Machine Learning

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University of Illinois at Urbana-Champaign, 2018

L25: Policy Gradient

Goals of this lecture

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- More about Reinforcement Learning Techniques

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- Getting to know Policy Gradient

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- More about Reinforcement Learning Techniques
- Getting to know Policy Gradient
- Understanding its relation to other methods

Recap so far: Known MDP

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- To compute V^*, Q^*, π^*: use **policy/value iteration or exhaustive**

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- Estimate transition probabilities using experience replay

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- Estimate transition probabilities using experience replay
- Q-learning

What else:

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Directly optimize parametric policy $\pi_\theta(a|s)$

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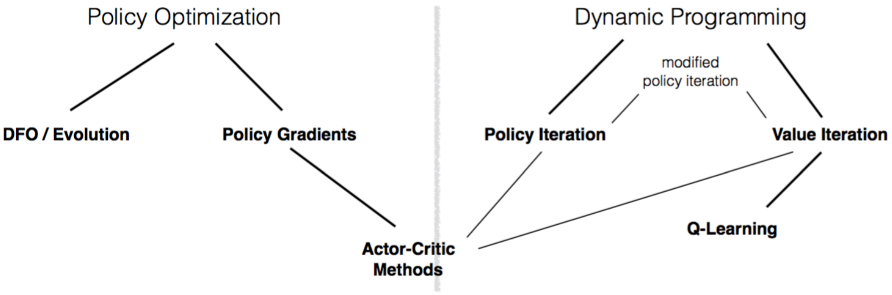
Directly optimize parametric policy $\pi_\theta(a|s)$

Why:

- π may be simpler than Q or V
- V doesn't prescribe actions: dynamics model + Bellman back-up needed
- Q requires efficient maximization: issue in continuous/high-dimensional action spaces



John Schulman & Pieter Abbeel – OpenAI + UC Berkeley

Variant: Likelihood ratio policy gradient

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- Expected reward: $R(\tau) = \sum_t R(s_t, a_t)$

$$U(\theta) = \mathbb{E}\left[\sum_t R(s_t, a_t); \pi_\theta\right] = \sum_\tau P(\tau; \theta) R(\tau)$$

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Goal:

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Related work:

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Related work:

- Aleksandrov, Sysoyev & Shemaneva; 1968
- Rubinstein; 1969
- Glynn; 1986
- Williams; 1992 –> Reinforce
- Baxter & Bartlett; 2001

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Gradient descent:

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$$\nabla_\theta U(\theta) \approx \hat{g} = \frac{1}{m}\sum_{i=1}^m \nabla_\theta \log P(\tau^{(i)};\theta)R(\tau^{(i)})$$ approx. impossible w/o trick

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Important property:

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Important property:

Valid even if R is discontinuous

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Important property:

Valid even if R is discontinuous

Intuition:

- Increase probability of paths τ with positive R
- Decrease probability of paths τ with negative R

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$$=$$

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Consequently:

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Practically important:

- Baseline
- Temporal structure

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Choices of b: e.g.,

$$b = \mathbb{E}\left[R(\tau)\right] = \frac{1}{m}\sum_{i=1}^{m} R(\tau^{(i)})$$

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Good choices for *b*:

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Good choices for b:

$$b(s_t) = \mathbb{E}\left[r_t + r_{t+1} + \ldots\right]$$

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$$R_t^{(i)} = \sum_{\hat{t} \geq t} \gamma^{\hat{t}-t} r_{\hat{t}}$$

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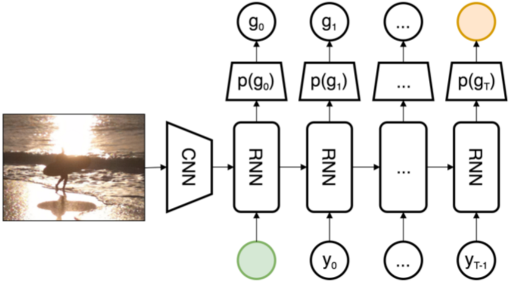
  - Re-fit the baseline b
  - Update the policy using the policy gradient estimate $\hat{g}$

Applications:

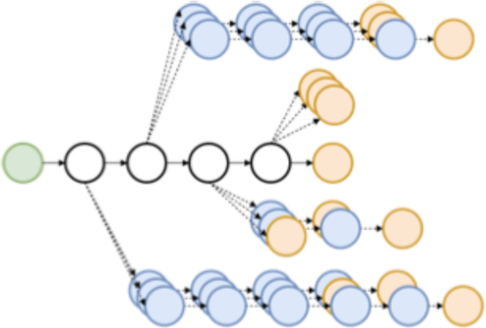
- S. Liu, Z. Zhu, N. Ye, S. Guadarrama, K. Murphy
- Improved Image Captioning via Policy Gradient optimization of SPIDEr
- 2016

Image Captioning

Image Captioning



Sampling a caption:



$$\nabla_\theta V_\theta(s_0) \approx \sum_{t=1}^{T} \sum_{g_t} [\pi_\theta(g_t|s_t) \nabla_\theta \log \pi_\theta(g_t|s_t) \times (Q_\theta(s_t, g_t) - B_\phi(s_t))] \quad (7)$$

$$L_\phi = \sum_t E_{s_t} E_{g_t} (Q_\theta(s_t, g_t) - B_\phi(s_t))^2 \quad (8)$$

Algorithm 1: PG training algorithm

Input: $\mathcal{D} = \{(\mathbf{x}^n, \mathbf{y}^n) : n = 1 : N\}$;
Train $\pi_\theta(g_{1:T}|x)$ using MLE on $\mathcal{D}$;
Train B_ϕ using MC estimates of Q_θ on a small subset of $\mathcal{D}$;
for each epoch **do**
    for example (x^n, y^n) **do**
        Generate sequence $g_{1:T} \sim \pi_\theta(\cdot|x^n)$;
        for $t = 1 : T$ **do**
            Compute $Q(g_{1:t-1}, g_t)$ for g_t with K Monte Carlo rollouts, using (6);
            Compute estimated baseline $B_\phi(g_{1:t-1})$;
        Compute $\mathcal{G}_\theta = \nabla_\theta V_\theta(s_0)$ using (7);
        Compute $\mathcal{G}_\phi = \nabla_\phi L_\phi$;
        SGD update of θ using $\mathcal{G}_\theta$;
        SGD update of ϕ using $\mathcal{G}_\phi$;

Quiz:

Quiz:

- Why Policy Gradient?

Quiz:

- Why Policy Gradient?
- Techniques to improve vanilla Policy Gradient?

Important topics of this lecture

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- Getting a feeling for reinforcement learning

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- Getting a feeling for reinforcement learning
- Understanding how to use reinforcement learning

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Thank you!