1) To prove that the E-STEP update on memberships achieves the minimum objective given the centroids, we have to look at how clusters are selected.

$$\pi_{iK} = \begin{cases} 1 & \text{then } K = \text{argmin } \|x_i - \mu_k\|^2 \\ 0 & \text{other} \end{cases}$$

- We know the obj. is to minimize SSE
- we know the E step selects the closest cluster per datapoint.

Therefore, by updating memberships to get the closest distance, it is inherently satisfying the obj. at the same time.