Machine Learning

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University of Illinois at Urbana-Champaign, 2018

Review: Discriminative Methods

Goals of this lecture

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- Review of what we learned so far

Recap: Empirical risk minimization

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$$\min_{\boldsymbol{w}} R(\boldsymbol{w}) + \sum_{i \in \mathcal{D}} \ell(\boldsymbol{y}^{(i)}, \hat{F}(\boldsymbol{w}, x^{(i)}))$$

where

$$\hat{F}(\boldsymbol{w}, x^{(i)}) =$$

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where

$$\hat{F}(\boldsymbol{w}, x^{(i)}) = \arg\max_{\hat{\boldsymbol{y}}} F(\boldsymbol{w}, x^{(i)}, \hat{\boldsymbol{y}})$$

So far:

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- kNN

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So far:

- kNN
- Least squares
- Logistic regression
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- Support vector machines
- Multiclass classification
- Deep nets
- Ensemble methods
- Structured prediction
- Learning theory

Nearest Neighbor

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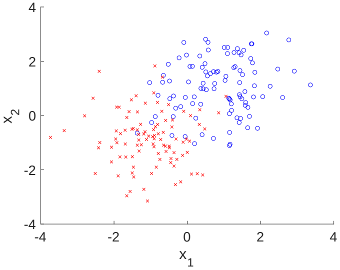

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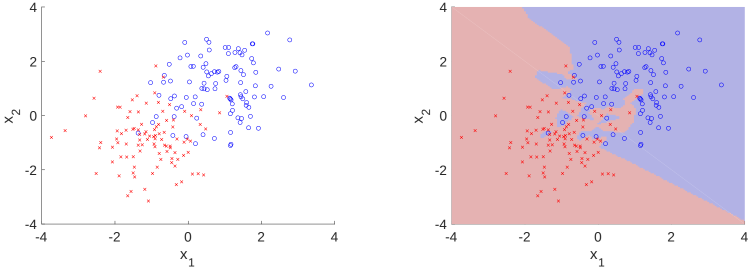

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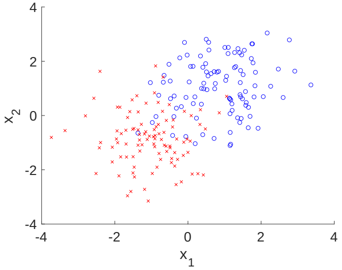

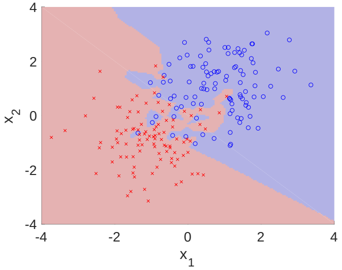

Shortcomings?

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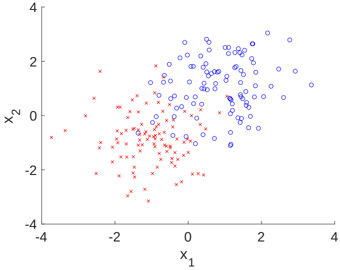
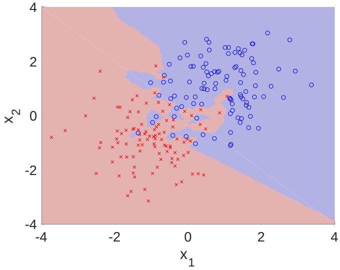
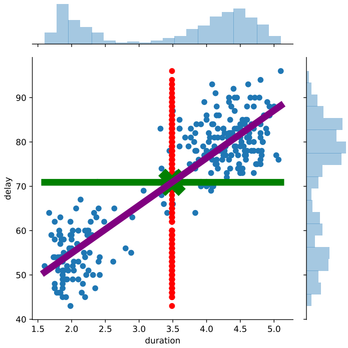




Shortcomings?
k-Nearest Neighbors

Least Squares



Least squares problem

$\arg\min_{\boldsymbol{w}\in\mathbb{R}^{d+1}} \frac{1}{2}\|\boldsymbol{X}\boldsymbol{w} - \boldsymbol{y}\|_2^2$.

OLS solution

$(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}^\top\boldsymbol{y}$.

Question:

still "why this line" ?

Three justifications/interpretations.

- Geometric interpretation.
- Probabilistic model.
- Loss minimization.

Logistic regression

Model:

$$p(y^{(i)}|x^{(i)}) = \frac{1}{1+\exp(-y^{(i)}\boldsymbol{w}^T\phi(x^{(i)}))}$$

Task:

$$\arg\max_{\boldsymbol{w}} \prod_{(x^{(i)},y^{(i)})\in\mathcal{D}} p(y^{(i)}|x^{(i)}) =$$

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Logistic Regression

Linear regression

Program:

Logistic regression

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Linear regression

Program:

$$\min_{\boldsymbol{w}} \sum_{(x^{(i)},y^{(i)})\in\mathcal{D}} \frac{1}{2}\underbrace{(1-y^{(i)}\underbrace{\boldsymbol{w}^T\phi(x^{(i)})}_{F(x^{(i)},w)})}_{F(x^{(i)},w,y^{(i)})}^2$$

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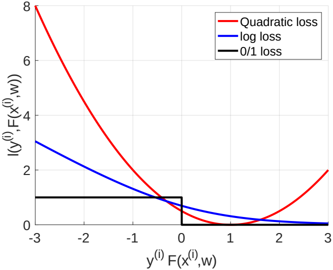

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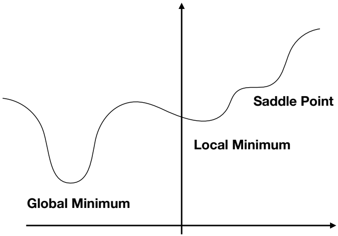

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  - Check whether we should stop (e.g., if $\nabla f(\boldsymbol{w}) \approx 0$)

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- Write down the Lagrangian L
- Minimize Lagrangian w.r.t. primal variables s.t. $\boldsymbol{w} \in \mathcal{W}$

Binary SVM:

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- Linear regression:

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$$\min_{\mathbf{w}} \frac{C}{2}\|\mathbf{w}\|_2^2 + \sum_{i\in\mathcal{D}} \frac{1}{2}(1 - y^{(i)} \underbrace{\mathbf{w}^T\phi(x^{(i)})}_{F(x^{(i)},\mathbf{w})})^2$$

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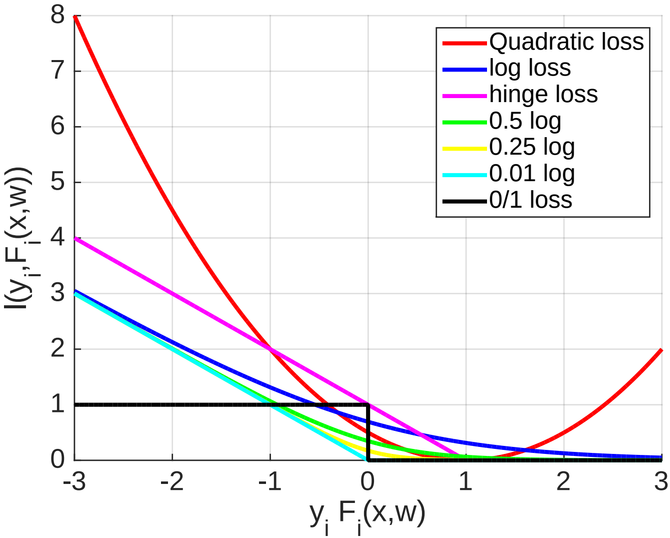
Binary SVM



Quadratic loss
log loss
hinge loss
0.5 log
0.25 log
0.01 log
0/1 loss
l(y_i,F_i(x,w))
y_i F_i(x,w)
8
7
6
5
4
3
2
1
0
-3
-2
-1
0
1
2
3

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Deep Learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2+\sum_{i\in\mathcal{D}}\left(\epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)},\hat{y}) + \boldsymbol{w}^\top \psi(x^{(i)},\hat{y})}{\epsilon} - \boldsymbol{w}^\top \psi(x^{(i)},y^{(i)})\right)$$

Deep Learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \left(\epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)}, \hat{y}) + F(\boldsymbol{w}, x^{(i)}, \hat{y})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, y^{(i)}) \right)$$

How to get to

Deep Learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \left(\epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)}, \hat{y}) + F(\boldsymbol{w}, x^{(i)}, \hat{y})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, y^{(i)}) \right)$$

How to get to

- **Logistic regression**

Deep Learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \left(\epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)}, \hat{y}) + F(\boldsymbol{w}, x^{(i)}, \hat{y})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, y^{(i)}) \right)$$

How to get to

- **Logistic regression**
- **Binary SVM**

Deep Learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \left(\epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)}, \hat{y}) + F(\boldsymbol{w}, x^{(i)}, \hat{y})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, y^{(i)}) \right)$$

How to get to

- **Logistic regression**
- **Binary SVM**
- **Multiclass regression**

Deep Learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \left(\epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)}, \hat{y}) + F(\boldsymbol{w}, x^{(i)}, \hat{y})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, y^{(i)}) \right)$$

How to get to

- **Logistic regression**
- **Binary SVM**
- **Multiclass regression**
- **Multiclass SVM**

Deep Learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \left(\epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)}, \hat{y}) + F(\boldsymbol{w}, x^{(i)}, \hat{y})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, y^{(i)}) \right)$$

How to get to

- **Logistic regression**
- **Binary SVM**
- **Multiclass regression**
- **Multiclass SVM**
- **Deep Learning**

Deep Learning:

What function $F(\boldsymbol{w}, x, y) \in \mathbb{R}$ to choose? ($y \in \{1, \ldots, K\}$)

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What function $F(\boldsymbol{w}, x, y) \in \mathbb{R}$ to choose? ($y \in \{1, \ldots, K\}$)

- Choose any differentiable composite function

$$F(\boldsymbol{w}, x, y) = f_1(\boldsymbol{w}_1, y, f_2(\boldsymbol{w}_2, f_3(\ldots f_n(\boldsymbol{w}_n, x)\ldots))) \in \mathbb{R}$$

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- More generally: functions can be represented by an acyclic graph (computation graph)

Deep Learning:

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Deep Learning:

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Deep Learning:

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Nodes are weights, data, and functions:

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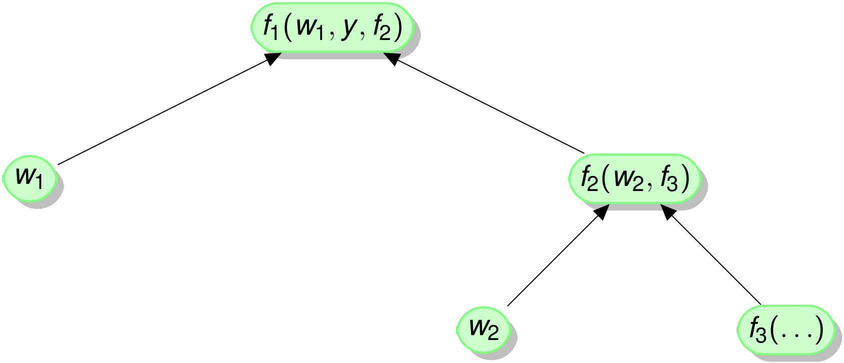

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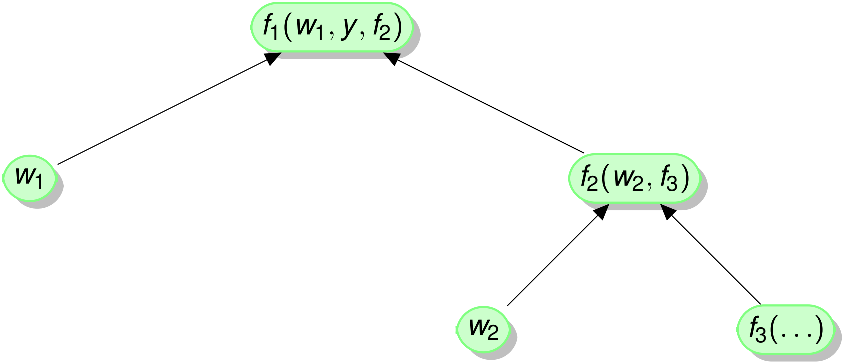

Internal representation used by deep net packages.

Deep Learning:

What are the individual functions/layers f_1, f_2 etc.?

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- Fully connected layers

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Deep Learning:

What are the individual functions/layers f_1, f_2 etc.?

- Fully connected layers
- Convolutions
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- Maximum-/Average pooling
- Soft-max layer
- Dropout

Ensemble methods:

Ensemble methods:

- Train multiple classifiers on subsets of the data

Ensemble methods:

- Train multiple classifiers on subsets of the data
- Average the results

Structured Prediction:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \epsilon \ln \sum_{\hat{\boldsymbol{y}}} \exp \frac{L(\boldsymbol{y}^{(i)}, \hat{\boldsymbol{y}}) + F(\boldsymbol{w}, x^{(i)}, \hat{\boldsymbol{y}})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, \boldsymbol{y}^{(i)})$$

Structured Prediction:

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Attention:

Structured Prediction:

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Attention:

- Scoring function

Structured Prediction:

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Attention:

- Scoring function $F(\boldsymbol{w}, x, y)$

Structured Prediction:

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Attention:

- Scoring function $F(\boldsymbol{w}, x, y)$
- Loss function

Structured Prediction:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \epsilon \ln \sum_{\hat{\boldsymbol{y}}} \exp \frac{L(\boldsymbol{y}^{(i)}, \hat{\boldsymbol{y}}) + F(\boldsymbol{w}, x^{(i)}, \hat{\boldsymbol{y}})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, \boldsymbol{y}^{(i)})$$

Attention:

- Scoring function $F(\boldsymbol{w}, x, y)$
- Loss function (log-loss, hinge-loss)

Structured Prediction:

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Attention:

- Scoring function $F(\boldsymbol{w}, x, y)$
- Loss function (log-loss, hinge-loss)
- Taskloss L

Structured Prediction:

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How to get to

- Binary Logistic regression

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How to get to

- Binary Logistic regression
- Binary SVM

Structured Prediction:

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How to get to

- Binary Logistic regression
- Binary SVM
- Multiclass regression

Structured Prediction:

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How to get to

- Binary Logistic regression
- Binary SVM
- Multiclass regression
- Multiclass SVM

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How to get to

- Binary Logistic regression
- Binary SVM
- Multiclass regression
- Multiclass SVM
- Deep Learning

Structured Prediction:

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- Loss function (log-loss, hinge-loss)
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How to get to

- Binary Logistic regression
- Binary SVM
- Multiclass regression
- Multiclass SVM
- Deep Learning
- Structured prediction

Prediction: Inference (how to find the highest scoring configuration):

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$$y^* = \arg\max_{\hat{y}} F(\boldsymbol{w}, x, y)$$

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$$\boldsymbol{y}^* = \arg\max_{\hat{\boldsymbol{y}}} \sum_r f_r(\boldsymbol{w}, x, \hat{\boldsymbol{y}}_r)$$

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Efficiency and accuracy of inference algorithms is problem dependent:

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Efficiency and accuracy of inference algorithms is problem dependent:

- Exhaustive search

Prediction: Inference (how to find the highest scoring configuration):

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Efficiency and accuracy of inference algorithms is problem dependent:

- Exhaustive search
- Dynamic programming

Prediction: Inference (how to find the highest scoring configuration):

$$y^* = \arg\max_{\hat{y}} F(\boldsymbol{w}, x, y)$$

Structured Prediction

$$\boldsymbol{y}^* = \arg\max_{\hat{\boldsymbol{y}}} \sum_r f_r(\boldsymbol{w}, x, \hat{\boldsymbol{y}}_r)$$

Efficiency and accuracy of inference algorithms is problem dependent:

- Exhaustive search
- Dynamic programming
- Integer linear program

Prediction: Inference (how to find the highest scoring configuration):

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Structured Prediction

$$\boldsymbol{y}^* = \arg\max_{\hat{\boldsymbol{y}}} \sum_r f_r(\boldsymbol{w}, x, \hat{\boldsymbol{y}}_r)$$

Efficiency and accuracy of inference algorithms is problem dependent:

- Exhaustive search
- Dynamic programming
- Integer linear program
- Linear programming relaxation

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Structured Prediction

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Efficiency and accuracy of inference algorithms is problem dependent:

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- Integer linear program
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- Message passing

Prediction: Inference (how to find the highest scoring configuration):

$$y^* = \arg\max_{\hat{y}} F(\boldsymbol{w}, x, y)$$

Structured Prediction

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Efficiency and accuracy of inference algorithms is problem dependent:

- Exhaustive search
- Dynamic programming
- Integer linear program
- Linear programming relaxation
- Message passing
- Graph-cut

Learning theory:

Why does learning on the training set generalize?

Learning theory:

Why does learning on the training set generalize?

Why does independently solving the homework help in the midterm?

We are sure you'll all make it!