$$\mathcal{L}_0 \xrightarrow{A} \mathcal{L}_1 \underset{L}{\overset{R}{\underset{\sim}{\rightleftarrows}}} Sh(\mathcal{L}_0, J)$$

$$\mathcal{L}_1 \underset{L}{\overset{R}{\rightleftarrows}} PSh(\mathcal{L}_0)$$

$$\mathcal{L}_0 \xhookrightarrow{PSh} PSh(\mathcal{L}_0)$$