Figure 4) because the schedule contains many Outside in boarding took the shortest amount of time and the Random Assigned Seat boarding took an amount of time in between BTF and OIB. OIB took the shortest amount of time for all airplane layouts and is therefore the optimal simulation solution. Out of the boarding schedules simulated, BTF was the least optimal.

An increase in the average delay time (d) caused a increase in the total boarding time for all seating schedules. An increase in the penalty (p) for moving around a seated passenger caused an increase in the average loading time as well for RBA and BTF. For all schedules the opposite case was true (decrease in parameter → decrease in average boarding time) except for OIB. OIB was not affected by an increase or decrease in p because no passenger ever needed to move around a seated passenger. An increase in the block size of BTF boarding caused a decrease in the average loading time for all airplane layouts. The opposite case is true as well (decrease in block size → decrease in average boarding time).

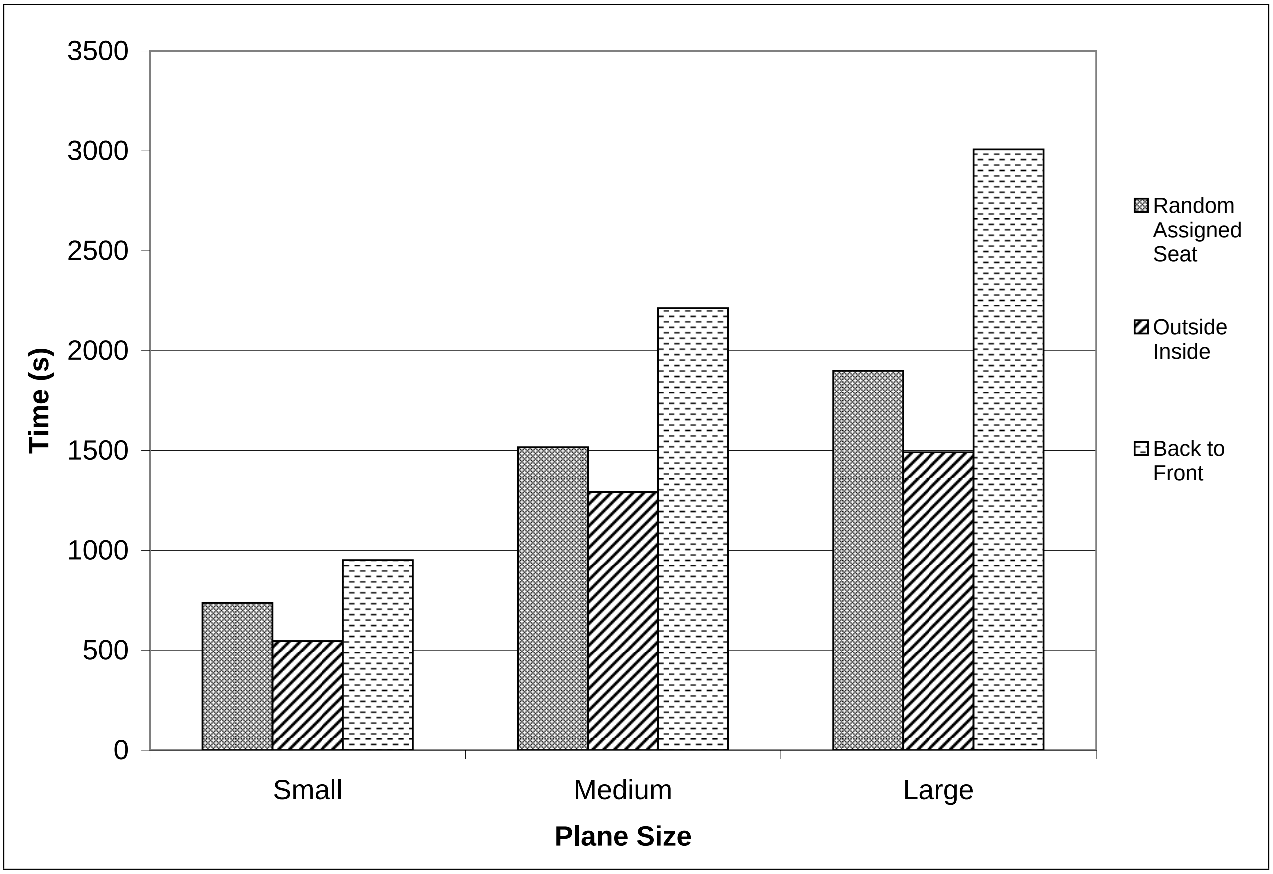


Figure 4: Average Loading times

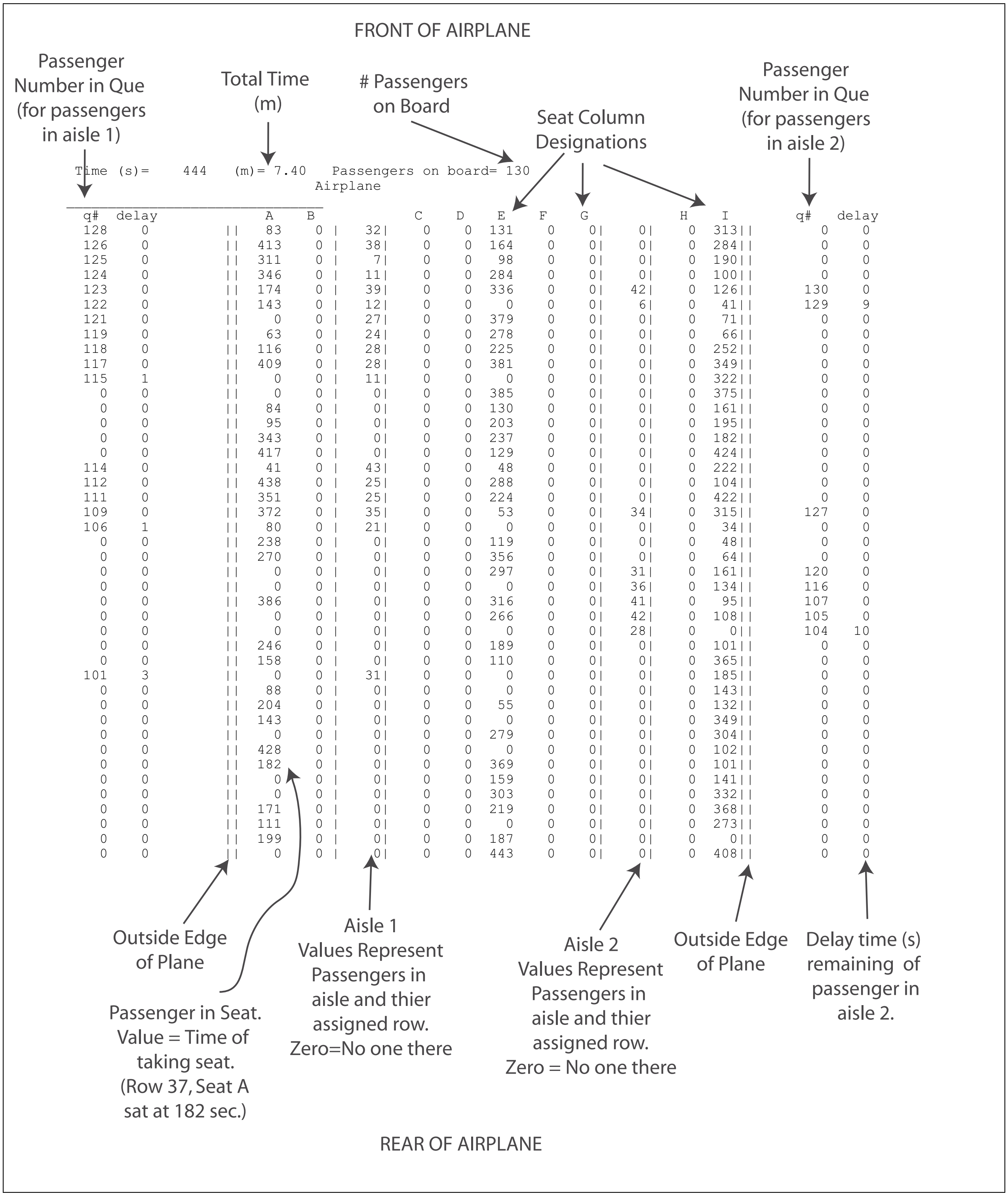


Figure 5: Description of Simulated Loading Techniques

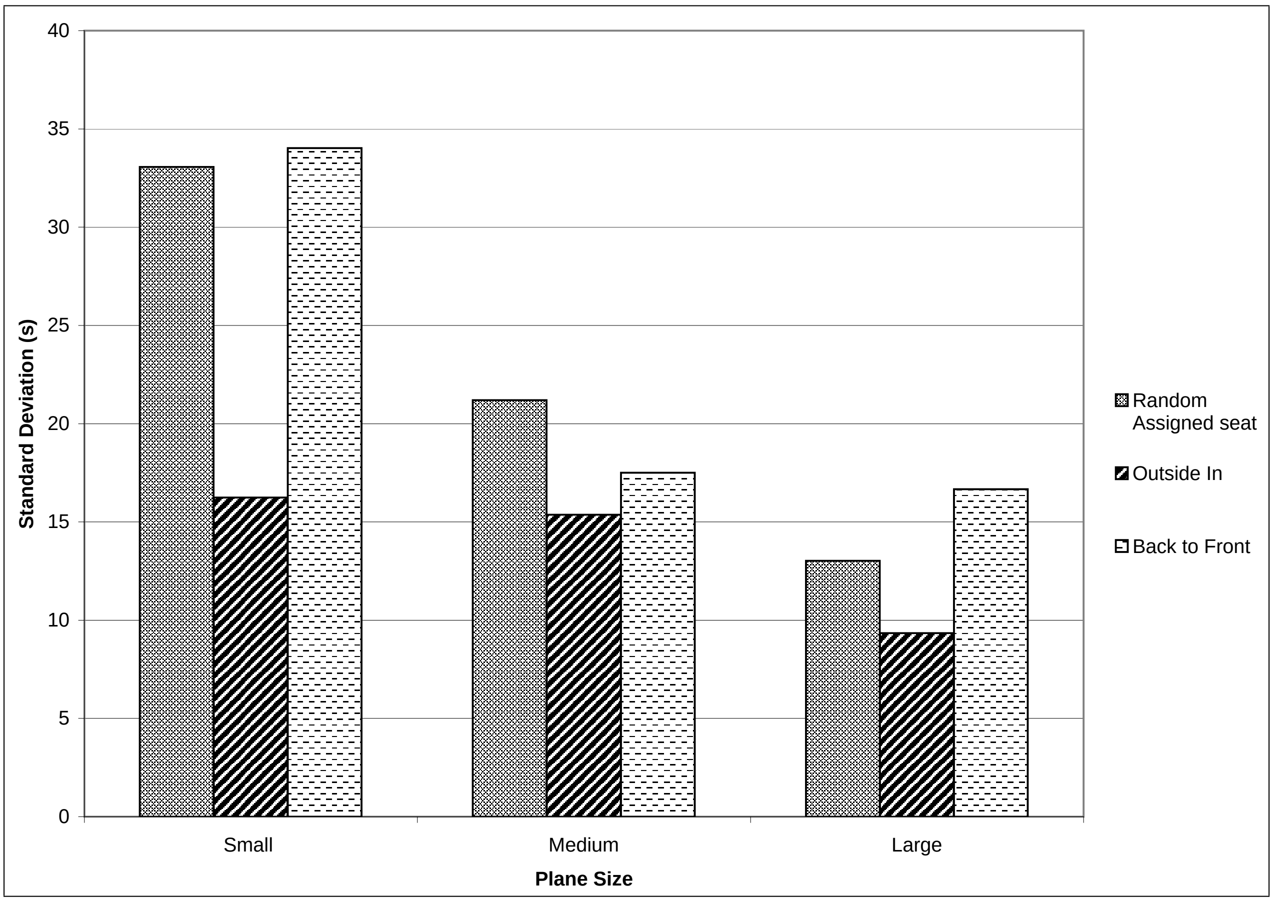


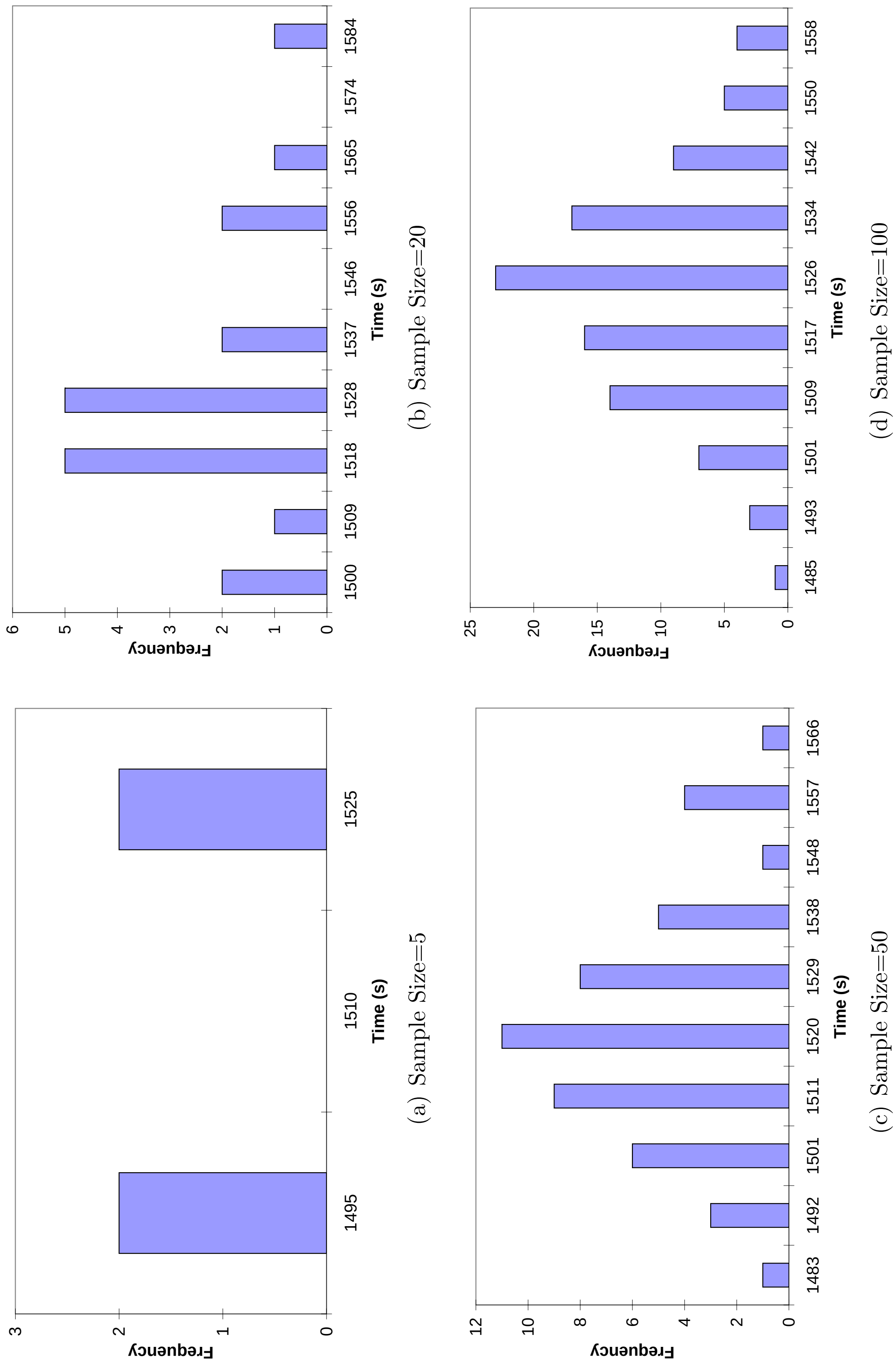
Figure 6: Standard deviation of average loading time

Table 10: Results for RBA and OIB Loading

Note: All units in seconds				Small Plane (85 Passengers)				Medium Plane (387 Passengers)				Large Plane (660 Passengers)			
				μ_x	σ_x	σ_x^2	95% C.I for μ_x	μ_x	σ_x	σ_x^2	95% C.I for μ_x	μ_x	σ_x	σ_x^2	95% C.I for μ_x
RBA	Standard			737.74	33.07	1093.95	64.83	1516.39	21.2	449.57	41.56	1899.31	13.02	169.56	25.52
RBA	Sensitivity	Delay	40%	927.44	38.71	1498.29	75.87	1827.97	20.2	408.18	39.6	2319.17	14.32	204.97	28.06
RBA	Sensitivity	Delay	20%	833.26	30.71	943.14	60.19	1670.54	20.04	401.64	39.28	2107.11	16.42	269.71	32.19
RBA	Sensitivity	Delay	-20%	642.56	28.24	797.61	55.35	1369.41	16.18	261.69	31.71	1697.28	13.03	169.88	25.55
RBA	Sensitivity	Delay	-40%	541.13	26.09	680.53	51.13	1222.62	16.34	266.87	32.02	1494.44	11.34	128.66	22.23
RBA	Sensitivity	Penalty	40%	805.38	36.9	1361.46	72.32	1609.65	20.57	423.15	40.32	2084.59	16.17	261.43	31.69
RBA	Sensitivity	Penalty	20%	765.63	35.58	1265.93	69.74	1566.73	23.38	546.75	45.83	1986.16	13.75	189.11	26.95
RBA	Sensitivity	Penalty	-20%	708.23	25.43	646.68	49.84	1471.69	18.66	348.06	36.57	1809.1	12.36	152.84	24.23
RBA	Sensitivity	Penalty	-40%	668.38	31.79	1010.74	62.31	1433.61	15.33	235.03	30.05	1725.92	12.91	166.78	25.31
OIB	Standard			950.8	34.02	1157.16	66.67	2212.41	17.5	306.1	34.29	3007.2	16.67	277.75	32.67
OIB	Sensitivity	Delay	20%	1906.44	31.66	1002.27	62.05	2474.24	20.58	423.37	40.33	3422.24	17.33	300.44	33.97
OIB	Sensitivity	Delay	10%	1234.15	29.76	885.62	58.33	2737.93	28.75	826.8	56.36	3835.53	1871	349.99	36.67
OIB	Sensitivity	Delay	-10%	817.22	27.92	779.34	54.72	1950.96	16.75	280.43	32.82	2607.75	17.09	292.15	33.5
OIB	Sensitivity	Delay	-20%	680.56	33.53	1123.98	65.71	1696.68	17.25	297.72	33.82	2193.11	11.7	136.94	22.94
OIB	Sensitivity	Penalty	20%	991.07	36.55	1336.13	71.64	2270.2	21.82	476.32	42.78	3158.73	15.81	250.07	30.99
OIB	Sensitivity	Penalty	10%	1031.38	35.18	1237.59	68.59	2340.66	21.3	453.55	41.74	3304.31	18.52	342.92	36.3
OIB	Sensitivity	Penalty	-10%	917.69	33.94	1151.87	66.52	2148.01	18.37	337.47	36.01	2870.1	20.1	404.17	39.4
OIB	Sensitivity	Penalty	-20%	881.74	27.31	745.67	53.52	2094.19	13.16	173.06	25.78	2734.68	14.92	222.65	29.25

Table 11: Results for BTF Loading

Note: All units in seconds				Small Plane (85 Passengers)				Medium Plane (387 Passengers)				Large Plane (660 Passengers)			
				μ_x	σ_x	σ_x^2	95% C.I for μ_x	μ_x	σ_x	σ_x^2	95% C.I for μ_x	μ_x	σ_x	σ_x^2	95% C.I for μ_x
BTF	Standard			950.8	34.02	1157.16	66.67	2212.41	17.5	306.1	34.29	3007.2	16.67	277.75	32.67
	Sensitivity	Delay	20%	1906.44	31.66	1002.27	62.05	2474.24	20.58	423.37	40.33	3422.24	17.33	300.44	33.97
			10%	1234.15	29.76	885.62	58.33	2737.93	28.75	826.8	56.36	3835.53	1871	349.99	36.67
			-10%	817.22	27.92	779.34	54.72	1950.96	16.75	280.43	32.82	2607.75	17.09	292.15	33.5
			-20%	680.56	33.53	1123.98	65.71	1696.68	17.25	297.72	33.82	2193.11	11.7	136.94	22.94
		Penalty	20%	991.07	36.55	1336.13	71.64	2270.2	21.82	476.32	42.78	3158.73	15.81	250.07	30.99
			10%	1031.38	35.18	1237.59	68.59	2340.66	21.3	453.55	41.74	3304.31	18.52	342.92	36.3
			-10%	917.69	33.94	1151.87	66.52	2148.01	18.37	337.47	36.01	2870.1	20.1	404.17	39.4
			-20%	881.74	27.31	745.67	53.52	2094.19	13.16	173.06	25.78	2734.68	14.92	222.65	29.25
		Block size	3 rows	1001.61	39.09	1528.04	1528.04	2532.2	16.63	276.59	32.6	3473.67	17.05	290.81	33.42
			8 rows	818.81	30.11	906.62	59.02	1876.97	21.35	455.82	41.85	2534.31	16.12	259.92	31.6



(a) Sample Size=5

(b) Sample Size=20

(c) Sample Size=50

(d) Sample Size=100

Figure 7: Histograms FINISH ME

6 Conclusion

The following can be concluded from the report:

- a simulation model was developed, using the Fortran programming language, to evaluate airplane boarding times;
- a number of assumptions were made in the development of the model
- three airplane layouts were used
- three boarding schedules were used;
- monte Carlo simulation was used to evaluate delay times;
- average boarding time was the least with the Outside In boarding schedule;
- average boarding time was the greatest with the Back to Front boarding schedule;
- average boarding time increased as airplane layout increased.
- standard deviation of average boarding time decreased as airplane layout increased.
- sensitivity analysis was preformed on the time taken to move past a seated passenger (p) and the passenger delay time(d);
- a change in the value of p had a negligible effect on the OIB average boarding time; and
- changes in the parameters had various effects on the average boarding time.

7 Further Research

Recommendations for further research include, but are not limited to:

- simulate Rotating-Zone, Reverse-Pyramid, and Block Boarding schedules;
-
- include first class and 'special needs' into simulation;
- include late passengers arriving late in simulation;
- add a 'passenger response' randomness to boarding schedule to simulate how well passengers would actually follow schedule;
- optimize boarding schedule by minimizing total boarding time; and
- optimize queue position to find optimal boarding schedule.

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Appendix

A. Data Tables

B. Model Source Code