Metrics-Math Bootcamp Day 4

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Overview

- Presentations
- Structural Modeling
- MoM and GMM

Presentations

Structural Modeling

Intro to Structural Modeling

- What is structural modeling?
- There is no formal definition but this is how I think about it
- Structural modeling consists of writing an economic model that specifies relationships (usually through equations) and then making statistical assumptions on those equations to allow one to use the data to give information
- Why might we use structural modeling?
- One common reason is because the effect(s) we are interested in cannot be captured by a single treatment X that we observe in the data
- Alternatively we may be interested in multiple effects occurring at one time, or thinking more carefully about the economic interpretation of the effects through a model

Structural Model Example I

- Suppose that I am the DOJ and I am interested in estimating the impact of a proposed merger on consumer welfare
- Difficult to do in reduced form: do we have similar mergers? What data to use?
- Instead, common methodology is to model and estimate both supply and demand to back out underlying preference and costs parameters, and then simulate in equilibrium how consumer welfare will change when we change the competition structure
- This is the foundation for a lot of work in Industrial Organization

Structural Model Example II

- Demand side: consumers may have preferences over the goods the firm produce j as

$$u_{ij} = x_j\beta + \epsilon_{ij}$$

  where x_j includes the price of the good
- Supply side: firms set prices in a Nash Equilibrium based on costs c and competition structure
- If we estimate β and costs c consistently, then we can simulate the impact of changing competition
- This specific circumstance and case has been studied very in-depth by IO economists

More Details on Structural Modeling

- The general formula is to first set up the economic model
- This will give relationships between the (potentially abstract) economic objects of interest in your study
- Then one needs to take this economic system and pick what parts of the data will be proxies or measures for different economic objects as well as pick appropriate statistical assumptions to allow for estimation
- For example if people have linear preferences over prices saying that consumpion $= \beta$price will be invalidated by data - need to add an error consumpion $= \beta$price $+ \epsilon$
- This provides $M(X, \theta)$ where X is data and θ are the economic parameters
- Then estimate by optimizing M with respect to some criteria (ex: OLS)

Pros of Structural Modeling

- Here is my take on structural modeling (and I think it is echoed by others, but people have varying opinions)
- First I'll go over pros
- Powerful in the sense that it opens up a lot more questions that can be answered with somewhat limited data
- More tightly links economic theory and econometrics
- Particularly useful for simulating policy changes and counterfactuals - allows one to take into account many different important features when thinking about policy changes, and also examine those features impacts on the policy
- Provides a lot of interesting econometric problems to study

Cons of Structural Modeling

- My major con: lack of transparency. Identifying assumptions become difficult to assess and think about in large models, and also difficult to think about threats to credibility and what would be impacts of these threats.
- Sensitive to functional form specifications, particularly when one uses an economic model (what type of utility function should I pick? Is quasi-linearity ok to assume?)
- This lack of transparency makes people in the profession generally prefer reduced form identification strategies for policy evaluation (when possible), although some work being made to bridge this gap and understand ID in structural models better

MoM and GMM

Method of Moments Intro

- Method of moments and its generalization generalized method of moments are popular ways to estimate structural models
- Another very popular way is maximum likelihood
- Method of moments starts by specifying moments from a function of a sample
- For example $E[x] = \mu$, $E[x^2] = \mu^2 + \sigma^2$
- The idea is to set up a system of equations that has k unknown parameters and k moments, and then solve for it using the sample analog of the moments - the means
- Great because distribution free

Method of Moments Properties

- Consistency: follows from law of large numbers and consistency of means
- Asymptotic distribution: Will have an asymptotic normal distribution as in OLS and other estimators (how to get? Taylor approximation to the system of equations)
- Thus, can get asymptotic var-covar and do inference as usual

Simple MoM Example

- Suppose want to estimate parameters from a normal distribition $x \sim N(\mu, \sigma^2)$
- 2 params so need 2 moments
- I'll use simple moments: $E[x] = \mu$, $E[x^2] = \mu + \sigma^2$
- Sample analogs are $\bar{x}$ and $\bar{x^2}$
- Then solving gives $\mu = E[x]$ and $\sigma^2 = E[x^2] - E[x]^2$
- Sample analogs are $\hat{\mu} = \bar{x}$ and $\hat{\sigma}^2 = \bar{x^2} - \bar{x}^2 = \frac{1}{n}\sum_i (x_i - \bar{x})^2$
- Note: biased but consistent!

OLS as MoM

- Recall OLS formula: $\hat{\beta} = (X'X)^{-1}X'y$
- The key identifying assumption in OLS is $E[X'\epsilon] = 0$
- Then

$$E[X'\epsilon] = 0 \Rightarrow E[X'(y - X\beta)] = 0 \Rightarrow \beta = E[X'X]^{-1}E[X'y]$$

- So OLS can be seen as a MoM estimator!
- Shows that really only important assumption in OLS estimation for recovering parameters is the exogeneity condition

IV as MoM

- Recall just identified IV: $\hat{\beta}_{IV} = (Z'X)^{-1}Z'Y$
- The key identifying assumption is $E[Z'\epsilon] = 0$
- Then doing same algebra get:

$$\beta_{IV} = E[Z'X]^{-1}E[Z'y]$$

Intro to GMM

- GMM is a natural extension of MoM when you have more moments than parameters to be estimated
- GMM refers to many different things in this case, but usually it refers to when you use the moments "in the most efficient way"
- The problem: now no exact solution, so idea is to maximize some function of the moments
- We choose the function: $M(X, \theta) = m(X, \theta)' W(\theta) m(X, \theta)$ where W is some weight matrix and $m(X, \theta) = 0$ is the vector of moment conditions
- This is quadratic form: natural generalization of minimizing $\sum w_i x_i^2$ over x_i
- Major metrics development: there is an "optimal" weight matrix W which depends on the moments (read Greene or other metrixs textbook for more details on this)

GMM Example

- Suppose that $x \sim \exp(\lambda)$
- Then single parameter
- Use $E[x] = \lambda$ and $E[x^2] = \lambda + \lambda^2$
- How to use both of these moments to estimate?
- One idea: minimize

$$(\bar{x} - \lambda)^2 + (\bar{x^2} - \lambda - \lambda^2)^2$$

  which uses equal weighting of the moments and does not look at any cross-relationship
- This uses the identity matrix as W
- Can use other weighting matrices, but there is an optimal one!

Basic Properties and Estimation of GMM

- Assume *all the moment conditions* specified hold
- For any weighting matrix W, the GMM estimator is consistent
- The GMM estimator has an asymptotic normal distribution (this is where optimal weighting matrix helps)
- To do optimal GMM or two-step GMM, usually do first step with $W = I$, then gives $\hat{W}$, and re-run

MoM and GMM in R

MoM and GMM in R

- I usually do MoM and GMM manually in R (helps me think through the model)
- Requires specifying the sample moments and the appropriate functions of the parameters, and then using some optimization function manually
- optim() works well in R for optimization - this requires a starting value, so might want to try different starting values (can also play around with different algorithms)
- There is also a gmm package and function that works well

Wrap-Up

- Structural modeling is cool but challenging
- MoM and GMM are other ways to estimate parameters, particularly useful for structural modeling
- Any other things we want to discuss as a group? RF program? Classes to take in GSB or Econ department?

Thanks!

Hope this bootcamp was helpful! Feel free to reach out over email if you ever want to meet to discuss logistics, graduate school, research, etc.!