2) Okay, so to prove that M-step update on centroids achieves min. obj. we have to start by differentiating w/ respect to μ_k (centroids)

$$\frac{\partial \mu_k}{\partial \mu_k} = \frac{\partial \sum_{i=1}^{N} \|x_i - \mu\| \pi_{ik}}{\partial \mu_k}$$

$$\partial \mu_k = \sum_{i=1}^{N} \partial(\mu_k - x_i)\pi_{ik} = 0$$

$$= \sum_{i=1}^{N} (\mu_k - x_i)\pi_{ik} = 0$$

$$= \sum_{i=1}^{N} \mu_i \pi_{ik} = 0$$

$$\mu_i \pi_{ik} = \sum_{i=1}^{N} x_i \cdot \pi_{ik}$$

$$= \mu_k = \frac{\sum_{i}^{N} x_i \pi_{ik}}{\sum_{i=1}^{N} \pi_{ik}}$$

Now we have proved μ_k equals a weighted average which essentially shows the summation of points in a cluster/number of points