Machine Learning

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University of Illinois at Urbana-Champaign, 2018

L9: Deep Neural Networks

Goals of this lecture

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- Understanding forward and backward pass

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- Learning about backpropagation

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Reading material

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- Understanding forward and backward pass
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Reading material

- I. Goodfellow et al.; Deep Learning; Chapters 6-9

Recap: Our earlier framework:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \left(\epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)}, \hat{y}) + \boldsymbol{w}^T \psi(x^{(i)}, \hat{y}))}{\epsilon} - \boldsymbol{w}^T \psi(x^{(i)}, y^{(i)}) \right)$$

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- More generally: functions can be represented by an acyclic graph (computation graph)

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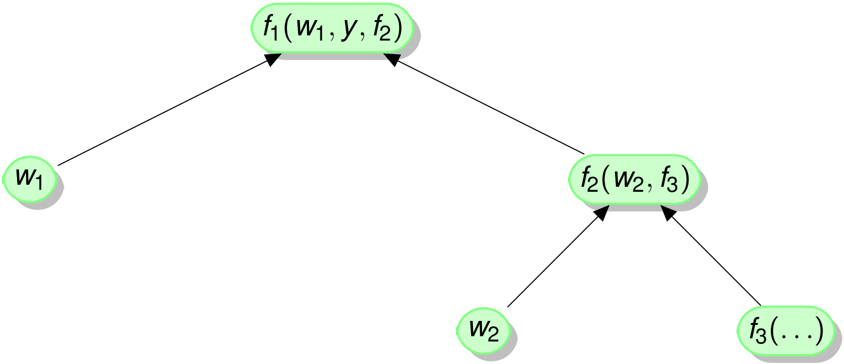

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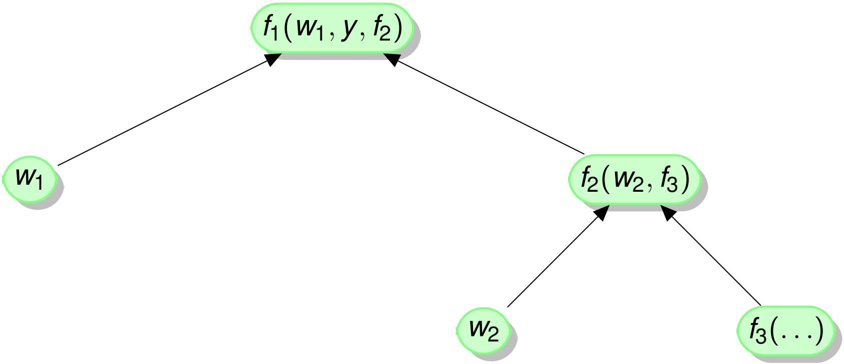

Internal representation used by deep net packages.

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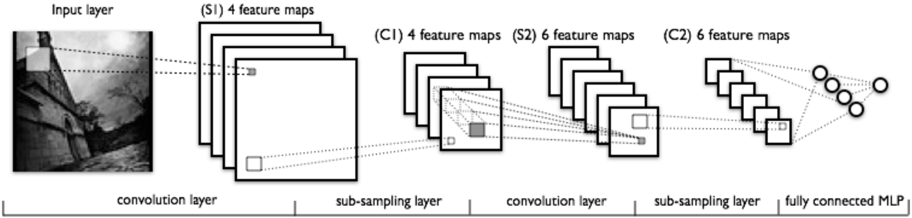

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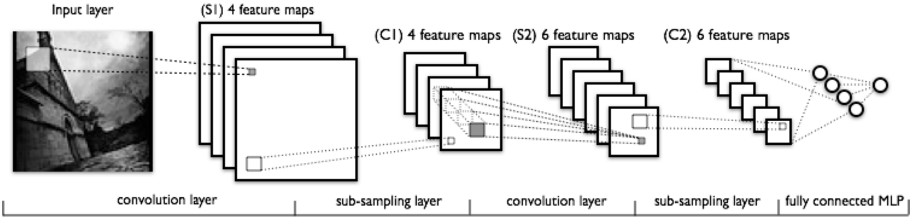

Decreasing spatial resolution and the increasing number of channels

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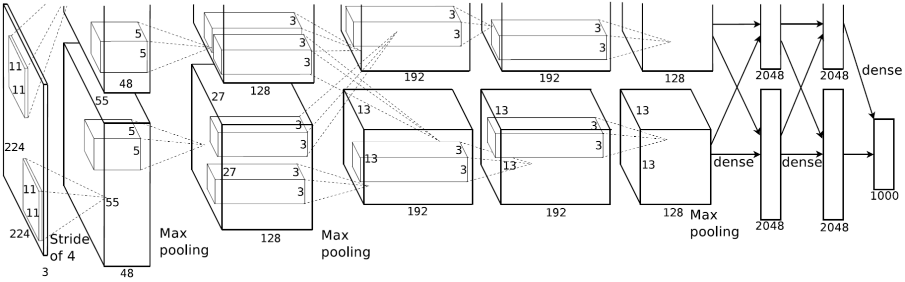

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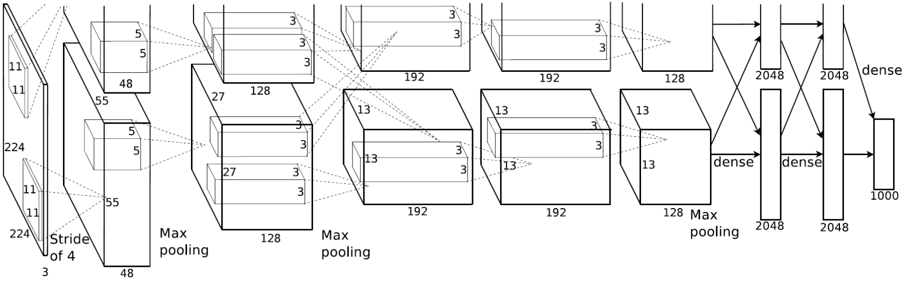

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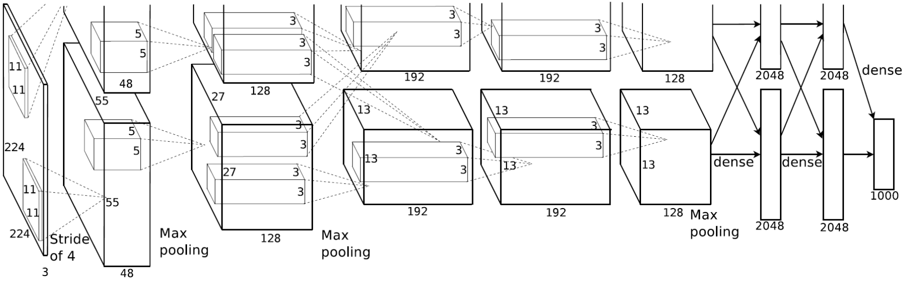
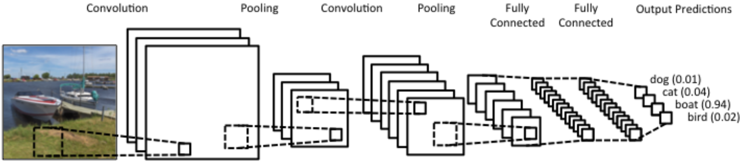


Decreasing spatial resolution and the increasing number of channels

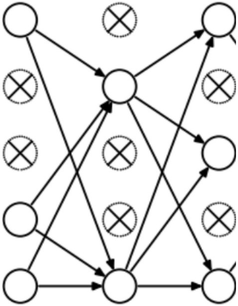
Why is the output 1000-dimensional?

Another deep net:



Those nets are structurally simple in that a layer's output is used as input for the next layer. This is not required.

Dropout layer:



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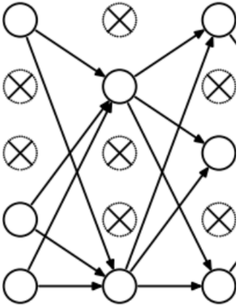

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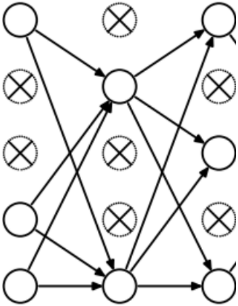

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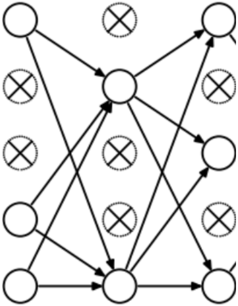

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What is C?

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Stochastic gradient descent with momentum: What was this again?

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Generally: To avoid repeated computation, backpropagation on an acyclic graph. Nodes in this graph are weights, data, and functions.

Composite function represented as acyclic graph

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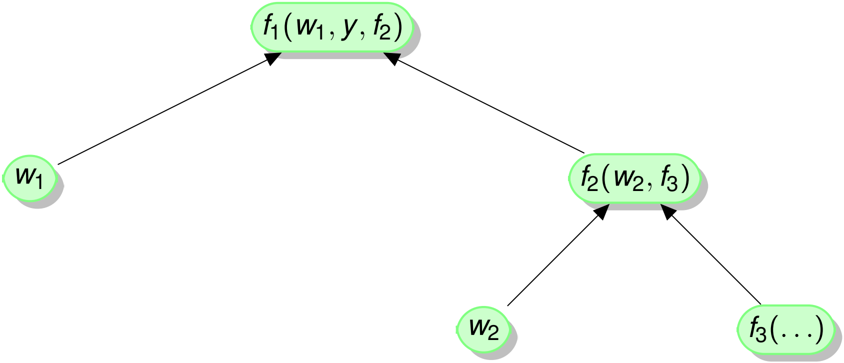

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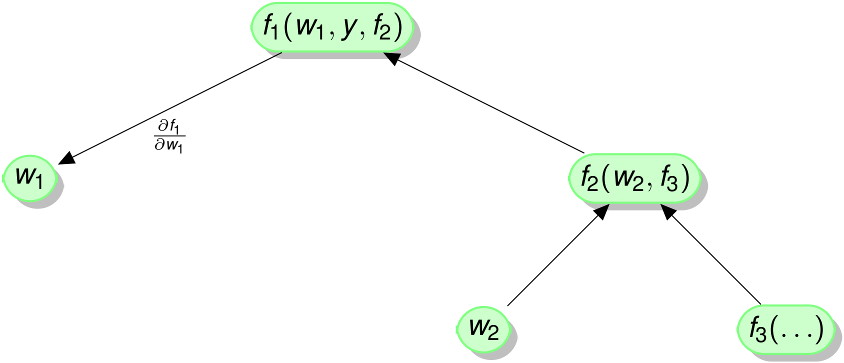

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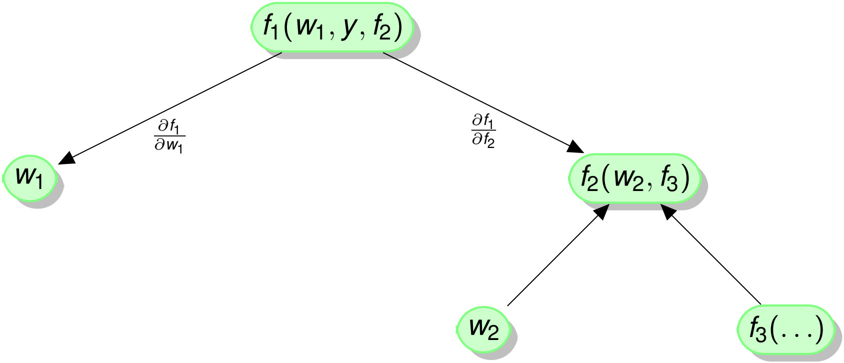

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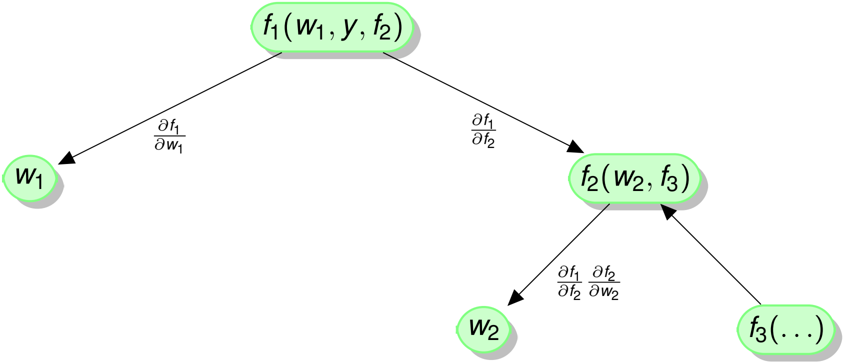

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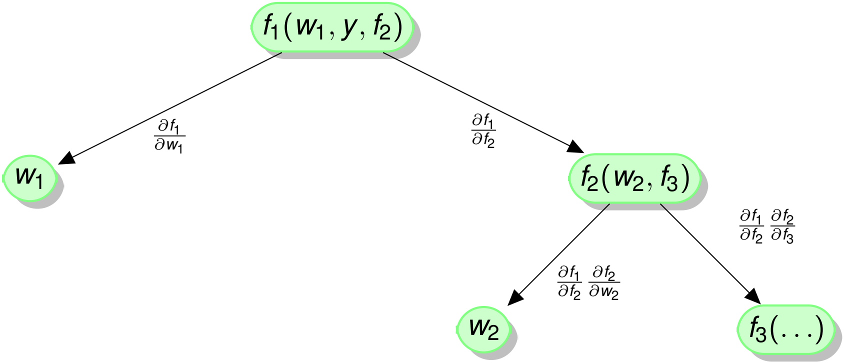

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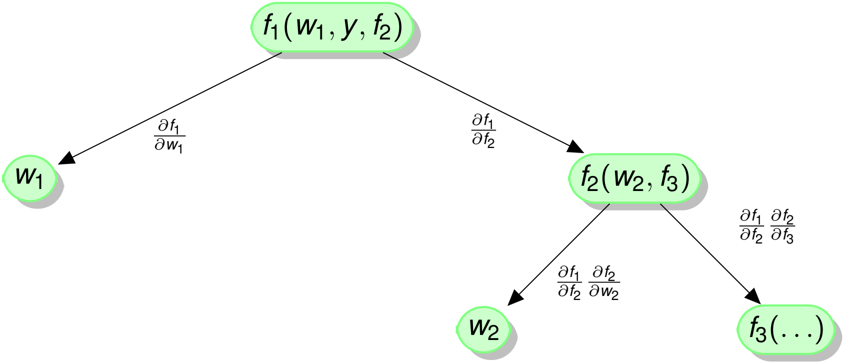

Repeated use of chain rule for efficient computation of all gradients

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Recommendation: implement a simple deep net framework yourself

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- Initialization of $\boldsymbol{w}$ matters

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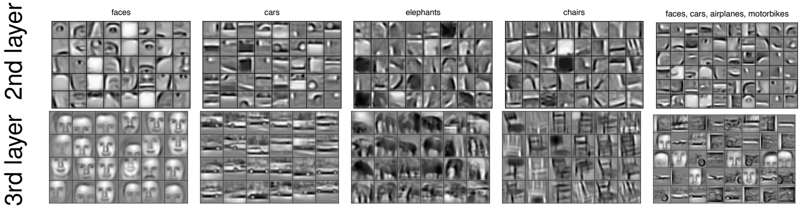

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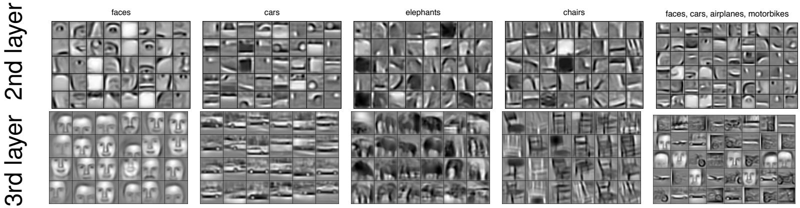

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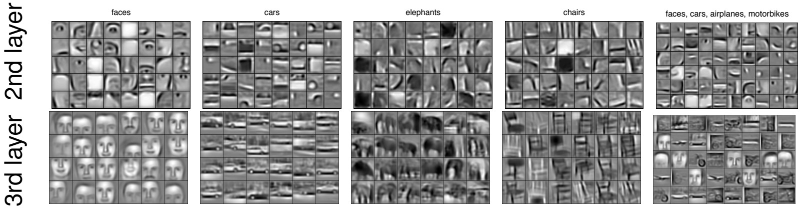

Disadvantage of deep nets compared to usage of features:

Deep nets are computationally demanding (GPUs) and require significant amounts of training data

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This combination lead to significant performance improvements on many datasets

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Know what you are doing, i.e., know all the dimensions.

Loss functions:

- CrossEntropyLoss

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Don't try without, it **will** fail!

Example (PyTorchCS446.py):

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        super(Net, self).__init__()
        self.conv1 = nn.Conv2d(1, 6, 5)
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What are the input dimensions?

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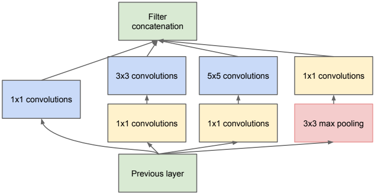

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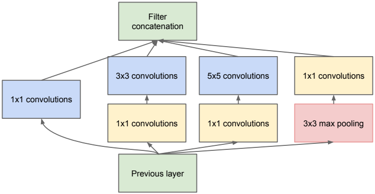
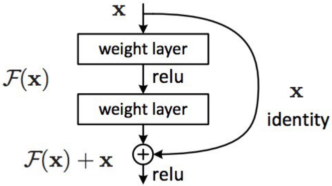


- ResNet (residual connections)



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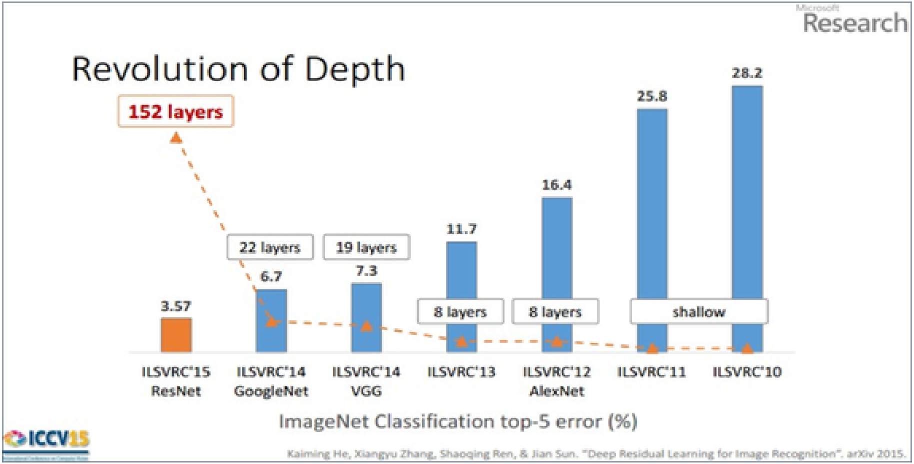
Microsoft
Research
Revolution of Depth
152 layers
22 layers
19 layers
3.57
6.7
7.3
11.7
16.4
25.8
28.2
8 layers
8 layers
shallow
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VGG
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Kaiming He, Xiangyu Zhang, Shaoqing Ren, & Jian Sun. "Deep Residual Learning for Image Recognition". arXiv 2015.

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Important topics of this lecture

- Deep nets
- Backpropagation

Up next:

- Ensemble methods