Machine Learning

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University of Illinois at Urbana-Champaign, 2018

L13: Conditional Random Fields, Structured SVMs, Deep Structured Nets

Goals of this lecture

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- Learning of structured distributions

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Reading material:

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- Learning of structured distributions

Reading material:

- D. Koller and N. Friedman; Probabilistic Graphical Models: Principles and Techniques;

Recap: General framework for learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)},\hat{y}) + F(\boldsymbol{w}, x^{(i)}, \hat{y})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, y^{(i)})$$

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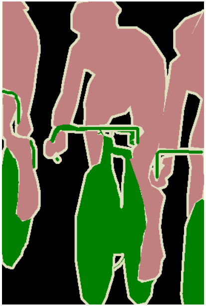
Example: Semantic segmentation



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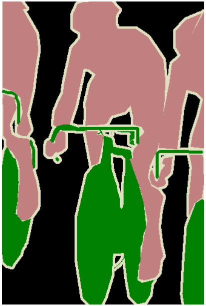

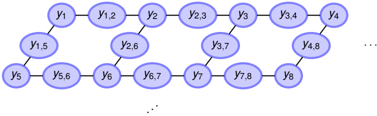

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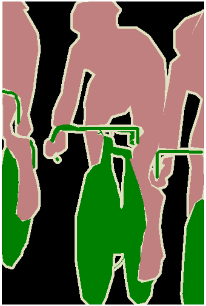

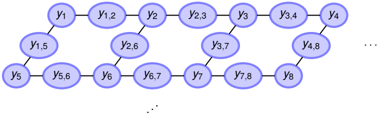

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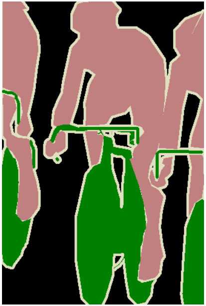

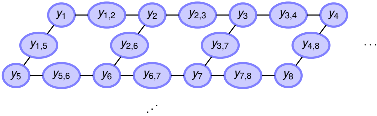

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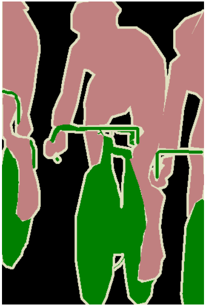

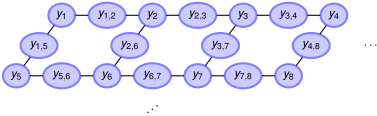

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How much weight should we give to each of the two cues?

Feature vector for structured objects:

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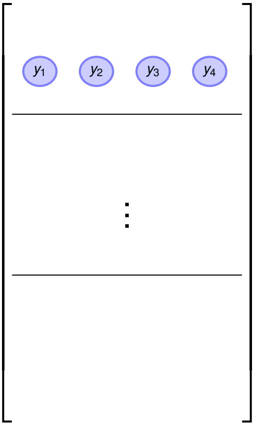

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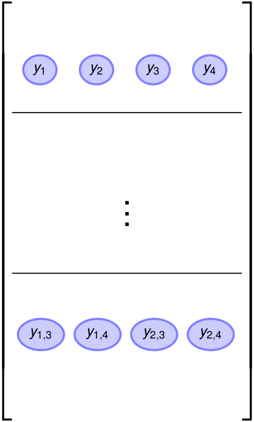

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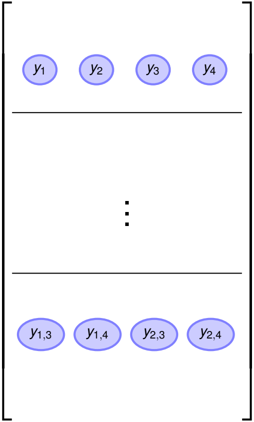

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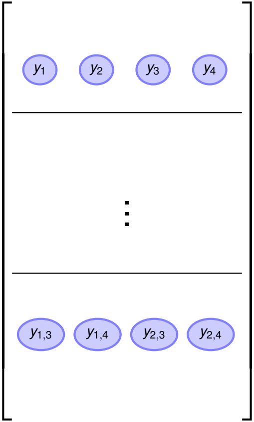

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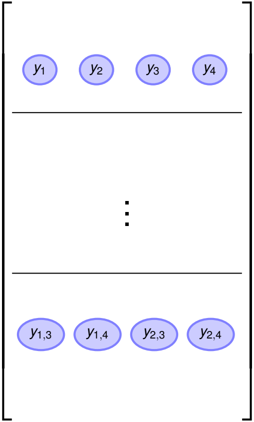

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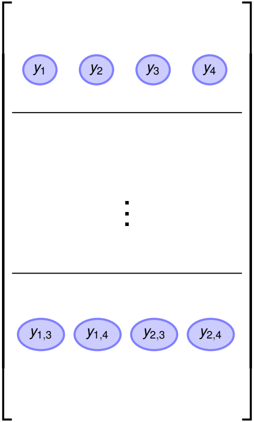

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Alternatively:

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- Structured Support Vector Machine [Tsochantaridis et al. 2004]

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$$\max_{\hat{\boldsymbol{y}}} \left(\boldsymbol{w}^\top f(x^{(i)}, \hat{\boldsymbol{y}}) + L(\hat{\boldsymbol{y}}, \boldsymbol{y}^{(i)}) \right) \geq \boldsymbol{w}^\top f(x^{(i)}, \boldsymbol{y}^{(i)})$$

- Max-Margin Markov Network [Taskar et al. 2003]
- Structured Support Vector Machine [Tsochantaridis et al. 2004]

$$\min_{\boldsymbol{w}} \frac{C}{2} \|\boldsymbol{w}\|_2^2 + \sum_{i \in \mathcal{D}} \left(\max_{\hat{\boldsymbol{y}}} \left(\boldsymbol{w}^\top f(x^{(i)}, \hat{\boldsymbol{y}}) + L(\hat{\boldsymbol{y}}, \boldsymbol{y}^{(i)}) \right) - \boldsymbol{w}^\top f(x^{(i)}, \boldsymbol{y}^{(i)}) \right)$$

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How complicated is this?

Solve one structured prediction task per sample per iteration.

Structured Learning

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \epsilon \ln \sum_{\hat{\boldsymbol{y}}} \exp \frac{L(\boldsymbol{y}^{(i)}, \hat{\boldsymbol{y}}) + \boldsymbol{w}^\top f(x, \hat{\boldsymbol{y}})}{\epsilon} - \boldsymbol{w}^\top f(x^{(i)}, \boldsymbol{y}^{(i)})$$

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Subsumes:

Structured Learning

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Subsumes:

- linear structured SVMs

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- linear structured SVMs
- linear conditional random fields (we condition on data x)

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- binary/multi-class SVM

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- linear structured SVMs
- linear conditional random fields (we condition on data x)
- binary/multi-class SVM
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What changed compared to our previous formulation?

Structured Learning

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What's the main limitation?

What happens if we make our score function non-linear in its parameters $\boldsymbol{w}$?

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$F(\boldsymbol{w}, x^{(i)}, \hat{\boldsymbol{y}})$ is a score function represented by a computation graph, e.g., a deep net

How to optimize this program:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i \in \mathcal{D}} \epsilon \ln \sum_{\hat{\boldsymbol{y}}} \exp \frac{L(\boldsymbol{y}^{(i)}, \hat{\boldsymbol{y}}) + F(\boldsymbol{w}, x, \hat{\boldsymbol{y}})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, \boldsymbol{y}^{(i)})$$

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Algorithm

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1. Forward pass to compute $F(\boldsymbol{w}, x^{(i)}, \hat{\boldsymbol{y}})$

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Algorithm

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Where are the challenges?

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Where are the challenges?

- How do we represent $F(\boldsymbol{w}, x, \hat{\boldsymbol{y}})$ if output space $\mathcal{Y}$ is large?

Output space size of some applications:

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Tag prediction

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$$|\mathcal{Y}| = 2^{\#tags}$$

Output space size of some applications:

Tag prediction Segmentation

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Output space size of some applications:

Tag prediction | Segmentation

$|\mathcal{Y}| = 2^{\#tags}$ $\quad$ $|\mathcal{Y}| = C^{\#pixels}$

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Tag prediction	Segmentation
	
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Observation:

- Interest in jointly predicting multiple variables $\boldsymbol{y} = (y_1, \ldots, y_D)$

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Solution:

- assume scoring function decomposes additively

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Every $f_r(\boldsymbol{w}, x, \boldsymbol{y}_r)$ is an arbitrary composite function, e.g., a CNN

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How to compute gradient:

$$\begin{aligned}
\frac{\partial}{\partial \boldsymbol{w}} \quad & \sum_{i \in \mathcal{D}} \left(\log \sum_{\hat{\boldsymbol{y}}} \exp F(\boldsymbol{w}, x^{(i)}, \hat{\boldsymbol{y}}) - F(\boldsymbol{w}, x^{(i)}, \boldsymbol{y}^{(i)}) \right) \\
& = \sum_{i \in \mathcal{D}, \hat{\boldsymbol{y}}} \left(p(\hat{\boldsymbol{y}}; x^{(i)}, \boldsymbol{w}) - \delta(\hat{\boldsymbol{y}} = \boldsymbol{y}^{(i)}) \right) \nabla_{\boldsymbol{w}} F(\boldsymbol{w}, x^{(i)}, \hat{\boldsymbol{y}})
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How to obtain marginals $p_r(\hat{\boldsymbol{y}}_r; x, \boldsymbol{w})$?

$$F(\boldsymbol{w}, x, \boldsymbol{y}) = F(\boldsymbol{w}, x, y_1, \ldots, y_D) = \sum_r f_r(\boldsymbol{w}, x, \boldsymbol{y}_r)$$

How to compute gradient:

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How to obtain marginals $p_r(\hat{\boldsymbol{y}}_r; x, \boldsymbol{w})$?

Approximate marginals $b_r(\hat{\boldsymbol{y}}_r; x, \boldsymbol{w})$ via:

- LP relaxation
- Message passing

General algorithm for learning:

repeat

  repeat

    update marginals b_r

  until convergence

  update parameters $\boldsymbol{w}$

until convergence

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What operation is used for computation of marginals in multi-class prediction?

General algorithm for learning:

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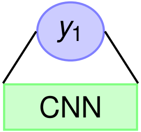
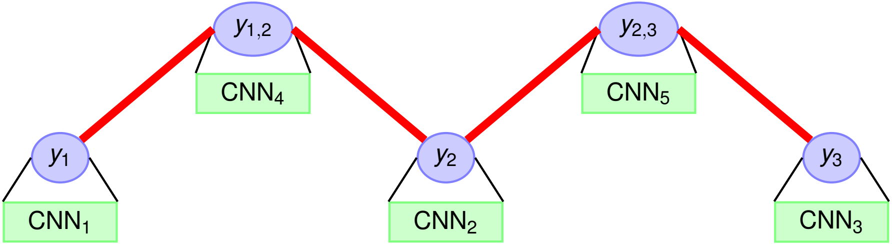
What operation is used for computation of marginals in multi-class prediction? Soft-max

Intuition: Standard CNN



Intuition: Independent Prediction

Intuition: Deep Structured Learning



Algorithm summary:

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1. Forward passes to compute $f_r(\boldsymbol{w}, x, \hat{\boldsymbol{y}}_r)$

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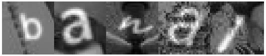
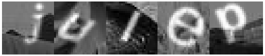
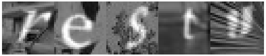
Example: Find five letters within distorted images



banal



julep



resty

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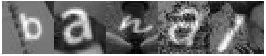
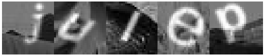


banal



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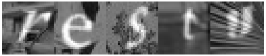
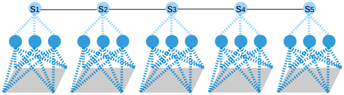
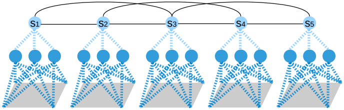


resty

- Graphical model: 1st or 2nd order Markov

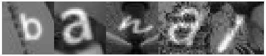


1st order Markov

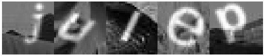


2nd order Markov

Example: Find five letters within distorted images



banal



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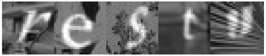

resty

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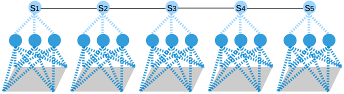
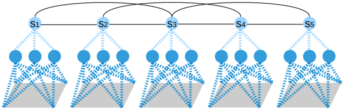
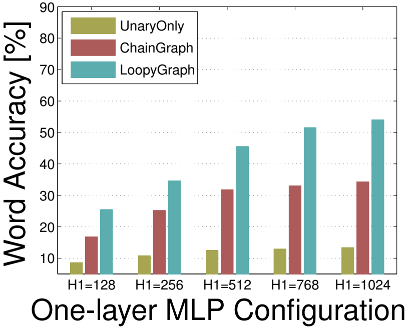


1st order Markov

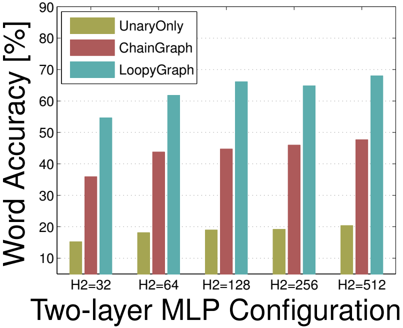


2nd order Markov

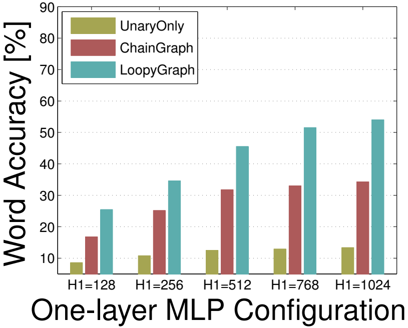
- Unary function: multi-layer perceptron (MLP)
- Pairwise function: linear or non-linear MLP
- $|\mathcal{Y}| = 26^5$

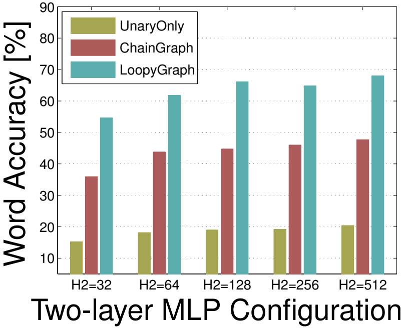


Word Accuracy [%]
UnaryOnly
ChainGraph
LoopyGraph
H1=128
H1=256
H1=512
H1=768
H1=1024
One-layer MLP Configuration



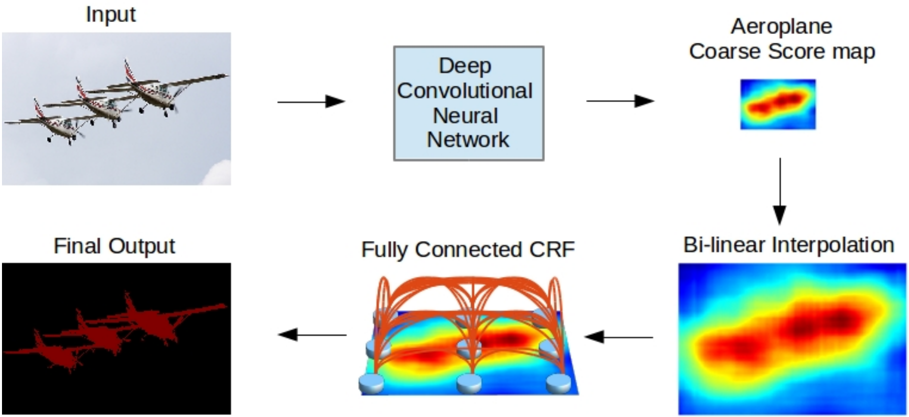
Word Accuracy [%]
UnaryOnly
ChainGraph
LoopyGraph
H2=32
H2=64
H2=128
H2=256
H2=512
Two-layer MLP Configuration





Deeper and more structured $\Rightarrow$ better performance

Example: Semantic segmentation with DeepLab



Results:


(a) Image
(b) Before CRF
(c) After CRF
(a) Image
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(c) After CRF
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(c) After CRF

Recap:

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Inference:

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$$\boldsymbol{y}^* = \arg\max_{\hat{\boldsymbol{y}}} F(\boldsymbol{w}, x, \hat{\boldsymbol{y}}) = \arg\max_{\boldsymbol{y}} \sum_r f_r(\boldsymbol{w}, x, \boldsymbol{y}_r)$$

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Learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \epsilon \ln \sum_{\hat{\boldsymbol{y}}} \exp \frac{L(\boldsymbol{y}^{(i)}, \hat{\boldsymbol{y}}) + F(\boldsymbol{w}, x^{(i)}, \hat{\boldsymbol{y}})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, \boldsymbol{y}^{(i)})$$

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repeat
- repeat
  - update marginals b_r
- until convergence
- update parameters $\boldsymbol{w}$

until convergence

Quiz:

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- What makes learning with structured output spaces hard?
- How can deep nets and structured output spaces be combined?

Important topics of this lecture

- Feature vectors for structured output space data
- Learning with structured output space data

Up next:

- Learning Theory