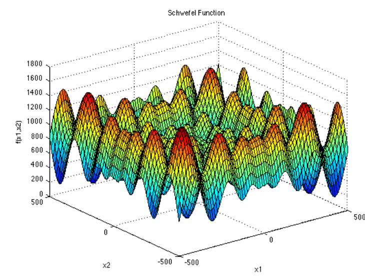
277B: Machine Learning Algorithms

Homework assignment #2: Metaheuristic global optimization
Assigned Feb. 6 and Due Feb. 16

1. Classical simulated annealing (15pt). We will use the Schwefel function for D=10 in order to find its global minimum using CSA



$$f(x_1, x_2 \ldots x_D) = 418.9829\text{x}D - \sum_i^D x_i \left(\sin\left(\sqrt{|x_i|}\right)\right)$$

$$x_i \in [-500{,}500] \text{ for } i = 1, \ldots, D$$

In which we use the visitation function of a random displacement along each dimension

$$x_i = x_i + (2 * URN - 1) \times \Delta, \text{ with } \Delta\text{=0.5 for } i = 1, \ldots, D$$

(a) (5pt) Fill in the blanks in the provided simulated annealing code. Use a linear (T_{t+1}=T_t-α) cooling schedule with α=0.5, and initializing T_{SA}=3000K, to perform CSA until the temperature reaches 30K and 10K, and record the function values. How long is your cooling schedule? Check against the debugging outputs. Given the stochastic nature of CSA, it would be best to report at least 3 runs for each lower bound temperature. Do you find better solutions when cooling to the lower temperature?

(b) (5pt) Choose logarithmic cooling (T_k=T_{SA}/(1+ T_{SA} log(1+k)/3σ_{curr}), where k is counter for number of cooling cycle) and σ_{curr} is an adjustable parameter, with two initial temperature T_{SA} = 3000K and 6000K. Use σ_{curr} = 1000 and k = 6000. Reconsider questions (a). Do these cooling schedules converge better than linear cooling?

(c) (5pt) Create your own annealing schedule (cooling and heating cycles) to see if you can find better solutions. Use a local optimization technique on your CSA answer, can you find even better solution?

2. Genetic Algorithms (40pt). Use GA to determine the maximum of the solution $f(x) = -x^2 + 8x + 15$ over the discrete range of x-values: [0,15], where $f(x)$ is the fitness function. Consider the following two possible encodings.

Encoding A					
Solution	**Fitness**	**Vector**	**Solution**	**Fitness**	**Vector**
0	15	1011	8	15	0100
1	22	0011	9	6	1100
2	27	1001	10	-5	0101
3	30	1000	11	-18	0110
4	31	0010	12	-33	0111
5	30	0001	13	-50	1101
6	27	0000	14	-69	1110
7	22	1010	15	-90	1111

Encoding B					
Solution	**Fitness**	**Vector**	**Solution**	**Fitness**	**Vector**
0	15	0001	8	15	1010
1	22	0010	9	6	0110
2	27	0100	10	-5	0111
3	30	1101	11	-18	0011
4	31	1011	12	-33	1000
5	30	1111	13	-50	1110
6	27	1001	14	-69	0000
7	22	1100	15	-90	0101

(a) (5pt) List all good solutions as those with $f(x) > 27$, and write out individual solutions for each encodings. Define schema by looking for perfect conservation along each column; if perfect conservation holds, give that value for that position, else represent that column with a *. What is length and order of the two schema? Which encoding will you choose?

(b) (5pt) Assume we draw candidate solutions $x = 10$, $x = 1$, $x = 15$, $x = 6$, $x = 0$, $x = 9$ as our initial population for GA optimization. Using your chosen encoding, list encoded solutions and their fitness. Pair the population so that the fittest members are paired with the least fit.

(c) (5pt) Use the cross-over operator by defining the cross-over point between the 1st and 2nd element. Exchange the last three elements between the pairs of strings. Do we have new solutions, and if so, what is their fitness? Have we increased fitness of population as a whole? What is the best solution?

(d) (5pt) Using population generated in (c), mutate 3rd element. Do we have new solutions, and if so, what are their fitness? Does mutation increase fitness of population as a whole? Did we find a better solution?

(e) (5pt) Using population generated in (d), eliminate least fit member, and replace with cloned best member. Do 2-point cross-over by exchanging inner two elements between pairs. Do we have new solutions and what are their fitness? Have we increased fitness of population as a whole? Did we find a better solution?

(f) (5pt) Using population generated in (e), eliminate least fit member, and replace with cloned best member. Do cross-over between 3rd and 4th elements and exchange first three elements between pairs. Do we have new solutions, and what are their fitness? Have we increased the fitness of population as a whole? Did we find a better solution?

(g) (10pt) Do you think that the encoding of the solution space was adequate? Why?