Machine Learning

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University of Illinois at Urbana-Champaign, 2018

L23: Markov Decision Processes

Goals of this lecture

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- Getting to know reinforcement learning

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- Getting to know reinforcement learning
- Getting to know Markov decision processes

Recap: What have we talked about so far?

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Machine learning paradigms

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- Discriminative learning and its applications
- Generative learning and its applications
- Now: Reinforcement learning and its applications

Machine learning paradigms:

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- Discriminative learning:

$$p(\boldsymbol{y}|x)$$

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- Discriminative learning:

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- Reinforcement learning (RL)

Examples

Reinforcement learning examples:

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- Fly stunt manoeuvres in a helicopter

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Reinforcement learning examples:

- Fly stunt manoeuvres in a helicopter
- Play Atari games
- Defeat the world champion at Go
- Manage investment portfolio
- Control a power station
- Make a humanoid robot walk

How are those tasks formulated?

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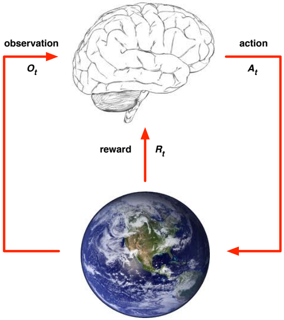

At each step t the agent

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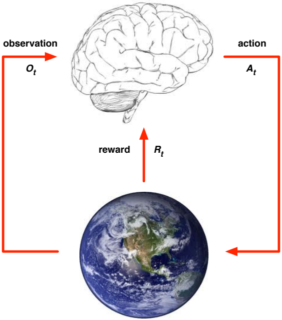

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- Thinks/Knows about being in state s_t

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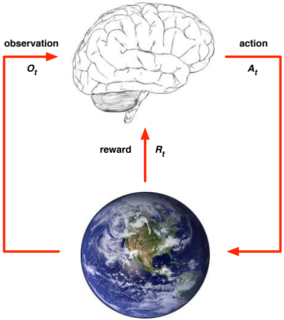

At each step t the agent

- Thinks/Knows about being in state s_t
- Performs action a_t

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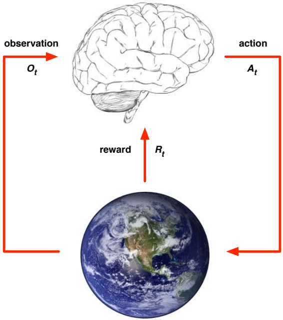

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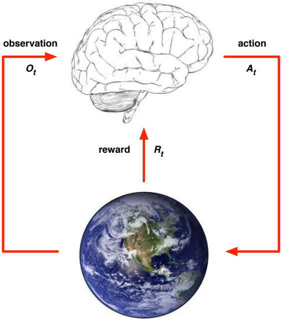

At each step t the agent

- Thinks/Knows about being in state s_t
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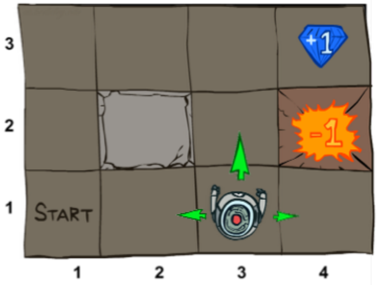
- Receives scalar reward $r_t \in \mathbb{R}$
- Finds itself in state s_{t+1}


3
2
1
START
+1
-1
1
2
3
4

Settings:

Settings:

Deterministic

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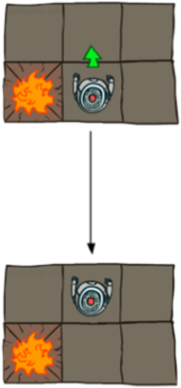
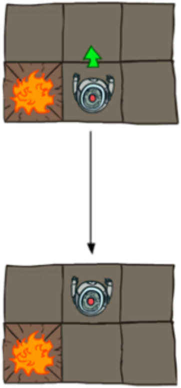
Deterministic



Settings:

Deterministic

Stochastic



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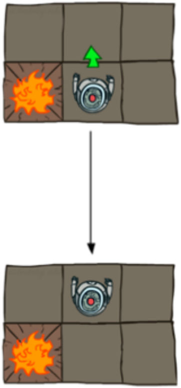
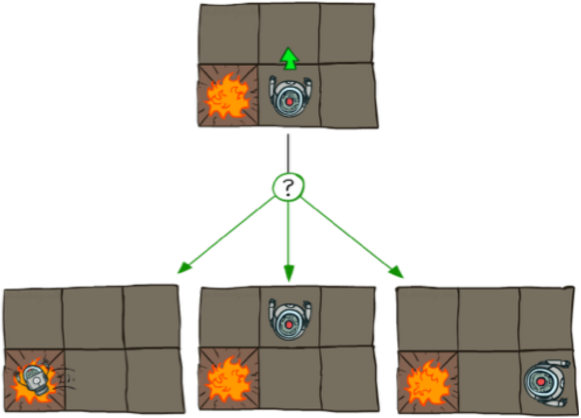


Stochastic



Formally:

Formally: Markov Decision Process (MDP)

- A set of states $s \in \mathcal{S}$

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$$
\begin{aligned}
&P(S_{t+1} = s' \mid S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1} = \ldots, S_0 = s_0) \\
&\quad = P(S_{t+1} = s' \mid S_t = s_t, A_t = a_t)
\end{aligned}
$$

Pictorial representation of MDP:

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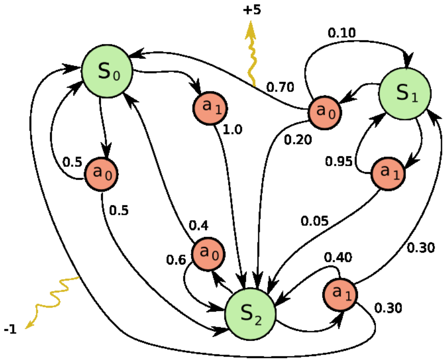

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- No supervisor, only **reward** signal
- Delayed feedback
- Actions affect received data

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How to encode the policy?

$$\pi(s) : \mathcal{S} \rightarrow \mathcal{A}_s$$

How to find the best policy π^*?

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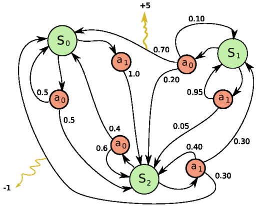
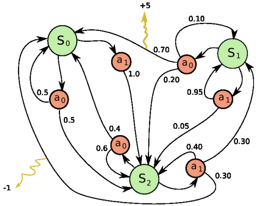


How to find the best policy π^*?

- Exhaustive search



How to find the best policy π^*?

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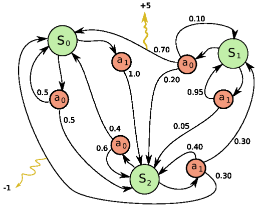
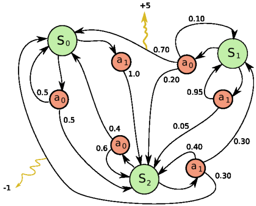


How to find the best policy π^*?

- Exhaustive search
- Policy iteration
- Value iteration



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Exhaustive search for best policy π^*:

- How many policies?

$$\prod_{s \in \mathcal{S}} |\mathcal{A}_s|$$

- How to evaluate quality of π? Compute expected future reward $V^\pi(s_0)$
- Choose policy π^* with largest expected future reward $V^{\pi^*}(s_0)$

Policy evaluation:

How to compute expected future reward $V^{\pi}(s)$ for a given policy?

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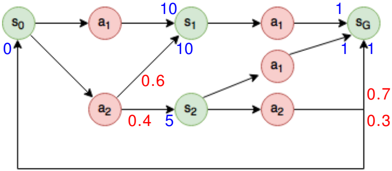
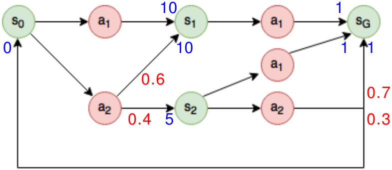
Example:

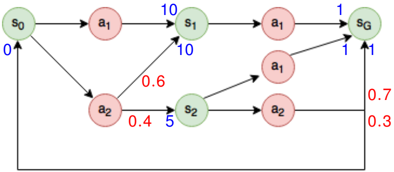
Policy evaluation:

How to compute expected future reward $V^{\pi}(s)$ for a given policy?

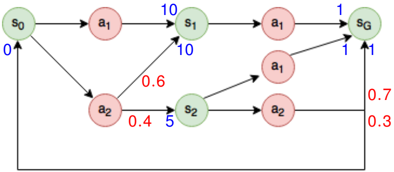
Example: rewards & transition probabilities




s0
a1
s1
a1
sG
a1
a2
s2
a2
10
10
0.6
0.4
5
0
1
1
1
0.7
0.3



$\pi(s_0) = a_1, \pi(s_1) = a_1$



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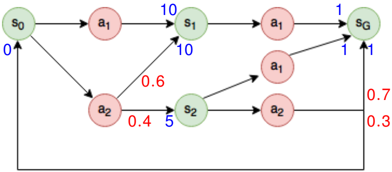
Policy graph:



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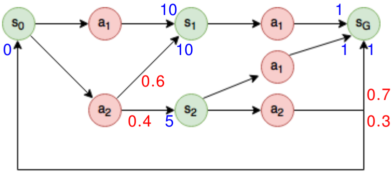

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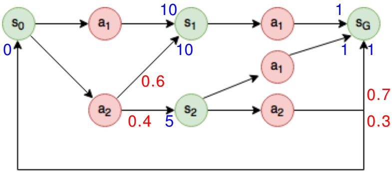
Policy graph:

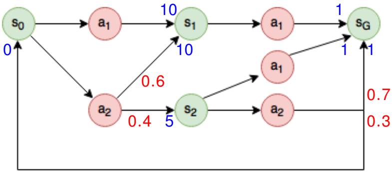
$V^\pi(s_1) = \quad , V^\pi(s_0) =$



$\pi(s_0) = a_1, \, \pi(s_1) = a_1$

Policy graph:

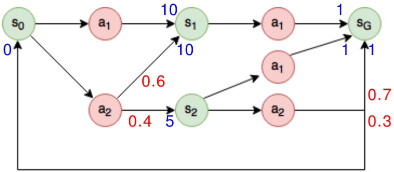
$V^\pi(s_1) = 1, \; V^\pi(s_0) =$



$\pi(s_0) = a_1, \pi(s_1) = a_1$

Policy graph:

$V^\pi(s_1) = 1, \; V^\pi(s_0) = 11$



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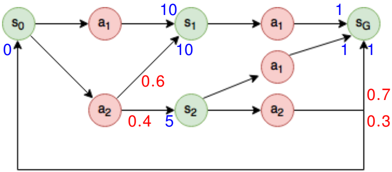
easy



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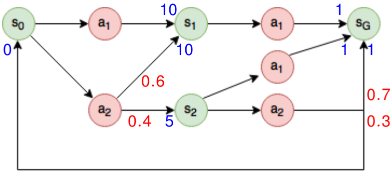
easy

$\pi(s_0) = a_2, \pi(s_2) = a_1$



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$\pi(s_0) = a_2, \pi(s_2) = a_1$

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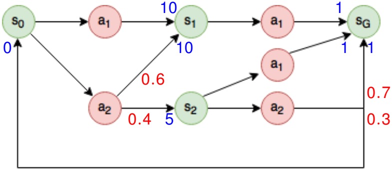

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Policy graph:

$V^\pi(s_1) = 1,\ V^\pi(s_0) = 11$

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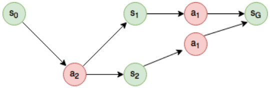
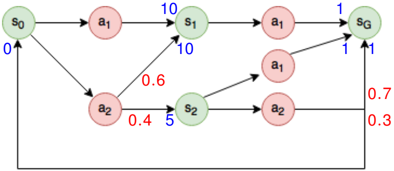
Policy graph:





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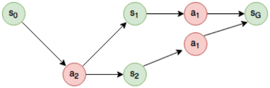
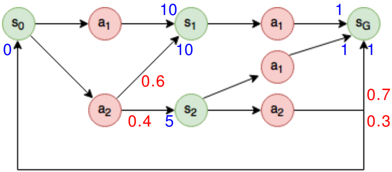
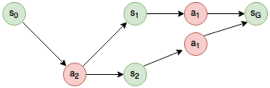
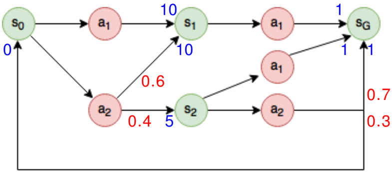


$V^\pi(s_1) = \quad , \; V^\pi(s_2) =$
$V^\pi(s_0) =$



$\pi(s_0) = a_1, \pi(s_1) = a_1$	$\pi(s_0) = a_2, \pi(s_2) = a_1$
Policy graph:	Policy graph:
	
$V^\pi(s_1) = 1,\ V^\pi(s_0) = 11$	$V^\pi(s_1) = 1,\ V^\pi(s_2) =$ $V^\pi(s_0) =$
easy	



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Policy graph:

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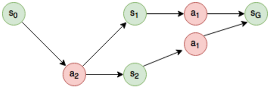
easy

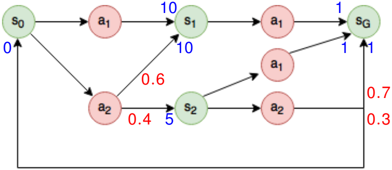
$\pi(s_0) = a_2,\ \pi(s_2) = a_1$

Policy graph:



$V^\pi(s_1) = 1,\ V^\pi(s_2) = 1$

$V^\pi(s_0) =$



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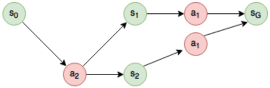
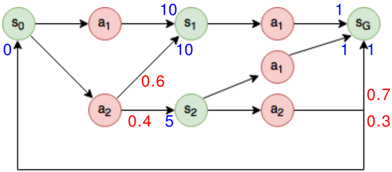
Policy graph:



$V^\pi(s_1) = 1,\ V^\pi(s_2) = 1$

$$V^\pi(s_0) = .6 \cdot (10 + 1) + .4 \cdot (5 + 1) = 9$$



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Policy graph:

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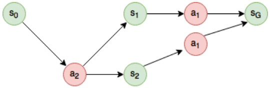
$\pi(s_0) = a_2, \pi(s_2) = a_1$

Policy graph:



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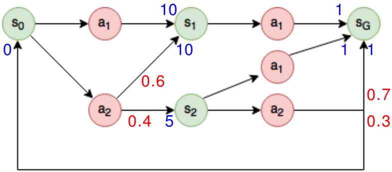
backpropagation



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Policy graph:

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easy

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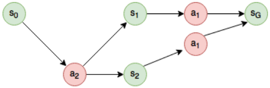
Policy graph:



$V^\pi(s_1) = 1, \; V^\pi(s_2) = 1$

$V^\pi(s_0) = .6 \cdot (10 + 1) +$
$.4 \cdot (5 + 1) = 9$

backpropagation

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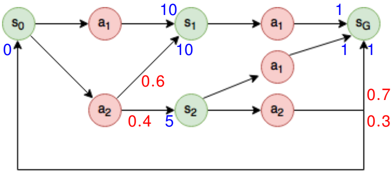

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Policy graph:

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easy

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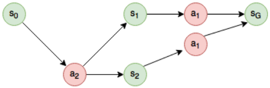

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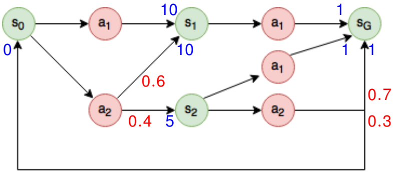

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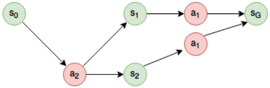

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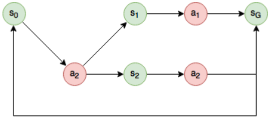

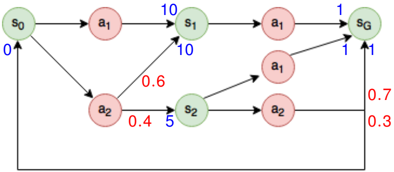

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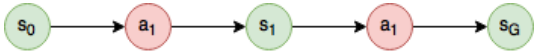

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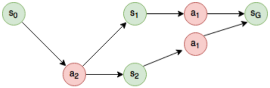

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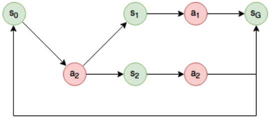

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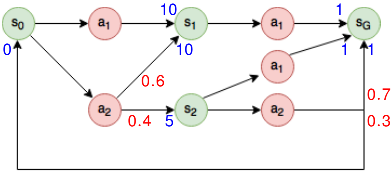

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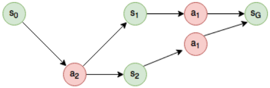

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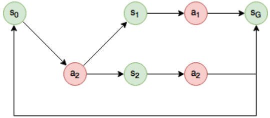

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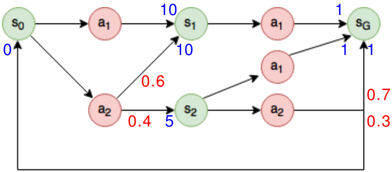

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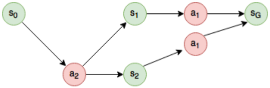

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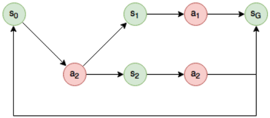

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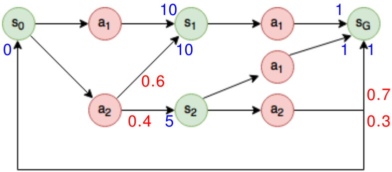

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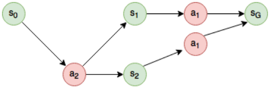

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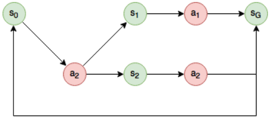

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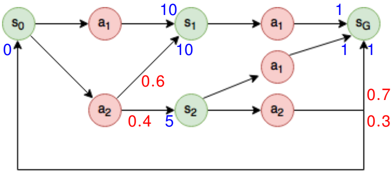

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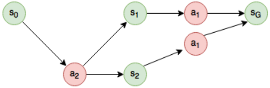

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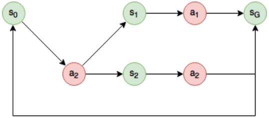

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linear system

Exhaustive search:

Exhaustive search: for each policy π

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Policy evaluation requires to solve linear system of equations:

$$\begin{array}{rcll} V^\pi(s) & = & 0 & \text{if } s \in \mathcal{G} \\ V^\pi(s) & = & \sum_{s' \in \mathcal{S}} P(s' \mid s, \pi(s)) \left[R(s, \pi(s), s') + V^\pi(s') \right] & \end{array}$$

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But searching over all policies is still expensive.

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  - Extract new policy π using

$$\pi(s) = \arg\max_{a \in \mathcal{A}_s} \sum_{s' \in \mathcal{S}} P(s' \mid s, a) \left[R(s, a, s') + V^{\pi}(s') \right]$$

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Shapley (1953), Bellman (1957)

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Recap so far: Known MDP

- To compute V^*, Q^*, π^*: use **policy/value iteration or exhaustive**
- To evaluate fixed policy π: use policy evaluation

Quiz:

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- What differentiates RL from supervised learning?

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- What is a MDP?

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- What differentiates RL from supervised learning?
- What is a MDP?
- What differentiates policy iteration from policy evaluation?

Important topics of this lecture

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What's next:

What to do if the MDP model is not known?