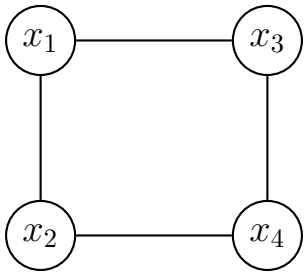
**University of Illinois** Spring 2018

# CS 446: Machine Learning
## Homework 7

Due on Tuesday, March 6, 2018, 11:59 a.m. Central Time

1. **[4 points]** Inference Methods for Discrete Markov Random Fields

   For this problem, consider the following Markov Random Field, where each node can be assigned one of the values in $\{1, 2, 3, 4, 5\}$:




   (a) To conduct MAP inference on this graph using exhaustive search, how many configurations must be checked?

   Your answer:

   (b) Can MAP inference be run on this graph using a dynamic programming algorithm? Why or why not?

   Your answer:

   (c) To run MAP inference on this graph using loopy belief propagation, how many messages must be computed?

Your answer:

2. **[7 points]** ILP Inference formulation in Discrete Markov Random Fields

   (a) Suppose we have two variables $x_1 \in \{0, 1\}$ and $x_2 \in \{0, 1\}$ and their local evidence functions $\theta_1(x_1)$ and $\theta_2(x_2)$ as well as a pairwise function $\theta_{1,2}(x_1, x_2)$. Using this setup, inference solves $\arg\max_{x_1,x_2} \theta_1(x_1) + \theta_2(x_2) + \theta_{1,2}(x_1, x_2)$. Using

$$\theta_1(x_1) = \begin{cases} 1 & \text{if } x_1 = 0 \\ 2 & \text{otherwise} \end{cases} \qquad \theta_2(x_2) = \begin{cases} 1 & \text{if } x_2 = 0 \\ 2 & \text{otherwise} \end{cases}$$

$$\theta_{1,2}(x_1, x_2) = \begin{cases} -1 & \text{otherwise} \\ 2 & \text{if } x_1 = 0 \ \& \ x_2 = 1 \end{cases}$$

   what is the integer linear programming formulation of the inference task? Make the cost function and constraints explicit for the given problem, i.e., do not use a general formulation.

Your answer:

   (b) What is the solution (value and argument) to the program in part (a).

Your answer:

(c) Why do we typically not use the integer linear program for reasonably sized MRFs?

| Your answer: |
| --- |