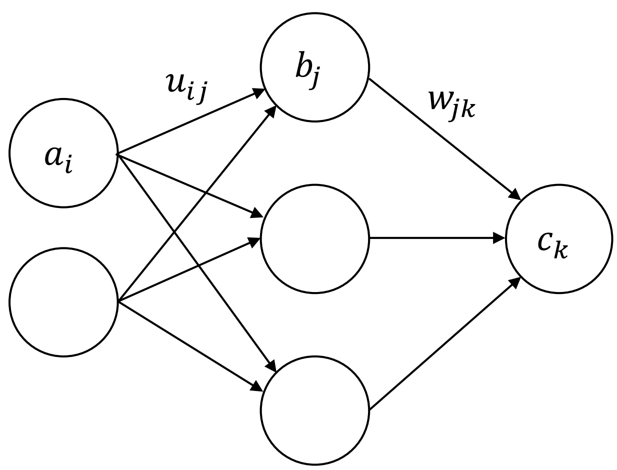
University of Illinois Spring 2018

CS 446: Machine Learning
Homework

Due on Tuesday, February 27, 2018, 11:59 a.m. Central Time

1. **[8 points]** Backpropagation

Consider the deep net in the figure below consisting of an input layer, an output layer, and a hidden layer. The feed-forward computations performed by the deep net are as follows: every input a_i is multiplied by a set of fully-connected weights u_{ij} connecting the input layer to the hidden layer. The resulting weighted signals are then summed and combined with a bias e_j. This results in the activation signal $z_j = e_j + \sum_i a_i u_{ij}$. The hidden layer applies activation function g on z_j resulting in the signal b_j. In a similar fashion, the hidden layer activation signals b_j are multiplied by the weights connecting the hidden layer to the output layer w_{jk}, a bias f_k is added and the resulting signal h_k is transformed by the output activation function g to form the network output c_k. The loss between the desired target t_k and the output c_k is given by the MSE: $E = \frac{1}{2}\sum_k (c_k - t_k)^2$, where t_k denotes the ground truth signal corresponding to c_k. Training a neural network involves determining the set of parameters $\theta = \{U, W, e, f\}$ that minimize E. This problem can be solved using gradient descent, which requires determining $\frac{\partial E}{\partial \theta}$ for all θ in the model.



(a) For $g(x) = \sigma(x) = \frac{1}{1+e^{-x}}$, compute the derivative $g'(x)$ of $g(x)$ as a function of $\sigma(x)$.

> **Solution:**
> (1 point) $g'(x) = \sigma(x)(1 - \sigma(x))$

(b) We denote by $\delta_k = \frac{\partial E}{\partial h_k}$ the error signal of neuron k in the second linear layer of the network. Compute δ_k as a function of c_k, t_k, g' and h_k.

> **Solution:**
> (1 point) $\delta_k = (c_k - t_k)g'(h_k)$

(c) Compute $\frac{\partial E}{\partial w_{jk}}$. Use δ_k and b_j.

> **Solution:**
> (1 points) $\frac{\partial E}{\partial w_{jk}} = \delta_k b_j$

(d) Compute $\frac{\partial E}{\partial f_k}$. Use δ_k.

Solution: (1 point) $\frac{\partial E}{\partial f_k} = \delta_k$

(e) We denote by $\psi_j = \frac{\partial E}{\partial z_j}$ the error signal of neuron j in the first linear layer of the network. Compute ψ_j as a function of δ_k, w_{jk} and $g'(z_j)$.

Solution:
(2 points) $\psi_j = \sum_{k \in K} \delta_k w_{jk} g'(z_j)$

(f) Compute $\frac{\partial E}{\partial u_{ij}}$. Use ψ_j and a_i.

Solution:
(1 points) $\frac{\partial E}{\partial u_{ij}} = \psi_j a_i$

(g) Compute $\frac{\partial E}{\partial e_j}$. Use ψ_j.

Solution:
(1 point) $\frac{\partial E}{\partial e_j} = \psi_j$