State Estimation for Pancake Robot:
Derivation of Theta From Gravity, Robot Acceleration and ϕ

Nicholas Hemstreet

1 Introduction

//TODO: Fill it out

2 Preliminaries

Let x_k be the robot state at time t_k defined as

$$\mathbf{x}_k = \begin{bmatrix} \phi_k \\ \theta_k \\ d_k \end{bmatrix}, \tag{1}$$

where ϕ_k is the angle that body x-axis makes with the flow direction, θ_k is the circumferential position of the robot and d_k is the linear displacement along the pipe. Let u_k be the control input (what we are applying to the system, for our case the velocity and angular velocity of the robot)

$$\mathbf{u}_k = \begin{bmatrix} v_k \\ \omega_k \end{bmatrix}. \tag{2}$$

Let $w_k \sim \mathcal{N}(0, \mathbf{Q}_k)$ be the zero-mean Gaussian process noise such that

$$\mathbf{w}_k = \begin{bmatrix} n_{v_k} \\ n_{\omega_k} \end{bmatrix}. \tag{3}$$

Given the state, control and process noise, state transition function or motion model is defined as $f(x_k, u_k, w_k)$.

We define the observation z_k, and observation model $h(x_k, \nu_k)$ such that $z_k = h(x_k, \nu_k)$ where $\nu_k \sim \mathcal{N}(0, \mathbf{R}_k)$ is zero-mean Gaussian observation noise.

The robot belief at time t_k is $b_k \sim \mathcal{N}(\hat{\mathbf{x}}_k^+, \mathbf{P}_k^+)$ where $\mathbf{x}_k^+$ is the robot state estimate at t_k and $\mathbf{P}_k^+$ is the state error covariance.

3 Motion Model

We can now define the state transition function or motion model $f(x_k, u_k, w_k)$ that takes x_k (the state variable), a control input u_k (what we are applying to the system, for our case the velocity and angular velocity of the robot) and w_k (the noise in each of the elements) such that :

$$\mathbf{f}(x_k, u_k, w_k) = \begin{bmatrix} \phi_k + \omega_k \Delta T + n_{\omega_k}\sqrt{\Delta T} \\ \theta_k + \frac{v_k \sin(\phi_k)\Delta T}{R} + n_{v_k}\frac{\sin(\phi_k)\sqrt{\Delta T}}{R} \\ d_k + v_k \cos(\phi_k)\Delta T + n_{v_k}\cos(\phi_k)\sqrt{\Delta T} \end{bmatrix} \tag{4}$$

With the motion model function defined we need to find the Jacobians so that we can build the Kalman Filter.

We define the motion model jacobian w.r.t state as

$$\mathbf{F}_k = \bar{\nabla}_x \mathbf{f} = \begin{bmatrix} 1 & 0 & 0 \\ v_k \frac{\cos(\phi_k)}{R}\Delta T & 1 & 0 \\ -v_k \sin(\phi)\Delta T & 0 & 1 \end{bmatrix} \tag{5}$$

and w.r.t noise as

$$\mathbf{L}_k = \bar{\nabla}_w \mathbf{f} = \begin{bmatrix} 0 & \sqrt{\Delta T} \\ \frac{\sin(\phi_k)\sqrt{\Delta T}}{R} & 0 \\ \cos(\phi_k)\sqrt{\Delta T} & 0 \end{bmatrix}. \tag{6}$$

4 Observation Model

We have the observation at time t_k as

$$\mathbf{z}_k = \begin{bmatrix} a_x \\ a_y \\ a_z \\ \phi_k \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{bp}^T \mathbf{g} \\ \phi_k \end{bmatrix}, \tag{7}$$

where $\mathbf{R}_{bp}$ is the rotation matrix from body frame to pipe frame and $\mathbf{g}$ is the gravity vector in pipe (inertial) frame. The gravity vector in pipe frame can be written out as

$$\mathbf{g} = \begin{bmatrix} -g \\ 0 \\ 0 \end{bmatrix}, \tag{8}$$

and $\mathbf{R}_{bp}$ as

$$R_{bp} = \begin{bmatrix} \sin(\theta)\sin(\phi) & -\sin(\theta)\cos(\phi) & -\cos(\theta) \\ -\cos(\theta)\sin(\phi) & \cos(\theta)\cos(\phi) & -\sin(\theta) \\ \cos(\phi) & \sin(\phi) & 0 \end{bmatrix}. \tag{9}$$

Now we may define the observation as

$$\mathbf{z}_k = \begin{bmatrix} -g\sin(\theta_k)\sin(\phi_k) + \nu_{k,a_x} \\ g\sin(\theta_k)\cos(\phi_k) + \nu_{k,a_y} \\ g\cos(\theta_k) + \nu_{k,a_z} \\ \phi_k + \nu_{k,\phi} \end{bmatrix}. \tag{10}$$

With the observation function defined we need to find the Jacobians so that we can build the Kalman Filter.

The observation jacobian w.r.t state is

$$\mathbf{H}_k = \bar{\nabla}_x \mathbf{h} = \begin{bmatrix} -g\sin(\theta_k)\cos(\phi_k) & -g\cos(\theta_k)\sin(\phi_k) & 0 \\ -g\sin(\theta_k)\sin(\phi_k) & g\cos(\theta_k)\cos(\phi_k) & 0 \\ 0 & -g\sin(\theta_k) & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{11}$$

The observation jacobian w.r.t noise is

$$\mathbf{M}_k = \bar{\nabla}_\nu \mathbf{h} = \mathbf{I}_{4\times 4} \tag{12}$$

where $\mathbf{I}_{4\times 4}$ is identity matrix of size 4×4.

5 Noise Covariance Matrices

At time t_0 the belief is $b_k \sim \mathcal{N}(\hat{\mathbf{x}}_0^+, \mathbf{P}_0^+)$.

$$\mathbf{P}_0^+ = \begin{bmatrix} 0.0001 & 0 & 0 \\ 0 & 0.0001 & 0 \\ 0 & 0 & 0.0001 \end{bmatrix} \tag{13}$$

$$\mathbf{Q}_k = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix} \tag{14}$$

$$\mathbf{R}_k = \begin{bmatrix} 0.0001 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0.0001 \end{bmatrix} \tag{15}$$