Problem 8.1:

Let $Y_v[n,k] = Y_v[n, \frac{2\pi k}{10})$ represent the discrete TDFT of a signal $y[n]$ with respect to an analysis window $v[n] = (0.5)^{|n|}\left(1 - \frac{1}{10}\sum_{k=0}^{9} e^{j\frac{2\pi k}{10}n}\right) + \delta[n]$. Is it true that $\alpha \sum_{k=0}^{9} Y_v[n,k] = y[n]$ for some real number α, whose value is independent of n?

Looking at this DTDFT, we have a temporal sampling factor, $L=1$ and frequency sampling factor, $M=10$. With this it seems we could be on our way to satisfying $\alpha \sum_{k=0}^{9} Y_v[n,k] = y[n]$ using Synthesis equation I

The FBS condition for this is $L=1$ and $v[kM]=0$ for $k \neq 0$, so we mainly need to prove the analysis window is zero at each frequency sampling interval and $\neq 0$ at $n=0$.

Focusing analysis on $\sum_{k=0}^{9} e^{j\frac{2\pi k}{10}n}$ =>
$$\begin{cases} 10 & \text{when } n = 0, \pm 10, \pm 20, \dots \\ \left.\dfrac{1-e^{j2\pi n} \leftarrow \text{always } 0}{1-e^{j\frac{2\pi n}{10}} \leftarrow \text{never } 0}\right\} 0 & \text{otherwise} \end{cases}$$

We see that $v[n] = (0.5)^{|n|}\left(1 - \frac{1}{10}\sum_{k=0}^{9} e^{j\frac{2\pi k}{10}n}\right) + \delta[n] = 0$ when $n = \pm kM$ where $k \neq 0$.

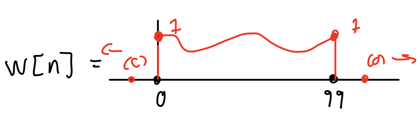
Special case that $k=n=0$, we have $v[0] = (0.5)^0\left(1 - \frac{1}{10}\sum_{k=0}^{9} e^{j(0)}\right) + \delta[0]$

$$= 1(0) + \delta[0]$$
$$= \delta[0]$$
$$= \boxed{1}$$

Synthesis Equation I: FBS: $\frac{1}{Mv[0]}\sum_{k=0}^{M-1} Y_v[n,k] = y[n]$ $\therefore$ $\boxed{\alpha = \frac{1}{10(1)} = \frac{1}{10}}$

Problem 8.2:

Throughout the problem, let $x[n]$ be an arbitrary 200-pt signal whose TDFT is given as $X_w[n, w)$. The analysis window is given by $w[n] = u[n] - u[n-100]$ and the DTDFT is specified as $X_w[n,k] = X_w[100n, \frac{2\pi k}{200})$ for $0 \le k \le 199$ and 0 otherwise. Is it true that $x[n] = \frac{1}{200}\sum_{k=0}^{199} X_w[n,k]$?



We see that we have a temporal sampling factor, $L = 100$, which > 1. This means we must do GFBS and satisfy the GFBS condition $\sum_{i=-\infty}^{\infty} f[n-iL]\,w[n-iL-pM] = \delta[p]$

Since we are looking at retrieving $x[n]$ from a signal that is not periodic, we must have $p=0$:

$$\sum_{i=-\infty}^{\infty} f[n-iL]\,w[n-iL] = 1$$ $\Rightarrow$ we must find a $g[n] = f[n]\,w[n]$ that equals 1 for all n

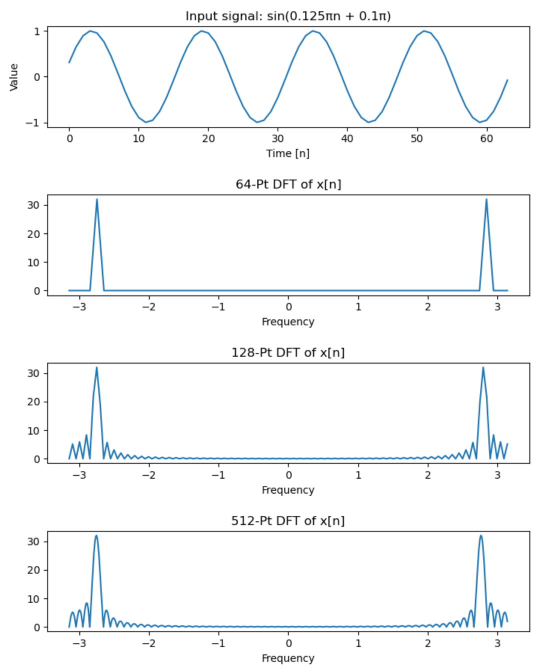
If we assign $f[n] = w[n]$, then this product will be 1 for all n, thus $g[n-100i]$ will be $f[n-100i]\,w[n-100i]$, and will always be 1.

With these things satisfied, we know that to retrieve the original signal back, we need more than just the discrete TDFT summed over each frequency for which it's defined. $\therefore$ We cannot claim that $x[n] = \frac{1}{200}\sum_{k=0}^{199} X[n,k]$

Problem 8.3:

Throughout this problem, use $x[n] = \sin(0.125\pi n + 0.1\pi)\{u[n] - x[n-64]\}$

(a) Generate a plot of the signal
(b) 64-Pt DFT
(c) 128-Pt DFT
(d) 512-pt DFT

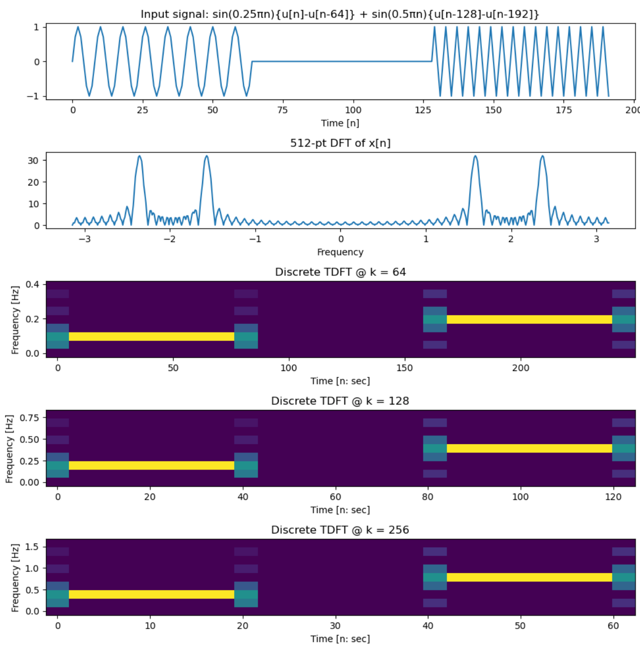


(e) The main differences between the three bottom subplots corresponding to the DFT are in the "resolution" of the underlying DTFT. Because the DFT corresponds to sampling the true DTFT, defined for all frequencies, we can only see a small portion of it, depending on our resolution (sampling interval). The more samples we include, such as the 512-pt vs the 64-pt, we see the DFT is better approximating the true DTFT.

Problem 8.4:

Throughout this problem, use

$$x[n] = \sin(0.25\pi n)\{u[n] - u[n-64]\} + \sin(0.5\pi n)\{u(n-128] - u[n-192]\}$$



For $X_w[n,64] = X_w[n, \frac{2\pi 64}{512}] = X_w[n, \frac{\pi}{4})$ is isolating frequencies around $\pi/4$.

For $X_w[n,128] = X_w[n, \pi/2)$, we are isolating frequencies around $\pi/2$

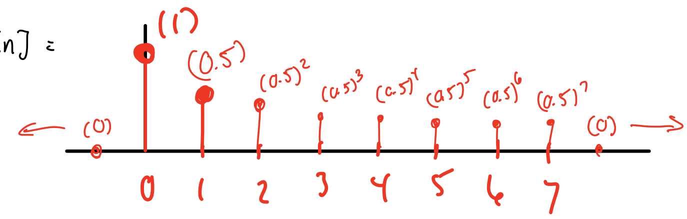
For $X_w[n,256] = X_w[n,\pi)$, we are isolating frequencies around π.

Problem 8.5

Assume that $X_w[n,w)$ is the TDFT of a signal $x[n]$ with respect to the analysis window $w[n] = (0.5)^n\{u[n]-u[n-8]\}$

if $x[n]=1 \quad \forall n$, is it true that $X_\omega[n,\omega) \neq 0$ for all finite values of n and ω?

$w[n] =$



Since $x[n]=1$ for all n, we have $X_\omega[n,\omega) = \sum_{m=-\infty}^{\infty} w[m]e^{-j\omega m}$

This becomes a typical FT that resembles a box:

$$\sum_{m=-\infty}^{\infty}(0.5)^m\{u[m]-u[m-8]\}e^{-j\omega m} = \sum_{m=0}^{7}(0.5)^m e^{-j\omega m} = \sum_{m=0}^{7}(0.5e^{-j\omega})^m$$

$$= \text{FSF} = \frac{1-0.5e^{-j8\omega}}{1-0.5e^{-j\omega}} \qquad \left(\frac{e^{-j\frac{8\omega}{2}}}{e^{-j\omega/2}}\right)\left(\frac{e^{j8\omega/2}-0.5e^{-j\frac{8\omega}{2}}}{e^{j\omega/2}-0.5e^{-j\omega/2}}\right)$$

$$e^{-j\frac{7}{2}\omega}\left(\frac{e^{j\frac{8\omega}{2}}-0.5e^{-j\frac{8\omega}{2}}}{e^{j\omega/2}-0.5e^{-j\omega/2}}\right)$$

Because these are complex exponentials that can be converted to sines and cosines, the numerator is guaranteed to have zero-crossings, thus $X_w[n, \omega)$ is guaranteed to have zeros for some finite values of ω.