3.3) $y_i = a + bx_i + e_i$

response variable; regression coefficients; predictor variable

minimize $SSres = \sum (y_i - a - bx_i)^2$

$$\frac{\partial SSres}{\partial a} = -2\sum(y_i - a - bx_i) = 0$$

$$\frac{\partial SSres}{\partial b} = -2\sum(y_i - a - bx_i)x_i = 0$$

$$a = \frac{(\sum y_i - b\sum x_i)}{n} \qquad b = \frac{(n\sum x_i y_i - \sum x_i \sum y_i)}{(n\sum x_i^2 - (\sum x_i)^2)}$$

$$b = \frac{cov(x_i, y_i)}{var(x_i)}$$

covariance between x_i and y_i

$var(x_i)$ is varience of x_i

3.4) Code

4.1) Description – An M14 gun is shot vertically upward with a muzzle velocity of 853 m/s. The tip of the gun bore is 3 meters from the ground. We want to predict the maximum height the bullet can reach and the time it takes to return to the ground.

Abstraction:

We can model the motion of the bullet using kinematic equations, we will neglect air resistance.

Let $h(t)$ be height of bullet at time t

V_0 be the muzzle velocity of bullet

g be the acceleration due to gravity ($-9.81\ m/s^2$

Equations: Using kinematic equation for vertical motion,

$h(t) = V_0 t - (1/2) g t^2$

@ max height, the velocity of bullet is 0

$\Rightarrow V = V_0 - gt = 0$

Max height $\rightarrow$ $t_{max} = V_0/g$

Substitute into height equation

max height $\rightarrow$ $h_{max} = V_0^2/2g$

Solution: $t_{max} = \dfrac{(853\,m/s)}{(9.8\,m/s^2)} = \underline{86.9\,s}$

$h_{max} = \dfrac{(853\,m/s)^2}{(2 \cdot 9.81\,m/s^2)} = \underline{38{,}873\,m}$

Interpretation: The maximum height the bullet can reach is 38, 873 m, which is higher than the height of the earths atmosphere. This result is unrealistic since we neglected air resistance. The time it takes for the bullet to return to the ground is twice the time to reach the maximum height, which is approximately 173.8 seconds.

Sensitivity Analysis:

Since we neglected air resistance, in our model, our predictions for the maximum height and time of flight are likely to be unrealistic. To obtain accurate predictions, we would need to account for air resistance. And also consider small variations in the muzzle velocity or initial height.

4.6| P = monthly payment required to achieve the annuity payments of $1,000 per month.

$$P = \frac{\left(A * 1 - (1+\frac{r}{n})^{(-n*t)}\right)}{\left(\frac{r}{n}\right)}$$

A = annuity payment, r = annual interest rate,
n = number of compounding periods per year,
t = number of yrs

A = $1,000 r = 0.05 (5%) n = 12, t = 38 (age 30 to 68)

$$P = \frac{\left(1000 * [1-(1+\frac{0.05}{12})^{(-12*38)}\right)}{\left(\frac{0.05}{12}\right)}$$

P = $273.57/mo.

4.7| 1) Find the solar constant (S) at the distance of the Moon,
⇒ 1/400th of solar constant at earth's distance
$S = (1361\ W/m^2)/(400^2) = 0.085\ W/m^2$

2) Using Stefan - Boltzmann Law

$$T_e = \left(\frac{S(1-A)}{4\sigma}\right)^{0.25}$$

A = moon's albedo
σ = Stefan constant

Assuming an albedo of 0.12,

$$T_e = \left(\frac{(0.085 * (1-0.12)}{(4 * 5.67\times10^{-8})}\right)^{0.25} = 269.2K$$

3) Calculate T at the latitude of interest using latitude factor (F), $T = T_e - F$

↳

$F = 60 \sin(\text{latitude})^2 + 40 \sin(\text{latitude})$

$F = 60 \sin(30)^2 + 40 \sin(30) = 54$

$T = 269.2 - 54 = 215.2\,k$

4.) Adjust the temp for time of day using the diurnal temperature range (DTR) factor.

The DTR varies depending on the lunar phase, but for a first approximation, we can assume 0.5

Thus, the estimated temperature at 3:00 PM is

$T = 215.2 - (0.5 * Te) = \underline{126.4\,K}$ ((ode)

8) <u>EBM + Code</u>

~~# Code~~

5.1 a) Code also

a) Use Kirchoffs Voltage Law:

$+12 - I_1 \times 9 - 6 \times I_2 = 0$

$9 \times I_1 + 6 \times I_2 = 12$

$+ I_2 \times 6 - I_3 \times 3 - I_4 \times 4 = 0$

$+6 \times I_2 + (-3) \times I_3 + (-4 \times I_4) = 0$

$+6 I_2 + (-3) I_3 + (-4) I_4 = 0$

$+4 \times I_4 - 9 \times I_5 - 3 \times I_5 = 0$

$4 \times I_4 - 12 \times I_5 = 0$

$+4 I_4 + (-12) I_5 = 0$

$I_1 = I_2 + I_3 \Rightarrow I_1 + (-1) I_2 + (-1) I_3 = 0$

$I_3 = I_4 + I_5 \Rightarrow I_3 + (-1) I_4 + (-1) I_5 = 0$

$9 \times I_1 + 6 I_2 = 12$

$9 I_1 + 6 I_2 + 0 + 0 + 0 = 0.012$

$0 + 6 I_2 + (-3) I_3 + (-4) I_4 + 0 = 0$

$0 + 0 + 0 + 4 I_4 + (-12) I_5 = 0$

$I_1 + (-) I_2 + (-) I_3 + 0 + 0 = 0$

$0 + 0 + I_3 + (-) I_4 + (-) I_5 = 0$

Using python $\Rightarrow I_1 = 0.001 A$

$I_2 = 0.0005 A$

$I_3 = 0.0005 A$

$I_4 = 0.000375 A$

$I_5 = 0.000125 A$

b) $V = IR$

$V_1 = I_1 \times 9 = 0.001 \times 9 = 9\,V$

$V_2 = I_2 \times 6 = 0.0005 \times 6 = 3\,V$

$V_3 = I_3 \times 3 = 0.0005 \times 3 = 1.5\,V$

$V_4 = I_4 \times 4 = 0.000375 \times 4 = 1.5\,V$

$V_5 = I_5 \times 9 = 0.000125 \times 9 = 1.125\,V$

$V_c = I_5 \times 3 = 0.000125 \times 3 = \underline{0.375\,V}$

c) $P = I^2R$ or $P = IV$

$P_1 = 9 \times 1\,mA = 9\,mW$

$P_2 = 3 \times 0.5 = 1.5\,mW$

$P_3 = 1.5 \times 0.5 = 0.75\,mW$

$P_4 = 1.5 \times 0.375 = 0.5625\,mW$

$P_5 = 1.125 \times .125 = 0.1406\,mW$

$P_c = 0.375 \times 0.125 = \underline{0.0468\,mW}$

d) Total power loss

$\Rightarrow \Sigma$ total power across the resistors $= 9 + 1.5 + 0.75 + 0.5625 + 0.1406 + 0.0468$

$= 11.99\,mW$

$= \underline{11990000\,W}$

In 10 minutes total work done:

$W = PT$

$\Rightarrow 11.99 \times 10 \times 60sec \times 10^{-3}$

$= 7.199\,J = \underline{7.2\,J}$

5.2 $2C_8H_{18}(l) + 25O_2(g) \rightarrow 16CO_2(g) + 18H_2O(g)$

5.4 a) Using the Leontif production model helps to express the final demand met by what is left after production.

If we suppose X is the production in each sector $X = [a \;\; m \;\; h]'$, where a = agricultural, m = manufacturing, h = household production.

Next, we can create a technology matrix A,

where $$A = \begin{bmatrix} 0.245 & 0.102 & 0.051 \\ 0.099 & 0.291 & 0.279 \\ 0.433 & 0.372 & 0.011 \end{bmatrix}$$

Then,

the demand matrix can be denoted, D,

where $D = [D_a \;\; D_m \;\; D_h]$.

The demand will be the net production in the economy written as $D = X - AX = (I - A)X$.

b) So, $D = (I - A)X \Rightarrow$ can also be in terms of X, where $X = (I - A)^{-1} D$, representing the production required to meet each given demand in each sector (D_a, D_m, D_h).

$X - AX$ represents the net production in the economy as stated in (a). The matrix Ax represents the requirements to produce the amount in each sector, X.

For example, we can use 100 units for agricultural

c) products, 200 units manufacturing, and 300 units of household. Then we know $D = [100 \;\; 200 \;\; 300]'$

Then we can show $X = (I - A)^{-1} D$,

$$X = \begin{bmatrix} 1 - 0.245 & 0 - 0.102 & 0 - 0.051 \\ 0 - 0.099 & 1 - 0.291 & 0 - 0.279 \\ 0 - 0.433 & 0 - 0.372 & 1 - 0.011 \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix} \rightarrow$$

After during calculations + matrix multiplication,
we obtain $X = \begin{bmatrix} 250.6 \\ 562.8 \\ 624.7 \end{bmatrix}$

Thus, in our given example we now have values of units for agricultural, manufacturing, + household products. (In billions).

c) According to the "Leontief - BLS Partnership", In 1932, Leontief arrived at Harvard and began the unusual project of constructing a 'tableau économique' for the U.S.
This was the first input - output tables, which helped to analyze how changes such as an increase in spending on luxuries, would affect the net product of France and its distribution among various French social classes (bls.gov).
In 1947, the Bureau's work with Leontief had a number of effects on the Agency, as the computer inverted 1947 matrix was still being assembled.

5.8 | Code