Machine Learning

A. G. Schwing & M. Telgarsky

University of Illinois at Urbana-Champaign, 2018

L22: Autoregressive Methods (RNNs/LSTMs/GRUs)

Goals of this lecture

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- Getting to know Recurrent Neural Nets (RNNs)

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Reading Material

- Goodfellow et al.; Deep Learning; Chapter 10
- Papers cited on the slides

Pixel Recurrent Neural Networks

Recap: Our models so far

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- Discriminative

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$$p(\boldsymbol{y}|x)$$

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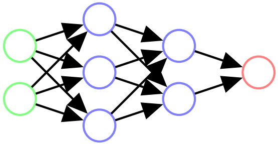

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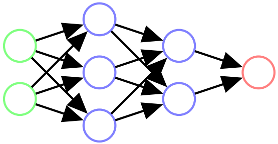

What's missing?

More flexibility regarding inputs and outputs:

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- Sequences of inputs

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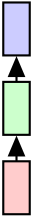

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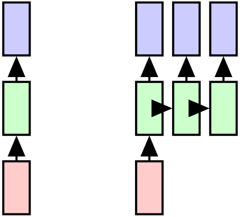

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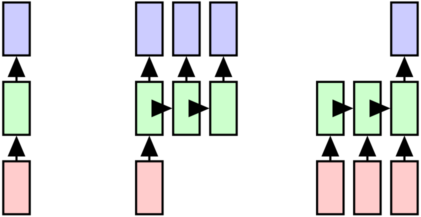

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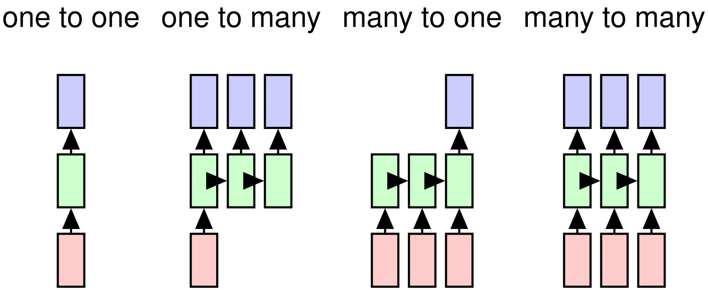

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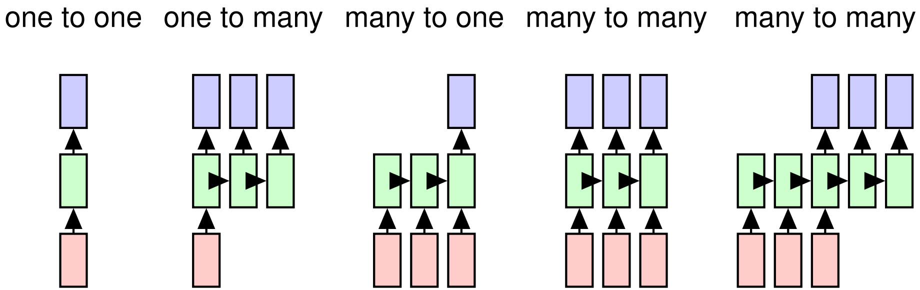
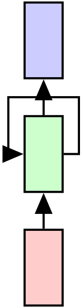


Recurrent Neural Nets (RNNs)



Recurrent Neural Nets (RNNs)

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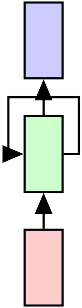

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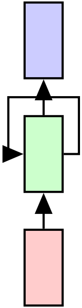
$$h^{(t)} = f(h^{(t-1)}, x^{(t)}, \boldsymbol{w})$$



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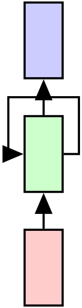

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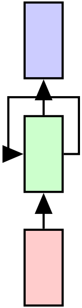

Applications:

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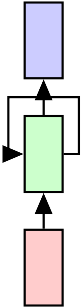

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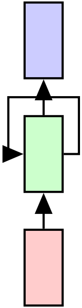

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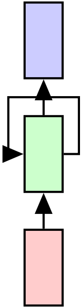

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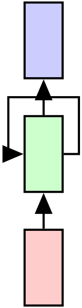

Applications:

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- Speech recognition
- Image processing
- Video processing

Important concept: Parameter sharing

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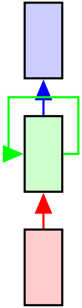

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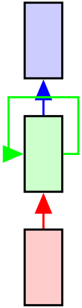

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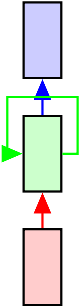

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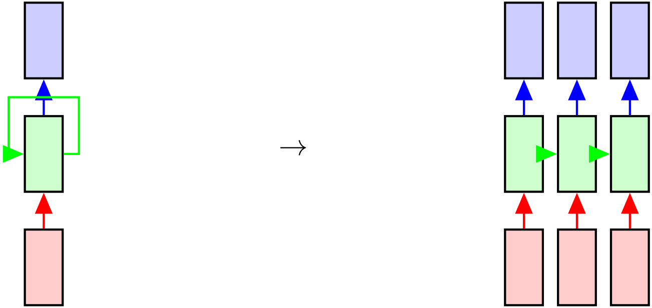

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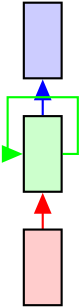

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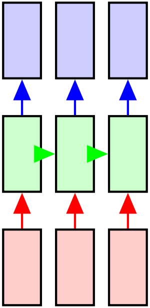

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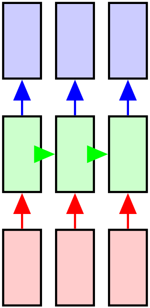
unfolded/unrolled network
performs identical operations
easier to understand

General structure for recurrence:



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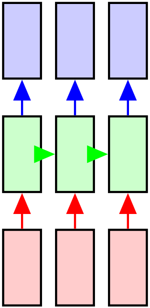

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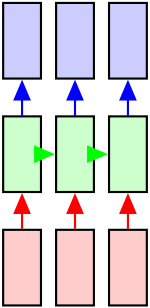

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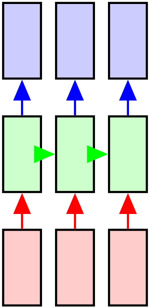

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Note that f and g are independent of time

What are f and g?

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(Jordan network is slightly different)

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What is σ_h and σ_y?

Activation functions: tanh, sigmoid

Affine transformations and point-wise non-linearity

What are the problems?

Problems with classical recurrent neural nets:

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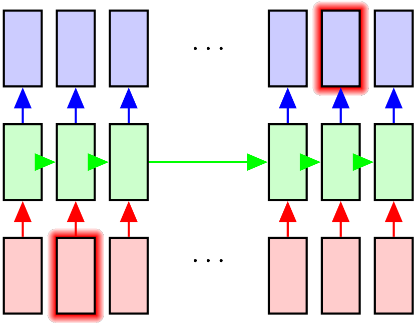

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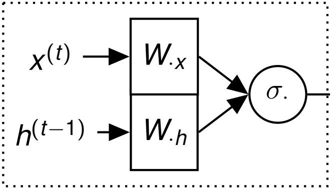

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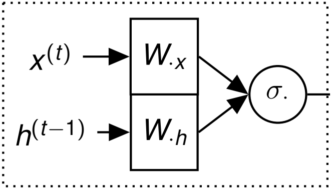

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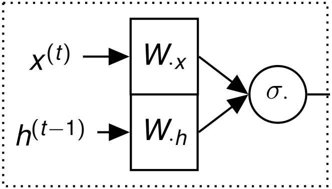

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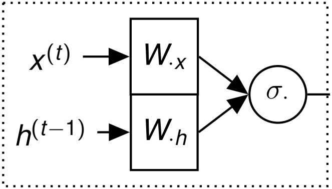

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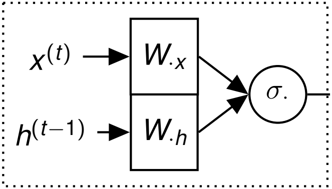

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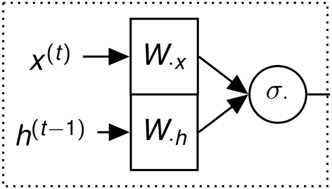

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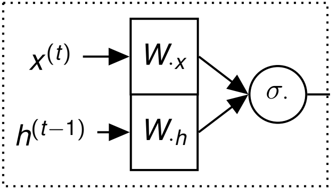

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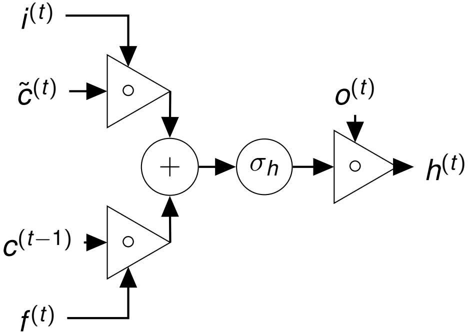

Long short term memory (LSTM):

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- Can be interpreted as a block in a neural net

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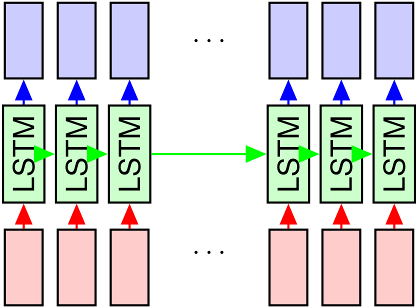

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Can again be interpreted as a block in the computation graph

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Intuition:

- $r^{(t)}$: Include $h^{(t-1)}$ in new memory?
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Recurrent nets generally:

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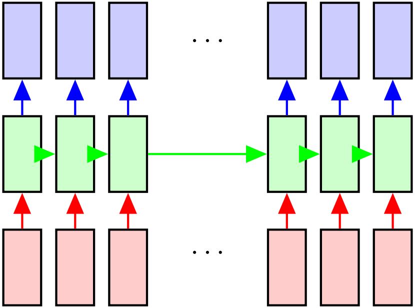

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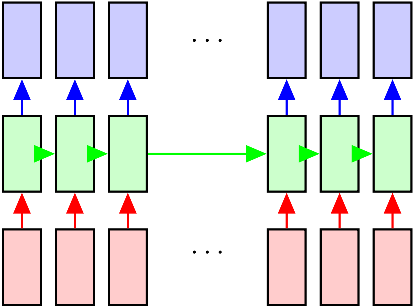

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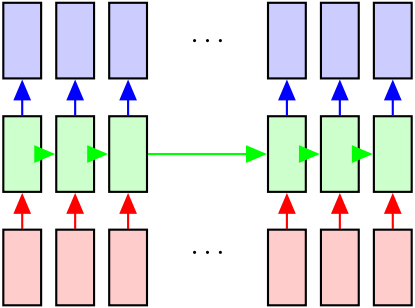

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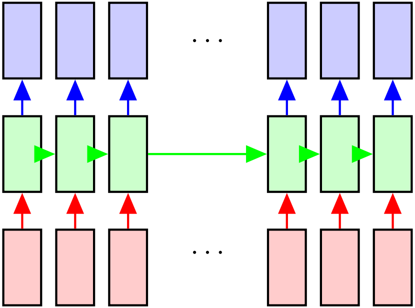

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- Bi-directional LSTMs [Schuster&Paliwal (1997), Graves&Schmidhuber (2005)]
- Continuous time RNNs

How do we learn the parameters in the network?

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Relation to structured models?

Training via gradient descent:

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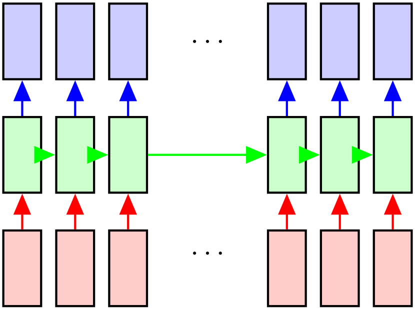

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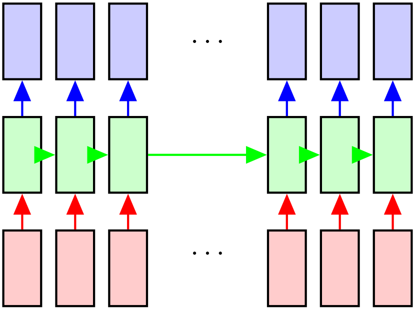
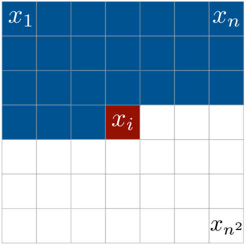


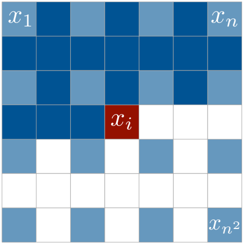
Backpropagation through time (BPTT)

Pixel Recurrent Neural Networks

PixelRNN model (Autoregressive model):



Context



Multi-scale context

Generative models overview:

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  - Pro:
  - Con:
- Generative Adversarial Nets (GANs):
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Generative models overview:

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  - Con:
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Generative models overview:

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Generative models overview:

- Variational Auto-encoders (VAEs):
  - Pro: probabilistic graphical model interpretation
  - Con: slightly blurry examples

- Generative Adversarial Nets (GANs):
  - Pro: generate sharp images
  - Con: difficult to optimize (unstable) [at least early variants]

- Autoregressive models (RNNs):
  - Pro: stable training & good likelihoods
  - Con: inefficient sampling & no low-dimensional codes

Very active research area

Quiz:

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- Describe the prediction process for an RNN?

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- Describe the prediction process for an RNN?
- Describe the training process for RNNs?

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- Describe the prediction process for an RNN?
- Describe the training process for RNNs?
- Contrast generative modeling techniques?

Important topics of this lecture

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- Getting to know RNNs and its variants

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- Getting to know their use

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- Contrasting RNNs to generative models

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Next up:

Reinforcement learning