(5 Q 3) (5 Q 3)

1.1

$F_g = G \frac{m_1 m_2}{r^2}$

$G = \frac{F_g \cdot r^2}{m_1 m_2}$

$\Rightarrow$ Unit (SI) of $G \Rightarrow \frac{N \cdot m^2}{(kg)^2}$

$= \frac{kg \cdot ms^{-2} \cdot m^2}{(kg)^2}$

$= \frac{m^3}{s^2 \cdot (kg)}$

$= (kg)^{-1} (m^3) (s^{-2})$

$\therefore$ Dimension of $G = M^{-1} L^3 T^{-2}$

1.4 a) falling distance h can be given as a function

$h = \alpha\, m^a g^b t^c$

$[h] = [\alpha] [m]^a [g]^b [t]^c$

$M^0 L^1 T^0 = M^0 [L^1 T^{-2}]^b T^c$

$M^0 L^1 T^0 = M^0 L^b T^{-2b+c}$

$\Rightarrow \underline{b = 1}$

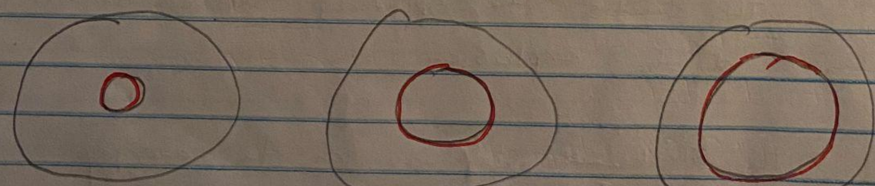
$-2b + c = 0 \rightarrow -2(1) + c = 0 \rightarrow \underline{c = 2}$

$\Rightarrow h = \alpha\, m^0 g^1 t^2$

$h = kgt^2$

1.6 b) Radius of nuclear shockwave

time $\longrightarrow$



These plots show the relationship of the radius' shock have ~~in~~ relation to time.

b) Radius of the surface shock wave is proportional to air density, energy released + time

$\Rightarrow R = \alpha E^a \rho^b t^c$

$[R] = [\alpha][E]^a [\rho]^b [t]^c$

$L = [ML^2T^{-2}]^a [ML^{-3}]^b T^c$

$L = 1 \times (ML^2T^{-2})^a (ML^{-3})^b T^c = M^{a+b} L^{2a-3b} T^{-2a+c}$

$a + b = 0$

$2a - 3b = 1$

$-2a + c = 0$

$\Rightarrow -2a - 2b = 0$

$+ \quad 2a - 3b = 1$

$-5b = 1$

$\underline{b = -1/5}$

$a + -1/5 = 0$

$\underline{a = 1/5}$

$-2(1/5) + c = 0$

$-2/5 + c = 0$

$\underline{c = 2/5}$

$R = \alpha E^{1/5} \rho^{-1/5} t^{2/5}$

$R = \alpha \left(\frac{Et^2}{\rho}\right)^{1/5} \rightarrow R = \alpha \left(\frac{E}{\rho}\right)^{1/5} t^{2/5}$

c) - The shockwave propagation depends on the particular medium / density of the air.

- The radiation of the energy is homogeneous. This creates the spherical surface in all directions of the explosion

- The shockwave depends on the energy radiated from the explosion

Continued.

→ It is important to understand the supersonic compression of air, which makes sense the crucial role density plays in the equation. With the given energy of the bomb and the calculated time after the explosion helps us to find the radius. Further, we can determine how far it can hit people in a specific distance away and at what particular speed/rate.

Q4| Derive $V = \sqrt{2gh}$ (1.56)

$V = \alpha m^a t^b g^c h^d$

$[V] = [\alpha][m]^a [t]^b [g]^c [h]^d$

The velocity v of a moving body is defined as the distance over time → $V = d/t$ ⇒ $[V] = LT^{-1}$

g = change of velocity / time → $[g] = LT^{-2}$

$[m] = M$

$[t] = T$

$[h] = L$

$[\alpha] = 1$ → constant

$LT^{-1} = 1 \cdot M^a T^b (LT^{-2})^c L^d = M^a T^{b-2c} L^{c+d}$

$a = 0$

$b - 2c = -1$ → we have 3 equations, but

$c + d = 1$ 4 different variables

So, drop h and keep t.

Assume $d = 0$

$c + 0 = 1$ → $c = 1$

$b - 2(1) = -1$

$b - 2 = -1$

$b = 1$

⇒ $V = \alpha g t$

or if we drop t + keep h,

$b = 0$

$0 - 2c = -1$ → $c = 1/2$

$1/2 + d = 1$ → $d = 1/2$

we get $V = \alpha \sqrt{gh}$

From physical law of energy conservation, $mgh = (1/2) mv^2$

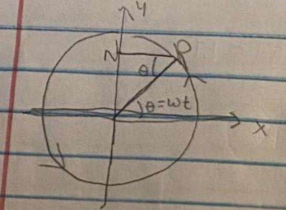
$$\frac{1}{2} m(\alpha\sqrt{gh})^2 = \left(\frac{1}{2}\right)\alpha^2 mgh$$

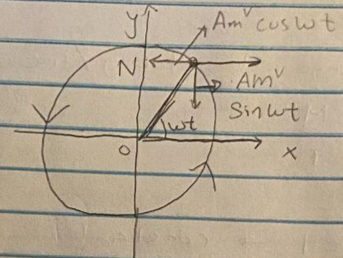
$$1 = \frac{1}{2}\alpha^2$$

$$\rightarrow \alpha = \sqrt{2}$$

Thus, $v = \sqrt{2gh}$

1.13 The shortest distance between mean position and position of a particle at any instant can be referred to as displacement of the particle in simple harmonic motion.





Refrence circles

Displacemnt (ON) = $A \sin\theta$

$Y = A \sin(\omega t + B)$

Particle @ mean position : $y_{min} = 0$

Particle @ extreme position $y_{max} = A$

Velocity $V \Rightarrow V = \frac{dy}{dt}$

$$= \frac{d}{dt}(A \sin(\omega t + B))$$

$$V = A\omega \cos(\omega t + B)$$

Accelaration is the derivative / rate of change of velocity

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}(A\omega \cos(\omega t + B))$$

$$= -A\omega^2 \sin(\omega t + B)$$

$\Rightarrow y = A \sin \omega t$ & $a = -\omega^2 y$

Time period is the time that it takes for a particle to complete an oscillation about the mean position.

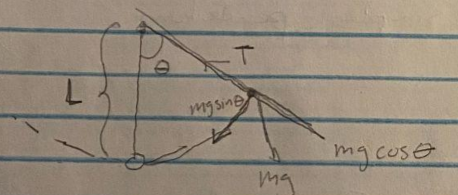
$$\text{Time period } (T) = \frac{2\pi}{\omega}$$

We know $a = \omega^2 y$

$$\Rightarrow \omega^2 = \frac{a}{y}$$

$$\omega = \pm\sqrt{\frac{a}{y}}$$

Plugging into T, $T = 2\pi\sqrt{\frac{y}{a}}$



L = length of pendulum.
We can put weight (mg) into two components.
There is a radial acceleration ($\omega^2 l$).
There is also tangential acceleration ($mg \sin\theta$)
There is also τ, representing Torque.

$\therefore \tau = -l(mg \sin\theta)$

By newtons law of rotational motion we have $\tau = I\alpha$.
If we equal both equations:

$$I\alpha = -l(mg \sin\theta)$$

Since $I = Ml^2$, $\sin\theta \simeq \theta$

$$I\alpha = -l \times mg\,\theta$$

$$Ml^2\alpha = -l \times mg\,\theta$$

So we get $\alpha = -\frac{g}{l} \times \theta$

We have $a = -\omega^2 y$

and $\alpha = \frac{-g}{l} \times \theta$.

Thus we get, $\omega^2 = \frac{g}{l}$

$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{g}{l}$

$\Rightarrow \therefore \boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

Hence, we have represented the time period of a simple pendelum.